MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Variationsrechnung

26.7. bis 1.8.1992

Die Tagung fand unter der Leitung von Herrn J. Jost (Bochum), Herrn L. Modica (Pisa) und Herrn E. Zeidler (Leipzig) statt. In Vorträgen und Gesprächen haben die Teilnehmer aus vielen verschiedenen Länder (z.B. Deutschland, Italien, USA, Australien ...) die neuesten Entwicklungen aus den Gebieten: Harmonische Abbildungen, Minimalflächen, Elastizität und allgemeine elliptische partielle Differentialgleichungen dargestellt und diskutiert.

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Vortragsauszüge

E. Acerbi: *Functionals with non-standard growth*

Many results are known concerning the regularity of minimizers of integral functionals whose simplest model is $\int_\Omega f(Du)dx$, under the growth assumptions:

$$|z|^p \leq f(z) \leq c(1+|z|^p), p > 1,$$

which are the standard ones. As for non standard growth assumptions:

$$|z|^p \leq f(z) \leq c(1+|z|^q), 1 < p < q,$$

after an example by Giaquinta who shows that

$$\int_\Omega (|Du|^2 + c|\frac{\partial u}{\partial x_n}|^4)dx$$

possesses highly irregular minimizers, some results began appearing. Among these, I quote two recent ones obtained in collaboration with N. Fusco.

<u>Statement 1:</u> For the function

$$\int_\Omega \sum_{i=1}^n |D_i u|^{p_i} dx, p_i > 1 \, \forall i = 1, \ldots, n, \Omega \subset \mathbb{R}^n$$

minimizers are partially $C^{1,\alpha}$ whenever all p_i's are strictly less than the Sobolev exponent of the harmonic mean of the p_i's.

This was a homogeneous, anisotropic case; an isotropic but inhomogeneous case is the following

<u>Statement 2:</u> Let Ω_1, Ω_2 be two contiguous open sets such that $\partial\Omega_1 \cap \partial\Omega_2 = \Sigma$ is a Lipschitz surface and set:

$$p(x) = \begin{cases} p_1 & \text{in } \Omega_1 \\ p_2 & \text{in } \Omega_2 \end{cases}.$$

The minimizers of $\int_{\Omega_1 \cup \Sigma \cup \Omega_2} |Du(x)|^{p(x)} dx$ are locally bounded, and Hölder continuous in $\Omega_1 \cup \Sigma \cup \Omega_2$, whenever $p_1, p_2 > 1$.

L. Ambrosio: *On the singularities of Convex Functions*

Viscosity solutions of Hamilton-Jacobi equations are not necessarily differentiable. Hence it is natural to study the properties of the singular set Σ (the set of all non differentiability points) of these solutions. We can naturally split Σ into n components $\Sigma^1, \dots, \Sigma^n$ (n being the dimension of the domain on which our viscosity solution u is defined) according to the dimension of the superdifferential $\partial^+ u(x)$ at the singular point x. In a joint work with G.Alberti and P. Carmorsa, I proved that the semi-concavity properties of u imply that Σ^k is countably $\mathcal{H}^{n-k}$-rectifiable for any integer $k \in [1, n]$. In particular the Hausdorff dimension of Σ^k does not exceed $(n-k)$ and Σ^n is at most countable. In a subsequent paper, written with P. Carmorsa and H.M. Soner, I study the propagation of singularities providing conditions on the Hamiltonian, which ensure that no singularity $x \in \Sigma \backslash \Sigma^n$ is isolated in Σ.

G. Anzellotti: *Functions of bounded variation over generalized surfaces*

Funktion of bounded variation over non regular surfaces [currents, varifolds] arise naturally as weak minimizers of functionals defined on surfaces, for instance functionals depending on curvatures. A study of such functions has been done by GMT methods.

P.U. Aviles: *A new proof of the regularity theory for minimizing harmonic maps and applications*

Here I will discuss the new proof of the regularity theory for minimizing harmonic maps and the extension of it to minimizing harmonic maps into some singular target, i.e. Lipschitz graphs.

G. Buttazzo: *Variations on a theme of Lavrentiev*

The term Lavrentiev phenomenon refers to a surprising result first demonstrated in 1926 by M. Lavrentiev who showed that it is possible for the variational integral of a two-point Lagrange problem, which is sequentially weakly lower semicontinuous on the admissible class of absolutely continuous functions to possess an infimum on the dense subclass of C^1 admissible functions that is strictly greater than its minimum value on the full admissible class. Since that time there have been additional works devoted to simplifying the original example, demonstrating that the phenomenon can occur even with fully regular integrands, devising conditions which forestall occurence of the phenomenon, sharpening the specification of the precise dense subclass of admissible functions for which the Lavrentiev gab occurs, presenting an analogous gap phenomenon in stochastic control and in certain (deterministic) Bolza problems.

Here we adopt the viewpoint that the Lvrentiev gap is actually a relaxation phenomenon assigning to each admissible function u a Lavrentiev term $L(u) \geq 0$ which specifies the magnitude of the gap between the value of the variational functional it self on u and the smallest sequential lower limit of the values it takes on Lipschitzian admissible functions converging weakly to u. Accordingly, given a sequentially weakly l.s.c. functional G defined on the class of all admissible functions, we proceed first to examine the function F which coincides with G on the Lipschitz class but is assigned value $+\infty$ on all non-Lipschitzian admissible functions. We seek the l.s.c. envelope $\overline{F}$ of F on the full class of absolutely continuous admissible functions. Then $L(u)$ is the nonnegative quantity defined by

$$\overline{F}(u) = G(u) + L(u).$$

We proved a characterization of $L(u)$ in terms of the value function V associated with the Lagrange problem: the quantity $L(u)$ is given as a limiting value of $V(x, u(x))$ as x converges to a critical abscissa for the integrand. This description is then utilized to provide a rather explicit calculation of $L(u)$ for integrands satisfying a homogeneity condition as well as for the far larger class of integrands which only satisfy the homogeneity condition in an asymptotic sense near the relevant critical abscissa. In particular, the Manilà integrands is fully analyzed by following this approach.

The presentation of cases in which the Lavrentiev term is identically zero, as well as of certain multidimensional problems, permits a clear discussion of the Lavrentiev phenomenon for general integral functionals of the calculus of variations.

G. Dal Maso: *Some new lower semicontinuity and relaxation results for polyconvex functionals*

For any matrix A, let $\mathcal{M}(A)$ be the vector whose components are the determinants of all minors of A of any order. For every bounded open set $\Omega \subseteq \mathbb{R}^n$ and for every $u \in W^{1,1}(\Omega, \mathbb{R}^k)$ let

$$F(u,\Omega) = \int_\Omega \sqrt{1 + |\mathcal{M}(\nabla u(x))|^2}dx,$$

where $\nabla u(x)$ denotes, as usual, the matrix whose entries are the partial derivatives of the components of u. Note that, if $u \in C^1(\Omega, \mathbb{R}^k)$, then $F(u,\Omega)$ coincides with the n-dimensional measure of the graph of the function u.

We can prove that $F(u,\Omega) \leq \liminf_{h\to\infty} F(u_h,\Omega)$ whenever $u, u_h \in C^1(\Omega,\mathbb{R}^k)$ and $u_h \to u$ in $L^1(\Omega,\mathbb{R}^k)$. The restriction $u, u_h \in C^1(\Omega,\mathbb{R}^k)$ is crucial. Indeed a simple example shows that $G(.,\Omega)$ is not lower semi-continuous in $W^{1,1}(\Omega,\mathbb{R}^k)$ with respect to $L^1(\Omega,\mathbb{R}^k)$-convergence.

This fact leads us to consider the relaxed functional $J(u,\Omega)$, defined for every $u \in L^1(\Omega,\mathbb{R}^k)$ as

$$J(u,\Omega) := \inf\{\liminf_{h\to\infty} F(u_h,\Omega) : u_h \in C^1, u_h \xleftarrow{L_1} u\}.$$

It is clear that $J(.,\Omega)$ is lower semi-continuous on $L^1(\Omega,\mathbb{R}^k)$ and that $J(u,\Omega) = F(u,\Omega)$ for every $u \in C^1(\Omega,\mathbb{R}^k)$. It is possible to prove that this equality can be extended to the case $u \in W^{1,\nu}(\Omega,\mathbb{R}^k)$, with $\nu := \min\{n,k\}$.

Moreover, if $J(u,\Omega_i) = F(u,\Omega_i)$ for two open sets $\Omega_1,\Omega_2) = F(u,\Omega_1 \cup \Omega_2)$.

Nevertheless, two counterexamples show that, in general, the set function $\Omega \mapsto J(u,\Omega)$ is not subadditive. In the first one, u is a piecewise constant function and $n = k = 2$. In the second one, $n = k = 3$ and $u \in W^{1,p}_{\mathrm{loc}}(\mathbb{R}^3,\mathbb{R}^3)$ for every $1 \leq p < 3$. This shows that, in general, for $u \notin W^{1,\nu}(\Omega,\mathbb{R}^k)$ we do not have $J(u,\Omega) = F(u,\Omega)$ and that J can not be represented by any other integral.

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All these results have been obtained in collaboration with Emilio Acerbi and can be extended, with the obvious modifications, to more general polyconvex integrals of the form

$$F(u,\Omega) = \int_\Omega f(\mathcal{M}(\nabla u(x)))dx,$$

where f is convex and has a linear behaviour at infinity.

U. Dierkes: *A Bernstein result for energy minimizing hpersurfaces*

In this talk I consider singular variational integrals of non-parametric type:

$$E_\alpha(u) = \int_\Omega u^\alpha \sqrt{1+|Du|^2}, \alpha > 0, u \geq 0, \Omega \subseteq \mathbb{R}^n$$

as well as the parametric counterpart

$$\mathcal{E}_\alpha(U) = \int |x_{n+1}|^\alpha |D\varphi_U|, \quad \text{where } U \subseteq \mathbb{R}^{n+1}$$

and φ_U denotes its characteristic function.
In the case $\alpha = 1$, the Euler equation corresponding to the non-parametric problem

$$\oplus \ \operatorname{div} \frac{Du}{\sqrt{1+|Du|^2}} = \frac{\alpha}{u\sqrt{1+|Du|^2}}$$

was derived by Lagrange and Poisson as a model equation for the so called "hanging roof" problem. Here we are primarily interested in Bernstein type results for equation $\oplus$ and for the corresponding parametric problem respectively.

Theorem 1: Suppose that $\alpha + n < 4 + \alpha\sqrt{\frac{2}{n+\alpha}}$ (i.e. $n + \alpha < 5,23\ldots$). then there is no entire, stable, positive solution $u \in C^2(\mathbb{R}^n)$ of equation $\oplus$.

Note that for all $n \geq \alpha, \alpha > 0$ there are entire, positive classical (and even rotationally symmetric) solutions of $\oplus$.

Theorem 2: Suppose that $\alpha + n < 4 + 2\sqrt{\frac{2}{n+\alpha}}$ and let $\mathcal{M} = \partial U$ be a smooth boundary of least α-energy $\mathcal{E}_\alpha(U)$ in $\mathbb{R}^{n+1}$. Then $\mathcal{M}$ is a hyperplane E which either is perpendicular to the coordinate plane $\{x_{n+1} = 0\}$ or identical to $\{x_{n+1} = 0\}$.

These results will be proved by first obtaining a generalized "Simons inequality" for the Laplacian of $|A|$ and the mean curvature H and then using this inequality to derive integral curvature estimates of the type

$$\int_{\mathcal{M}} |x_{n+1}|^{\alpha} \left(\frac{1}{\sqrt{\alpha}} H^2 + \sqrt{\alpha}|A|^2\right)^{2+q} \xi^{4+2q} d\mathcal{H}_n$$
$$\leq C(n,\alpha,q) \int_{\mathcal{M}} |x_{n+1}|^{\alpha} |\nabla \xi|^{4+2q} d\mathcal{H}_n$$

$\forall \xi \in C_c^1(\mathcal{M},\mathbb{R})$ and all positive $q < \sqrt{\frac{2}{n+\alpha}}$. Finally a suitable choice of ξ and approximate energy estimates show that $H = |A| = 0$, provided $n + \alpha < 4 + 2\sqrt{\frac{2}{n+\alpha}}$.

F. Duzaar: Joint work with Klaus Steffen

Boundary regularity for minimizing curents with prescibed mean curvature

We consider the following situation: Given an oriented submanifold Γ of dimension $n-1$ in $\mathbb{R}^{n+1}, \partial\Gamma = \emptyset$, and a smooth function $H : \mathbb{R}^{nM} \to \mathbb{R}$ we are interested in boundary rgularity properties of energy minimizing rectifiable n-currents with prescribed mean curvature H and boundary $\partial T = B := \mathbb{C}\Gamma D$. Thenotion of energy minimality refers to the energy functional associated with the prescribed mean curvature function H; i.e. $\mathbb{E}_h(T) = \mathbb{M}(T) + \mathbb{V}_H(T)$ where $\mathbb{M}(t)$ is the mass of T and $\mathbb{V}_H(T)$ in the weighted H-volume enclosed by T and some fixed rectifiable n-current T_0. then we have:

Theorem: Suppose T is a rectifiable n-current in $\mathbb{R}^{n+1}$, $H : \mathbb{R}^{n+1} \to \mathbb{R}$ in locally Lipschitz, T is energy minimizing in some neighborhood of a point $a \in supp\partial T$ and the boundary ∂T is represented in this neighborhood by an oriented $n-1$ dimensional submanifold Γ of class $C^{1,\alpha}, 0 < \alpha \leq 1$, with multiplicity one. Then either:

(i) $\Theta^n(\|T\|, a) = \frac{1}{2}$ and $suppT$ is a $C^{1,\beta}$ submanifold with boundary Γ locally at a for any $0 < \beta < \frac{1}{2}\alpha$; or

(ii) $\Theta^n(\|T\|, a) = m - \frac{1}{2}$ for some integer $m \geq 2$ and $supp\, T$ is a $C^{1,\beta}$ submanifold locally at a for any $0 < \beta < \frac{1}{2}\alpha$, which in separated by Γ into two parts on which T has constant multiplicity m and $m-1$ respectively.

Moreover, in both cases the $C^{1,\beta}$ submanifold *supp* T has (with the orientation induced by T) mean curvature $H(x)$ for any point x close to a (in the sense of distributions).

The case $H \equiv 0$ corresponds to absolutely mass minimizing currents for which boundary regularity was proved by Hardt and Simon.

Klaus Ecker: *Local estimates of mean curvature flow*

We study local behaviour of the flow of hypersurfaces by their mean curvature. A comparison argument yielding the formation of singularities in finite time for certain initial surfaces (e.g. dumbbell-surface) is given.

We furthermore present (joint work with G. Huisken) interior estimates for all geometric quantities on the evoluing surfaces, given that we have local curvature control (or local uniform gradient estimates).

Finally, in the case of surfaces in 3-manifolds we prove an ε-regularity theorem involving a local curvature integral.

Martin Flucher: *A variational approach to Bernouilli's free-boundary problem*

We study the interior problem on a planar domain Ω, i.e. given $Q > 0$, find $A \subset \Omega$ and a function u solving

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \backslash A \\ u = 0 & \text{on } \partial\Omega \\ u \equiv 1 & \text{on } A \\ |\nabla u| = Q & \text{on } \partial A \end{cases}$$

Particular solutions are found by minimization of the capacity $\text{cap}_\Omega(A)$ among all $A \subset \Omega$ with $|A| = \varepsilon$. The so called elliptically ordered branch of solutions tending to $\partial\Omega$ as $Q \to \infty$ and $\varepsilon \to |\Omega|$ is fairly well understood. However very few is known on hyperbolic solutions (large Q and small ε). Using our asymptotic formula for the minimal capacity among sets of equal area (to appear in Calculus of Variations and PDE's), we can show that a subsequence of the minimizer's concentrates at a maximum point of the conformal radius on Ω as $\varepsilon \to 0$. Moreover, we conjecture that a hyperbolically disk-shaped solution concentrates at any critical point of the conformal radius.

Nicola Fusco: *Regularity results for a model problem from non-linear elasticity*

Consider the functional $\int_\Omega |Du|^2 + |\det Du|^2$ where $u : \Omega \subseteq \mathbb{R}^2 \to \mathbb{R}^2$ belongs to the space $\Lambda_2 W^{1,\ell} := \{v \in W^{1,2}(\Omega, \mathbb{R}^2) : \det Dv \in L^2\}$. We prove in a joint paper with J. Hutchinson, that if u is a minimizer, then $u \in C^{1,\alpha}$, hence C^∞, in an open set $\Omega_0 \subset \Omega$ such that $L(\Omega \backslash \Omega_0) = 0$. The difficulty here is that the term $|\det Du|^2$ can grow in certain directions like $|Du|^4$ which is not controlled by the functional. As a consequence, the space $\Lambda_2 W^{1,2}$ is not a linear space. Moreover it is not known if smooth functions approximate any function $v \in \Lambda_2 W^{1,2}$ in the norm naturally induced by the functional.

We prove also a maximum principle for minimizers of the above functional and combining it with the Courant-Lebesue lemma we can deduce that u is indeed continuous in the whole Ω.

Finally the $C^{1,\alpha}$ partially regularity result can be also extended to a model functional of the form

$$\int_\Omega |Du|^p + \sum_{i=1}^{k} |\Lambda_i Du|^p, p > n-1,$$

where $u : \Omega \subseteq \mathbb{R}^n \to \mathbb{R}^N, 1 \leq k \leq \min\{n, N\}$ and $|\Lambda_i Du|$ is the norm of the map $\Lambda_i Du(x) : \Lambda_i \mathbb{R}^n \to \Lambda_i \mathbb{R}^n$ canonically induced by $Du(x)$.

Mariano Giaquinta: *Some geometric and analytic aspects of variational problems for vector valued maps*

After showing a few simple examples which naturally lead to the notion of Cartesian currents, I discuss a few results obtained in collaboration with Giuseppe Modica (Firenze) and Jiži Souček (Praha). In particular I report on a serie of results concerning the problem of minimizing harmonic maps between Riemannian manifolds and the problem of approximability of Cartesian currents of finite mass by graphs of smooth maps.

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Bob Gulliver: *The Fredholm index for branched H-surfaces of higher genus*

Let $\mathcal{F}$ be a Riemann surface of genus $g \geq 1$ having one boundary component. Let H be a constant $\in [-1, 1]$. A small H-surface is a mapping $f : \mathcal{F} \to B_{\frac{1}{2}} \subseteq \mathbb{R}^3$ satisfying in local conformal coordinates (x, y),

$$\begin{cases} f_{xx} + f_{yy} = 2H f_x \wedge f_y \\ |f_x|^2 - |f_y|^2 = 0 = f_x \cdot f_y \end{cases}$$

Given integers $\gamma_1, \ldots, \gamma_p \geq 1$, we show that the set $\mathcal{M}_H(f, \gamma_1, \ldots \gamma_p)$ of all small H-surfaces having branch points of orders $\geq \gamma_1, \ldots, \gamma_p$ at p distinct interior points, and no boundary branch points, is a manifold. We further show that the projection of $\mathcal{M}_H(f, \gamma_1, \ldots \gamma_p)$ onto the manifold of immersed curves is a Fredholm mapping of index $2 \sum\limits_{\nu=1}^{p} (1 - \gamma_\nu)$, using the natural Sobolev norms (joint work with Reinhold Böhme).

Pedro M. S.G. Henriques: *Noether's theorem and the reduction procedure introduced by Weinstein and Marsden, in the context of differential systems*

During the last few years I have been working on an extension of Phillip Griffith's work. I generalized the study of variational problem for multiple integrals in a large framework which allows the consideration of mixed boundary conditions using exterior differential systems defined on C^∞-manifolds. The purpose of my recent work is to complete this study proving the Noether's theorem and by establishing an adequate reduction procedure to generalize the work of Weinstein and Marsden. The version of the Noether's theorem for multiple integrals shows that there exists a closed form defined on integral manifolds of the Euler-Lagrange differential system with infinitesimal symmetries.

The reduction procedure permits to find the integral manifolds of the Euler-Lagrange system through lifts of the integral manifolds of a reduced system defined on reduced manifolds. The procedure is based upon the properties of a Lie-group's action on C^∞ manifolds.

Gerhard Huisken: *An evolution equation for hypersurfaces of constant mean curvature*

Let $F_0 : M^n \to N^{n+1}$ be a smooth immersion of a closed hypersurface in some compact Riemannian manifold N^{n+1}. Then we consider the evolution equation

$$\frac{d}{dx}F(p,t) = (h - H)\nu(p,t), p \in M^n, t > 0$$

with $F(p,0) = F_0(p)$, where ν is the unit normal and H the mean curvature of the hypersurface. Also, h is the average of the mean curvature, $h := \fint H d\mu$. This flow is decreasing the area of the moving hypersurface while keeping the enclosed volume constant it is the gradient flow of isoperimetric problem. It is shown that the solution M_t of this problem remains regular and converges in subsequences to constant mean curvature surfaces, provided the initial hypersurface is small and convex, depending on the geometry of N. In joint work with S.T. Yau (Harvard) this technique was also applied to construct regular foliations by constant mean curvature spheres near infinity in asymptotically flat 3-manifolds of positive mass. These manifolds model isolated gravitating systems in general relativity and the so constructed(unique) foliations by constant mean curvature surfaces can be interpreted as the centre of mars for the infinitely far observer.

John E. Hutchinson: *The "Oil-Water-Surfactant" Problem*

Suppose oil (0), water (W), and surfactant (S) (i.e. soap or detergent) are thoroughly mixed in a container Ω. Then S forms an oriented surface which can be written in the form $S = S_1 \cup S_2^+ \cup S_2^-$ where $S_1 = \partial 0$ and $S_2^+ \cup S_2^-$ is a belayer of two oppositely oriented sheets.
From physical chemical considerations the energy

$$E(S) = \int_S \alpha(H - H_0)^2 - \beta K,$$

where H_0 is the prefered mean curvature, $\alpha > 0$, $0 < \beta < \alpha$, H is the mean curvature, and K is the Gauss curvature (under these conditions on α and β, the integrand is symmetric and positive definite in the principal curvatures).
The problem can be naturally modelled in the setting of oriented integral curvature varifolds (Hutchinson, Ind. Journal of Math. 1986). Thus

(i) $\Omega \subset \mathbb{R}^3$

(ii) 0 is a set of bounded parameters with spt $0 \subset \Omega$.

(iii) S is an oriented 2-varifold with $\int |A|^2 < \infty$, where $|A|^2$ is the squared of the second fundamental form taken in the approximate weak sense.

(iv) $\partial 0 = c(S)$, where $c(S)$ is the current associated to S (by "cancellation").

(v) $\mathbb{M}(S), \mathbb{M}(0)$are prescribed.

We discuss various existence and regularity results for minimizers as well as for the class of competing surfaces.

Norbert Jakobowsky: *A perturbation result concerning a second solution to the dirichlet problem (DP) for the equation of prescribed mean curvature*

$$(DP) \text{ :Find } X : B_1(0) \to \mathbb{R}^3 \text{ such that}$$
$$\begin{cases} \Delta X = 2H(X)X_u \wedge X_v & \text{in } B_1(0) \\ X = Z & \text{on } \partial B_1(0) \end{cases}$$

Well known results (Struwe, Brezis-Coron, Steffen, Wente) state the existence of a second solution to (DP) for constant (curvature) $H \neq 0$ and a class of (non constant) boundary data Z.
In 1989, Struwe extended his results to (a dense set of) non constant curvature functions near a constant $\neq 0$.
The purpose of this talk is to give a related statement: Assume $Z \in W^{1,2} \cap C^0(\overline{B_1(0)})$, $Z \neq$ constant, $\|Z\|_\infty < 1$, $H_0 \in]-1,1[\setminus\{0\}$. Then, there exists some $\alpha > 0$ such that DP) admits at least two solutions whenever $H \in C^1(\mathbb{R}^3, \mathbb{R})$, $\nabla H \in C^\infty$, $\sup\limits_{X \in \mathbb{R}^3} \{(1+|X|)|H(X) - H_0| + |Q(X) - H_0 X|\} < \alpha$, where

$$E_H(X) = \int\limits_B \frac{1}{2}|\nabla X|^2 + \frac{2}{3}Q(X)X_u \wedge X_v dudv,$$

$$Q(X) = (\int\limits_0^{X^1} H(s, X^2, X^3)ds, --, --).$$

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As an essential tool convergence in $W^{1,2}_{\mathrm{loc}}(B_1(0)\backslash\{q_1,\ldots,q_N\})$ of a subsequence of $\{X_n\} \subset \{Z\} + W_0^{1,2}(B_1(0))$ is derived if $\|X_n\|_{1,2} \leq M < \infty$ and

$$\sup_{0\neq\Phi\in W_0^{1,2}\cap C^\infty} \|\Phi\|_{1,2}^{-1}|dE_H(X_n)\Phi| \to 0;$$

moreover strong convergence in $W^{1,2}(B_1(0))$, if $\liminf\limits_{n\to\infty} \int\limits_{B_\rho(\omega)} |\nabla X_n|^2 dudv < \mu(H)$ for all $\omega \in B_1(0)$ with $\rho = \rho(\omega)$.

Jürgen Jost

Michael Struwe: *Minimal surfaces of varying topological type*

A global parametric approach to the Plateau problem for oriented minimal surfaces spanning a given set of smooth Jordan curves $\Gamma = (\Gamma_1, \ldots, \Gamma_m)$ is presented.
By considering parametrisations of Teichmüller space in terms of Fenchel-Nielson coordinates the Plateau problem for genus g surfaces may be phrased as a variational problem for a smooth functional E on a convex set M_g of a suitable Banach space. A partial completion of M_g is achieved by allowing certain geodesics on the model surface G to shrink and by allowing degenerations of the boundary maps under the action of the conformal group on the boundary maps, giving rise to discs splitting off at the boundary. The augmented space $\hat{M}_g$ constructed in this way is stratified into surfaces of varying genus.
Moreover, E and its partial derivatives continuily extend to $\hat{M}_g$ and respect the stratification in the sense that a pseudo gradient vector field may be constructed which is tangent to the strata. Travelling along the corresponding pseudo gradient flow we may enter strata of lower genus but the topological complexity is never increased.
Finally a Palais Smale condition holds if we suitably factor out the modular group and the conformal group action on the disc. Thus the problem is amenable to Ljusternik-Schnirelman and Morse-Conley theory. As an application we obtain Morse inequalities for embedded surfaces and various new results for stable and unstable minimal surfaces.
Recently, the first author extended the theory to the non-orientable case. As an application, an example of a configuration of two disjoint arcs in exhibited spanning infinitely many (oriented or non-orientable) minimal surfaces.

Bernhard Kawohl: *New results on an old nonconvex variational problem*

More than 300 years ago Isaac Newton developed a model to predict the resistance of a body which moves through a rare medium. His model used Newtonian particle mechanics. He found the resistance of a ball to be half as large as that of a cylinder of same diameter and length and "reckoned that this proposition will not be without applications in the building of ships".
To find a body of minimal resistance, one has to minimize a functional of type

$$R(u) = \int\limits_{\Omega} \frac{1}{1+|\nabla u|^2} dx$$

over a suitable class of admissible functions. Newton's contemporaries had used a different but equivalent formulation for rotational bodies.
In the lecture I derive the functional and discuss various aspects of the problem: What is the right class of admissible functions? How about existence, uniqueness, symmetry etc.? One aspect are interesting compactness results in function spaces. Another aspect is the validity of the Euler equations, which are of mixed (ellptic-hyperbolic) type and which seem to call for entropy-type conditions.
Most of the results were obtained in joint work with G. Buttazzo (Pisa), but some also with V. Ferone (Napoli).

Rolf Klötzler: *The multiplier rule for multiple integrals and inequality constraints*

This paper deals with the problem: in which sense can we generalize Pontryagin's maximum principle for problems of optimal control with multiple integrals on a domain Ω? Under certain goodness conditions it will be shown that this kind of Lagrange multiplier rule can be obtained at least in approximate sense as en ε-maximum principle in integrated form. From this we get in many cases a correct maximum condition with multipliers in $L_\infty(\Omega)$. In comparison of the deposit problem (P) and its dual transportation flow problem (D) is illustrated, that each optimal solution of (P) satisfies a such rule with multipliers which are optimal for (D), elements of $L^*_\infty(\Omega)$ but not always belong to $L_1(\Omega)$.

Yaroslav Kurylev: *On some inverse boundary value problems for elliptic operators on a Riemannian manifold*

Let Ω be an n-dimensional ($n \geq 2$) compact smooth Riemannian manifold with non empty border $\Gamma = \partial\Omega$. We denote by $-\Delta$ its Laplace-Beltrami operator, by $-\Delta_N$ the operator $-\Delta$ with Neumann boundary condition, by $-\Delta + q$ the Schrödinger operator on Ω with potential q, by $(-\Delta + q)\gamma$ the Schrödinger operator on Ω with boundary condition of the 3rd type: $\partial_\nu u - \gamma u|_\Gamma = 0$.

Definition: The set $(\Gamma, \{\lambda_k\}, \{\varphi_{k|\Gamma}\}), k = 1, 2, \ldots$ is called boundary spectral data (BSD) of $-\Delta_N$ (resp. $(-\Delta + q)_\gamma$) if (λ_k) is the spectrum of $-\Delta_N$ (resp. $(-\Delta + q)_\gamma$) and $\{\varphi_{k|\Gamma}\}$ the traces on Γ of the corresponding eigenfunctions of $-\Delta_N$ (resp. $(-\Delta + q)\gamma$).

Theorem: (Belishev-Kurylev 91): Given a BSD of $-\Delta_N$ the Riemannian manifold Ω may be recovered uniquely.

Theorem: (Kurylev 92): Given BSD of $(-\Delta + q)\gamma$ the Riemannian manifold Ω, the potential q and the impedance γ may be recovered uniquely. Both theorems take place under some conditions on $\Omega(q, \gamma)$ formulated in terms of the corresponding wave operator.
Let $\Omega \subseteq \mathbb{R}^n$ and $M_{<a>}$ an operator of the form

$$M_{<a>}u = -\partial_i(a^{ij}(x)\partial ju), a^{ij}\xi_i\xi_j \geq q_0|\xi|^2; \partial_\nu u|_\Gamma = 0$$

where ∂_ν is the normal derivative in the metric associated to $\{a^{ij}\}$. Denoting $(\{\lambda_k\}, \{\varphi_k|_\Gamma\})$ the spectrum and traces of eigenfunctions of $M_{<a>}$ as BSD we have

Theorem: (Kurylev 92): Given BSD, $M_{<a>}$ may be covered uniquely modulus the group of unimodular diffeomorphism of Ω, identical on Γ.

Ernst Kuwert: *Exterior domain problems for the minimal surface equation* $div\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = 0$.

We consider boundary value problems for the nonparametric minimal surface equation on the exterior of a uniformly convex, bounded set K. We show that there is a unique variational solution with a given normal at infinity and (varying) contact angle along ∂K. In contrast we give two-dimensional example of non-existence for the Dirichlet problem and prove that any possible solution with given normal at infinity must satisfy an oscillation bound on ∂K.

Joachim Lohkamp: *Metrics of negative Ricci curvature*

The relation between curvature and topology of a Riemannian manifold is a classical problem in Riemannian geometry. Hadamard-Cartan, resp. Preissman theorems (for instance) imply that S^n and T^n cannot carry a negative sectional curvature metric. On the other hand results due to Aasn, Bland and Kalka imply that each manifold admits a complete metric of (constant) negativ scalar curvature. Hence the existence of negative Ricci curvature metrics on manifolds, a problem taking formally position between those above, was an open problem for a long time.
The author proved that each manifold M^n, $n \geq 3$ admits a complete metric with negative Ricci curvature, indeed with $-a(n) < r(g) < -b(n)$ for some constants $a(n) > b(n) > 0$ depending only on n. Furthermore there are results concerning the so called "Gromov h-principle".

Stephan Luckhaus: *An explicit Harnack type estimate for almost minimizers of the area in half space*

The Harnack inequality for harmonic functions can be easily used to get an estimate for the defect of the Dirichlet integral

$$d_\Delta(V) = \int_{B_1} |\nabla V|^2 - \int_{B_1} |\nabla V_n|^2 \text{ where}$$

$$\Delta V_n = 0 \text{ in } B_1, V_n - V|_{\partial B_1} = 0$$

The estimate can be stated as follows:

There are explicit constants $c(n), \varepsilon(n)$ such that

$$\int_{B_1 \setminus B_{\frac{1}{2}}} v > c\delta, v \geq 0 \text{ in } B$$

implies: $d_\Delta(v) \geq \varepsilon\delta \int\limits_{B_{\frac{1}{2}}} [\delta - v]_+$.

This estimate can be carried over to the area functional on sets M of finite perimeter in halfspace, X_M denoting their characteristic functions.

Proposition: Let $M \subset B_1 \times [-\infty, \delta] \subseteq \mathbb{R}^n$ with $\int |\nabla X_M| < \infty, \delta \leq 1$,

$$d_{ar}(M) = \int\limits_{B_1 \times \mathbb{R}} |\nabla X_M| - \inf\{ \int\limits_{B_1 \times \mathbb{R}} |\nabla X| / X \to \{0,1\}, X - X_M|_{\partial B_1 \times \mathbb{R}} = 0\}.$$

There exist explicit constants $\varepsilon_1(n), \varepsilon_2(n)$ such that:

$\int\limits_{(B_1 \setminus B_{\frac{1}{2}}) \times [0,\delta]} X_M < \delta, \gamma^{\gamma_n}, \gamma_n = 2 - \frac{4}{3n-2}$ implies $d_{ar}(M) \geq \varepsilon_2 \delta \int\limits_{B_{\frac{1}{2}} \times [\frac{\delta}{2}, \delta]} X_M$.

The proof uses Harnack's principle for the Laplacian and the explicit excess decay estimate for minimal surfaces. It is important to have $\gamma_n < 2$. As an application one has the following estimate for the parametric capillarity problem.

Lemma: Let $M_0 \Subset \mathbb{R}^n, \partial M_0 \in C^{1,\alpha}, |\beta| < 1$. Suppose $x \in \partial M_0, M \subset M_0$ minimizer for $\int\limits_{M_0} |\nabla X_M| + \beta \int\limits_{\partial M_0} X_M$ with prescribed volume. Then there exist $\varepsilon_1(n,\beta), \varepsilon_2(n,\beta)$ and $\rho_0(M_0, n, \beta)$ such that for $\rho < \rho_0$

$$\int\limits_{B_{\varepsilon,\rho}(x_0)} X_M = 0 \text{ or } \int\limits_{B_\rho(x_0) \cap \partial M_0} X_M > \varepsilon_2 \rho^{n-1}.$$

Erich Miersemann: *On an old problem of B. Taylor in capillarity*

We will discuss two asymptotic expansions for capillary problems:

1. it is shown that there exists an asymptotic expansion of the height rise of the surface in a wedge when $\alpha + \gamma < \pi/2$ where 2α is the corner angle and $0 \leq \gamma < \pi/2$ the contact angle between the surface and the container wall. The asymptotic does not depend on the particular solution considered.
2. It is shown the asymptotic correctness of a formal expansion given by Laplace in 1806 of the rise height of a fluid in a circular capillary tube. It is of special interest that the expansion is uniform with respect to the boundary contact angle although the governing quasilinear elliptic equation becomes singular on the boundary if the contact angle tends to zero. The reason for this uniform behaviour is the special nonlinearity of the problem.

The proofs are completely based on the comparison principle of Concus and Finn which applies to the particular nonlinearity of the problem.

Luciano Modica: *Non-uniqueness in the non-parametric Plateau problem in the disk*

Consider the weak form of the Cartesian Plateau problem on the disk $B = \{x \in \mathbb{R}^2 : |x| < 1\}$: $\min\limits_{v \in BV(B)} [\int\limits_B \sqrt{1 + |\nabla v|^2} + \int\limits_{\partial B} |v - \varphi| d\mathcal{H}^1]$ where $\varphi \in L^1(\partial B)$ is the given boundary value. If φ has at least one continuity point in ∂B, then the solution is unique.

In a recent paper in collaboration with S. Baldes we have constructed an example of a $\varphi \in L^1(\partial B)$ nowhere continuous such that the corresponding Cartesian Plateau problem has infinitely many solutions. The idea is to have $|\varphi| \equiv 1$ on ∂B, but φ is so rapidly oscillating between -1 and $+1$ that the corresponding solution u "prefers" to oscillate between $-a$ and a with $a < 1$. Then $u + \lambda$ is a minimizer for every $\lambda \in (a, 1)$.

John Pitts: *Applications of variational methods in the large*

We discuss new constructions for obtaining compact embedded minimal surfaces in Riemannian 3-manifolds by variational methods in the large. These constructions augment the basic approach which involves saddle point methods developed jointly with Prof. Hyam Rubinstein of Melbourne University. As an application, we sketch a proof of the existence of a new family of minimal surfaces in $S^3 (= \{x \in \mathbb{R}^4 : |x| = 1\})$. Here is a sequence $M_1, M_2, \ldots,$ of these minimal surfaces such that $\lim\limits_{k \to \infty}$ genus $(M_k) = \infty$ and the sequence converges (as varifolds) to a totally geodesic 2-sphere with multiplicity two.

Friedrich Sauvigny: *A variational problem with partially free singular boundary*

We take a singular support surface ∂S_α, consisting of two halfplanes which meet in a singular line L with an angle $\pi + 2\pi$, where $\alpha \in (-1, 1)$ holds true and a Jordan arc Γ emanating from ∂S_α. Solving an adequate variational problem we obtain a minimal surface for this configuration $\{\Gamma, \partial S_\alpha\}$ with a regular behaviour near L. For special arcs Γ we prove that the configuration $\{\Gamma, \partial S_\alpha\}$ bounds only one stable minimal surface. These results have been achieved in my joint work with Professor Stefan Hildebrandt.

Friedrich Tomi: *Nonexistence and instability in the exterior Dirichlet problem for the minimal surface equation in the plane*

For a bounded domain $\Omega \subseteq \mathbb{R}^2$ it is well known that convexity of Ω is both necessary and sufficient for the unrestricted solvability of the Dirichlet problem for the minimal surface equation. The situation for exterior domains was unclear until very recently when E. Kuwert constructed examples of boundary data which do not extend as minimal graphs to the exterior domain. These boundary data are smooth but have large oscillation. In a joint work N. Kutev and the present author were able to sharpen this result considerably by showing that even the smallness of the $C^{0,\alpha}$-Hölder norm $0 < \alpha < \frac{1}{2}$, is not sufficient for the solvability of the Dirichlet problem. Similarly, we can also show the instability of arbitrary solutions with respect to small perturbations on the boundary data in the $C^{0,\alpha}$-norm.

N.S. Trudinger: *On the regularity of viscosity solutions of fully nonlinear elliptic equations of quasilinear type*

In this talk, we are concerned with regularity, gradient estimates and uniqueness for Lipschitz solutions of fully nonlinear elliptic equations, satisfying structur conditions modelled on uniformly elliptic quasilinear equations. In particular, Lipschitz viscosity solutions have Hölder continuous first derivatives and the corresponding estimates for classical solutions extend to the viscosity solutions. Our techniques involve careful examination of the semiconvex regularizations of sub-solutions, due to Jensen and Lions, Souganidis, as functions of the associated parameters. Prospects for approximation using second order flows, such as mean curvature flow are also discussed.

Henry C. Wente: *Exact solutions to some capillarity problems*

The class of constant mean curvature immersions of Joachimsthal type (one set of curvature lines planar) on Enneper type (one set spherical) yield nice solutions to certain capillarity problems: 1) Fluid in a infinite trough (as studied by Thomas Vogel) and 2) Immersed cmc annuli inside a sphere which meet the walls of the sphere at a constant angle. We describe the solutions.

Brian White: *The Bridge principle for minimal surfaces*

In this talk I described a method for proving that arc can join minimal surfaces by suitable thin bridges. The method applies, for instance, to arbitrary smooth strictly stable surfaces with boundary, in arbitrary dimension and codimension. An extension to the method allows one to also connect unstable surfaces of nullity 0.
Using those methods, one can prove, for example, that there is a simple closed curve C in ∂B^3 such that C is smooth except to arc point and such that C has the following property: For every real number α greater than some $A \in \mathbb{R}$ (A depending on C) and every pair of nonnegative integers f and i, there exist uncountably (continuous) many embedded minimal surfaces, each of which has boundary C, area α, genus g, and index i. This does not seem to follow from other bridge theorem (Meeks – Yau, Almgren, Smale).

Rugang Ye: *Finite time blow-up for solutions of the harmonic map heat flow*

It is a delicate issue to determine whether finite time blow-up can occur along the heat flow for harmonic maps when the domain dimension is two. On the other hand, in higher dimensions, finite time blow-ups are caused by a kind of conflict between topological obstruction and analytic convergence. We demonstrate examples of finite time blow-up's in dimension two.

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