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Hyperbolic Systems of Conservation Laws
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Nonlinear hyperbolic systems of conservation laws arise in modeling conservative physical systems, such as fluid dynamics and elasticity. The nonlinear structure gives rise to interesting features such as shock waves and oscillations, and has been the subject of intense mathematical study for over 40 years. Questions of existence, uniqueness and regularity are still being actively investigated using vanishing viscosity and entropy condition techniques as well as newer approaches such as compensated compactness. There is currently much interest in the kinetic theory of gases, the relation of Boltzmann equations and Broadwell models to conservation laws and the development of new mathematical techniques based on these connections.

New applications are being studied where the structure is more complicated than in the classical genuinely nonlinear homogeneous case. Source terms are of particular importance in combustion problems. Applications to general relativity, magnetohydrodynamics, elastic-plastic solids and traffic flow were also discussed.

The study of numerical methods for conservation laws is also an active area of research, with emphasis on methods that capture shocks sharply while respecting entropy conditions and giving highly accurate smooth solutions. Several talks concerned the development of genuinely multi-dimensional methods for gas dynamics on structured or unstructured grids. Applications to combustion and enhanced oil recovery also received attention, as did new techniques for estimation errors and proving convergence.

Vortragsauszüge

Yann Brenier

Conservation laws derived from the Vlasov-Poisson System

The singular limit of the Vlasov-Poisson system

$$\begin{cases} f = f^{\varepsilon}(t, x, \xi) \geq 0, \ (x, \xi) \in \mathbf{R}^d \times \mathbf{R}^d, \ t \geq 0 \\ \partial f + \mathrm{div}_x(\xi f) + \mathrm{div}_{\xi}(Ef) = 0 & (1) \\ E = E^{\varepsilon}(t, x) = - \nabla_x \phi^{\varepsilon}(t, x) & (2) \\ -\varepsilon \Delta_x \phi = \int f d\xi - 1 & (3) \end{cases}$$

is considered when $\varepsilon > 0$ goes to zero. The limit system is formed of (1), (2), and

$$-\Delta_x \phi = \sum_{ij = 1, d} \partial_{x_i} \partial_{x_j} \int \xi_i \xi_j f d\xi \qquad (4)$$

instead of (3). It is a highly singular system for which the new kinetic theory of DiPerna, Lions, Golse, Penthame etc. does not seem to provide satisfying answers because of the loss of 2 derivatives from (3) to (4). In the one dimensional case (d = 1), constant solutions of the form:

$$f(t, x) = \begin{cases} c_{i - \frac{1}{2}} \text{ if } u_{i-1}(t, x) > \xi > u_i(t, x) & i = 2, ..., N \\ 0 & \text{otherwise} \end{cases}$$

can be considered, at least locally in time, where N is a fixed integer, $c_{3/2}, ..., c_{N - 1/2}$ are fixed >0 constants. The unknown $u_2, ..., u_{N-1}$ form a system of conservation laws

$$\partial_t u_i + \partial_x\left(\frac{u_i^2}{2}\right) + \partial_x \phi = 0, \quad i = 2, ..., N - 1 \qquad (5)$$

where

$$\phi = - \frac{1}{3} \sum_{i = 2}^{N} c_{i - \frac{1}{2}} (u_i^3 - u_{i-1}^3)$$ and u_i, u_N can be algebraically eliminated by

$$\sum_{i = 2}^{N} c_{i - \frac{1}{2}} (u_i - u_{i-1}) = 1, \qquad \sum_{i = 2}^{N} c_{i - \frac{1}{2}} (u_i^2 - u_{i-1}^2) = 0 \qquad (6).$$

It can be shown (joint work with B. Engquist and E. Grenier) that the resulting system of conservation laws is strictly hyperbolic in the domain $u_1 > u_2 > ... > u_N$ for any choice of N and $c_{1/2}, ..., c_{N/2-1}$ provided. Here is $j \in \{2, ..., N-1\}$ such that

$$c_{3/2} \leq ... \leq c_{j-1/2} \geq c_{j+1/2} \geq ... \geq c_{N-1/2}. \qquad (7)$$

Conversely, there are cases (typically $N = 4$, $c_{3/2} = c_{7/2} > 0$, $c_{5/2} = 0$) where the systems is not hyperbolic. Therefore, the continuation of such systems after appearance of singularities is entirely open.
New tools, as Tartar's H-measures and DiPerna-Majda generalized Young's measure (L. T., Proc. Roy. Soc. Edinb. 1991, R. D.-A.M., Comm. Math. Phys. 1987) allow us to pass to the limit, when $\varepsilon \to 0$, in the 2 first moment equations:

$$\begin{cases} \partial_t \int f^\varepsilon d\xi + \sum_i \partial_{x_i}\left(\int \xi_i f^\varepsilon d\xi\right) = 0 & (8) \\ \partial_t \int \xi_i f^\varepsilon d\xi + \sum_{ij} \partial_{x_j}\left(\int \xi_i \xi_j f^\varepsilon d\xi - \varepsilon \partial_{x_i}\phi^\varepsilon \partial_{x_j}\phi^\varepsilon \right) & (9) \\ = \partial_x\left(\phi - \frac{\varepsilon}{2}|\nabla_x \phi|^2\right) & (10) \end{cases}$$

by using the conservation of total energy

$$\frac{d}{dt}\left[\frac{1}{2}\int\int |\xi|^2 f^\varepsilon d\xi dx + \frac{\varepsilon}{2}\int |\nabla\phi^\varepsilon|^2 dx\right] = 0.$$

Indeed, the limit of f^ε and $\sqrt{-\varepsilon\Delta\phi^\varepsilon}$ can be defined in terms of positive measures ν_{DM} and ν_T acting in the phase space (x, ξ) a test functions of the type: $\varphi(x)\Psi\left(\frac{\xi}{|\xi|}\right)$, $\varphi \in C_0(\mathbf{R}^d)$, $\Psi \in C(S^{d-1})$.
Then the measure (ν_{DM}, ν_T) can be seen as a DiPerna-Majda measure valued solution to the incompressible (!) Euler equations.

Gui-Qiang Chen
Hyperbolic Systems of Conservation Laws

The effect of relaxtion is important in many physical situations such as the kinetic theory of gases, phase transition, elasticity with memory, and water waves. In this talk I will first describe the structure of general N x N hyperbolic systems of conservation laws with stiff relaxation terms, and discuss the relationship between the stability and the existence of a convex entropy. Then I will discuss the behavior of the stiff relaxation limit and the weakly nonlinear limit for general 2 x 2 systems, which corresponds the compressible Euler limit and the incompressible Navier-Stokes limit. Some physical models including some 3 x 3 systems will also be studied if possible. (Joint work with T.-P. Liu and D. Levermore).

Björn Engquist

Large Time Behavior for Periodic Solutions fo Scalar conservation Laws

Convergence of space-periodic solutions to a constant steady state were proved under certain nonlinearity condition on the flux functions. The result in two dimensions generalizes earlier work by Dafermos in one space dimension. The related homogenization problems were also discussed.

Heinrich Freistühler

Magnethydrodynamic shock waves and their stability

I plan to

(a) make some remarks about the discrepancy between ideal and dissipative frameworks, and

(b) present some results about viscous profiles for intermediate MHD shock waves.

James Glimm

Stochastic Solutions of Conservation Laws

Turbulence and multiphase flow are important in fluid mixing and boundary layers. Modeling of this phenomena leads to an enlargement of systems of conservation laws. These mixing theories define an internal structure for constant discontinuities, slip surfaces and material (fluid) interfaces.

From a microscopic point of view, the same phenomena can be studied using two fluid Euler equation with an unstable interface. Direct numerical simulation of these equations provides new information concerning the mixing process. For acceleration driven instabilities (the Ragleigh-Taylor problem), we present results showing that the compressible mixing rate can be more than twice its value in the incompressible limit. This result is surprising, because the mixing rate has an universal value, in earlier (but purely incompressible) studies.

J. M. Greenberg

Dispersive Difference Schemes and Continuum - Limits of Chain Equations

I discussed the $h = 0^+$ limits of solutions of the chain equations

$$h\ddot{\varphi}_k = \sigma\left(\frac{\varphi_{k+1} - \varphi_k}{h}\right) - \sigma\left(\frac{\varphi_k - \varphi_{k-1}}{h}\right) \text{ and}$$

$$\frac{h}{8}(\psi_{k-1} + 6\psi_k + \psi_{k+1}) = \sigma\left(\frac{\varphi_{k+1} - \varphi_k}{h}\right) - \sigma\left(\frac{\varphi_k - \varphi_{k-1}}{h}\right).$$

Where the interparticle force law was a finite range repulsive force or a Langrange force of the form $\sigma = \frac{1}{2}(\gamma - 1/\gamma),\ \gamma > 0$.

In the former case it was shown that the weak limits of these solutions were the same as for a point system of elastic point masses experiencing elastic collisions and in the latter case that the weak limits satisfied a system of mean field conservation laws whose solution successfully reproduced the results of numerical simulations. Results will appear in Comm. on Pure and Appl. Math. and Physica D.

Eduard Harabetian

The viscosity condition for Hamilton-Jacobi equations is shown to be equivalent to the condition that the speed of the level sets is minimal.

A. Heibig

Errors estimates for oscillating system of conservation laws

Our system is:

$$\begin{cases} u_t^\varepsilon + f(u^\varepsilon)_x = 0 & (1) \\ u^\varepsilon(x,0) = u_0^\varepsilon(x), \quad u_0^\varepsilon \text{ smooth} & (2) \end{cases}$$

The p^{th} field is supposed to be linearly degenerate; the other are GNL. Following Serre, we make the change of variable:

$$\begin{cases} D \subset \mathbf{R}^n \to \mathbf{R}^n \\ u \leftrightarrow \phi(u) \end{cases}$$

defined by: $d\phi_i(u) \cdot r_p(a) \equiv 0,\ i \leq n-1$ and set:
$v = |\phi_1(u), \ldots, \phi_{n-1}(u)|,\ w = \phi_n(u)$. (1) can be written:

$$\begin{cases} \partial_t v^\varepsilon + B(v^\varepsilon, w^\varepsilon)\partial_x v^\varepsilon = 0 & (3) \\ \partial_t w^\varepsilon + q(\cdot\ , w)\ \partial_x v^\varepsilon + \mu(v^\varepsilon)\partial_x w^\varepsilon = 0 & (4) \end{cases}$$

Ansatz: (Serre):

$$\begin{cases} v^\varepsilon(x,t) = v_0(x,t) + \varepsilon V_1,(\alpha, t,\ \varphi(\alpha,t)/\varepsilon) + 0(\varepsilon^2) \\ w^\varepsilon(x,t) = w_0(x, t\,\varphi(x,t)/\varepsilon) + \varepsilon w_1(x, t,\ \varphi(x,t)/\varepsilon) + 0(\varepsilon^2) \end{cases}$$

Place into (3, 4) and set:

$$\begin{cases} \varphi_t + \mu(v_0)\varphi_x = 0 \\ \partial_t v_o + B(v_0, w_0)\partial_x v_o + \varphi[B(v_0, w_0) - \mu(v_0)]\partial_y V_1 = 0 \\ \partial_t w_o + q(v_0, w_0)(\partial_x v_o + \varphi_x \partial_y V_1) + \mu(v_0)\partial_x w_0 + \varphi_x(\Delta_u(v_0)\cdot V_1)\partial_y w_0 = 0 \end{cases} \qquad (5)$$

Our goal is to prove local existence th. for eqs (5) and (1) and to estimate $(v^\varepsilon, w^\varepsilon) - (V_0 + \varepsilon V_1\, w_0 + \varepsilon W_1)$.

Under the assumption:

$\exists K: \mathbf{R}^n \to GL_{n-1}(\mathbf{R})$ regular such that

$\forall (v, w) \in \mathbf{R}^{n-1} \times \mathbf{R}$, $K(v, w)$ is symmetric positive definite

$\forall (v, w) \in \mathbf{R}^{n-1} \times \mathbf{R}$, $K(v, w)\, B(v, w)$ is symmetric

$\forall v \in \mathbf{R}^{n-2}$, function: $w \to K(v, w)\,[B(v, w) - \mu(0)]$ is constant (depending on v),

we prove existence of a local smooth sol. for eq. (5).
Moreover, introducing the flows ϕ^ε and ϕ_1^ε of $\partial_t + \mu(v^\varepsilon)\partial_x$ and $\partial_t + \mu(V_1^\varepsilon(0, 0, \varphi(0, 0)/\varepsilon)$ and taking suitable initial datas (oscillations only on the CD field) we set:
$\exists T > 0 / \exists M > 0 / \forall \varepsilon > 0,\ \forall \tau \in [0, T[,\ \|[\phi^{\varepsilon *}(u^\varepsilon) - \phi_1^{\varepsilon *}(u_1^\varepsilon, (0, 0, \varphi(0, 0))](\cdot, \tau)\|_2 \le M\varepsilon$.

Andreas Heidrich

Global weak solutions of initial boundary value problems to the onedimensional quasilinear wave equation with large data

Traces and sections of weak solutions to general systems of balance laws are defined and investigated a priori. With the help of artificial viscosity and compensated compactness, the existence of global admissible weak solutions to the onedimensional quasilinear wave equation

$$\left.\begin{aligned} u_t - \sigma(v)_x = 0 \\ v_t - u_x = 0 \end{aligned}\right\} \text{ on } (0, \infty) \times (a, b),$$

$$\left.\begin{aligned} u(0, \cdot) = u_0 \\ v(0, \cdot) = v_0 \end{aligned}\right\} \text{ on } (a, b),$$

$$\sigma \in C^2(\mathbf{R};\mathbf{R}),\ \sigma(0) = 0,\ \sigma' < 0,\ y\ \sigma''(y) < 0\ [y \ne 0]$$

with large data $(u_0, v_0) \in L^\infty((a, b) \infty\, \mathbf{R}^2)$ is proved such that the traces

$\gamma^a u$, $\gamma^a\sigma(v)$, $\gamma^b u$, $\gamma^b\sigma(v)$ of u and $\sigma(v)$ at the respective boundary components $(0, \infty) \times \{a\}$ and $(0, \infty) \times \{b\}$ of $(0, \infty) \times (a, b)$ satisfy the damping boundary conditions

$$\left.\begin{aligned} r(a)\gamma^a u - \gamma^a\sigma(v) = 0 \\ r(b)\gamma^b u + \gamma^b\sigma(v) = 0 \end{aligned}\right\} \text{on } (0, \infty)$$

of Greenberg and Li [J. Differential Equations 52, 66-75 (1984)] where $r \in C(\{a, b\}; [0, \infty])$ is the damping factor and the condition reads $\gamma^x u = 0$ in case of $r(x) = \infty$ $[x \in \{a, b\}]$

Helge Holden (joint work with N. H. Risebro, Oslo)
<u>A mathematical model for traffic flow on a network of roads</u>

We introduce a model that describes heavy traffic on a network of unidirectional roads. The model consists of a system of initial-boundary value problems for nonlinear conservation laws. We formulate and solve uniquely the Riemann problem for such a system and based on this, we then show existence of a solution to the Cauchy problem

John Hunter
<u>Global existence of weak solutions of hyperbolic variational principles</u>
(joint work with Yuxi Zheng and Ralp Saxton)

We analyze a canonical asymptotic equation for weakly nonlinear solutions of hyperbolic variational equations of the form $\delta\int A^{jk}_{pq}(x, u)\dfrac{\partial u^p}{\partial x^j}\dfrac{\partial u^q}{\partial x^k}dx = 0$. Such variational principles were suggested by a simple model of director fields in a mematic liquid crystal with director $\underline{n}(x, t) \in S^2$, kinetic energy $T = \frac{1}{2}\underline{n}_t \cdot \underline{n}_t$ and potential energy $V(\underline{n}, \nabla\underline{n}) = \tilde{A}^{jk}_{pq}(\underline{n})\dfrac{\partial u^p}{\partial x^j}\dfrac{\partial u^q}{\partial x^k}$ = Oseen-Frank potential energy.

The asymtotic problem is:

$$\text{(1)} \qquad \begin{aligned} &v_t + (uv)_x = \tfrac{1}{2}v^2 \qquad &&u_x = v, \qquad &&x > 0, t > 0 \\ &v(x, 0) = v_0(x) &&u(0, t) = 0. && \end{aligned}$$

We show that:
(a) if $v_0(x)$ is smooth, and negative at some point, then $v(x, t) \downarrow -\infty$ in finite time;
(b) if $v_0(x)$ is a step function, then (1) has globally defined $(t > 0)$ weak solutions which are also step functions;

(c) if $v_0(x)$ has bounded variation, and compact support, then (1) has global weak solutions (but $v(x, t)$ is not in BV).

An open question is to show that (1) has a global weak solution when $v_0(x) \in L^2$.

Rolf Jeltsch

A new multidimensional scheme for Euler flows

In multidimensional flow calculations most finite-volume and finite difference methods use dimensional splitting. A new scheme is presented which does not associate fluxes with borders of cells. In principle one computes the flow from one cell to any other point in space by decomposing it into three waves i) transport by velocity ii) spherical scalar wave expanding radially with the speed of second iii) a vector valued wave again expanding radially with the speed of second. This last wave describes how momentum and energy is distributed due to the pressure in the original cell. Integration over the receiving cell gives the numerical flux. The resulting scheme is first order if we use piecewise constant values. It is robust, i. e. one can compute strong shocks without a reduction in step size. No unphysical values occur. It can be used for unstructured and structured grids and the mesh can be refined even with hanging nodes. The scheme can be accelerated by approximating these integrals to compute the fluxes. This work has been done as a Ph. D. thesis of M. Fey at ETH Zürich.

Rupert Klein (joint work with D. S. Stewart and J. B. Bdzil)

Weakly Nonlinear Dynamics of Fast Combustion Waves

Fast combustion waves are modeled as gasdynamic shocks followed by a (thin) zone of chemical activity (Zel'dovic, von Neumann, Döring). Steady plane detonation solutions exist for any wave speed $D > D_{cj}$, where D_{cj} is a limit velocity depending on the combustible medium, the "Chapman-Jouguet"-speed. At the CJ-limit the detonation decouples from the background flow acoustically, which is why CJ-waves are dominantly observed in realistic applications.

A study of different scaling regimes for the detonation front geometry in multi-dimensions leads to a hierachy of simplified asymptotic equation systems describing the interaction of the shock front evolution with a weakly nonlinear transonic flow in the back of the wave.

These systems describe, of increasing complexity, an interplay of weakly nonlinear acoustics, chemical reactions, curvature effects and the coupling of a weakly nonlinear multi-dimensional transonic flow with the simultaneous evolution of the detonation shock. They may serve as simplified models for hyperbolic systems of conservation laws in multi-dimensions and as a basis for generating new test problems for high quality direct simulation codes for combustion problems.

Christian Klingenberg
On regularity of solutions to the scalar equation $u_t + f(u)_x = 0$ without convexity

We prove that for piecewise constant initial data with finitely many constants the solution generically is piecewise smooth, i. e. it is continuous except on a finite set of smooth arcs. The proof is via an explicit construction in the phase plane, u - f(u), by initially taking all the convex or concave hulls for the local Rieman problems and then deforming the union of these hulls to obtain the solution for all time. The genericity condition is on f(u), in particular polynomials yield piecewise smooth solutions.

Dietmar Kröner
Finite volume methods for conservation laws

We consider a finite volume discretization for general scalar nonlinear hyperbolic conservation laws in two space dimensions on an unstructured grid. Essentially there are three different numerical methods which are used in general for solving these equations numerically. There are the finite difference methods on cartesian grids in the form of dimensional splitting, the finite volume and the finite element methods. The methods on unstructured grids can be adapted much better to general geometries and local mesh refinement than for structured grids. In the talk we have presented a proof for the convergence of a general class of upwind finite volume schemes on unstructured grids. The results are more general than those of Coquel et al.

For getting a global control of the entropy dissipation we choose suitable numerical entropy fluxes. These arguments can be used in particular for the Lax-Friedrichs and the Engquist-Osher finite volume schemes but also for a more general class of finite volume schemes. The method we are using in the proof is based on the concept of measure valued solutions in the sense of DiPerna. For this method we do not need any BV-estimates. This is important since in general for systems in 2-D we cannot expect uniform estimates of the variation of an approximating sequence. The method of measure valued solutions assumes less reguarity than is needed for BV-estimates.

S. N. Kruzkow
Uniqueness of the Solution to Cauchy problem for $u_t + (u^2/2)_x$ in L_∞

It is well known that the Cauchy problem $u_t + (\frac{u^2}{2})_x = 0$, $(\frac{u^2}{2})_t + (\frac{u^3}{3})_x \leq 0$ (in the sense of distributions)

$u/_{t=0} = u_0(x) \in L_\infty(R^1)$ $(u(t,x) \rightarrow u_0(x)$ in $L_{1,loc})$ as $t \rightarrow +0$ has a unique solution in the class $L_\infty \cap BV_{loc}$.

Is the uniqueness theorem true in the class L_∞?

Randy LeVeque

Multi-dimensional Numerical Methods

A simplified flux limiter method for conservations laws in multiple space dimensions is presented. The flux is computed by solving a one dimensional Riemann problem at each interface. The resulting wave strengths are limited before being used for the second order correction and are also distributed to update neighboring fluxes in an upwind manner for the transverse derivative terms. Rotated schemes and applications to boundary conditions for Cartesian grid methods are also briefly described.

Arnon Levy

On Majda's model for dynamic combustion

Majda's model of combustion consits of the system

$$(u + q_0 z)_t + f(u)_x = 0 \qquad\qquad z_t + K\varphi(u)_z = 0.$$

We consider the Cauchy problem for this system. A weak entropy solution for this system is defined, existence, uniqueness and continuous dependence on initial data are proved, as well as finite propagation speed, for initial data in L^∞. The existence is proved via the "vanishing viscosity method". Furthermore it is proved that the solution to the Rieman problem converges as $t \rightarrow \infty$ to the traveling wave solution in the case of strong detonation conditions and the speed of the front tends to the C-J speed in the case corresponding to weak detonation.Some numerical results are presented.

Ling Hsiao

Global existence and qualitative behavior of discontinuous solutions for a nonlinear hyperbolic system with lower order dissipation

Consider the following system which may be viewed as isentropic Euler equations with friction term added to the momentum equation to model gas flow through a porous media.

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = -\alpha u, \quad \alpha > 0 \end{cases}$$

where $p'(v) < 0$ and $p''(v) > 0$ for $v > 0$.

We show the existence and the construction of the globally defined discontinuous solutions and we show the qualitative behavior of the solutios as well for the discontinuous initial data which are certain pertubations of the corresponding Riemann data.

Pierangelo Marcati

Convergence of the Pseudo-Viscosity Approximation for Conservation Laws.

(joint work with Roberto Natalini)

In a paper in J. Appl. Phys. 21 (1950), 380-385, Von Neumann and Richtmyer introduced the method of "pseudo viscosity", namely a nonlinear gradient-dependent artificial diffusion, in order to approximate shock wave of large amplitude.

We show the rigorous proof of the convergence of the pseudo viscosity approximation via weak convergence methods. We consider in particular

$$(1)\begin{cases} u_t + f(u)_x = \varepsilon\beta(u_x)_x \\ u(x, 0) = u_0(x) \end{cases}$$

Where the β diffusion is active only in the compressive regions.

The main problem is to deal with the existence of weak solutions to (1) by adding an additional viscosity δu_{xx} and sending $\delta \to 0$. The gradient bound independent on δ is the main estimate. Then we conclude with the limit as $\varepsilon \to 0$, by using the method of Compensated Compactness by L. Tartar.

Claus D. Munz

Godunov-Type Methods for Lagrangean Gas Dynamics

Godunov-Type schemes are considered for the equations of gas dynamics using Lagrangean coordinates. A Roe-linearisation of this equation is constructed and shown that the Roe mean values do not coincide with those obtained for Eulerian coordinates. It is shown that this linearisation fails in the vicinity of strong compressions, in the sense that the corresponding approximate Riemann solution contains unphysical states of negative specific volume. An algorithm to caculate a priori bounds for the smallest and largest signal velocity is obtained by the correction of the signal velocities of the Roe linearisation. These bounds are used to present a simple Godunov-type scheme which captures strong compressions very well.

Robert Natalini

Uniqueness and Stability for Solutions for Balance Laws

We consider the Cauchy problem for a scalar balance law:

$$\partial_t u + \sum_{i=1}^{n} \partial_{x_i} f^i(u) = f(u).$$

We are interested in the existence, uniqueness and qualitative behavior of the entropy solutions in the sense of Kruzkov. In fact even simple 1-d examples show that solutions become in general unbounded in a finite time, with different behavior depending on the balance between the convection and the reaction term.

A monotone approximation of the R. H. S. is proposed to continue solutions after the blow-up time (in the L^∞-norm), and some uniqueness and stability arguments are given for such continuations. Occurrence and qualitative properties of blow-up sets are also studied.

Sebastian Noelle

Convergence of higher order upwind finite volume schemes

We prove convergence of a class of higher order upwind finite volume schemes on unstructured triangular grids for scalar conservation laws in two space dimensions. The result also applies to the discontinuous Galerkin finite element method (see [CHS]).

The proof refines two techniques: first a projection based on B-triangulations (see [CHS]), which generalizes flux-limitors to two space dimensions. Second, and elegant new estimate of the entropy dissipation of first order schemes due to all numerical fluxes which are convex combinations of the Engquist-Osher and the Lax-Friedrich flux, and more. (Joint work with D. Kröner and M. Rokyta)

[CHS] B. Cockburn, S. How, C. W. Shu: Math. Comp. 54 (1990)
[KR] D. Kröner, M. Rokyta: SFB 256 preprint, Bonn, Germany (1992)

Olga Oleinik

Asymtotic for travelling waves

Many problems about asymptotic of travelling waves lead one to study the behaviour of solutions of nonlinear elliptic equations in cylindrical domains. In the lecture some classes of semilinear second order elliptic equations are considered. In particular, for the equation

$$\Delta u - e^u = 0 \qquad (1)$$

in the halfcylindrical domain $S(0, \infty) = \{x: x' \in w, 0 < x_n < \infty\}$ where w is a smooth bounded domain in $\mathbf{R}^{n-1}$, $x' = (x_1, \dots, x_{n-1})$, the following theorems are proved.

Theorem 1.

For any solution $u(x)$ of equation (1) in $S(0, \infty)$ with the boundary condition

$$\frac{\partial u}{\partial \nu} = 0 \text{ on } \sigma(0, \infty) = \{x : x' \in \partial\omega, 0 < x_n < \infty\} \qquad (2)$$

the inequality

$$c_1 x_1 + c_2 \leq u(x) \leq -2\ln\left(\frac{\sqrt{2}}{2} x_n\right), \ c_1, c_2, = \text{const}, \ c_1 < 0$$

holds.

Theorem 2.

Let $u(x)$ be a solution of equation (1), (2). Then

$$u(x) = -2\ln x_n + 0(\ln x_n)$$

or

$$u(x) = Cx_n + 0(x_n), \ C = \text{const} < 0.$$

Theorem 3.

Let $u(x)$ be a solution of equation (1) with the boundary condition $u = 0$ on $\sigma(0, \infty)$. Then

$$|u - u_0| \leq C_1 \exp\{-\alpha x_n\}, \ C, \ \alpha = \text{const} > 0,$$

where u_0 is a solution of the problem

$$\sum_{j=1}^{n-1} \frac{\partial^2 u_0}{\partial x_j^2} - e^{u_0} = 0 \ \text{ in } \omega,$$

$u_0 = 0$ on $\partial\omega$ or

$u(x)$ is negative for $x_0 > X_n$ and

$$|u(x^k)| \geq \exp\{\beta x_n^k\}, \ \beta = \text{const} > 0,$$

$x_n^k \to \infty$ as $k \to \infty$. These theorems are proved joint with V. A. Kondratiev.

B. Perthame

Kinetic formulation of conservation laws

We present a new formulation of conservation laws in two examples, scalar conservation laws and isentropic gas dynamics and the p-system. The formulation is obtained adding a "velocity" free variable to x and t and writing a "kinetic" equation $X(V(x, t), v)$, where $V(x, t)$ is the solution to the conservation law. Various applications are given: we recover invariant regions, give new "dispersion" type of estimates, compactness results. This is a common work with P. L. Lions and E. Tadmor.

Robert Peszek

Time periodic solutions of quasilinear wave equations

I presented a construction of spatially and temporally periodic solutions to a class of quasilinear wave equations of the type

$$u_t - v_x = 0$$
$$v_t - c^2(u)u_x = 0,$$

where c satisfies

$$c(-u) = c(u) \geq c_0 = c(0) > 0$$

and $\quad c'(u) > 0 \;\text{ if }\; u > 0.$

presented a construction which generalizes the Greenberg-Rascle construction. Constructed solutions are shock free and the only discontinuities are created by expanding and focusing simple waves.

M. Rascle

Non trivial oscillations in systems of conservation laws

We construct a 2 x 2 hyperbolic system:

$$\begin{cases} \partial_t u - \partial_x \sigma(v) = 0 & \sigma'(v) \geq c > 0 \\ \partial_t v - \partial_x u = 0 & v.\sigma''(v) > 0 \qquad \forall v \neq 0 \end{cases}$$

nonlinear easticity system (u: velocity, v: strain, σ: stress), with only one inflection point. This system has the surprising property of admitting large amplitude (non trivial) oscillations, although it is not linearly degenerate. Therefore, this example contradicts the conventional wisdom, which is: "the only systems which admit such propagating oscillations are linearly degenerate". The example would also be a perfect counter example to the well-known result of R. J. DiPerna, if the stress strain relation $\sigma(\cdot)$ was C^2, which is true at any point $v \neq 0$, but not at the origin. The mechanism is that $\bar{\sigma}(\cdot)$ is chosen in such a way that the interaction of 2 centered rarefaction waves converts them into 2 centered compression waves.

Steve Schochet

Resonant Nonlinear Geometric Optics For Weak Solutions of Conservation Laws

The $0(\varepsilon)$ part of solutions obtained is Glimm's scheme to strictly hyperbolic conservation laws with $0(\varepsilon)$ periodic BV initial data tends as $\varepsilon \to 0$ to the unique entropy solution of the modulation equations of weakly-nonlinear geometric optics.

D. Serre

Explicit formula for the Cauchy problem

The system $u_t + f(u)_x = 0$ is endowed with a Temple's characteristic field if there exists functions $\lambda(u)$, $w(u)$ satifying both

H1 $dw(df - \lambda) \equiv 0$

H2 Level sets $\pi_\alpha =: w^{-1}(\alpha)$ are affine hyperplanes.

One assumes morever the genuine non linearity of the field: $d\lambda.r > 0$ ($\geq$ is enough) as $(df - \lambda)r = 0, r \neq 0$ continuous.

Given a linear equation $e_\alpha(f(u))$ is a constant q_α of π_α, one findes that $e_\alpha(f(u))$is a constant q_α on π_α. Then the Cauchy problem $u(x, 0) = u_0(x)$ is rewritten as a Hamilton-Jacobi system $v_t + f(v_x) = 0, v(x, 0) = v_0(x)$.
One find the following explicit formula:

The graph $G = \{(x, t, v(x, t)); x \in \mathbf{R}; t \geq 0\}$ is included in the envelope of the one-parameter family of hyperplanes

$$e_\alpha(V - v_0(y)) + (y - X)p_\alpha + q_\alpha T = 0,$$

as $y \in \mathbf{R}$ and $\alpha = w(u_0(y))$.

M. Slemrod & A. E. Tzavaras

Self-similar fluid dynamic limits for the Broadwell system

The Broadwell system of discrete kinetic theory is given by the system

$$\frac{\partial f_1}{\partial t} + \frac{\partial f_1}{\partial x} = \frac{1}{\varepsilon}(f_3^2 - f_1 f_2)$$

$$\frac{\partial f_2}{\partial t} - \frac{\partial f_2}{\partial x} = \frac{1}{\varepsilon}(f_3^2 - f_1 f_2)$$

$$\frac{\partial f_3}{\partial t} = \frac{1}{\varepsilon}(f_1 f_2 - f_3^2).$$

Here we assume symmetry and independence of z. y. The model describes a gas of particles with identical masses moving along three coordinate axes with speed 1. Results of a particular collision have the same probability and only binary are considered. In our work we give a new approach to the resolution of the fluid dynamics limit problem. The main idea is to replace the Knudsen number ε by εt and obtain self-similar soltions in $\xi = \frac{x}{t}$ and then let $\varepsilon \to 0+$. The limit thus obtained is a solution of the Riemann problem for the dynamic limit equation.

J. Smoller
Global Solutions for the Relativistic Euler Equation

Consider the relativistic Euler equations in Minkowski spacetime, $\operatorname{div} T^i_j = 0$, where T_{ij} is the stress-energy tensor of a perfect fluid and the pressure γ satisfies $\gamma(p) = \sigma^2 p$ where $|\sigma| < c$. We construct solutions of the initial-value problem for arbitrary initial data in the class BV when $p(t = 0, x) \geq \bar{p} > 0$ and $|v(t = 0, x)| > c$. We show that for this system, the ideas of Nishida (1968) carry over and we get a solution via the Glimm method. Our solution is shown to satisfy the Lorentz-invariant estimates

$$\operatorname*{Var}_{x} \ \ln p(t, x) \leq V_0$$

$$\operatorname*{Var}_{x} \ \ln \frac{v(t, x) + c}{v(t, x) - c} \leq V_i$$

when V_i and V_0 are Lorentz-invariant constants depending only on the variation of the initial data.
(This work is joint with Blake Temple.)

P. E. Souganidis
Nonlinear hyperbolic pde's

We consider here the resolvent equations for a scalar, one-dimensional conservation law and its approximations by MUSCL methods i. e. second-order, TVD, finite-differences approximation. We prove the convergence towards the entropic weak solution in the case of a strictly convex flux. The proof relies upon the theory of viscosity solutions.

D. Scott Stewart
Multi-Scale Aspects of Detonation Dynamics

Combustion problems in complex engineering applications contain many important yet disparate length scales. The device size is typically many orders of magnitude larger then the heat-release zone size. Accurate rendering of the physics requires adequate modeling of all scales. We discuss three related problems: By direct numerical simulation we show how an asymptotic model (The relationship between normal delation velocity and intrinsic shock curvature $D_n = D_n(x)$), provides a simple and accurate description of a stable, curved detonation. Next we describe a detailed numerical study of a highly unstable detonation zone. Finally we dissuss computational costs associated with high-resolved solutions to multi-scale problems.

Anders Szepessy

Stability of viscous shocks and error control

I talked on asymptotic stability of weak shocks of viscous conservation laws of strictly hyperbolic systems, which is joint work with Zhouping Xin; and on error control and aposterior estimates which is joint with Claes Johnson.

The stability result is a continuation of T.-P. Lin's work in mermoirs AMS 1985. Our contribution is to include general localized perturbations. A new phenomenon occurs in this case: a perturbation of order $t^{-1/2}$ moves in the shock region. In the second part on error control a theorem was shown that estimates the error of finite element method in terms of the approximate solution. The basic assumption is that the computed solution satisfies entropy conditions and the result is an almost optimal error estimate.

Eitan Tadmor

Error estimates for approximate solutions of nonlinear conservation laws

Convergence analysis of approximate solutions to nonlinear conservation laws is often accomplished by BV or compensated-compactness arguments, which lack convergence rate estimates. An L^1-error estimate is available for monotone approximations.

We present an alternative convergence rate analysis. As a stability condition we assume *Lip*$^+$-stability in agreement with Oleinik's E-condition. We show that a family of approximate solutions, $\{v^\varepsilon(x,t)\}$, which is *Lip*$^+$-stable satisfies

$$\|v^\varepsilon(\cdot\,,t) - u(\cdot\,,t)\|_{Lip'} \leq \text{Const}\,[\|v^\varepsilon(\cdot\,,0) - u(\cdot\,,0)\|_{Lip'} + \|v^\varepsilon_t + f(v^\varepsilon)_x\|_{Lip'}.$$

Consequently, familiar L^p and new pointwise error estimates are derived. We demonstrate these estimates for viscous and kinetic approximations, finite-difference and Glimm's schemes and spectral methods.

Luc Tartar

Compensated Compactness and H-measures

For a scalar conservation law $u_t + f(u)_x = 0$ the classical method of compensated compactness needs to characterize probability measures ν satisfying the relation $\langle\nu, \varphi_1\psi_2 - \varphi_2\psi_1\rangle = \langle\nu, \varphi_1\rangle\langle\nu, \psi_2\rangle - \langle\nu, \varphi_2\rangle\langle\nu, \psi_1\rangle$ for all pairs (φ, ψ) of an entropy of class C^2 and its corresponding flux ψ, i. e. $\psi' = \varphi' f'$. The work of P. L. Lions & B. Pedhame & E. Tadmor suggested to use the relation for discontinuous entropies. For

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$v \in \mathbf{R}$ let $\varphi_v(u) = \begin{cases} 1 & \text{if } u < v \\ 0 & \text{if } u \geq v \end{cases}$ and $\psi_v(u) = f'(v)\, \varphi_v(u)$. One can approach φ_v by a sequence of C^2 functions $\varphi_{v,\varepsilon}$ whose fluxes $\psi_{v,\varepsilon}$ converge to ψ_v, the sequences staying uniformly bounded and converging everywhere. The functional relation for ν extends then to pairs (ψ_v, φ_v) and (ψ_w, φ_w) with $v<w$ and as $\varphi_v\psi_w - \varphi_w\psi_v = (f'(w) - f'(v))\, \varphi_v\psi_w$ and $\varphi_v\varphi_w = \varphi_v$ one obtains $\langle \nu, \varphi_v\rangle(f'(w) - f'(v)) = \langle \nu, \varphi_v\rangle\langle \nu, \varphi_w\rangle(f'(w) - f'(v))$ and therefore either $\langle \nu, \varphi_v\rangle = 0$ or $\langle \nu, \varphi_w\rangle = 1$ or $f'(w) - f'(v) = 0$. If ν is not Dirac mass and $[\alpha, \beta]$ is the smallest interval containing the support of ν, then by choosing $\alpha < v < w < \beta$ one finds $f'(u) = f'(v)$.

The same method should be successful for multidimensional scalar equations, but the div-curl lemma cannot be used in that situation and the full compensated compactness theory could be enough to answer this case, but this still has to be done.

H-measures have been described, but at the moment, they have not helped doing anything new for quasilinear equations.

Blake Temple (joint work with Joel Smoller)
Shock waves in general realtivity

Einstein's field equation for the graviational field are given by

$$G = 8\pi T,$$

where G is the Einstein curvature tensor, and T is the stress energy tensor defined on spacetime. We prove a general theorem which states that if g_{ij} (the unknown gravitational metric) is Lipschitz continuous across a co-dimension one shock surface Σ, then the jump conditions

$$[G^i_j]n_i = 0$$

hold identically across the surface. Here [G] denote the jump in G across the surface, and $n_i dx^i$ is the i-form normal to the surface. As a consequence solutions of $G = 8\pi T$ automatically conserve mass and momenturm across such a surface because then $[T^i_j]n_i = 0$. As a consequence, we find a generalization of the Oppenheimer-Snyder model for gravitation collaps, which we construct by matching the Robertson-Walker metric to the interior Schwarzschild metric across a "shock surface" in a Lipschitz continuous sense. We conclude that conservation must hold across the surface. In this theory, the original Oppenheimer-Snyder model has a "contact discontinuity", which prevents mass and momentum flux across the matched surface with the condition p = 0, as is well known. Our generalization allows for an arbitrary equation of state p =

p(ρ). This introduces a general procedure for constructing dynamical solutions of the Einstein equations by matching simple metrics across co-dimension one surface. This is facilitated by the covariance properties of the equation.

John Trangenstein

Shocks in elastic-plasic solids, or simulation of enhanced oil recovery

The hyperbolic wave structure of nonlinear solids is significantly more complicated than gas dynamics. Most of the literature on multidimensional waves is devoted to linear elasicity.

Godunov methods have been used with success in gas dynamics, expecially in combination with adaptive mesh refinement. The extension of these techniques to solid mechanics is non-trivial, due to the form of the kinetic equation of state (rate of stress = function of rate of strain).

I will review the develpment of Godunov methods for 1-D problems, present the 2-D algorithm and its extension to adaptive mesh refinement, and discuss some preliminary work on shock refaction at a solid-solid interface.

Gerald Warnecke

Conservation Laws of Mixed Type

The talk focussed on systems related to transonic flow. The transonic small disturbance system is formally the same as the p-system. But, the physical model requires different admissibility condition. Some well known criteria like the entropy inequality, viscousity method, Lax inequalities were discussed. There is a different version of each for both models. Finally, joint work with Barbara Keyfitz on admissibility for steady transonic potential flow was presented. The existence of mixed type solutions is still an open problem, since L^∞-bounds for regularizations are not known.

Zhouping Xin

Uniqueness of Weak Solutions for Isentropic Compressible Flows

This is a joint work with P. LeFlock on the uniqueness of weak solution for compressible isentropic flows. We propose a new form of entropy condition on this system, which not only selects the physical jump discontinuities, but also yields the precise rate of development of singularity at the centers of rarefaction waves and centers of elementary wave interactions. This generalizes the Oleinik's entropy condition for convex scalar conservation laws. Making use of the sharp forms of entropy condition enables us to show, in particular, that piecewise smooth solutions with nonaccumulating centers of centered rarefaction waves are unique. This result is proved by studying the corresponding uniqueness of Lipschitz solutions for a linear system with only L^∞ coefficients.

Robin Young (joint work with Blake Temple)
<u>Reorderings in Glimm's Scheme</u>

We consider the system of conservation laws,

$$u_t + f(u)_x \geq 0, \qquad u(x,0) \geq u_0(x), \qquad u, f \in \mathbf{R}^N,$$

where we are interested in the case $N \geq 2$. By identifying second order effects of wave interactions exactly, we obtain third order estimates. These are used in Glimm's scheme to obtain a stability estimate for the L^2-norm, and an explict constant depending only on the flux, for which the Glimm approximation is stable in the total variation norm, and thus converges to a solution of the conservation law. As immediate consequences, we have L^2-decay and L^2-stability, shown by Temple with the assumption

$$\|u(0,t)\|_{L^\infty} \leq C\,\|u\|_{L^\infty},$$

which is obtained here.

Kevin Zumbrun
<u>Hyperbolic Systems of Conservation Laws</u>

I will discuss joint work with Tai-Ping Liu on stability of undercompressive shocks. Stability is known for inviscid shocks, but up to now there is only numerical evidence for the stability of viscous undercompressive shocks.

We show nonlinear stability for a particular viscous undercompressive shock, namely that occurring in the complex Burgers' equation. We observe that the linearized equations about the shock wave can be explicitly solved by transformating similar to Hopf-Cole. This allows a careful study of the behavior under perturbation and a fairly detailed pointwise description of the asymptotic behavior.

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