MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 31/1992

Arithmetic algebraic geometry

12.–18.07.1992

Die Tagung fand unter der Leitung von Herrn Prof. G. Faltings (Princeton), Herrn Prof. G. Harder (Bonn) und Herr Prof. N.M. Katz (Princeton) statt. In den Vorträgen wurde über neueste Entwicklungen in der arithmetischen Geometrie berichtet, wobei die Spanbreite von der Theorie der Motive und den Beilinsonvermutungen über L–Funktionen, Shimuravarietäten, Modulräumen, Arakelovtheorie bis hin zur Darstellungstheorie über lokalen Körpern reichte.

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Vortragsauszüge

Yu.I. Manin (z.Z. Bonn):

Absolute Motives?

C. Deninger und N. Kurokawa recently stressed and developed analogies between the geometry over finite field and that over Spec $\mathbb{Z}$. Apart from the fact that the theory of L–functions of schemes over $\mathbb{Z}$ remains mostly conjectural, there is one major drawback in this analogy: the absence of "absolute direct product" $V \times W$ and "absolute point". In the talk Yu.I. Manin proposed some absolute constructions in particular for absolute Tate's motive $\mathbf{T}$ with zetafunction $L(\mathbf{T}^n, s) = \frac{s-n}{2\pi}$. This makes apparent the presence of $\mathbb{P}^1_{\mathbb{F}_q}$, resp. $\mathbb{P}^1_{\mathbb{F}_1=\bullet}$, resp. $\mathbb{P}^1_{\mathbb{C}}$ in the weight decompositions of zetas over $\mathbb{F}_q$, resp. $\overline{\mathrm{Spec}\,\mathbb{Z}}$, resp. Selberg's zeta. Some speculations about absolute motives were added.

C. Soulé (Paris):

On the height of resultants

C. Soulé reported on joint work with J.–B. Bost and H. Gillet. Let $N \geq 0$ be an integer and $d_0, \ldots, d_n$ positive integers. Denote by R the resultant of homogeneous polynomials $P_0, \ldots, P_N$ in $N+1$ variables of degrees $d_0, \ldots, d_N$ respectively. This is a polynomial with integer coefficients, well defined up to sign, in the coefficients a_I^i where $i = 0, \ldots, N$ and I runs over $\binom{N+d_i}{d_i}$ indices. Let Σ be the product of sheres $\sum\limits_I (a_I^i)^2 = 1$, $i = 0, \ldots, N$ and dv the invariant Haar measure on Σ. C. Soulé and his coworkers obtain an exact formula for $\int_\Sigma \log |R| dv$ in terms of $d_0, \ldots, d_N$. The method is to view the divisor of R as the (generalized) Chow divisor $\mathrm{div}\,(R) = g_* f^* v_* (\mathbb{P}^N)$ in the diagram

$$\begin{tikzcd} & & \Gamma \arrow[dl, "f"'] \arrow[dr, "g"] & \\ \mathbb{P}^N \arrow[r, "v"] & \prod_{i=0}^N \mathbb{P}^{N_i} & & \prod_{i=0}^N (\mathbb{P}^{N_i})^\vee \end{tikzcd}$$

where $N_i := \binom{N+d_i}{d_i} - 1$, v is the product of Veronese embeddings and Γ is the product of the incidence correspondences between $\mathbb{P}^{N_i}$ and its dual $(\mathbb{P}^{N_i})^\vee$. To compute the height of $\mathrm{div}\,(R)$ they use the theory of heights of projective varieties based on Arakelov theory (and initiated by G. Faltings in his work on abelian varieties).

A. Scholl (Durham):

L–functions of modular forms and higher regulators

Let f, g be newforms of weight $p+2, q+2$, where $p, q \geq 0$, and some level.

Assume $f \neq \overline{g}$; then the L–series $L(f \times g, s)$ has a simple zero at the points $s = 1 + l$ where $0 \leq l \leq \min(p, q)$. The same L–series occurs as a factor of $L(h^{p+q+2}(X_p \times X_q), s)$, where X_p, X_q are suitably compactified Kuga–Sato varieties attached to forms of weight $p+2, q+2$. He constructs elements of the motivic cohomology $H^{p+q+3}_{\mathcal{M}}(X_p \times X_q, \mathcal{O}(p+q+2-l))$, such that the $(f \times g)$–component of the regulator is non–zero on these elements, and equals (up to a rational factor) the derivative $L'(f \times g, 1+l)$. The elements are constructed using certain mappings between the Kuga–Sato varieties, together with an explicit knowledge of their boundaries. A. Scholl explains how the constructions – and the corresponding construction of regulators interpreting $L'(f, m)$ for $m \leq 0$ – becomes very natural in the fantasy–world of motivic sheaves.

M. Rapoport (Wuppertal):

On non–archimedian uniformization

M. Rapoport reported on joint work with T. Zink. In the first part of the talk, a moduli problem of rigidified p–divisible groups was formulated and was shown to be representable by a formal sheme $\mathcal{M}$. In the second part, the formal completion of a PEL–Shimuravariety along the sublocus of the reduction modulo p corresponding to an isogeny class was shown to be isomorphic to the disjoint sum of formal schemes $\Gamma \setminus \mathcal{M}$ where $\mathcal{M}$ is one of the formal schemes of the first part and where Γ is a discrete group operating on $\mathcal{M}$. In the third part the rigid aspect of the situation was considered. In particular, there is a period morphism from the rigid space associated to $\mathcal{M}$ to a suitable Grassmannian. This construction generalizes constructions due to Dwork, Drinfeld and Gross and Hopkins.

S. Bloch (Chicago):

Mixed Tate Motives

Algebraic cycles on products of $\mathbb{P}^1 - \{1\}$ (cubes) lead to a commutative graded DGA $\mathcal{N}^\bullet = \bigoplus_{r \geq 0} \mathcal{N}(r)^\bullet$, where $\mathcal{N}(r)^p$ = alternating projection of codim r algebraic cycles on $(\mathbb{P}^1 - \{1\})^{2r-p}$. In joint work with I. Kriz, S. Bloch proposed $H^\circ(\mathrm{Bar}(\mathcal{N}^\bullet))$ as a candidate for the Hopf algebra associated to the category of mixed Tate motives. Writing $\mathcal{L}$ for the (pro–)dual of the space $H^\circ(\mathrm{Bar}(\mathcal{N}^\bullet))^+ / H^\circ(\mathrm{Bar}(\mathcal{N}^\bullet))^{+^2}$ of indecomposables, one has that the category of mixed Tate motives should be given by the category of finite dimensional graded $\mathcal{L}$–modules. These modules are shown to admit Hodge and l–adic realizations and to include the polylogarithm motives.

H. Gillet (Chicago):

Algebraic Arakelov Geometry

H. Gillet reported on joint work with S. Bloch and C. Soulé. In his talk he proposed an analog of the archimedian Arakelov theory of varieties over fields with complex embeddings for varieties over discretely valued fields. If X is a smooth projective variety over such a field K, one defines "forms" and "currents" on X in terms of the Chowgroups of cycles modulo rational equivalence, and the operational Chow groups, of the special fibres $\mathcal{X}_0$ of models $\mathcal{X}$ of X over the valuation ring $\mathcal{O} \subset K$. Using these one can give a definition of $\widehat{\mathrm{CH}}^*(X)$ analogous to the definition in the archimedian case given by H. Gillet and C. Soulé. One proves an analog of the regularity of the dd^c lemma, giving:

Theorem: Assuming resolution of singularities

$$\widehat{\mathrm{CH}}^*(X) = \varinjlim \widehat{\mathrm{CH}}^*(\mathcal{X}).$$

H. Gillet and his coworkers also show that if $f : X \to Y$ is a flat map, then there is a well defined map $f_* : \widehat{\mathrm{CH}}^*(X) \to \widehat{\mathrm{CH}}^*(Y)$.

T. Oda (Kyoto):

Local monodromy on the fundamental groups of algebraic curves

Joint work with M. Asada, M. Matumoto.

Let C_0 be a maximally degenerate stable curve of genus $g \geq 2$ with associated dual graph Y. Let $f : C \to D$ be a local universal deformation of C_0 in the analytic context. One may assume that D is a product of $3g-3$ copies of a small complex disk. Let D^* be the locus of smooth fibres of f. Then D^* is a product of $3g-3$ copies of the punctured disk. The goal is to describe the nonabelian monodromy representation $\rho_f : \pi_1(D^*, t) \to \mathrm{Out}\pi_1(C_t)$, C_t being the fibre at $t \in D^*$. First T. Oda describes this monodromy homomorphism in terms of the graph of groups and "edge" twists, in a completely combinatorial way. Second he describes the relation between the action on the truncated quotient of $\pi_1(C_t)$ by the weight filtration and some invariants of the graph Y.

G. Laumon (Paris):

Zeta functions of Drinfeld modular varieties

Let F be a function field, let ∞ be a place of F, let I be a nonzero ideal of the ring A of functions in F which are regular outside ∞ and let d be a positive integer. Using the non–invariant Arthur trace formula for Gl_d over F, G. Laumon computes the zeta function of the Drinfeld modular variety M_I^d in terms of automorphic L–functions for $\mathrm{Gl}_{d',F}(1 \leq d' \leq d)$. He also formulated a conjecture about the intersection complex of the Satake compactification of M_I^d. This conjecture is compatible with the previous computation and the function field analog of Arthur's L^2–Lefschetz–numberformula for Gl_d.

B. Dwork (Princeton):

Does the equation $\frac{dy}{dx} = \left(\frac{1}{x}\cdot A + \frac{B}{1-x}\right)\cdot Y$ *come from geometry?*

Let W be the algebraic set of $n \times n$ matrices A, B, such that $A^n = 0 = B^n$ and such that the eigenvalues of $A - B$ are fixed elements of $\mathcal{O}$. If the equation "came from geometry" then, excluding a finite set of primes, the reduction V_p of any irreducible component V of W would parametrize a family of differential equations with nilpotent p–curvature. B. Dwork shows for $n = 3$ that this would lead to a 1–parameter family of Fuchsian DE on the sphere (over $\mathbb{F}_q$) with fixed singular points. This contradicts a rigidity theorem for scalar equations with nilpotent p–curvature.

S. Lichtenbaum (Providence):

Suslin complexes and Deligne 1*–motives*

Let X be a variety over an algebraically closed field k. Let $\Delta^n = \mathrm{Spec}\, k[t_0, \ldots, t_n]/(\Sigma t_i - 1)$. Suslin introduced some years ago the complex Sus $^\bullet(X)$, where Sus $^\bullet(X)$ = the group of cycles in $\Delta^n \times X$ finite and surjective over Δ^n. As $\Delta^\bullet$ is a cosimplicial scheme, Sus $^\bullet(X)$ is a simplicial abelian group, and hence a chain complex. Suslin conjectured that if $k = \mathbb{C}$, $H^{*\,\mathrm{top}}(X, \mathbb{Z}/n\mathbb{Z}) = H^*(\mathrm{Sus}\ {}^\bullet(X) \otimes \mathbb{Z}/n\mathbb{Z})$. If X is a curve S. Lichtenbaum shows that Sus $^\bullet(X)$ has a filtration in the drived category sense whose pieces are the homology motives attached to X. More specifically the pieces are $h_0(X) = \mathbb{Z}^e$, e = the number of connected components of X, $h_1(X)$ = the dual 1–motive to the one attached by Deligne to the curve X and $h_2(X) = (k^*[-1])^f$, f = the number of complete irrducible components of X. The speaker closed with the question: if X is arbitrary can Sus $^\bullet(X)$ be interpreted as giving the homological motive of X?

J. Nekovar (Berkeley):

p–adic Gross–Zagier formula for weight > 2

For a newform $f = \sum_{n=1}^{\infty} a_n q^n \in S_{2k}(\Gamma_0(N))$ $(a_n \in \mathcal{O})$, $2k > 2$, and a suitable imaginary quadratic field $K := \mathcal{O}(\sqrt{-D})$, there exist Heegner cycles x in the Chow group $\mathrm{CH}^k(V/K)_0 \otimes \mathcal{O}$ of a suitable smooth compactification of the

Kuga–Sato variety V associated to forms of weight $2k$. For a prime $p \not| N$ the Abel–Jacobi map

$$\begin{array}{ccc}\Phi : \mathrm{CH}^k(V/K)_0 \otimes \mathcal{O} & \longrightarrow & H^1(K, H^{2k-1}_{et}(V \otimes \overline{K}, \mathcal{O}_p)(k)) \\ & & \downarrow \\ & & H^1(K, M_p(k))\end{array}$$

(where M_p is the p–adic Galois representation associated to f) has image in the Selmer group $S := H^1_f(K, M_p(k))$. If $p \not| N \cdot D \cdot a_p$, there is a canonical p–adic height (in the cyclotonic direction) $\langle , \rangle_p : S \times S \to \mathcal{O}_p$.

Theorem: $\langle \Phi(x), \Phi(x) \rangle_p = \mathrm{const} \cdot L'_p(f \otimes K, s)$ (derivative of a p–adic L–function in the cyclotinic direction). Together with a descent result, this implies that $\mathrm{ord}_{s=k} L_p(f \otimes K, s) = 1 \Rightarrow S = \mathcal{O}_p \cdot \mathrm{Im}(\Phi) = \mathcal{O}_p \cdot \Phi(x)$.

G. Faltings (Princeton):

Proof of the Verlinde Formula

The Verlinde formula expresses the dimension of the space of global sections of line bundles on the moduli space of G–torsors on a curve C, G = simply connected simple algebraic group. The result is stated in terms of conformal field theory on a degenerate curve. To show it one expresses torsors on the general fibre of a degenerate curve in terms of loop group quotients. The global sections of the line bundle on these loop group quotients is given by integrable representations, and elements invariant under some global Lie algebra give invariant functions.

P. Schneider (Köln):

Sheaves on Bruhat–Tits buildings

P. Schneider reported on joint work with U. Stuhler, about a new technique to use the geometry of the Bruhat–Tits building for the representation theory of the general linear group $G := \mathrm{GL}_{d+1}(K)$ over a local field K. Fix a principal congruence subgroup U in G and consider a smooth G–representation V which is generated by its U–fixed vectors V^U. The representation V in a natural way gives rise to a sheaf $\underline{V}$ on the topological realization $|BT|$ of the Bruhat–Tits building. A first main result is that this sheaf has no higher cohomology. In order also to compute its cohomology with compact support one constructs an extension $j_{*,\infty}\underline{V}$ to the Borel–Serre compactification $\overline{|BT|}$ and shows that this extension has trivial cohomology, too. Under mild assumptions on V the cohomology on the boundary can be computed by imitating a technique by Deligne / Lusztig over finite fields. In this way P. Schneider and U. Stuhler prove the second main result that the cohomology with compact support of

$\underline{V}$ is zero except in a single degree. From these results they obtain canonical projective resolutions of V with good finiteness properties in the category of all smooth representations (with a fixed central character). In particular they get a description of the Zelevinski involution as a certain exact EXT–functor which shows that this involution respects irreducible representations.

D. Blasius / D. Ramakrishnan (Los Angeles / Pasadena):

Tate's conjecture for divisors on Shimura varieties

This lecture described a proof of Tate's conjecture concerning algebraic cycles for the case of H^2 of Shimura varieties. The work, based heavily upon that of earlier workers, has two branches. In one, undertaken by Murty and Ramakrishnan, the conjecture is proved for all Shimura surfaces defined by quaternion algebras, and forms of Gl (2), over totally real fields. In the second, the conjecture is proved for all other Shimura varieties by various techniques, usually by reducing to previously known cases, but also employing the Eichler–Shimura congrunence relation in a novel way.

The case of sufaces above includes many interesting new ideas concerning the Hodge structures, attached to modular forms. Overall, one uses the existence of a Tate class to produce a geometric involution which produces some period relations and hence a Hodge class.

G. Wüstholz (Zürich):

Effective Poincaré reducibility

G. Wüstholz reported on joint work with D.W. Masser. Let A, B be abelian varieties over a number field K such that A is isogeneous to B. Then let $\delta(A, B)$ be the minimal degree of an isogeny from A to B and $\delta(A)$ the minimal degree of an isogeny from A to a product of simple abelian varieties. Then G. Wüstholz and D.W. Masser proved some time ago that $\delta(A, B) \leq c \cdot h^{\kappa}$ where $h = \max(1, h_F(A)), h_F(A) =$ Faltings height of A, and c, κ are positve constants depending on the dimension of A, the degree of the field K and the degrees of polarizations on A and B. In the talk G. Wüstholz gave a sketch of the proof how one can eliminate the polarizations of A and B. One of the tools is the Zarkin trick by which one can transfer the problem to $Z(A) = (A \times \hat{A})^4$ and $Z(B) = (B \times \hat{B})^4$. A secons tool is an effective version for the classical Jordan–Zassenhaus theorem. It uses techniques from the goemetry of numbers and the description of the endomorphism ring of a simple abelian variety as a cyclic algebra. Finally one uses cross–discriminants in order to bound homomorphisms from A to B in terms of a certain discriminant.

A. Goncharov (Bonn):

Polylogs, algebraic K–theory and $\zeta_F(3)$

Let $Li_n(z) = \sum_{m=1}^{\infty} \frac{z^m}{m^n}$, $|z| \leq 1$ be the classical n–logarithm. It can be continued to a multivalued analytical function on $\mathbb{CP}^1 - \{0,1,\infty\}$:

$$Li_n(z) = \int_0^z Li_{n-1}(t) d\log t .$$

There is a single valued version ($B_k = k-th$ Bernoulli number)

$$\mathcal{L}_n(z) = \left\{ \begin{array}{l} \mathrm{Re}\, n \text{ odd} \\ \mathrm{Im}\, n \text{ even} \end{array} \right. \text{ of } \sum_{k=1} \frac{B_k \cdot 2^k}{k!} Li_{n-k}(z) \log^k |z| .$$

Let $\mathbb{Z}[\mathbb{P}^1_F]$ be the free abelian group generated by the F–points of $\mathbb{P}^1$. Define $\delta_3 : \mathbb{Z}[\mathbb{P}^1_F] \to B_2(F) \otimes F^*$ by $\{x\} \mapsto \{x\}_2 \otimes x$, where

$$B_2(F) := \frac{\mathbb{Z}[\mathbb{P}^1_F]}{\langle \{0\}, \{\infty\} \sum_{i=1}^5 \{r(x_1, \ldots, \hat{x}_i, \ldots, x_5)\} \rangle} .$$

Theorem (Zagier's conjecture):
For any number field F, there exists $y_1, \ldots, y_{r_1+r_2} \in \ker \delta_3$, $[F : \mathcal{O}] = r_1 + 2 \cdot r_2$, such that

$$\zeta_F(3) = q \cdot \pi^{3 \cdot r_2} \cdot |d_F|^{\frac{1}{2}} \cdot \det |\widetilde{\mathcal{L}}_3(\tau_i(y_j))|$$

where $1 \leq i,j \leq r_1 + r_2, \widetilde{\mathcal{L}}_3 : \mathbb{Z}[\mathbb{P}^1_{\mathbb{C}}] \to \mathbb{R}, \{x\} \mapsto \mathcal{L}_3(x)$ and $\tau_i : F \hookrightarrow \mathbb{C}$. Let $C_n(m)$ be a free abelian group generated by configurations of $(n+1)$ vectors in V_m (an m–dimensional vector space). One conjectures that there is a homomorphism of the Bigrassmanian complex, whose components are the $C_n(m)$, to the complex

$$0 \longrightarrow B_n \longrightarrow \cdots \longrightarrow B_2 \otimes \Lambda^{n-2} F^* \longrightarrow \Lambda^n F^*$$

where $B_n = \frac{\mathbb{Z}[\mathbb{P}^1_F]}{R_n(F)}$, $R_n(F)$ is the group generated by $\alpha(0) - \alpha(1)$ with $\alpha \in \ker \delta_n$, $\delta_n : \mathbb{Z}[\mathbb{P}^1_{F(t)}] \to B_n \otimes F(t)^*$.

J. de Jong (z.Z. Bonn):

Cohomology of moduli spaces of two dimensional abelian varieties with $\Gamma_0(p)$*–level structure*

Let (A, λ) be a principally polarized abelian scheme of relative dimension 2 over the scheme S. By a $\Gamma_0(p)$–level structure on $(A,\lambda)/S$ we mean a chain, $0 \subset H_1 \subset H_2 \subset A[p]$ = kernel of multiplication by p, of finite locally free groupschemes $H_i, i = 1,2$ of rank p^i such that H_2 is totally isotropic for the Weil pairing on $A[p]$. Let n be a positive integer ≥ 3 and let $\mathbb{Z}_p \subset \mathcal{O}$ be finite unramified extension which contains an n–th root of unity. Finally let M be the moduli space over $\operatorname{Spec} \mathcal{O}$ classifying two dimensional principally polarized abelian varieties with a $\Gamma_0(p)$–structure and a symplectic level n structure. One first describes the singularities of the scheme M, and a blow up $\widetilde{M} \to M$ such that the special fibre of $\widetilde{M}$ is a divisor with normal crossings in $\widetilde{M}$ and let $\widetilde{M} \hookrightarrow \overline{M}$ be a compactification such that the same is true for $\overline{M}$. The theorem is that the $\mathrm{Gal}(\overline{K}/K)$–module ($K = \mathrm{frac}(\mathcal{O})$) $H^2_{et}(\overline{M}_k, \mathbb{Q}_l)$ is unramified and of weight 2. This is proved using the Steenbrink spectral sequence and the explicit structure of the special fibre of $\overline{M}$.

P. Colmez (Paris):

A product formula for periods of CM abelian varieties

The talk proposed an extension of the product formula for algebraic numbers to certain transzendental numbers. In the easiest case of $2\pi i$ the formula reads $v_p(2\pi i) = \frac{1}{p-1}$ and

$$\log\left(\prod_p |2\pi i|_p\right) = \log 2\pi + \sum_{p<\infty} \frac{-\log p}{p-1}$$

$$= \log 2\pi + \frac{\zeta'(1)}{\zeta(1)}" =" \log 2\pi - \frac{\zeta'(0)}{\zeta(0)}.$$

The generalization of these formulas to the case of periods of CM abelian varieties gives rise to a new conjecture about the logarithmic derivative of Artin L–functions at $s = 0$ which one can prove in the case of abelian functions. As a corollary one obtains a geometric proof of the Chowla–Selberg formula.

P. Vojta (Berkeley):

Division points on semiabelian varieties

P. Vojta reported on work of M. McQuillen. Let A be a seminabelian variety, and let X be a closed subvariety. Assume X, A are both defined over a number field K. Then one has the following theorem:

Theorem (McQuillen): Let Γ be a finitely generated subgroup of $A(\overline{K})$ and let $\overline{\Gamma} = \{a \in A(\overline{K}) \mid n \cdot a \in \Gamma \text{ for some } n \in \mathbb{N}\}$. Then $X \cap \overline{\Gamma} \subseteq \bigcup_i B_i(\overline{K})$, where the union is finite and each B_i is a translated sub–semiabelian variety of A contained in X.

The proof draws heavily on earlier work of M. Hindry, as in his work it reduces the problem to the corresponding problem for $X \cap \Gamma$ via arguments on the action of Galois on points of $A(\overline{K})$. The theorem was originally conjectured by S. Lang, building upon conjectures of Mordell and Manin–Mumford.

M. Harris (Waltham):

Mixed Hodge structure on the boundary cohomology of Shimura varieties

M. Harris reported on joint work with S. Zucker.

Let $(G, \mathfrak{X})$ be the data defining a Shimura variety $Sh(G, \mathfrak{X})$, and let $p : G \to \mathrm{Gl}(V)$ be a $\mathbb{Q}$–rational representation of the reductive $\mathbb{Q}$–group G. Let $\tilde{V}/Sh(G, \mathfrak{X})$ be the corresponding local system, which underlies a polarized variation of Hodge structures. By the theorem of M. Saito, the cohomology groups $H^*(Sh(G, \mathfrak{X}), \tilde{V})$ and $H^*_c(Sh(G, \mathfrak{X}), \tilde{V})$ have mixed Hodge structures. On the other hand, there is a long exact sequence

$$\begin{array}{lllllll} \ldots & \longrightarrow & H^*_c(Sh(G,\mathfrak{X}),\tilde{V}) & \longrightarrow & H^*(Sh(G,\mathfrak{X}),\tilde{V}) & \longrightarrow & \\ & \longrightarrow & H^*(\partial Sh(G,\mathfrak{X}),\tilde{V}) & \longrightarrow & H^{*+1}_c(Sh(G,\mathfrak{X}),\tilde{V}) & \longrightarrow & \ldots \end{array}$$

where ∂Sh is the boundary of the Borel–Serre compactification of $Sh(G, \mathfrak{X})$. A spectral sequence studied by Harder and others, computes $H^*(\partial Sh, \tilde{V})$ in terms of cohomology of Levi factors of rational parabolic subgroup of G. The E_1–terms of this spectral sequence possess a natural mixed Hodge structure as well. One proves that the spectral sequence in fact respects the mixed Hodge structures and computes the mixed Hodge structure on $H^*(\partial Sh, \tilde{V})$.

N.M. Katz (Princeton):

Rigid local systems on open sets of $\mathbb{P}^1$

Over $\mathbb{C}$ let U be $\mathbb{P}^1$ – {finite set of points}. A local system of finite dimensional $\mathbb{C}$ vector spaces $\mathcal{F}$ on U^{an} is called "rigid" if for any local system $\mathcal{G}$ on U^{an} which is isomorphic to $\mathcal{F}$ over a (complex analytic) punctured neighbourhood of each point "at ∞", i.e. each point of $X - U$, there exists an isomorphism $\mathcal{F} \cong \mathcal{G}$ of local systems on U^{an}. For $\mathcal{F}$ irreducible, a sufficient condition for rigidity is $\chi(\mathbb{P}^1, j_* \mathrm{End}(\mathcal{F})) = 2$, for $j : U^{an} \hookrightarrow \mathbb{P}^1$ the inclusion (the condition is probably equivalent). The main result is that all irreducible $\mathcal{F}$'s with $\chi(\mathbb{P}^1, j_* \mathrm{End}(\mathcal{F})) = 2$ are built out of rank one local systems by finitely ($\leq \mathrm{rk}\, \mathcal{F}$) iterating two operations, $\mathcal{F} \mapsto \mathcal{F} \otimes$ (rank one) and (a variant of) $\mathcal{F} \mapsto F *_{!+} \mathcal{L}_\chi$ = additive convolution with a Kummer sheaf $\mathcal{L}_\chi$, both of which preserve both rigidity and irreducibility. The proof depends heavily on Laumon's results on Fourier transform of l–adic sheaves on $\mathbb{A}^1$ on char $p > 0$.

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