

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 35/1992

Jordan-Algebren
9.8. bis 15.8.1992

An der Tagung über Jordan-Algebren, die unter Leitung von W. Kaup (Tübingen), K. McCrimmon (Charlottesville) und H. P. Petersson (Hagen) stand, nahmen 57 Mathematikerinnen und Mathematiker aus Deutschland, England, Frankreich, der GUS, Irland, Israel, Kanada, Österreich, Rumänien, Spanien und den USA teil. Die während der Tagung gehaltenen Vorträge sind den Gebieten

- Algebraische Theorie der Jordan-Strukturen
- Jordan-Strukturen und Analysis
- Jordan-Strukturen und Geometrie
- Jordan-Strukturen und Ausnahme-Lie-Algebren
- Lie-Algebren der Charakteristik p
- Allgemein nichtassoziative Algebra

zuzuordnen.

Die Tagung gliederte sich in

- 15 einstündige Übersichtsvorträge, deren Ziel es war, den aktuellen Stand der Forschung in speziellen Teildisziplinen darzustellen und offene Probleme sowie zukunftsweisende Entwicklungstendenzen aufzuzeigen.
- 18 halbstündige und 14 20-minütige Spezialvorträge in Parallelsektionen, wo über spezielle Forschungsergebnisse berichtet wurde.

Vortragsauszüge

Allison, Bruce: Structurable Algebras and D_4

This talk surveys some recent work in the study of structurable algebras including an application to the classification of Lie algebras of type D_4 over number fields. We first introduce structurable algebras as unital nonassociative algebras with involution that occur naturally as coefficient algebras in 3×3 -unitary Lie algebras. We then describe the recent list given by O. Smirnov of finite dimensional central simple structurable algebras. Finally, we show that a particular class of 8-dimensional structurable algebras can be used as coefficient algebras to classify central simple Lie algebras of type D_4 over number fields.

Arazy, Jonathan: A Jordan-theoretic approach to isometries of Banach algebras which satisfy the von-Neumann inequality

Let A be a Banach algebra, let $D = \{z \in A : \|z\| < 1\}$ and H be the Hermitian elements of A . Let A_s be the symmetric part of A (in the sense of holomorphy). The **von-Neumann inequality** holds in A if $\|f(z)\| \leq \|f\|_\infty := \max\{|f(\lambda)| : |\lambda| = 1\}$ for every polynomial f and every $z \in D$.

Proposition *The von-Neumann inequality holds in A if and only if the vector field $h(z) := I - z^2$ is complete in D .*

Proposition $A_s \subset H + iH$, and $A_s = H + iH$ if and only if $I \in A_s$.

Proposition *If the von-Neumann inequality holds in A then $A_s = H + iH$ is a C^* -algebra. Moreover, the (abstract) partial Jordan triple product is $(x, y, z) = (xy^*z + zy^*x)/2$ for $x, z \in A$ and $y \in A_s$.*

Since isometries preserve the Jordan triple product, we get:

Theorem *Let A, B be complex unital Banach algebras satisfying the von-Neumann inequality, and let ϕ be a linear isometry of A onto B . Then $\phi(z) = u\psi(z)$ where ψ is a self-adjoint Jordan isomorphism of A onto B and u is a unitary element of $H + iH$.*

Ayupov, Sh. A.: Jordan and Lie Operator Algebras

Let M be a factor, α its $*$ -antiautomorphism of order 2, and $M^\alpha(\pm 1) = \{a \in M : \alpha(a) = \pm a\}$ the spectral subspaces of α . It follows that $M^\alpha(+1)$ is a Jordan algebra and $M^\alpha(-1)$ is a Lie algebra. We classify the pairs of Jordan and Lie algebras which can occur in this manner in the case when M is a finite factor. The main problem considered in this talk is the following isomorphism problem:

The Jordan algebras $M^\alpha(+1)$ and $M^\beta(+1)$ (respectively the Lie algebras $M^\alpha(-1)$ and $M^\beta(-1)$) are isomorphic if and only if the antiautomorphisms α and β are conjugate, i.e. $\alpha = \theta^{-1}\beta\theta$ for a *-automorphism θ of the factor M .

The result is completely proved for Jordan algebras. What for Lie algebras, the problem is solved for the cases of type I, type II₁ and σ -finite type III factors.

Benkart, Georgia: Commuting Algebras

Let G be a reductive Lie group (or its associated Lie algebra), and let V be a G -module. Consider the G -module $T = (\otimes^r V) \otimes (\otimes^s V^*)$ obtained by tensoring r copies of V and s copies of its dual space V^* . In this talk we discuss the centralizer algebra $\text{End}_G(T)$ of transformations on T commuting with G (that is the G -module homomorphisms) for various choices of G and V . In particular, when $G = \text{Gl}(V)$ or $\text{gl}(V)$ for $V = \mathbb{C}^n$, we identify $\text{End}_G(T)$ with a subalgebra $B_{r,s}^n$ of the Brauer algebra B_{r+s}^n . When $G = \text{so}(n, \mathbb{C})$ (the orthogonal Lie algebra), S = its open representation and $T = \otimes^r S$, we identify $\text{End}_G(T)$ with the associative envelope of $\text{so}(r, \mathbb{C})$ acting on T . We describe representation and combinatorial theoretical implications of these results and their quantum versions.

Block, Richard E.: Differentiably Simple Graded Algebras

Let N be a not-necessarily-associative superalgebra, finite-dimensional over an algebraically closed field K (the results extend to arbitrary base fields), and g a Lie superalgebra acting on N by superderivations. Suppose N is g -simple (i.e., the only g -invariant ideals of N are 0 and N , and $N^2 \neq 0$). Let P be the (necessarily unique) maximal ideal of N , $S = N/P$, and $h = \{x \in g : xP \subset P\}$ or, at characteristic p , $h = \{x \in \hat{g} : xP \subset P\}$ where \hat{g} is the p -closure of g in Ug . The coinduced module $\text{Hom}_{U_h}(Ug, S)$ has a superalgebra structure (as a subalgebra of $\text{Hom}_K(Ug, S)$).

Theorem *At characteristic 0, g/h is odd, and (canonically, by the g -superalgebra map $n \mapsto (y \mapsto yn + P)$) $N \cong \text{Hom}_{U_h}(Ug, S) (\cong S \otimes \Lambda(g/h))$. At characteristic p , (canonically) $N \cong \text{Hom}_{U_h}(U\hat{g}, S) (\cong S \otimes B(n) \otimes \Lambda(m))$ where $B(n) = K[X_1, \dots, X_n]/(X_1^p, \dots, X_n^p)$, $n = \dim(\hat{g}/h)_0$, $m = \dim(\hat{g}/h)_1$.*

The proof also extends to a determination of a certain class of infinite-dimensional \mathbb{Z} -graded g -simple (super)algebras, and of g -simple linearly compact (super)algebras.

Dorfmeister, Josef: Algebraic Structures and Differential Geometry

We start with three geometric examples and obtain differential equations (like the equations for special moving trihedron) which depend on a parameter λ . We interpret

these equations as equations in loop groups. Using a generalized Riemann-Hilbert splitting in the spirit of the Zakharov-Shabat method we show that part of a loop group acts on the solutions to these differential equations. Moreover, a special orbit for the modified KdV-equation is investigated. Projecting the MKdV manifold down (with fiber $\mathbb{C}P^1$) we obtain a variety associated with the potential KdV-equation. Finally we embed this latter variety into a much larger variety (Sato-Segal-Wilson variety). We discuss how this latter setting can be generalized by using Jordan Banach pairs. Throughout we list a number of open problems.

Edwards, C. M.: Inner Ideals in JB^* -triples

This talk is devoted to a discussion of some of the results obtained since 1988 in the identification of inner ideals in JB^* -triples.

In 1988 it was shown that there exists an order isomorphism $(e, f) \mapsto eAf$ from the complete lattice of centrally equivalent pairs of projections in a W^* -algebra A onto the complete lattice $J(A)$ of weak $*$ -closed inner ideals in A . Every continuous JBW^* -triple is isomorphic to one of the form $pC \oplus_M H(B, \alpha)$ where B and C are continuous W^* -algebras, p is a projection in C , and $H(B, \alpha)$ is the JBW^* -algebra of elements of B invariant under a $*$ -antiautomorphism α of B of order two. It is shown that every weak $*$ -closed inner ideal I in $pC \oplus_M H(B, \alpha)$ is of the form $fCg \oplus_M eH(B, \alpha)\alpha(e)$ for some projections f and g in C with $f \leq p$ and e a projection in B . (Research done jointly with G. T. Rüttimann and S. Yu. Vasdovsky.)

A general geometric result along slightly different lines shows that a norm closed subtriple B of a JB^* -triple is an inner ideal if and only if B has the unique Hahn-Banach extension properly. (Research done jointly with G. T. Rüttimann.)

Elduque, Alberto: Mutation Algebras, Algebras related to Connections in Homogeneous Spaces, Malcev Algebras and Superalgebras

Let A be an alternative algebra with multiplication xy over a field of characteristic $\neq 2$. For fixed $p, q \in A$, a new algebra $A(p, q)$, called the (left) (p, q) -mutation of A , is defined under the product $x * y = (xp)y - (yq)x$. The mutations of all alternative algebras form a class of Jordan and Malcev admissible algebras. If A is associative, then $A(p, q)$ is Lie-admissible and originated from a generalization of quantum mechanics. The structure of $A(p, q)$ has recently been determined for any artinian alternative algebra A . The main concern in this talk is to determine $\text{Aut } A(p, q)$ and $\text{Der } A(p, q)$ for any central simple artinian alternative algebra A . From the associative case, it turns out that $\text{Aut } A(p, q)$ is isomorphic either to a certain subgroup G of $(A^* \times A^*)/Z(A)$ or to an extension group of G by index two, and that $\text{Der } A(p, q)$ is isomorphic to a subalgebra of $(A^- \times A^-)/Z(A)$. Here, A^*

denotes the group of units in A . Similar results are obtained for mutations of octonion algebras. The description of $\text{Aut } A(p, q)$ and $\text{Der } A(p, q)$ becomes more concrete for generalized quaternion algebras A .

Faraut, Jacques: Jordan Algebras and Special Functions

In 1957 Koecher defined the gamma function of a domain of positivity. This function was generalized in 1962 by Gindikin who computed it. As an application one obtains a Bernstein identity for the reduced norm Δ of a Jordan algebra V . By using the decomposition of the space of polynomials on V under the action of the structure group, one obtains a generalized Taylor expansion for $\Delta(e - a)^{-\lambda}$, the binomial expansion. As an application one determines the Wallach set. On the model of the binomial expansion one defines the Gauss hypergeometric function, for which there is an analogue of the Euler integral representation. Recently H. Dib proved that it satisfies a differential system generalizing the hypergeometric differential equation.

Farnsteiner, Rolf: Representations of Reduced Enveloping Algebras

Let $(L, [p])$ be a finite dimensional restricted Lie algebra over an algebraically closed field F of characteristic p . In this talk we shall be concerned with indecomposable representations of the reduced enveloping algebras $u(L, S)$, $S \in L^*$. For a finite dimensional $u(L, S)$ -module M , we let $c_{u(L, S)}(M)$ denote its complexity, that is the rate of growth of a minimal projective resolution of M .

Theorem *Let $L = L^- \oplus L_0 \oplus L^+$ be a restricted Lie algebra with triangular decomposition, $S \in L^*$ a linear form such that $S(L^+) = (0)$. Suppose that $u(L, S)$ has finite representation type. Then the following statements hold:*

- (1) $c_{u(L_0, S|L_0)}(M) \leq 1$ for every finite dimensional $u(L^- \oplus L_0, S|L^- \oplus L_0)$ -module M .
- (2) Suppose there exists a $u(L^- \oplus L_0, S|L^- \oplus L_0)$ -module M such that $p | \dim_F M$, then $u(L_0, S|L_0)$ has finite representation type.

The foregoing result is applicable in the context of classical semisimple Lie algebras.

Faulkner, John R.: Algebraic Structures and Geometry

This talk was a survey of the role of nonassociative algebra in geometry, including recent results and open questions. In addition the following result was announced:

A connected symmetric space M is rotational if

- (1) M has a symmetric subspace N isomorphic to the sphere or elliptic plane.
- (2) If $x, y, z \in N$ with $y, z \perp x$ and $S_y, S_z \neq I$ on M , then x is an isolated fixed point of $S_y S_z$ in M .

Theorem *A connected symmetric space M is rotational iff its tangent space is isomorphic to 2×1 matrices over a real structurable algebra $(A, -)$ with the curvature tensor given by*

$$[xyz] = \langle xyz \rangle - \langle yxz \rangle - \langle zxy \rangle + \langle zyx \rangle$$

where $\langle xyz \rangle = (x\bar{y}^t)z$.

Fernández López, Antonio: Prime Nondegenerate Jordan Algebras satisfying local Goldie Conditions

Based on ideas from semigroup theory, Fountain and Gould introduced a notion of local order in a ring which need not have an identity, and gave a Goldie-like characterization of local orders in a simple ring with minimum condition on principal one-sided ideals.

Inspired by these ideas we introduce a notion of local order in a Jordan algebra and prove that a prime nondegenerate Jordan algebra satisfying local Goldie conditions is a local order in a simple Jordan algebra satisfying dcc on principal inner ideals and which is not a quadratic factor containing an infinite dimensional isotropic subspace. Actually our result is a natural extension of a celebrated Goldie theorem for Jordan algebras due to Zel'manov.

Friedman, Yaakov: Applications of the Theory of Bounded Symmetric Domains to Special Relativity, Gyrogroup Structure on Bounded Symmetric Domains

- (1) Velocity addition formula in special relativity as a consequence of boundedness of speed by speed of light.
- (2) Motion of symmetric velocity and connection of $\text{Aut}(D)$ with the conformal group.
- (3) Gyrogroup structure of the translations in $\text{Aut}(D)$ is the gyrosemidirect product group of the gyrogroup of translations and the rotation group.

Gindikin, Simon: Jordan Algebras, Homogeneous Nonconvex Cones and Pseudo-Hermitian Symmetric Manifolds

(Joint work with J. Faraut.)

We consider linear homogeneous cones which are semisimple affine symmetric spaces. They are corresponding to real Jordan algebras. Corresponding tube domains can be realized as Zariski open parts of pseudo-Hermitian symmetric spaces.

Gradl, Hans: Continuous Time Models in Genetic and Bernstein Algebras

Algebraic structures arise in population genetics in a natural way. The underlying models assume discrete, partially overlapping or continuously overlapping generations. The latter case leads to the differential equation

$$\dot{x} = x^2 - x, \quad x(0) = y,$$

where x^2 denotes the square in a commutative algebra. We investigate the long-time behavior in the case of a genetic and Bernstein algebra using algebraic structure theory. The main results are limit formulas for this system.

González, Santos and Martínez, Consuelo: On Nuclear Jordan-Bernstein Algebras

The notion of "free nuclear Jordan-Bernstein algebra" of rank r is introduced and some properties of it are studied. The main result is: A free nuclear Jordan-Bernstein algebra of rank r has $\text{Ker } \omega$ nilpotent of nilpotency index $\leq 2r + 1$, consequently is finite dimensional.

It is also proved that every nuclear Jordan-Bernstein algebra is a quotient of some free nuclear Jordan-Bernstein algebra and in the general case, a Jordan-Bernstein algebra is a subalgebra of a quotient of a "free algebra".

These results may be used to get some bounds of the nilpotency index of $\text{Ker } \omega$ for a Bernstein algebra in the cases Jordan and nuclear. Also they may be used in the study of inner derivations of a Jordan-Bernstein algebra.

Hopkins, Nora C.: Isomorphism Classes of Noncommutative Matrix Jordan Algebras

In joint work with Robert B. Brown we consider the isomorphism question for degree two noncommutative Jordan algebras $J(S, B)$ over an algebraically closed field k ,

char $k \neq 2, 3$, constructed from a finite dimensional anticommutative algebra S having a nondegenerate symmetric associative bilinear form B . Here

$$J = J(S, B) = \left\{ \begin{pmatrix} a & \alpha \\ \beta & a \end{pmatrix} \mid a, b \in k, \alpha, \beta \in S \right\}$$

with

$$\begin{pmatrix} a & \alpha \\ \beta & b \end{pmatrix} \begin{pmatrix} c & \gamma \\ \delta & d \end{pmatrix} := \begin{pmatrix} ac + B(\alpha, \delta) & a\gamma + d\alpha - 2\beta\delta \\ c\beta + b\delta + 2\alpha\gamma & bd + B(\beta, \gamma) \end{pmatrix}.$$

Let $\text{Der}(S, B) = \{D \in \text{Der } S \mid B(\alpha D, \beta) = -B(\alpha, \beta D)\}$. We prove that if $J(S, B) \cong J(\tilde{S}, \tilde{B})$ where S is an irreducible $\text{Der}(S, B)$ -module and $\text{Der}(S, B) = \text{Der } J$, then $\tilde{S} \cong S_R$ for some $R \in \text{Aut } S$ such that $R^2 = \text{id}$ and $B(\alpha R, \beta) = B(\alpha, \beta R)$ where S_R is S with the multiplication \cdot_R defined by $\alpha \cdot_R \beta = (\alpha\beta)R$. We prove similar results when the construction of $J(S, B)$ is iterated using as the anticommutative algebra J' , the trace zero elements of J , with the bracket product $[x, y] := xy - yx$ and using the bilinear form $-\frac{1}{2}C(x, y) := -\frac{1}{2}\text{trace}(xy)$.

И'т'ков, А.: Identities of Representations of Algebras

In 1986 A. Kemev proved that every associative algebra over a field F of characteristic zero has a finite basis of identities.

In this work it was managed to obtain the following results.

Theorem *Every finite dimensional representation (or birepresentation) of a Lie (or Jordan or alternative) algebra over F has a finite basis of identities.*

Corollary *Every finite dimensional Lie (or Jordan or alternative) algebra has a finite basis of identities.*

Iordanescu, Radu: Geometrical Applications of Jordan Structures

We connect older and recent Romanian geometrical results with the study of Jordan structures pointing out how the research can proceed. A series of such results are those concerning quaternionic Grassmann manifolds. Another series of results refer to spaces with constant affine connection. A third series of results are those devoted to the theory of manifolds (the so-called "preringed" manifolds and "SH-manifolds"). Finally a fourth series of results on projective geometries are commented. For details see Iordanescu, R., "Jordan structures with Applications" (mimeographed), 495 pp., Bucharest, 1990, ISSN 02503638.

King, Daniel: The Kantor Doubling Process on Dot-Bracket Superalgebras

Superalgebras $J(F)$ of finite or infinite dimension obtained through the Kantor doubling process from unital dot-bracket superalgebras (F, \cdot, \times) with an associative, supercommutative bilinear product \cdot and a superskew-symmetric bilinear product \times are examined. Such a superalgebra $J(F)$ is Jordan if and only if (F, \cdot, \times) is a Jordan dot-bracket superalgebra and $J(F)$ is simple if and only if (F, \cdot, \times) is a simple dot-bracket superalgebra (contains no proper dot-bracket ideals and \times is not a trivial product). Superalgebras of vector type are special whereas the generalized Kantor superalgebras are exceptional except in the case of a single odd variable and no even variables. Finally, a superalgebra in Jordan folklore believed to be the Kantor superalgebra (defined in terms of the Clifford algebra) is shown not to be a Jordan superalgebra.

Kleinfeld, Erwin: Right Alternative Rings

(Joint work with Harry F. Smith.)

We study two types of problems:

- (1) When does right alternative and an additional assumption imply alternative?
- (2) When is a result known to hold for alternative rings valid for certain varieties of right alternative rings, but not for right alternative rings in general?

In particular we take up simple, prime and semiprime rings and additional assumptions such as $[R, R] \subset N_l$, $[R, R] \subset N_\beta$, where $[R, R]$ is the usual commutator, $N_l = \{x \in R \mid (x, R, R) = 0\}$, $N_\beta = \{y \in R \mid (z, z, y) = 0 \forall z \in R\}$, $N_r = \{z \in R \mid (R, R, z) = 0\}$, $U = \{w \in R \mid [w, R] = 0\}$, $C = N_r \cap N_l \cap U$.

Some results have appeared in *Communications in Algebra* 19 (1991), 1593-1601 and in *Bulletin Austral. Math. Soc.* 46 (1992), 81-90. The following is a sample of results not yet published.

Theorem *In a simple ring which is right alternative $U = C$, if simple is weakened to prime then either $U = C$ or $N_r = N$, $\text{char} \neq 2, 3$. In a semiprime strongly $(-1, 1)$ ring $N_l = C$.*

Theorem *If R is right alternative, $\text{char} \neq 2$, then*

- (1) $[R, R] \subset N_l$ and left nilpotent \Rightarrow nilpotent.
- (2) $[R, R] \subset N_r$ and right nilpotent \Rightarrow nilpotent.

Theorem *If R is prime, right alternative, $\text{char} \neq 2, 3$ such that $[R, R] \subset N_\beta$ then either R is alternative or strongly $(-1, 1)$.*

Loos, Ottmar: Finiteness Conditions in Jordan Pairs

A survey of recent results related to finiteness conditions. Main results:

- (1) Conjugacy of frames, improving on work by H. P. Petersson.
- (2) A direct proof that Jordan pairs of finite capacity have descending chain condition on principal inner ideals.
- (3) A Jordan version of the module-theoretic characterization of semiprime Artinian rings. This involves the concept of complemented inner ideals and characterizes the nondegenerate Jordan pairs with dcc on all inner ideals (joint work with E. Neher).
- (4) The socle of a Jordan pair has principal dcc and is an inductive limit of finite capacity subpairs imbedded as inner ideals.

Magnus, John: Lie Ideals closed under Non-Lie Polynomials

Over an arbitrary ring without nilpotent elements, it is shown that there are nonzero formal ideals of Lie polynomials which are closed under an associative-yet-non Lie polynomial product. In particular this implies that all Lie algebras L satisfying $[\text{skew}(A, *), \text{skew}(A, *)] \subset L \subset \text{skew}(A, *)$ for some associative algebra A with involution $*$ cannot be characterized by Lie polynomials, as E. Zel'manov was able to do in Jordan theory with his tetrad-eating polynomials, since these Lie algebras are closed under non-Lie products. An interesting development in the proof of this result was the construction of a Lie algebra over an arbitrary field which was both simple and semisimple and contained a subalgebra free on a countable number of generators.

Magnus, Teresa: Faulkner Geometry

Axioms are presented for a Barbilian geometry of dimension $n \geq 2$ over a ring for which $ab = 1$ implies $ba = 1$. It is shown that any Faulkner geometry of dimension $n \geq 3$ is coordinatized by a unique associative two-sided units ring R and that the group generated by all transvections is a group of Steinberg type over R . Whether a geometry of dimension $n \geq 3$ can be constructed over a given associative two-sided units ring R depends on the behavior of groups of Steinberg type over the ring, particularly $St_n(R)$ and $E_n(R)$.

Martínez Moreno, J.: An Allison-Kantor-Koecher-Tits Construction for Lie H^* -Algebras

In this paper we show that the restriction of finite-dimensionality can be dropped in the modified Kantor-Koecher-Tits construction, by means of Hilbert space methods,

in order to obtain all Lie H^* -algebras with zero annihilator from structurable H^* -algebras.

McCrimmon, Kevin: Kaplansky Superalgebras

Kaplansky's 3-dimensional Jordan superalgebra with basis e_0, η_1, ζ_1 has specialization

$$\alpha e_0 + \beta \eta_1 + \gamma \zeta_1 \mapsto \begin{pmatrix} \alpha I & 2(\beta I + \gamma L_t) \frac{d}{dt} \\ -(\beta I + \gamma L_t) & 0 \end{pmatrix}$$

in the 2×2 matrices with entries in the differential operators on the polynomials $\Phi[t]$. More generally, if A is an associative algebra with derivation D then the Jordan superalgebra $J(A, D) = J_0 \oplus J_1$ ($J_0 = A$, $f_0 \cdot g_j = (fg)_j$, $f_1 \cdot g_1 = (D(f)g - fD(g))_0$) is special with specialization

$$f_0 + g_1 \mapsto \begin{pmatrix} L_{f_0} & 2L_{f_1} D \\ -L_{f_1} & 0 \end{pmatrix}.$$

We investigate general "Kaplansky superalgebras" which are half-unital, have J_0 commutative associative with J_1 as commutative associative (half-unital) bimodule, and have skew product on J_1 to J_0 . We show the simple such superalgebras are all subalgebras of $J(A, D)$'s, hence all are special.

Medvedev, Yuri: J-special Identities in Jordan Triple Systems

O. Loos and K. McCrimmon [1] investigated imbeddings of Jordan Triple Systems in associative and Jordan algebras. They proved that all Jordan Triple Systems with one generator are a-special (can be imbedded to associative algebras). Then, they gave an example of a Jordan Triple System with two generators which cannot be imbedded in any Jordan algebra (j-exceptional). There was constructed a Glennie type a-special identity, valid in all a-special but not all Jordan Triple Systems, and there was raised a question if there exist j-special identities, valid in all j-special but not all Jordan Triple Systems.

Theorem *There exist j-special identities.*

To prove this result we use the structure of free Jordan algebras obtained in [2] and the Loos-McCrimmon j-special Peirce identity [1].

[1] O. Loos, K. McCrimmon: *Speciality of Jordan Triple Systems*, Comm. in Algebra 5 (1977), 1057-1082.

[2] Yu. Medvedev: *Free Jordan Algebras*, Algebra i Logika 27 (1988), 172-200.

Meyberg, Kurt: \mathcal{R} -Algebras

\mathcal{R} -algebras generalize commutative power-associative algebras. They arise very naturally from the study of the Bernoulli equation $\dot{x} = x^2$ in an arbitrary commutative K -algebra. We discuss some properties of these algebras, idempotents and Peirce decomposition. Moreover we survey some results for \mathcal{R} -algebras with a certain non-degenerate trace form.

Neher, Erhard: k -forms of Jordan Pairs and Algebraic Groups

In this talk, the following result was explained:

Theorem *Let V be a Jordan pair over a ring, let $\mathcal{G} \subset V$ be a finite covering division grid and let $\mathcal{E} \subset V$ be a grid. Then there exist an elementary automorphism φ of V and for every $e \in \mathcal{E}$ an orthogonal system $\mathcal{O}_e \in \mathcal{G}$ such that $\varphi e \approx \sum_{g \in \mathcal{O}_e} g$ for every $e \in \mathcal{E}$.*

As a consequence of this, one obtains a conjugacy theorem for k -forms of finite dimensional semisimple Jordan pairs over an algebraically closed field K (k a subfield of K). A similar conjugacy theorem holds for self-dual orders in Jordan pairs.

Given a k -form V' of a Jordan pair V as above, let G be the algebraic K -group associated to V (Koecher, Loos, Springer). It was explained, how to obtain the index of this semisimple k -group in terms of combinatorial data associated with a properly chosen grid base.

Osborn, J. Marshall: Novikov Algebras

We call a nonassociative algebra A **Novikov** if it satisfies the two identities $(x, y, z) = (y, x, z)$ and $(xy)z = (xz)y$, where $(x, y, z) = (xy)z - x(yz)$. Novikov algebras were introduced by Balinskii and Novikov in 1985. More recently Zel'manov found the simple algebras in characteristic zero. We have shown that if A is a simple finite dimensional Novikov algebra of dimension greater than one over a field F of characteristic $p > 2$, then A^- is a simple Lie algebra of dimension p^n , and some finite scalar extension of A^- is a simple Lie Algebra of Cartan type $W(l : n)$. Over a perfect field F with more than three elements and of characteristic $p > 2$, we have found a basis and constructed a multiplication table for each simple finite dimensional Novikov algebra which contains an idempotent.

Petersson, Holger P.: Composition Algebras over Rings

We sketch a theory of composition algebras over an arbitrary commutative associative ring R of scalars. We describe a twisted version of the Zorn vector matrix algebra and the Cayley-Dickson Doubling Process. The latter involves an associative composition

algebra D over R as well as a projective right D -module P of rank one, subject to certain norm conditions. Then there exists a nondegenerate quadratic form $n : P \rightarrow R$, unique up to a factor in R^* , such that $n(w \cdot u) = n(w)n_D(u)$ ($w \in P, u \in D$), n_D being the norm of D . Also, fixing n , there is a unique bilinear map $P \times P \rightarrow D$, written multiplicatively and having $(wu)(wv) = n(w)v^*u$ ($w \in P, u, v \in D$). Then $C = D \oplus P$ becomes a composition algebra under the multiplication

$$(u, w)(u', w') = (uu' + ww', w'u + wu'^*).$$

Conversely, any composition algebra containing D as a composition subalgebra having half its rank arises in this way.

Petersson, Holger P.: Max Koecher's Work on Jordan Algebras

This lecture contains a brief outline of the mathematical work of Max Koecher, the initiator of the Oberwolfach Conferences on Jordan algebras, who died after a long illness approximately two years ago.

The talk focuses on

- his approach to the theory of Jordan algebras via domains of positivity,
- his construction of the Tits-Kantor-Koecher algebra,
- the connection of this construction with Jordan triple systems and bounded symmetric domains.

Racine, Michel L.: Albert Algebras and Conjectures of Serre

(Joint work with Holger P. Petersson.)

The state of knowledge on Albert algebras (central simple exceptional Jordan algebras over a field k) up to 1990 was surveyed. That year, in his work on Galois cohomology, Serre introduced two cohomological invariants in $H^3(k, \mathbb{Z}/2\mathbb{Z})$ and $H^3(k, \mathbb{Z}/2\mathbb{Z})$ for reduced Albert algebras, and conjectured the existence of a third in $H^3(k, \mathbb{Z}/2\mathbb{Z})$ for all Albert algebras. He also asked whether the first two exist in the general situation and wondered whether all these would determine the algebra up to isomorphism. (We denote $H^i(k, C)$ the i th cohomology group of $\text{Gal}(k_0/k)$ in C , where k_0 is the separable closure).

Rost has proved that the element of $H^3(k, \mathbb{Z}/3\mathbb{Z})$ is indeed an invariant. Rost and the authors, independently and by different methods, have shown that the "mod 2" invariants can be defined in general.

Rodríguez Palacios, Angel: Jordan Structures in Analysis

In this survey we have collected several favorite recent results on the following topics:

- A. Some geometrical conditions on nonassociative normed algebras giving rise to Jordan algebras.
- B. Nongeometric theory of normed Jordan algebras.
- C. Jordan-Banach triple systems.
- D. Selected topics on JB^* -algebras and triples.
- E. Results in H^* -theory.
- F. Looking for normed versions of Zel'manov's Prime Theorem.

Russo, Bernard: Structure of JB^* -Triples

Two structure theorems obtained jointly with Yaakov Friedman:

Theorem (Gelfand-Naimark Theorem) (1986 Duke MT): *Embedding a JB^* -triple into a direct sum of Cartan factors.*

Theorem (Classification of Atomic Facially Symmetric Spaces) (1992/93 Can. J. M.): *A geometric characterization of the unit ball of the predual of an atomic JBW^* -triple.*

Ingredients of the proofs:

- (a) *Contractive Projection Theorems and their role in representation theory*
- (b) *Structure of discrete and continuous JBW^* -triples.*
- (c) *Geometry of the dual ball of the complex spin factor (1992 Proc. Lon. Math. Soc.).*

Rüttimann, Gottfried T.: Structural Projections on JBW^* -Triples

A linear projection P on a Jordan * -triple A is said to be **structural** provided that $\{PabPc\} = P\{aPbc\} = P\{aPbc\}$, $a, b, c \in A$. A subtriple B of A is said to be **complemented** if $A = B + \text{Ker}(B)$, where $\text{Ker}(B) = \{a \in A \mid \{BaB\} = 0\}$. A subtriple of a JBW^* -triple is complemented if and only if it is the range of a structural projection.

A weak * -closed subspace B of the dual space E^* of the Banach space E is said to be an N^* -ideal if every weak * -continuous linear functional on B has a norm preserving extension to a weak * -continuous linear functional on E^* and the set of elements in E which attain their norm on the unit ball in B is a subspace of E . It is shown that a subtriple of a JBW^* -triple A is complemented if and only if it is an N^* -ideal. It follows that complemented subtriples of A are weak * -continuous and norm non-increasing. (Joint work with C. M. Edwards.)

Schafer, R. D.: A Generalization of the Algebra of Color

The "algebra of color" is a 7-dimensional quadratic non-commutative Jordan algebra defined by Domokos and Kövesi-Domokos in *J. Math. Phys.* 19 (1978), 1477-1481. It satisfies

$$(1) \quad 2[[[x, y], y], y] = 3yxy^2 - 3y^2xy - [x, y^3],$$

$$(2) \quad D_{x,y} = R_{[x,y]} - 3L_{[x,y]} + 2[R_x, R_y] + 6[L_x, R_y] \text{ is a derivation, and}$$

$$(3) \quad 8[[R_x, R_y], R_z] = 4R_{[[x,y,z]]} - 12L_{[[x,y,z]]} + D_{[x,y],z}$$

for all x, y, z , where $[[x, y, z]] = (zx)y - (zy)x - [x, y]z$. Let A be a finite dimensional noncommutative Jordan algebra of characteristic $\neq 2$. Using results of Elduque and standard theorems on noncommutative Jordan algebras, we determine all semisimple algebras A satisfying (1). We prove that, if both (1) and (2) are satisfied in A , the radical (= maximal nilideal) is nilpotent, and that, if also (3) is satisfied and the characteristic is $\neq 3, 5$, the Wedderburn principal theorem (= radical splitting theorem) holds.

Shestakov, Ivan: On Some Questions in the Theory of Superalgebras

Theorem *Let $M = M_0 + M_1$ be a prime Malcev superalgebra of characteristic $\neq 2, 3$. If $M_1 \neq 0$, then M is a Lie superalgebra.*

Example. Let Φ be a field of characteristic $\neq 2$, V a subgroup of $\langle \Phi, +, \rangle$, $c \in V$. Consider Φ -vector spaces J_0 and J_1 with the bases $\{a_v | v \in V\}$ and $\{x_v | v \in V\}$, respectively, and define multiplication on $J = J_0 + J_1$ by the rules: $a_u \cdot a_v = a_{u+v}$, $a_u \cdot x_v = x_v \cdot a_u = x_{u+v}$, $x_u \cdot x_v = (u-v)a_{u+v+c}$. Let further $G(J) = G_0 \otimes J_0 + G_1 \otimes J_1$ be the Grassmann envelope of J , and \mathcal{F}_J be a free algebra in the variety, generated by $G(J)$.

Theorem *J is a simple special Jordan superalgebra. If $|V| = \infty$, then \mathcal{F}_J is a special degenerated prime Jordan algebra.*

The analogue constructions exist for prime degenerated $(-1, 1)$ -algebras and alternative algebras of characteristic 3.

Theorem *The free Jordan superalgebra on 2 generators, one of which is odd, is not special.*

Slinko, Arkadi: Bounded Degree of Weakly Algebraic Topological Lie Algebras

(Joint work with Bienvenido Cuartero and José E. Gale.)

We shall say that a Lie algebra L over a field K is weakly algebraic if for any $x, y \in L$ there exists a polynomial $f(t) \in K[t]$ without constant term and depending on both such that $x \cdot f(\text{ad } y) = 0$.

Theorem *A complete metric weakly algebraic Lie algebra L over a complete non-discrete valuable field K is algebraic of bounded degree, that is, there exists a positive integer n such that for every $x \in L$ there exist functions $\alpha_i : L \rightarrow K$ provided*

$$(\text{ad } x)^n = \sum_{i < n} \alpha_i(x) (\text{ad } x)^i.$$

These functions $\alpha_i(x)$ can be chosen continuous when L is a Banach Lie algebra.

Strade, Helmut: Simple Modular Lie Algebras and Related Topics

We present the structure of the proof for the following

Classification Theorem *Let L denote a finite dimensional simple Lie algebra over an algebraically closed field of characteristic $p > 7$. Then L is classical or Cartan type.*

Recent results towards an extension of this theorem for $p = 5, 7$ are given.

The Classification Theorem allows to determine some classes of infinite dimensional simple Lie algebras.

Theorem *Let L be an infinite dimensional locally finite simple Lie algebra over an algebraically closed field of characteristic $p > 7$. Assume that there exists $d \in \mathbb{N}$ such that every simple quotient G/I of any finite dimensional subalgebra is either Cartan type or satisfies $\dim G/I \leq d$. Then L is of Cartan type.*

Theby, Armin: Left Jordan Algebras

As common generalization of left alternative algebras and noncommutative algebras we consider the class of left Jordan algebras R defined by the identities $(x, x, x) = 0$, $(x, x, xy) = x(x, x, y)$, $(x, x, yx) = (x, x, y)x$ and $(x, yx, x) = x(x, y, x)$ holding strictly. Such an algebra R is strictly power-associative and carries the structure of a quadratic Jordan algebra R^+ with U -operator $U_x := L_x V_x - L_{x^2} = R_x V_x - R_{x^2}$, and $V_x = L_x + R_x$. Defining recursively operators $A_x^{(n)} \in \text{End } R$ by $A_x^{(0)} := 0$, $A_x^{(1)} := id$

and $A_x^{(n)} := V_x A^{(n-1)} - U_x A_x^{(n-2)}$ for $n \geq 2$ one has $R_{x^n} = R_x A_x^{(n)} - U_x A_x^{(n-1)}$, $L_{x^n} = L_x A_x^{(n)} - U_x A_x^{(n-1)}$. If x is invertible these formulas hold for all $n \in \mathbb{Z}$. The 3-dimensional example with basis e, x, y , where e is unit element and $x^2 = y^2 = e$, $xy = -yx = x$ is left and right Jordan, simple and neither left alternative nor noncommutative Jordan (if $\text{char} \neq 2$). Mutations of two-sided Jordan algebras are two-sided Jordan algebras, too. If $c = c^2 \neq 0$ satisfies $(c, R, c) = 0$ then the Peirce spaces multiply as in noncommutative Jordan algebras. It is hoped that a structure theory can be developed in case there are no absolute zero divisors.

Torrence, Eve: Jordan Algebras and Hexagonal-Barbilian Planes

A projective plane admitting all elations (Moufang plane) is coordinatized by an alternative division algebra. A Barbilian plane is a set of points and lines with two symmetric relations between objects, incidence and neighboring. A Faulkner plane is a Barbilian projective plane with axioms that allow coordinatization by an alternative ring.

An n -gon geometry is a set of points and lines for which every pair of objects lies in an n -gon and there are no k -gons for $2 \leq k < n$.

A hexagonal geometry with Moufang conditions is coordinatized by a quadratic Jordan division algebra over a field. A hexagonal-Barbilian plane has axioms similar to a Faulkner plane that allow coordinatization by a quadratic Jordan algebra over a ring. Progress made toward showing the coordinatization and the construction of a hexagonal-Barbilian plane from a quadratic Jordan algebra over a ring will be discussed.

Upmeyer, Harald: Jordan Algebras, Complex Analysis and Quantization

Every formally real Jordan algebra (or, more generally, a Jordan triple) gives rise to a complex phase space (the open unit ball Ω or the unbounded Siegel domain Π) with non-euclidian hermitian geometry of rank r . We consider two types of quantization $T_\lambda(f)$ (Toeplitz-Berezin) and $W_\lambda(f)$ (Weyl) for classical observables $f \in C^\infty(\bar{\Omega})$, acting on a Hilbert space $H_\lambda^2(\Omega)$ of holomorphic functions, and discuss algebraic properties (structure of Toeplitz C^* -algebras and symbolic properties (Berezin and Weyl transform, classical limit and quantum groups). The results should carry over to the supersymmetric case.

Walcher, Sebastian: Algebraic Structures and Differential Equations

L. Markus (1960) exhibited the connection between differential equations $\dot{x} = p(x)$ with homogeneous quadratic right-hand side and commutative non-associative algebras. While differential equations of this type constitute a much too complicated class to be easily accessible by any method, the use of algebras is efficient in various cases; for instance in finding the general solution. We discuss a few aspects of this algebraic treatment, including semi-invariants and application of S. Lie's classical work on differential equations to the given situation (this was initiated by H. Röhl and M. Koecher). In the unital case there is an "inversion" naturally associated with the general solution of $\dot{x} = x^2$, and there are strong relations between the structure algebra of this inversion and a Jordan subalgebra first introduced by Koecher in the Lie context.

Zel'manov, Efim: Applications of Jordan Algebras in Lie Algebras and Group Theory

We shall consider applications of Jordan algebra theory to

- (1) some Lie algebras related to the Burnside problem on periodic groups;
- (2) Recognition Theorems for Lie algebras graded by root systems;
- (3) Abstract homomorphism of linear groups and Lie algebras over arbitrary rings of coefficients.

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