MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 26/1992

FREIFORMKURVEN UND FREIFORMFLÄCHEN

14.06. bis 20.06.1992

Die fünfte internationale Tagung zum Thema "Freiformkurven und Freiformflächen" stand unter der Leitung von R.E. Barnhill (Arizona State University, Tempe), W. Böhm (TU Braunschweig) und J. Hoschek (TH Darmstadt).

Ziel dieser Tagung war die Entwicklung neuer mathematischer Methoden zur Kurven- und Flächenerzeugung. Besonderer Wert wurde hierbei jeweils auf die geometrische Interpretierbarkeit sowie die Umsetzbarkeit in effiziente Algorithmen gelegt. Dieser Anwendungsbezug wurde auch durch die Tatsache belegt, daß sich unter den ca. 50 Teilnehmern nicht nur an Universitäten lehrende Wissenschaftler befanden, sondern auch in der Industrie tätige Mathematiker. Diese Mischung aus Theoretikern und Anwendern stellte sich als besonders produktiv heraus, da es trotz der Vielzahl an Vorträgen zu zahlreichen, intensiven Gesprächen kam, die eine Fülle von Anregungen für alle Beteiligten lieferten.

Zu folgenden Themenkreisen wurden u.a. die neuesten Forschungsergebnisse vorgestellt: formerhaltende Interpolation mit rationalen Splinekurven, Modellierungsaspekte von NURBS, Übertragung des Blossoming-Prinzips auf verallgemeinerte Splinekurven, Anwendungen von multivariaten B-Splines und Box Splines, Approximation mit polynomialen sowie rationalen Bézier-Kurven, Modellierung mit impliziten Flächen, allgemeine baryzentrische Koordinaten, Berechnung von Singularitäten, Flächenverschneidungsmethoden, Erzeugung von optimalen sowie bedingten Triangulationen, Darstellung von Kurven- und Flächenstücken auf beliebigen Quadriken. Vorträge von in der Industrie tätigen Teilnehmern wiesen darüberhinaus auf eine Reihe offener Fragen und Problemstellungen hin und stellten somit eine wichtige Ergänzung dar.

Vortragsauszüge

J. GREGORY :

Surface modelling with rational spline curves

A network of design curves in $\mathbb{R}^3$ is constructed using a parametric rational cubic spline method. The rational form provides interval and point tension weights which can be used to control the shape of the spline curves. Two special cases are considered, namely that of interpolating convex planar data and that of conic spline interpolation of planar data. The curves determine a polygonal patch topology over which the curves and cross boundary tangents are blended to give a C^1 surface description.

G. FARIN :

Geometric shape handles

The shape parameters of γ–, ν–, β– etc. splines are real numbers which have little intuitive meaning to a designer. The same is true for the weights of NURB curves. This talk shows a way to replace the standard shape handles by more geometric ones: "weight points" and tangents. The curves can then be constructed by projectively invariant cross ratio constructions.

H. POTTMANN :

On the geometry of Tchebycheffian splines

It is shown that many properties of Bézier and B–spline curves hold for a much wider class of curves. Using a "normal curve" associated with an extended Tchebycheff space, we derive a Bézier like representation of Tchebycheffian spline curve segments. These curves are affine or projective images of the normal curve which is a curve of geometric order m in affine or projective m–space. A generalization of the blossoming method is a main tool in the study of Tchebycheffian spline curves and their segments. The basic algorithms such as knot insertion and the construction of the Bézier points are developed in a purely geometric way. Whereas the generation of tensor product surfaces is straightforward, some preliminary studies indicate that a similarly natural generalization of Bézier triangles does not exist.

H. PRAUTZSCH :

Convergence of subdivision and degree elevation

A short, simple, and general proof of 4 lines has been presented showing that the 'subdivided' and 'degree elevated' control polygons of a spline, box–spline, or multivariate polynomial in Bézier representation converge to the spline, box–spline, or simplicial polynomial volume, respectively. The proof is based only on a Taylor expansion. All other known proofs also need a Taylor expansion besides other concepts.
A second and even shorter but as simple proof was presented then for the fact that subdivided Bézier nets converge to the underlying multivariate polynomial patch.

M. BERCOVIER, E. TISHEL :

Enhancement of Gordon–Coons interpolations by Bubble functions

A popular method for generating surfaces in design packages is the boolean sum method of Gordon–Coons surfaces. The resulting surface has two major flaws:

- A very little flexibility in design, since the definition of the boundary determines completely the surface.
- C^1 connection of adjacent surfaces requires cubic blending functions.

In this conference we present a new surface construction, based on the Gordon–Coons method, but enriched by Bubble functions. Such functions can be viewed as the "interior shape functions" added to a classic Finite Element of Serendipity type. Noting that these functions are actually the missing monomials that will bridge between the full tensor product approximation and the Serendipity one we get a method to enhance the Gordon–Coons patches. And for polynomial boundary curves (Bézier curves), we show what is the minimal polynomial enhancement needed to define C^1 surfaces.

T.D. DEROSE, A. DAHL :

Weyl: An interactive programming environment for geometry

Weyl is an interactive programming environment designed to facilitate the rapid prototyping of programs that perform geometric computations. Novel features of Weyl include a powerful programming language, an extensive set of geometric data types in an arbitrary number of dimensions, and a user–extensible graphics mechanism.

T. JENSEN :

A rigid, n–sided surface matching G^1 constraints

An algorithm for building a G^1 surface across an n–sided region is described. The direction of blending of the surface is arbitrary and independent of the boundary discontinuities; nevertheless, the surface is C^2 internally. The surface is constructed from an initial surface which may have C^1 discontinuities along lines of constant parameter. These are filleted across by means of univariate quintic Hermite interpolation. Examination of the properties of the fillet boundaries establishes C^2 internal continuity and satisfaction of the G^1 boundary conditions.

T.N.T. GOODMAN :

Bézier nets, convexity and subdivision

We explore the relationship between Bézier nets, convexity and subdivision for polynomials on simplices. In particular we give explicit conditions for convexity of the Bézier net under regular subdivision and show that this subdivision process preserves the convexity of the Bézier net. All these results are new for simplices in more than two dimensions.

H. MCLAUGHLIN :

Interpolation properties of parametric curves

It is known that $n + 1$ points in the plane can be interpolated uniquely by a scalar valued polynomial (in one variable) of degree n. Similar questions arise for vector valued polynomials (in a parameter t). Here the analysis is nonlinear and interpolation results are not well understood. The talk will include remarks on these problems with an emphasis on curves of shortest "time" between two points in the plane. Specifically, one considers the class of planar cubic curves with initial and terminal points specified as well as initial and terminal tangent directions (not magnitudes). From this class one seeks pairs of points which can be interpolated. The analysis leads to the question: from this class what is the curve of shortest "time" from the initial point to a given point?

T.A. FOLEY :

Monotonicity preserving curves and surfaces in Bézier form

Necessary and sufficient monotonicity conditions for a piecewise cubic in Hermite form are given in [Fritsch and Carlson '80] that involve a two-dimensional region bounded by an ellipse. Equivalent bounds can be easily constructed for the Bézier representation by simple substitution or by a direct argument. By using a subdivision technique, simple linear constraints on the Bézier control points are given that converge to the elliptic bounds. For monotone bicubic surfaces, the conditions are much more complicated in Hermite form. Using the Bézier representation, a larger monotonicity region is given that has a simple geometric interpretation. A new algorithm for bivariate monotone surfaces is given that is based on constraining the Bézier control points.

W. DAHMEN :

Modeling and visualization with implicit surfaces

Design and modeling of freeform surfaces is commonly based on parametric representations which faciltate meeting smoothnes as well as accuracy requirements. However, such representations are less suitable for high quality (photorealistic) visualization. Instead typical rendering schemes work on polyhedral representations of geometry which are less accurate and require enormous storage. This talk reports on attempts to develop surface representations that are suitable for both, modeling and visualization. Specifically, a scheme for constructing piecewise algebraic surfaces of degree three is described which has the following properties:

1. It handles arbitrary topologies;

2. it produces tangent plane continuous surfaces;

3. it interpolates positional data as well as normal directions;

4. it is completely local.

It is pointed out that these surface representations are well suited for efficient high quality rendering techniques with a significant storage reduction. In this context it is essential to have implicit patches of low degree which are enclosed by simple polytopes.

W.L.F. DEGEN :

Rational approximations of parametric curves

DeBoor, Hoellig and Sabin (CAGD 4, 1987) proved that to each planar curve segment there exists (under weak assumptions) a polynomial cubic which generalizes the Hermite interpolation scheme and has 6–th order of accuracy. This result can be improved to 8–th order of accuracy using rational cubics instead of polynomial ones.
Moreover, applying methods of affine differential geometry, much more insight concerning the geometric meaning of the assumptions and the asymptotic behavior will be gained. In particular, the tripel bifurcation point playing a crucial role in that former paper turns out to simply correspond to the osculating parabola at the limit point.
Similar settings of rational approximants (e.g. a 5–th order conic), their orders of accuracy and their limits will be discussed.

M.J. PRATT, R.J. GOULT :

Rational approximation of curves

A method is presented for the rational approximation of scalar or vector–valued functions over the interval $[0,1]$. The error functional is chosen so that its minimisation leads to a linear system of equations whose solution gives a 'good' but not 'best' approximation. The method is therefore suitable where computational efficiency is important, where optimal accuracy is not a major consideration or where a good starting point is needed for the iterative computation of a 'best' approximation. The use of an orthonormal basis reduces the scale of the computational problem, and it is shown how certain sets of constrained orthonormal polynomials can be used to preserve position and derivative values up to any desired order at the end–points of the interval of approximation. This allows the construction of a C^k continuous rational approximation to a C^k continuous piecewise defined curve by application of the method to each segment successively.

M. ECK :

Degree reduction of Bézier curves

Representing an n–th degree Bézier curve segment $\overline{X}$ as a Bézier curve X of degree $n-1$ is in general not exactly possible. Therfore mostly an approximation process is carried out by minimizing the L_∞–norm. The solution of this problem is called 'best' approximation and can be obtained by the help of Chebyshev polynomials of the first kind. In our talk we present an easy formula how to determine the control–points of X directly from the ones of the given curve $\overline{X}$. Furthermore we point out how to modify this formula in order to obtain $C^{\alpha-1}$–continuity of the resulting curve X in relation to the given curve $\overline{X}$ at the two boundaries. Here so-called 'constrained' Chebyshev polynomials having α-fold zeroes at the boundaries are helpful.

P. DE CASTELJAU :

La Logique du Paramétrage — Artifices de Dècoupage et d'Affectation du Paramètre

Les dangers d'un paramètrage indadapté les courbes polynomiales par morceaux — formes à poles ou B-splines, courbes en pointillé. Deux problèmes distincts: les poles ou points retenus et leurs paramètres. Au lieu de troiter les équations différentielles par des mèthodes approchèes (Runge Kutta), poser directement des èquations aux diffèrences finies et pourquoi pas aux poles. Artifices possibles: calcul des variations — rèpartition uniforme de l'erreur — gèodèsiques — iterations physiques — un exemple: le "ricochet" permettant de dèfinir un système de miroirs aplanètiques comparaison de la solution "exacte" avec "l'approximation". Problème del'initialisation.

H.-P. SEIDEL :

An implementation of multivariate B-splines over arbitrary triangulations

We describe the results of a test implementation that implements the new multivariate B-spline scheme as recently developed by Dahmen, Micchelli and Seidel for quadratics and cubics. The surface scheme is based on blending functions and control points and allows to model C^{n-1}-continuous piecewise polynomial surfaces of degree n over arbitrary triangulations of the parameter plane. The surface scheme can be used to manipulate the shape of the surface locally. Additional degrees of freedom in the underlying knot net allow for the modeling of discontinuities. Explicit formulas are available for the representation of polynomials and piecewise polynomials as linear combinations of B-splines. This work is incorporated into a surface editor, and several examples will illustrate our implementation.

J. WARREN :

Barycentric coordinates for convex polytopes

A generalization of barycentric coordinates for convex polytopes is introduced. Given a convex polytope of dimension d with n facets, the coordinate functions are rational of degree $n-d$. Under this degree restriction, the functions are shown to be unique. Several examples from a 3D implementation are given.

P. BRUNET :

Approximate geometric continuity for modeling closed bounded objects

We study the modeling of closed surfaces (bounding a solid volume) defined by a set of triangular patches. This problem has been studied from several points of view by many authors. In previous works (in the inception of the face octrees) we explored approximate C^0 conditions. De Rose has also shown that relaxing the VC^1 condition may give surfaces of better global quality.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Here we propose the notion of approximate geometric continuity where different tolerances may be set to control the "acceptable discontinuity" for each differential order. We propose a scheme to compute approximately continuous approximating surfaces to a net of points in space, and discuss its properties.

S.-S. JIANG :

The number of independent conditions of G–continuity between Bézier surfaces

Geometric continuity is one of the most important research topics in CAGD and different types of G^1 and G^2 necessary and sufficient conditions between Bézier surface patches have been developed. In the face of these different G^1 necessary and sufficient conditions, we may put such a question naturely: how many independant conditions can determine the G^1 connection of Bézier surface patches totally and is the number of the independant conditions fixed? If the number is fixed and known, we can choose these conditions flexibly to avoid giving unnecessary conditions and to reduce blindness.

N.M. PATRIKALAKIS :

Computation of singularities for computer aided design

The computation of singularities or critical points of polynomial and other more complex vector fields in a finite subdomain of the n–dimensional Euclidean space is the underlying fundamental process behind several important engineering and scientific problems. These include, for example, design, analysis, scientific visualization, and manufacture of complex objects in a computer environment. This lecture starts with a review of extant solution techniques and focuses on recent research by the MIT Design Laboratory in this general area. Specifically, we summarize the algorithmic techniques we have developed on computation of solutions of systems of non–linear polynomial equations and other implicit equations involving more complex functions. Such equations arise in shape interrogation problems including intersections of sculptured objects, symmetry transforms, distance function computations, visualizations of rational and offset or parallel surfaces, stationary point computations and in differential geometry interrogation of complex free–form surfaces. Examples illustrate our techniques.

F.-E. WOLTER :

Foundations of medial axis and cut locus in the Euclidean space

The cut locus C_A of a closed set A in a Euclidean space E is defined as the closure of the set containing all points which have at least two shortest paths to A. The medial axis of a solid D in E is defined as the union of all centers of all maximal discs which fit in this solid. We assume in the medial axis case that D is closed and that the boundary ∂D of D is a topological (not necessary connected) hypersurface of E. Under these assumptions the medial axis of D equals that part of the cut locus of ∂D which is contained in D. The concepts of cut locus and medial axis have recently been found to be important as tools for global shape interrogation and representation in CAGD. There exist some computational methods to compute the medial axis and the cut locus in a variety of practically relevant cases. However statements on fundamental topological relations between the shape of a solid and its medial axis mainly exist as conjectures although these relations are crucial for global shape interrogation and representation. We present several basic topological results on medial axis and cut locus which answer open questions in this area.

R. GOLDMAN :

Bounding arclength and surface area of Bézier curves and surfaces

The perimeter of the control polygon is an upper bound for the arclength of a Bézier curve. We study the convergence properties of this bound under recursive subdivision and degree elevation.
The surface area of the control polyhedron is not a bound for the surface area of a Bézier surface. We will provide examples to illustrate this problem. We go on to provide a time upper bound for the surface area of a Bézier patch, and we study the convergence of this bound under recursive subdivision and degree elevation.

R. SCHABACK :

Remarks on surface intersection algorithms

A "divide–and–conquer" method and a "marching" method are combined into a multistage (and parallelizable) algorithm for computing surface–surface intersections at a trade off between speed and safety. The (global) divide–and–conquer method relies on early disposal of patch pairs with nonintersecting bounding boxes, and tries to find a starting point for a marching method as early as possible. The (local) marching method combines stepsize estimation and relaxation into a single (parametrized) Newton iteration. By application of a Newton–Kantorovich convergence theorem we provide domains of uniqueness which can be used to discard patch pairs with a domain contained in a uniqueness box. Proofs are given for the "classical" observations:

a) Parallelepiped boxes are better than min–max coordinate boxes, if they make use of quadratic convergence of subdivision. Asymptotically (for accuracy $\varepsilon \longrightarrow 0$), the computational effort reduces to the square root of the original effort.

b) Marching algorithms are faster than divide–and–conquer methods, the difference being $O(\log\log\varepsilon)$ and $O(\varepsilon^{-\alpha})$ behaviour for $\varepsilon \longrightarrow 0$.

Examples show the practical feasibility of this approach; the number of path pairs to be treated stays within reasonable bounds.

D. HANSFORD, G. FARIN, H. HAGEN :

Gauss frame offsets

We describe a method to approximate the offset of a rectangular surface by a bicubic Bézier patch. The method takes advantage of the geometric controls of the Bernstein Bézier form, using the Gauss frame and a least square fit to sampled exact offset points.

R.K.E. ANDERSSON:

Surfaces with prescribed curvature

In the interactive design of freeform surfaces, assessments of curvatures is an important tool for the analysis of surface shape. Commonly used measures include Gaussian and mean curvatures as well as normal curvatures along a given field of directions and curvatures of planar non–normal sections.
In the talk, we will discuss the process of surface design based on direct modifications of these curvatures. Mathematically, this amounts to the solution of certain nonlinear partial differential equations, some of which have attracted considerable attention during the preceding 25 years. The problem will be considered both from a mathematical and numerical point of view.

J. PETERS :

Constructing C^1 surfaces of arbitrary topology using biquadratics and bicubics

Given a polyhedral mesh of arbitrary topology, a C^1 surface is constructed that interpolates the average of the vertices of each mesh cell. The surface is generically biquadratic and locally in the convex hull of the mesh points.

L. PIEGL :

Surface triangulation via constrained Delaunay triangulation

This talk presents a method of triangulating trimmed parametric surfaces based on triangulation in the parametric space. The trimmed 2–D region is triangulated using constrained Delaunay triangulation and consists of the following steps:

1. discretise boundary curves and create a grid structure,
2. establish constraints along the boundary, and
3. compute constrained Delaunay triangulation.

The constrained triangulation breaks the constraining polygon such that the resulting triangulation will always satisfy the Delaunay criterion. A shelling procedure is applied to put triangles together such that the triangulation is valid and complete, and no triangles are computed on the invalid region of the parametric space.

M. DAEHLEN, E. ARGE, A. TVEITO :

Computing smooth box spline approximations to regulary scattered data

We consider the problem of solving systems of linear equations arising from a particular scattered data interpolation problem with box splines. We present a method for solving these equations and study some features of the method from a computational point of view.

R. BARNHILL :

Geometry processing and surfaces on surfaces

A key problem in geometric modeling is the development of a robust surface–surface intersection algorithm. We present a new feature of our marching algorithm for this problem, namely, improved initial approximations for intersection curves using an enhanced bounding box technique. This research is joint with Todd Frost and Ai Zhang. The other part of our talk concerns a trivariate approach to triangle–based interpolation to data defined on a smooth surface. This new algorithm has significant advantages over known algorithms such as the restriction of scattered data interpolants. This research is joint with Helmut Pottmann and Karsten Opitz.

G. GEISE :

The idea of shift curves for free form curve design

Piecewise defined polynomial curves of some smoothness, not necessary maximal with respect to the degree of the polynomials used, are named (polynomial) subspline curves. The most famous subspline curves are the B–spline curves. They have maximal degree of smoothness, and this hold, in substance, independent of the choice of control points. Subspline curves composed by Bézier segments in general have degree 0 of smoothness, otherwise some control points take place in G^r–configurations ($r > 0$). A comparison of these two sorts of subspline curves leads to the shift curve idea. It is possible to use the control points sucessively subset by subset, organized by the shifts, that the most important property of B–spline curves, namely the independence of smoothness on the choice of control points, is preserved, but the degree of smoothness is proper for the technique of design. The content of the lecture is to illustrate the shift curve idea. This is a work in common with G. Beyer, Th. Nestler and G. Meinl.

T. LYCHE :

Rate of convergence for subdividing exponential B–splines

Splines in tension were introduced by Schweikert in 1966 as a means of eliminating wiggles in cubic spline interpolation. These splines have smoothness C^2 and have one shape parameter per interval. A B–spline basis for these functions was introduced by Koch and the author in 1989. In this talk we study the convergence of the polygon formed by connecting the B–spline coefficients by straight lines. We show that the convergence is quadratic in the knot spacing and usually exponential in the size of the tension parameters.

A. ROCKWOOD :

Topological design of sculptured surfaces

Topology is primal geometry. Our design philosophy embodies this principle. We report on a new surface design perspective based on a "marked" polygon for each object. The marked polygon captures the topology of mappings from polygon to sculptured surface. The mappings arise naturally from the topology and other design parametrization for surfaces with handles. Examples demonstrate the design of sculptured objects and their manufacture.

B. WOERDENWEBER :

Light play with free form

With the rising influence of aerodynamic and styling considerations on vehicle design the task of producing vehicle lights has become more changing. In order to produce optically functional and manufacturable lights a number of free form surface types have to be exploited:

1. Functional free form surface — headlight reflectors are computed to generate a given light distribution,
2. "fuzzy" free form surface — the reflector surfaces are randomly perturbed and unwanted violations of the light distributions detected in order to indicate sensitive regions on the reflector surface,
3. "fancy" free form surface — free form surfaces are overlayed with smaller optical primitives (eg. torii) in a styling compliant pattern.

The talk illustrates practical examples of free form surface design and verification as they arise in vehicle lighting and indicates the neccessary computing environment.

L. SCHUMAKER :

Computing optimal triangulations by simulated annealing

It is shown how simulated annealing can be used to compute optimal triangulations whenever the global optimality criterion can be defined in terms of local quantities so that edge swapping can be applied. Examples involving geometric measures (such as minmax or maxmin angle) and surface measures (data dependent triangulation) are presented.

L. BARDIS :

Generalized boundary representation for free form shapes

Large engineering structures, such as marin and aerospace vehicles, are bounded by sculptured surfaces and are characterized by complex internal subdivision. The objective of the talk will be to present a topological structure for n–dimensional manifolds involving general parametric free form curves and surfaces, based on the boundary representation paradigm. Some existing approaches will be briefly reviewed and the proposed model, based on a graph theoretic cell tuple structure, will be described in some detail. Some basic theorems underlying the proposed structure and a procedure for incremental construction of complex models will be addressed. Finally, a methodology for implementation by means of adjacency graphs in an object–oriented environment, which simplifies the task of processing and interrogating general procedural parametric free form curves and surfaces, will be discussed.

J. HOSCHEK :

Bézier / B–spline curves and surfaces on quadrics

An algebraic approach to construct curves and surfaces on quadrics is developed. This algebraic representation can be interpreted as a stereographic projection combined with a hyperbolic map. With help of product formulas for the Bernstein and the B–spline basis functions one get arbitrary Bézier and B–spline curves and surface patches on the quadrics. The degree elevates from n to $2n$. The problem to determine all biquadratic tensor–product patches on a sphere can be easily solved.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft