MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 27/1992

Porous Media

21.-27 Juni 1992

Die Tagung fand unter der Leitung von Jim Douglas (West Lafayette), Cornelis J. van Duijn (Delft) und Ulrich Hornung (Neubiberg) statt. Im Mittelpunkt des Interesses standen Probleme der mathematischen Modellierung von Strömungs- und Transportprozessen in porösen Medien. Zu den wichtigsten Themen gehörten: Skalenaspekte (mikro, meso, macro, mega), räumliche Heterogenitäten, Homogenisierung, gekoppelte nichtlineare Systeme, freie Randwertaufgaben, Identifizierung von Parametern, singuläre Störungen, effektive numerische Verfahren, stochastische Modelle und statistische Methoden. In den zahlreichen Diskussionen wurden unter anderem die prinzipiellen Schwierigkeiten angesprochen, zuverläßliche Modelle aufzustellen, die mit den Grundgesetzen der Physik und Chemie verträglich sind, aber auch mit den Beobachtungen und Experimenten auf Feld-Ebene. Dies bezieht sich insbesondere auf Mehr-Phasen-Strömungen und auf die Einbeziehung biologischer Vorgänge.

Vortragsauszüge

Myron B. **Allen** *Parallelizable methods for modeling flow and transport in heterogeneous porous media*

Groundwater flow and contaminant transport present several challenges to the mathematical modeler. Among these are the need to compute accurate water velocities as well as accurate hydraulic heads, the need to accommodate fine-scale geologic heterogeneities without destroying the efficiency of the flow-equation model, and the need to track sharp contaminant concentration fronts in advection-dominated flows.

For the groundwater flow equation, mixed finite-element models yield approximate velocities whose accuracy is comparable to that of the computed heads. However, mixed methods share with standard formulations two difficulties associated with aquifer heterogeneities. First, large variations in hydraulic conductivity can cause slow convergence of iterative linear solvers. Second, the fine grids needed to resolve the heterogeneities lead to poor conditioning. We present an iterative scheme for the lowest-order mixed finite-element equations. This method largely overcomes both sources of poor behavior through the use of an inner-outer iteration strategy incorporating a highly parallelizable multigrid scheme.

To solve the equation governing contaminant transport, we use a finite-element collocation method that has high-order spatial accuracy. The timestepping scheme combines two ideas: a modified method of characteristics, which reduces the temporal truncation error in advection-dominated transport, and an alternating-direction formulation, which is "embarassingly parallel" in addition to having a favorable operation count.

Todd **Arbogast** *Computational validation of a homogenized model for flow in fractured reservoirs*

Single-phase, compressible flow in a naturally fractured porous medium can be homogenized by taking the matrix block size ε to zero. This results in the usual equations of flow with some averaged porosity and permeability. Such a model is inadequate to model the true flow. Homogenization of the equations scaled by ε produces a dual-porosity model. Gravitational effects are lost in the matrix unless the equations are posed in terms of the pseudo-potential rather than the pressure. Computational results demonstrate that the dual-porosity solution agrees with the microscopic description of the flow; whereas the averaged parameters model does not. Similar results hold for two-phase, immiscible flow.

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Alexander G. **Belyaev** *Homogenization of elliptic boundary value problems in periodically perforated domains with small shape-varying perforations*

Consider the following nonlinear Dirichlet problem in $\Omega \setminus T^\varepsilon$

$$\begin{cases} -\nabla \cdot (|\nabla u_\varepsilon|^{p-2} \nabla u_\varepsilon) &= f \text{ in } \Omega \setminus T^\varepsilon \\ u_\varepsilon \in W_0^{1,P}(T^\varepsilon) \end{cases}$$

where $p \geq 2$, Ω is a given open bounded domain with smooth boundary $\partial\Omega$, T^ε is the union of small inclusions, which are ε-periodically distributed and similar to a reference inclusion T_ε, which is $p-cap(T_\varepsilon) \to 0$ as $\varepsilon \to 0$, $f \in L^{p'}$ (with $1/p + 1/p' = 1$) is a given function. It is proved that there exist a critical ratio between the first eigenvalue of some special nonlinear spectral problem and the period of perforations ε, which determine the first term of the asymptotic expansion for the problem of homogenization. Similar problems are also considered for the elasticity system and when the inclusions are ε-periodically distributed along the boundary $\partial\Omega$.

Alain **Bourgeat** S. M. Kozlov, A. Mikelić *Effective equations of two-phase flow in random media*

We consider the behavior of incompressible two-phase flows in heterogeneous reservoirs with randomly placed heterogeneities; that is, in media with permeability K and porosity ϕ which are stochastically homogeneous random fields. We assume viscosity and diffusion flux being a given nonlinear function of concentration. Using the tools of stochastic homogenization we get the nonlinear effective equations which govern flow behavior in a homogeneous medium, being equivalent in the sense of homogenization theory to the original one. Using the rescaling parameter ε, corresponding to the characteristic scale of heterogeneities, we prove the convergence of the homogenization process when $\varepsilon \to 0$.

John C. **Bruch, Jr.** *Efficient iterative parallel computational methods for free surface seepage*

Results will be presented from two studies in which a parallel computer was used to solve a free surface seepage problem. The first study highlights an interface relaxation domain decomposition technique using finite elements on an iPSC/2 Hypercube. On the interfaces between two adjacent subdomains are imposed a continuity condition on one side and an equilibrium condition on the other side. SOR (Successive Over-Relaxation) iterative processes with projection are then carried out for all subdomains with a relaxation process imposed on the interfaces. With this domain decomposition technique, the problem can be solved parallelly until convergence is reached both in the interiors and on the interfaces of all subdomains. Furthermore, it is shown through numerical experiments how the values of the relaxation parameters, the choice of imposed boundary conditions, and the number of subdomains (i.e. the number of processors used) affect the solution convergence in this parallel computing environment.

The second discussion will focus on a simple and efficient parallel SOR iterative algorithm for both the finite element and the finite difference methods. Implementation on an iPSC/2 Hypercube concurrent computer will be given along with relevant results, such as speedups, efficiencies, and number of iterations.

John **Chadam** *Reactive flows in porous media*

When reactive fluids flow through a porous medium porosity/permeability changes may occur which can lead to shape destabilization of the reaction front (reaction-infiltration instability). This phenomenon is studied mathematically in the context of the simplified model equations in $-\infty < x < \infty, 0 < y < L$:

$$\begin{aligned} \varepsilon(\varphi\gamma)_t &= \nabla \cdot (\varphi D(\varphi) \nabla \gamma + \varphi k(\varphi) \gamma \nabla p) + \varphi_t \\ \varepsilon \varphi_t &= \nabla \cdot (\varphi k(\varphi) \nabla p) \\ \varepsilon \varphi_t &= -(\varphi_f - \varphi)^{\frac{2}{3}} (\gamma - 1) \end{aligned}$$

where φ is the porosity, $\gamma = c/c_{eq}$ is the scaled solute concentration, p is the pressure and $\varepsilon = c_{eq}/\rho$ ($\simeq 10^{-5}$) is the ratio of the equilibrium concentration to the density of the dissolving material. These equations are to be solved subject to initial and no-flow boundary conditions on $y = 0, L$ and the asymptotic conditions $\varphi = \varphi_f$, $\gamma = 0$ and $p_x = p'_f$ at $x = -\infty$ and $\varphi = \varphi_0 < \varphi_f$, $\gamma = 1$ and p_x to be determined at $x = +\infty$.

We show the existence of a unique weak solution for all time in $[-M, M] \times [0, L]$ and up to time $t = \tau_0/\varepsilon$ in $(-\infty, \infty) \times [0, L]$. We prove the existence of a unique planar travelling wave solution for all $\varepsilon > 0$ and show that as $\varepsilon \to 0$, this solution tends to the travelling wave solution of the free boundary problem obtained formally as the limit of the above problem. The shape stability of the moving free boundaries are examined using a linear and weakly nonlinear analysis. Computer simulations are presented to show the wide variety of structured fronts (tip-splitting, budding, meandering) which can be obtained when many unstable modes compete in selecting the pattern. The analysis is extended to consider viscosity effects (competition of the reaction-infiltration instability with the Saffman-Taylor instability) and the effects of layering in the medium.

Michel **Chipot** *On the dam problem*

We consider the dam problem with so-called leaky boundary condition, i.e. find $(p, \chi) \in H^1(\Omega) \times L^\infty(\Omega)$ such that

(i) $p \geq 0, 0 \leq \chi \leq 1,\ \chi = 1$ on $[p > 0]$,

(ii) $p = 0$ on S_2,

(iii) $\int_\Omega \nabla p \nabla \xi + \chi \xi_y dx - \int_{S_3} \beta(x, \varphi - p) \xi d\sigma(x) \leq 0 \quad \forall \xi \in H^1(\Omega),\ \xi \geq 0$ on S_2.

(φ is the hydrostatic pressure on $S_2 \cup S_3$ due to the reservoirs, β is some appropriate function.)

We prove existence, uniqueness in various cases and indicate properties of the solution.

Gedeon **Dagan** *Representative averaging volume and dispersivity: Are they meaningful for heterogeneous formations?*

Flow and transport in heterogeneous formations, of spatially variable permeability, are considered. The basic problem is whether one can recover the Darcy law with an effective permeability and the convection-dispersion equation with constant effective dispersivity by a suitable space averaging. It is shown that the constant effective dispersivity concept implies ergodicity. In turn, the latter is ensured only if the transverse site of solute plumes is much larger than the permeability integral scale. This condition is generally not met for point sources and regional flows.

Clint **Dawson** *Numerical simulation of contaminant transport in groundwater with adsorption*

We consider the numerical approximation of nonlinear, degenerate parabolic equations arising in contaminant transport. The prototype equation is

$$u_t + u_t^p + qu_x - \varepsilon u_{xx} = 0$$

where $0 < p < 1$. The numerical method used is based on combining a Godunov-type procedure for advection with a mixed method for diffusion. Error estimates for two different adsorption models are derived. Numerical results are presented which demonstrate the convergence of the method. The method has also been used to study the long-time behavior of solutions; these results are joint work with Hans v. Duijn and Robert Grundy.

Jim **Douglas**, Jr. *Immiscible flow in multi-scale porous media*

Consider a porous medium with a heterogenous periodic structure, and then consider immiscible, incompressible, two-phase flow in this medium. Apply homogenization to the family of scaled replicas of the problem to derive a practical model for the flow. A dual porosity model results in the simplest case, while multi-porosity models can result if there exists a multi-scaling in the inhomogeneity. A fractal scaling of the inhomogeneneities leads to an infinite-porosity model. Examples arising from fractured reservoirs will be illustrated.

Paul **DuChateau** *Indirect determination of hydraulic properties of porous media*

Single and multi-phase flow in unsaturated porous media may be modelled by non-linear PDE's whose coefficients are related to the hydraulic properties of the porous medium. We often assume that these coefficients are functions of the (unknown) dependent variable only. In order to model flow in a specific porous medium, it is necessary to determine these coefficients, either directly by means of a usually very difficult physical experiment, or indirectly. The indirect method requires a simple physical experiment whose data is then the input to a quite difficult mathematical problem called an inverse problem.

A common practical method for solving inverse problems involves characterizing the unknown coefficient function(s) in terms of a finite number of parameters and then adjusting these parameters until the computed output from the model matches the experimentally measured output of the physical experiment. Such methods are referred to as "history matching" and while they lead to solutions, there is no easily established connection between the computed solution of the history matching problem and a solution to the inverse problem.

An alternative procedure for solving the inverse problem involves reconstructing the unknown coefficient(s) explicitly in terms of the data by using the over-specified condition. This reconstruction procedure is illustrated for a simple example. For this example, we define the notion of polygonal equivalence on the class of admissible coefficients and prove that two admissible coefficients in the same equivalence class produce equal outputs for a given input. This establishes the uniqueness of the solution to the inverse problem. An algorithm is described which generates a monotone sequence of elements in an equivalence class of admissible coefficients. If a solution to the inverse problem is assumed to exist, then the sequence generated by the algorithm can be shown to converge monotonically to their solution. Finally, some numerical examples are given to illustrate the working of the algorithm.

P. Donato, D. Cioranescu, Horia I. **Ene** *Stokes Problem with Non-homogeneous Slip Boundary Conditions*

We study the Stokes problem in a periodical model of porous media using a boundary condition of the form

$$\sigma_{ij} \cdot n_j + \alpha\varepsilon^\gamma u_i = g_i$$

which means that the stress vector induces a slowing effect on the motion, expressed by the coefficient $\alpha\varepsilon^\gamma$, and also under the action of exterior surface forces.

In the limit $\varepsilon \to 0$ we obtain three cases:

(i) $\gamma < 1$ a Darcy's law,

(ii) $\gamma = 1$ Brinkmann's equation, and

(iii) $\gamma > 1$ the Stokes equation.

Phillip Binning, Michael A. Celia, Richard **Ewing** *Multiphase transport of contaminants in groundwater*

The ability to numerically simulate single- and multiphase flow of fluids in porous media is extremely important in understanding the flow and in developing remediation strategies for groundwater contamination. The flow is complicated by the presence of heterogeneities in the reservoir and by phenomena such as diffusion, dispersion, and viscous instabilities. The determination of effective parameters and physical models for various length scales is discussed. Permeability averaging techniques coupled with dispersion models are used for both single- and multiphase flow problems at large scales.

Jürgen **Fuhrmann** *Multigrid methods for special convection-diffusion problems*

The paper deals with the convergence of algebraically defined multigrid methods. Based on the theory for multigrid methods with nonnested spaces and noninherited quadratic forms a convergence proof for an algebraically defined multigrid method is given. Further, a method is suggested which allows to construct multigrid algorithms for special nonsymmetric problems arising from exponential fitting discretizations.

James **Glimm** *Scaling laws for dispersion induced by rock heterogeneities*

Anomalous (non Fickean) diffusivity is observed in field data, so that laboratory measurements of diffusion constants do not scale to predict diffusion constants appropriate for field studies. A systematic study has been performed to understand anomalous diffusivity scaling as a consequence of rock heterogeneity correlations which contain variation on all length scales. Fractal and multifractal correlations are examples. Theoretical analysis and numerical experiments provide a consistent understanding of anomalous diffusivity scaling.

S. P. K. Sternberg, M. M. Botz and Robert A. **Greenkorn** *Mixing during miscible flow in fractal porous media*

Mixing during miscible flow in porous media appears to be scale dependent. The objectives at this work are to investigate the effect of heterogeneity on mixing experimentally and to explain the scale dependence with a random walk model and fractal analyses at the experimental and model results. Concentration versus time at a given distance was measured in three homogeneous non-uniform models with average permeabilities. These models were connected in all combinations 12, 21, 13, 31, 23, 32, 123, 231, 321 and 132 and their overall dispersion measured. Dispersion depended on advective velocity, permeability, viscosity, length and order. A random walk model containing a constant velocity mode and a random dispersive mode provided calculated dispersion values for models 1,2 and 3 by varying the size of the dispersive mode. The trace of the streamlines of the random walk model was analyzed in each instance to determine its fractal dimension. The size of the dispersive mode correlates with the fractal dimension. The fractal dimension is inversely proportional to velocity. Knowing the fractal dimension-size at dispersive mode allowed calculation at the dispersive (and dispersively) of the heterogeneous models. The results correlate well with the experimentally measured results. However, the random walk model did not show order dependence.

Allan **Gutjahr** *Conditional simulation and joint use of data applied to flow modelling*

Models for joint conditioning on both head and transmissivity have been developed for steady two-dimensional stochastic flow. First-order perturbation analysis was used to develop the conditioning equations in both the spectral and space domain. Monte-Carlo simulations using a Fast Fourier Transform Spectral method were used to study aspects of linearization and conditioning. The first example showed the effects of different numbers of data points on travel paths and arrival time. For small input (transmissivity) variance as few as twenty observations can substantially reduce the uncertainty in both travel paths and arrival times at a boundary. The second example compared linearized conditioned solutions with an exact multi-grid solution. With appropriate conditioning one can get good agreement between the two solutions. The final example showed some results for a large variable (variance of the log of transmissivity equal to one) case where the log of transmissivity had a trend. Added measurements reduced the travel path uncertainty but had a small effect on the travel time distribution.

Michael **Hauhs** *Problems of water transport through forest ecosystems*

The functional relation between terrestrial ecosystems and their abiotic surrounding can be expressed by the irreversible fluxes of matter between them. Ideally, in undisturbed ecosystems matter input fluxes do not depend on the state of the system and output fluxes not on the state of the environment.

The study of transport through ecosystems was often initiated by a real or anticipated loss of predictability within their experience-based management. The scientific task is seen as an attempt to compensate this loss through an increased understanding. For this research no theory is available that allows to abstract from the historical or regional context of field observations. Therefore, ecosystems cannot be put into defined initial states. However, a hypothetical phenomenology allows to define the undisturbed ecosystem as an information-closed entity from wich output fluxes of matter are redundant with respect to the thermodynamically corresponding input fluxes.

A transport problem is considered for which the boundaries of the porous flow region coincide with those of an undisturbed ecosystem. They are of Neumann type along the whole boundary. In this extreme case the prediction of the ecosystem's effect on its environment is trivial for a given input, whereas the identification of model parameters for any point inside the flow region is impossible. Disturbance of such a system shows a complementary relation between the degree of "understanding" and its functional predictability. This example reflects typical epistemological problems in ecosystem research. It is argued that such systems would form a principally different class of objects for scientific study rather than a class of high structural complexity.

George **Homsy** *Viscous fingering in porous media*

We study the problem of viscous fingering in displacement processes involving miscible fluids, with a focus on two areas:

(i) the effect of anisotropic dispersion in determining the nature of fingering and mechanisms of finger interactions in two dimensions, and

(ii) the nature of three-dimensional fingering.

Our approach is large scale simulation by spectral methods based on Hartley transforms. We find that anisotropic disperson can *substantially* alter the mechanisms of finger interaction. In addition to subharmonic pairing and tip splitting described earlier, we observe coalescence of neighboring fingers, a phenomenon that is absent in the case of isotropic dispersion, (Tan., C.-T. & G. M. Homsy *Physics of Fluids* **30**, 1239 (1989)). In addition, we find the growth of the mixing zone to be linear in time with a growth rate that is nearly *independent* of both Peclet number and dispersion anisotropy, suggesting that non-linear finger interactions are controlled by their corresponding pressure fields.

We report recent simulations of *three-dimensional* fingering, believed to be the first reliably accurate simulations at high Peclet number. The mechanism observed earlier, including tip splitting, persist in three dimensions and the mixing zone grows at nominally the same rate as before, indicating little difference in fingering in two and three dimensions.

Jérôme **Jaffré** *Riemann solvers for two-phase flow at the interface between two rock types*

When discretizing a first order nonlinear hyperbolic equation, one has to face the calculation of numerical flux function at the boundaries of the discretized cells. This calculation involves generally the solution, exact or appropriate, of a one-dimensional Riemann problem. We consider the equation modelling a one-dimensional two-phase flow in porous media with negligeable capillary effects. Two numerical flux functions are of interest: the Godunov flux function which is given by the exact solution to the Riemann problem, and the upstream mobilities flux function commonly used by petroleum engineers. In practical situations, the flux function associated with the continuous problem may vary discontinuously from one cell to the other due to heterogeneities in the porous medium. Using the special form of the flux function in the case of two-phase flow, we show how to calculate a numerical flux associated to the Riemann problem with a discontinuous flux function.

Barbara Lee **Keyfitz** *Conservation laws that change type; multidimensional conservation laws*

The systems of evolution equations that model two-phase flow or multi-phase porous medium flow are often simplified by closure or zero-viscosity assumptions that result in convervation laws that are not everywhere hyperbolic. We have a number of results on shock formation and evolution, structural stability of shock patterns, and stability under perturbation of such ill-posed systems. These lead to some advice for computation and further modelling.

Peter **Knabner** *Contaminant transport through porous media with mobile and immobile sorbents*

For dissolved contaminants in porous media, e.g. soils, the mobility is a major concern, as a reduced mobility may prevent leaching to groundwater or an enhanced mobility may be of importance for sanitation strategies. Immobile sorbents like the soil matrix reduce mobility, but there may be also mobile sorbents like dissolved organic carbon (DOC), being a solute itself, possibly also undergoing adsorption to the soil matrix. The sorption of the contaminant to DOC increases its mobility, but the sorption of the "contaminant-DOC-complexes" being formed may again cancel this effect or even enhance mobility. We set now a model for the transport of a contaminant, taking into account the effects described. A transformation reduces this model to a multiple-site-adsorption model with space and time dependent isotherms and rate functions. We point out some properties not present in the situation without mobile sorbents and investigate the possibility of travelling wave solutions.

John H. **Knight** *Some source-obstacle solutions of the linearized equation for steady unsaturated soil water flow*

The movement of unsaturate soil water under the combined influence of capillarity and gravitational forces is governed by Richard's equation, which can be approximated by a linear diffusion-convection equation. For steady flow in two or three dimensions this linear equation has multipole solutions, of which the line and point sources are special cases. The Stokes stream function for a point source is simple and well known, but there is no simple expression known for the stream function corresponding to a line source. A useful new and rapidly convergent Bessel function series for this stream function will be derived.

Infinite series of multipole solutions can be used to solve exterior problems for flow in regions outside cylinders with axes horizontal or outside spheres, with the series coefficients determined by the boundary conditions on the surface. The form of the boundary condition will determine which analytical solution is easier to find - the moisture distribution or the stream function. It will be shown how to find one of these once the other is known.

These solution methods will be used to derive analytical solutions for the moisture distribution and stream function corresponding to an impermeable cylindrical obstacle in the flow field of a line source.

Sergej M. **Kozlov** *Central limit theorem for multiscaled permeability*

Recent experiments of Noetinger and Jacquin (IFP) showed high accuracy of the effective 3D permeability formula which is the cube of the average of the third root of local permeability. Here I propose a model of the local multiscaled log-normal permeability for which this formula is asymptotically exact. The model reflects a real situation of many (asymptotically infinite) lengthscales of heterogeneities.

David O. **Lomen** *Two dimensional solute transport*

We have successfully found perturbation solutions to the one dimensional solute transport problem, for either continuous or step inputs. In this paper we extend the techniques used in the one dimensional case to the case of solute transport in two dimensions. The type of scenario we have in mind is that of having a line source for both water and solute. The water flow is already steady by the time solute is put into the water stream. The amount of solute is sufficiently small that its presence does not change the steady flow. We are interested in describing the solute movement.

The basic idea is to recognize that, once diffusion has been neglected, the only way solute can be transported is along the streamlines present in the given water flow. Therefore it is natural to follow the solute along these streamlines and since streamlines do not cross, we can concentrate on a single streamline and find the concentration of the solute here as a function of time. Even though we have two dimensional streamlines, a suitable change of coordinates makes the problem one dimensional and we can use the technique developed in the solution of the one dimensional case.

Stefan **Luckhaus** *Singular perturbation problems in hydrology*

Two singular perturbation problems for the equation of filtration flow are presented:

$$\Theta\partial_t s - \nabla(aK(s)(\nabla p + g)) = 0 \text{ in } \Omega, \tag{1}$$

where Θ is porosity, $aK(s)$ permeability, with $K(1) = 1$, p pressure, g gravity, and $s = s(p)$ the saturation, with $s \equiv 1$ for $p \geq 0$.

The first problem is the boundary condition for the infiltration of rainwater. Actually rainwater stands in shallow furrows and pools distributed over the surface. This is modelled by the boundary conditions ($\partial\Omega = \partial_1\Omega \cup \partial_2\Omega,\ \partial_2\Omega \supset \Gamma_{2,\varepsilon}$):

on $\partial_1\Omega$: Neumann or Dirchlet boundary conditions

on $\Gamma_{2,\varepsilon}$, the part covered by water: $p = 0$

on $\partial_2\Omega \setminus \Gamma_{2,\varepsilon}$: the outflow condition $p \leq 0, \partial_\nu p + g \cdot \nu \leq 0,\ p \cdot (\partial_\nu p + g \cdot \nu) = 0$.

The estimate $\int\int ak|\nabla_p|^2 \leq$ const is enough to show that if $\chi_{\Gamma_{2,\varepsilon}} \rightharpoonup \chi$ weakly on $\partial_\varepsilon\Omega$ and $\chi > 0$, then the solutions p_ε converge to p_0 which is the solution of equation (1) with $p_0 = 0$ on all of $\partial_2\Omega$. The open problem is to get the next order in the ε expension, e.g. for periodic $\Gamma_{2,\varepsilon}$, to get an estimate on how the infiltration is overestimated by the values for p_0.

The second problem is to model groundwater flow in domains of small soil depth versus diameter ratio

$$\Omega = \Omega_\varepsilon = \left\{(x, y, z)|\ g_1(\varepsilon x, \varepsilon y) < z < g_2(\varepsilon x, \varepsilon y),\ (\varepsilon x, \varepsilon y) \in \hat{\Omega}\right\}$$

with impermeable boundary $\{z = g_1(\varepsilon x, \varepsilon y)\}$. The aim is to find an equation valid in $\hat{\Omega}$ for lateral flow, which is of order ε. Formally for the second term in the ε expression one finds, for a=constant:

$$\Theta\partial_t\tilde{h} = \nabla(a\tilde{\tilde{h}}(\nabla h + \nabla g_1)) \text{ in } \hat{\Omega}$$

where

$$\hat{h} = h + \int_{g_1+h-g_2}^{0} s, \quad \tilde{\tilde{h}} = h + \int_{g_1+h-g_2}^{0} K(s).$$

This is a modified Dupuit equation, for $g_1 + h$, the level zero of p. The open problem is to make this rigorous.

Gunter M. **Meyer** *Comments on the Hele-Shaw problem*

The results of some numerical experiments on the extraction of fluid from a horizontal Hele-Shaw cell will be described. The mathematical model is a free boundary problem for the Laplacian of pressure where the free boundary moves with the Darcy velocity. It is known that its solution is unstable unless surface tension is taken into account. A front tracking algorithm based on a method of linear discretization is used to solve the free boundary problem with a curvature dependent pressure on the free boundary. As in the related undercooled Stefan problem it is shown that increasing surface tension regularizes the problem.

Pierre Fabrie, Andro **Mikelić** *Regularity and uniqueness result for two-phase miscible flows in porous medium*

We consider the Peaceman model for nonstationary two-component miscible flows through porous media. It is known that there exists a global weak solution and we address the question of its regularity, and after that we are in a situation to prove uniqueness. Our results hold under the assumption that the viscosity is not too oscilating, or - speaking from the physical point of view - that the mobility ratio is not too much larger than 1. Furthermore, we address the regularity and uniqueness for a case of nonturbulent dispersion tensor.

Marek **Niezgódka** *Nonlinear non-isothermal diffusion in multi-phase systems*

A class of problems in non-isothermal diffusive systems involving several phases is considered, with standard phase-field model and diffusive phase separation as references. New features accounted to cover time-dependent constraints imposed on the systems and couplings within evolutionary terms. In Hilbertian setting, the problems reduce to the evolution systems of the form

$$\begin{cases} u' + \mu w' + \partial\varphi^t(u, w; u(t)) \ni f(t, u(t), u(t)) \\ xu' + w' + \partial\psi^t(u, w; w(t)) \ni g(t, u(t), w(t)), \; t > 0 \end{cases}$$

with $\varphi^t(u, w; z)$, $\psi^t(u, w; z)$ strictly convex, l.s.c., proper components that are subject to some regularity hypotheses for t-variations, and with Lipschitz components f, g for $u(0), w(0)$ prescribed. The systems are shown to admit weak solutions, unique in a number of special cases. A perturbation result is given for φ^t, ψ^t, f, g and $u(0), w(0)$ variations. (Joint work with N. Kenmochi, Chiba)

Bernt **Øksendal** *Stochastic partial differential equations with applications to fluid flow in porous media*

A fundamental problem with the mathematical treatment of the partial differential equations describing the fluid flow in porous media, is that some of the parameters, such as the permeability, are varying rapidly from point to point and therefore basically unknown. This makes it natural to represent these quantities by some kind of "noise", thereby leading to stochastic partial differential equations (SPDE's). We present a program for solving such SPDE's with special emphasis on the *stochastic pressure equation*

$$\begin{cases} \nabla \cdot (k \nabla p) = f, & x \in D \subset \mathbf{R}^n \\ p = 0, & x \in \partial D \end{cases}$$

where $p = p(x)$ is the pressure, $f(x)$ is the given source rate, and $k = k(x)$ is "positive noise" representing the permeability.

Olga A. **Oleinik** *On Solutions of semilinear elliptic equations in unbounded domains*

Many problems of mathematical physics lead one to consider solutions of linear and nonlinear equations in cylindrical and other unbounded domains (stationary states, travelling waves, homogenization, boundary layer problems, Saint-Venant's principle and so on). In this lecture we consider solutions of boundary-value problems for some classes of nonlinear second order elliptic equations in unbounded domains, asymptotic behavior of these solutions at infinity. For some boundary-value problems the existence and uniqueness theorems are proved without any restrictions on the behavior of solutions at infinity. We formulate here the following results.

Theorem Let $u(x)$ be a solution of the equation

$$\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x')\frac{\partial u}{\partial x_j}\right) + \sum_{i=1}^{n} a_i(x')\frac{\partial u}{\partial x_i} - |u|^{p-1}u = 0,\ p > 1$$

in $S(0,\infty) = \{x : x' \in w, 0 < x_n < \infty\}$, $x' = (x_1, \ldots, x_{n-1})$, ω is a smooth bounded domain in $\mathbf{R}_x^{n-1}$, with the boundary condition

$$\frac{\partial u}{\partial \gamma} \equiv \sum_{i,j=1}^{n} a_{ij}(x')\frac{\partial u}{\partial x_j}\gamma_i = 0 \text{ on } \{x : x' \in \partial w, 0 < x_1 < \infty\}$$

where $\alpha_1|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x')\xi_i\xi_j \le \alpha_2|\xi|^2$, $\gamma = (\gamma_1, \ldots, \gamma_n)$ is a normal, $x' \in \omega$, $\xi \in \mathbf{R}^n$, α_1, α_2=const> 0. Here a_{ij}, a_i are bounded measurable functions, $a_{in}(x') \equiv 0$ for $i < n$, $a_{nn} \equiv 1$, a_n= const. If $u(x)$ changes sign for $x_1 > k$ for any $k > 0$, then

$$|u(x)| \le C\exp\{-\beta x_n\},\ C,\ \beta = \text{const} > 0.$$

If $a_n = 0$, $u(x)$ is a positive in $S(0,\infty)$, then

$$u(x) = M_p(x_n + \gamma)^{2/(1-p)} + O(\exp\{-h, x_n\})$$

as $x_n \to \infty$, h_1 = const $> 0, h$ does not depend on u, γ = const with $M_p = (2(1+p)(p-1)^{-2})^{1/(p-1)}$.

Paulo Jorge **Paes-Leme** *Darcy's law revisited*

By now, there is sufficient evidence that Darcy's law for multiphase flow, as in current use by petroleum engineers and groundwater hydrologists needs to be revised. In this talk, we identify some of these problems and, for two-phase flow, analyze the effect of considering the relative permeability as a tensor in the the corresponding Buckley-Leverett problem.

Peter A. C. **Raats** *Kinematics of water in unsaturated soils*

Two approaches can be used to describe the movement of soil water, namely

i) a spatial approach describing what happens in the course of time at specific places, and

ii) a material approach describing what happens in the course of time to specific parcels of water.

The latter approach provides a basis for describing the movements of solutes relative to the soil water. The key concepts in kinematics of continua are the deformation gradient tensor and the velocity gradient tensor. The application of these two concepts to water in unsaturated soils is described in detail. Special attention is given to flows with time-invariant flow patterns, including unsteady rectilinear flows and steady multi-dimensional flows. The influence of uptake by plant roots will also be considered.

Ralph **Showalter** *Distributed microstructure models of porous media*

Laminar flow through through fissured or otherwise highly inhomogeneous media leads to very singular problems with rapidly oscillating coefficients for porous media equations. The limiting case (by homogenization) is a continuous distribution of model cells which represent a valid approximation of the finite (singular) case, and we survey some recent results on the theory of such systems. This is developed as an application of continuous direct sums of Banach spaces which arise rather naturally as the energy or state spaces for the corresponding (stationary) variational or (temporal) dynamic problems. We discuss the basic models for a totally fissured medium, the extension to include effects of secondary flux, and the classical model systems which are realized as limiting cases of the microstructure models.

Su Ning *Multidimensional degenerate diffusion problems with evolutionary boundary conditions: existence, uniqueness, and approximation*

This work studies an initial boundary value problem of the following nonlinear degenerate parabolic equation

$$\partial_t \Theta(u) - \nabla \cdot (\nabla u + b(\Theta(u))) = f(\Theta(u)) \text{ in } \Omega \quad (2)$$

with a special attention to the evolutionary boundary condition

$$\partial_t \beta(u) + (\nabla u + b(\Theta(u))) \cdot \nu = g(\beta(u)) \text{ on } \Gamma, \; u = u_D \text{ on } \partial\Gamma \setminus \Gamma$$

where the functions $\Theta(z)$ and $\beta(z)$ are nondecreasing. This problem arises in the study of rainfall infiltration through soils. Existence and uniqueness for the problem are established through a semi-descrete scheme combined with parabolic regularization. Some L^2-error estimates for this scheme as well as some other ones are derived.

Sjoerd van der **Zee** *Transport modeling of reactive solute in random porous media*

Nonlinearity of adsorption may give rise to traveling waves (dependent on boudary/initial conditions) (Van Duijn & Knabner, 1991, 1989, Van der Zee, 1990). Such behavior may be very important in the case of layering. In the presentation I will assess whether for small retardation factors the traveling wave solutions, derived for homogeneous columns, also describe the fronts in heterogeneous columns (random, exp. autocorrelated adsorption coefficient). The applicability of traveling wave solutions for large retardation factors and heterogeneity was demonstrated by Bosma and Van der Zee (1991). Of special interest is the injection of a pulse of solute subject to nonlinear adsorption. Such pulses will not lead to traveling waves. Their behavior in homogeneous and heterogeneous (random) columns will be shown, using among others moment theory (Aris, 1953).

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