MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 36/1992

Reelle Analysis

16.8. bis 22.8.1992

Die Tagung fand unter der Leitung von D. Müller (Bielefeld), E.M. Stein (Princeton) und H. Triebel (Jena) statt. Es wurden 34 Vorträge über neue Forschungsergebnisse gehalten. Ferner wurde die Tagung zu regem Gedankenaustausch und zur Zusammenarbeit genutzt.

In diesem Tagungsbericht sind die Vortragsauszüge gemäß der zeitlichen Reihenfolge der Vorträge zusammengestellt. Schwerpunkte waren Themen der klassischen Fourieranalysis auf euklidischen Räumen (Fouriersche Multiplikatoren, singuläre Integraloperatoren, Regularitäts- und Konvergenzprobleme) und der Theorie der partiellen Differentialgleichungen (lineare und nichtlineare PDE, Systeme von PDE). Einige Vorträge stellten auch Resultate aus der Theorie der Funktionenräume dar beziehungsweise behandelten verschiedene Fragestellungen unter Nutzung der Theorie der Pseudodifferentialoperatoren. Des weiteren wurden Ergebnisse auf dem Gebiet der harmonischen Analysis auf nilpotenten Gruppen und symmetrischen Gruppen sowie auf dem Gebiet der komplexen Analysis vorgestellt.

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Vortragsauszüge

D.H. PHONG

Fourier Integral Operators with Degeneracies

This is a report on a recent joint work with E.M. Stein.
Let S(x,y) be a homogeneous polynomial of degree n in $(x,y)\in\mathbb{R}^2$.
We give necessary and sufficient conditions for L^2 estimates for oscillatory integrals of the form

$$\left\| \int_{-\infty}^{\infty} e^{i\lambda S(x,y)} \chi(y)\varphi(y)\,dy \right\|_{L^2(\mathbb{R})} \leq C|\lambda|^{-1/n} \|\varphi\|_{L^2(\mathbb{R})}$$

$$\left\| \int_{-\infty}^{\infty} e^{i\lambda S(x,y)} |S''_{xy}(x,y)|^{1/2}\chi(y)\varphi(y)\,dy \right\|_{L^2(\mathbb{R})} \leq C|\lambda|^{-1/2} \|\varphi\|_{L^2(\mathbb{R})} .$$

Here $\chi \in C_0^\infty(\mathbb{R})$ is a fixed cut-off function. We also discuss L^2 regularity and L^p-L^q estimates for Radon transforms and related Fourier integral operators.

M. CHRIST

Analytic (Non-) Hypoellipticity of $\bar\partial_b$

Let X,Y be real analytic (denoted C^ω), real vector fields in an open subset of $\mathbb{R}^3$, and $\bar\partial_b$ = X + iY. Assume X,Y are independent at each point. Set $\lambda(x)$ = determinant (X,Y, [X,Y])(x). Suppose $\bar\partial_b \bar\partial_b^* f \in C^\omega$ in some open set Ω. Under what geometric hypotheses can one conclude that $\bar\partial_b^* f \in C^\omega(\Omega)$?

<u>Proposition</u>: For $X = \partial_x$, $Y = \partial_y + (x^{m-1} + xt^M)\partial_t$, there exists f as above with $\bar{\partial}_b^* f \notin C^\omega$, provided m,M are even <u>and</u> sufficiently large.

The significance of these examples is that $\{\lambda = 0\} = \{x = 0 = t\}$ has dimension only one, whereas it had dimension two in all previous examples. This supports a conjecture of Treves: any <u>curve</u> with certain geometric properties, contained in $\{\lambda = 0\}$, should pose an obstruction to analyticity.
The proof combines a model case, previously treated by D. Geller and the speaker, with a perturbation argument.

E. DAMEK

<u>Admissible convergence for the Poisson-Szegö Integrals</u>

This is a report on a joint work with Andrzej Hulanicki and Richard C. Penney.
We prove almost everywhere semirestricted admissible convergence of the Poisson-Szegö integrals of L^p functions $(1 < p \leq \infty)$ to the Bergman-Shilov boundary of a Siegel domain. In the case of symmetric domains the result is a consequence of a theorem by Peter Sjögren.
Given a regular cone Ω in $\mathbf{R}^{n_1}$ let a Hermitian bilinear map

$$\Phi : \mathbf{C}^{n_2} \times \mathbf{C}^{n_2} \to \mathbf{C}^{n_1}$$

be Ω-positive i.e. $\Phi(z_2, z_2) \in \bar{\Omega}$ for all $z_2 \in \mathbf{C}^{n_2}$ and if $\Phi(z_2, z_2) = 0$ then $z_2 = 0$.
The domain

$$D = \{(z_1, z_2) \in \mathbf{C}^{n_1} \times \mathbf{C}^{n_2} : Im\, z_1 - \Phi(z_2, z_2) \in \Omega\}$$

is called a Siegel domain determined by Φ, Ω. The Bergman-Shilov boundary of D is the set

$$B = \{(z_1, z_2) \in \mathbf{C}^{n_1} \times \mathbf{C}^{n_2} : Im\, z_1 - \Phi(z_2, z_2) = 0\}.$$

There is a step two nilpotent group $N(\Phi)$ acting on D and acting simply transitively on B.

We do not assume that D is homogeneous but we consider an abelian group $A \subset GL(n,\mathbb{R})$ acting on D in a diagonal way and having a "positive Weyl chamber". An example of such A is $\mathbb{R}^+$ with the action

$$r(z_1,z_2) = (r\, z_1, \sqrt{r}\, z_2)$$

but there are also more sophisticated examples.
Given a group $A, y \in N(\Phi)$ and a compact set $K \subset D$ we define an approach region by

$$\Gamma_y(K) = \{yaz : a \in A,\ z \in K\}.$$

The Poisson kernel for D is the function $P(\omega,z)$ on B x D defined by

$$P(\omega,z) = \frac{|S(\omega,z)|^2}{S(z,z)} \qquad \omega \in B,\ z \in D$$

where S is the Szegö kernel.
Given $F_o \in L^p(B) = L^p(N(\Phi))$ the Poisson integral of F_o is

(1) $$F(z) = \int_{N(\Phi)} F_o(y)\, P(y,z)\, dy$$

If $F \in H^p(D), 1<p\leq\infty$, then F is a Poisson integral of a function $F_o \in L^p(B)$. If D is symmetric the Poisson integral (1) is harmonic with respect to the Laplace-Beltrami operator. On a general Siegel domain the Poisson integral may not be harmonic.

<u>Theorem:</u> Let $F_o \in L^p(N(\Phi)), 1 < p \leq \infty$ and F be the Poisson integral (1) of F_o. Then for every compact $K \subset D$

$$\lim\{F(z) : z \in \Gamma_y(K),\ dist(z,B) \to 0\} = F_o(y) \text{ for a.e } y \in B = N(\Phi).$$

Remark: The nontangential convergence in the sense of the regions $\Gamma_y(K)$ seems to be somewhat more general then the notions of convergence considered before in the context of Siegel domains by A.Korányi and E.M.Stein in 1968-72.

H.-G. LEOPOLD

Pseudodifferentialoperators and function spaces of variable and generalized smoothness

In the classical function spaces of Besov type $B^{s}_{p,q}(\mathbf{R}^n)$ norms can be defined via resolution of unity in the Fourier image $\mathbf{R}^n_\xi$ of the function u(x), which is connected with the symbol $|\xi|^2$ of the Laplacian. To get scales of function spaces where special degenerate elliptic partial differential operators or $\psi DO's$ of variable order of differentiation have similar properties as the elliptic operators in the classical function spaces of Besov-Triebel-Lizorkin-type, we replace the Laplacian by a suitable hypoelliptic ψDO $A(x,D_x)$. Then, the definition of the Besov spaces of variable order $B^{s,a}_{p,q}(\mathbf{R}^n)$ based on decomposition of $\mathbf{R}^n_x \times \mathbf{R}^n_\xi$ which are induced by the symbol $a(x,\xi)$ of this ψDO. This means that we may have different resolutions of $\mathbf{R}^n_\xi$ for different $x \in \mathbf{R}^n_x$. In order to belong to one of these spaces, now a function will have to satisfy smoothness assumptions that can vary on different regions in $\mathbf{R}^n_x$.

W. HANSEN

A converse to the mean value theorem for harmonic functions

It is shown (joint work with N. Nadirashvili) that for every domain $U \subsetneq \mathbf{R}^d$, $d \geq 1$, every continuous function f on U which is bounded by some harmonic function $h \geq 0$ is harmonic provided for every $x \in U$ there exists a ball B_x contained in U and centered at x such that $f(x) = 1/\lambda(B_x) \int_{B_x} f \, d\lambda$ (a result which is new even for continuous bounded functions on the unit ball in $\mathbf{R}^d$, $d \geq 2$). This result holds as well for more general means. In addition, known results on Lebesgue measurable functions having the restricted mean value property are improved.

In contrast to preceding work on the problem the proof is given in a purely analytic way. It uses the Martin compactification, in particular the minimal fine topology, and exploits properties of the Schrödinger equation $\Delta u - \delta d(\,\cdot\,, \partial U)^{-2} 1_A u = 0$ ($\delta \approx 10^{-60}$, A a suitable subset of U).

F. TREVES

Parametrices for Schrödinger equations

The equations under study are of the type

$$L = \frac{1}{i}\frac{\partial}{\partial t} - \Delta_x - a(x,D_x),$$

where $a(x,D_x)$ is a pseudodifferential operator of the following kind

$$a(x,D_x)u(x) = (2\pi)^{-n}\int e^{ix\xi}\, a(x,\xi)\, u(\xi)\, d\xi \qquad (u \in \mathcal{S})$$

and there is $C_{\alpha,\beta} > 0$ for each $\alpha,\beta \in \mathbb{Z}_+^n$,

$$|\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| \le C_{\alpha,\beta}\langle x\rangle^{p-|\alpha|}\langle\xi\rangle^{q-|\beta|} \quad (\langle x\rangle = \sqrt{1+|x|^2}).$$

This is expressed below by saying $a \in S^{p,q}(\mathbb{R}^{2n})$.

The approach is to write

$$e^{-it\Delta}L\, e^{it\Delta} = \frac{1}{i}\frac{\partial}{\partial t} - e^{-it\Delta_x}a(x,D_x)\, e^{it\Delta_x}.$$

Theorem 1: $e^{-it\Delta_x}a(x,D_x)e^{it\Delta_x} = A(t,x+2itD_x,D_x)$, where

$$A(t,x,\xi) = C_n \int_{\mathbb{R}^n} e^{-i|z|^2} a(x+z\sqrt{t},\xi)\, dz \qquad (C_n = (2\sqrt{\pi})^{-n} e^{i\frac{n}{4}\pi}).$$

We have, modulo $S^{-\infty,q}(\mathbb{R}^{2n})$,

$$A(t,x,\xi) \approx \sum \frac{1}{\alpha!}\left(\frac{t}{4i}\right)^{|\alpha|}(\partial_x^{2\alpha}a)(x,\xi) \quad (\approx e^{\frac{1}{4i}t\Delta_x}a(x,\xi)).$$

(In Theorem 1, $a(x,\xi) \in S^{p,q}(\mathbb{R}^{2n})$.) In the sequel, one assumes p = q = 0. Then one seeks an operator K s.t.

$$(*) \quad \frac{1}{i}\frac{\partial K}{\partial t} - A(t,x+2itD_x,D_x)K \approx 0, \quad K|_{t=0} = I.$$

The parametrix of L will then be the compose $e^{it\Delta}K$.

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To solve (*) one write

$$K(t)u(x) = (2\pi)^{-n} \int e^{ix\cdot\xi} k(t,x,\xi)\hat{u}(\xi)\,d\xi,$$

and $k(t,x,\xi)$ is determined by transport equations. Actually one write

$$k(t,x,\xi) = e^{i\int_0^t a(x+2s\xi,\xi)\,ds} g(t,x,\xi).$$

The transport equations for g are some simpler than those for k. One gets the following estimates:

$$|\partial_x^\alpha \partial_\xi^\beta g_r(t,x,\xi)| \le C_{r\alpha\beta}(T)\, t\langle x\rangle^{-r-\langle\alpha\rangle}\langle\xi\rangle^{-r-\langle\beta\rangle} M_\pm(x,\xi)^{-2r-|\alpha+\beta|},$$

with

$$M_\pm(x,\xi) = \left(1 \pm \frac{x\cdot\xi}{\langle x\rangle|\xi|}\right)^{\frac{1}{2}},$$

where the signum ± is that of t.

The error term in the first equation (*) is an operator $\mathcal{S}' \to \mathcal{O}_M$ (space of C^∞ functions that grow at most polynomially at infinity), and $\mathcal{O}'_M \to \mathcal{S}$. ($\mathcal{O}'_M$ is a space of distributions that decay rapidly at infinity).

J.P. ANKER

Multipliers on noncompact manifolds

This is a report on a joint work with Andreas Seeger. We are interested in obtaining minimal smoothness conditions of Hörmander type ensuring L_p boundedness for functions of the Laplacian Δ on Riemannian symmetric spaces G/K of noncompact type and more generally on complete Riemannian manifolds M with C^∞ bounded geometry. Recall that an analogue of the Hörmander-Mikhlin theorem was obtained in this context by Michael Taylor in 1989, achieving a long series of efforts by several authors, and that we extended and sharpened this theorem in the particular case M = G/K in 1990. In order to state our latest result, let us introduce some notations. The infimum of the L_2 spectrum of $(-\Delta)$ is denoted by σ^2. The volume growth is uniformly controlled by $|B(x,r)| \le C(1+r)^{\delta} e^{2\kappa r}$. Notice that $\sigma = \kappa = |\rho|$ and $\delta = \frac{rank-1}{2}$ in the case M = G/K.

<u>Theorem:</u> Assume that $\delta < \frac{n-1}{2}$, where n is the dimension of M. Let $1 < p < 2\frac{n+1}{n+3}$ and let $m(\lambda)$ be an even holomorphic function in the strip $|Im\lambda| \le \kappa_p := 2\kappa(\frac{1}{p} - \frac{1}{2})$ such that $C_p := \|\varphi_0\, m\,(.\pm i\kappa_p)\|_{B_{21}^{\alpha_p}} + \sup_{t\ge 1} \|\varphi\, m(t.\pm i\kappa_p)\|_{B_{21}^{\alpha_p}} < \infty$, where $\varphi_0(\lambda)$, $\varphi(\lambda)$ are bump functions around $\lambda = 0$, resp. $|\lambda| = 1$ and $B_{21}^{\alpha_p}$ denotes the usual Besov space of smoothness index $\alpha_p = n(\frac{1}{p} - \frac{1}{2})$. Then $m(\sqrt{-\Delta-\sigma^2})$ is a bounded operator on $L_p(M)$ with norm $\ll C_p$.

Analogous results were proviously obtained by Andreas Seeger in the Euclidean case and on compact manifolds. Let us make some comments about our theorem. A slightly weaker result is available in the case $\delta \ge \frac{n-1}{2}$. The holomorphy condition is known to be necessary for M = G/K. The condition $p < 2\cdot\frac{n+1}{n+3}$ comes from the following version of the Sogge-Stein-Tomas restriction theorem.

<u>Lemma:</u> Let $1 \le p \le 2\cdot\frac{n+1}{n+3}$, $A_\xi = \sqrt{-\Delta-\sigma^2+\xi^2}$ with $\xi > \kappa_p$ and let $\Psi \in C_c^\infty(\mathbb{R})$ be an even function with small support. Then

$$\|\hat{\Psi}(A_\xi - t) + \hat{\Psi}(A_\xi + t)\|_{L_p - L_2} \le C(1+t)^{\alpha_p - \frac{1}{2}}.$$

Some more comments. When $2\cdot\frac{n+1}{n+3} \le p < 2$, our theorem holds with $B_{21}^{\alpha_p}$ replaced by $B_{q1}^{\alpha_p}$ $(\frac{1}{q} < \frac{n+1}{2}(\frac{1}{p} - \frac{1}{2}))$. It also holds more generally for the Laplacian on forms, the square of Dirac-type operators ... and small perturbations of them. As far as applications are concerned, it yields for instance good $L_p \to L_p$ bounds for A_ξ^{it} and good $L_p \to L_p$ boundedness results for the strongly singular operators $A_\xi^{-b} e^{iA_\xi^a}$ $(0 < a < 1)$.

J. BOURGAIN

Applications of multiple Fourier series to nonlinear PDE

C. SADOSKY

Interpolation in the polydisk

It is well-known that the classical interpolation problems of Nevanlinna-Pick, Carathéodory-Fejér, and other, can be solved through the Nehari theorem in the circle.
Abstract lifting theorems for bounded invariant form acting in scattering systems with several evolution groups provide several-dimensional versions of the Nehari theorem [Cotlar-Sadosky, 1990,1991].
Using a formulation of that theorem in $\mathbf{T}^d$ expressed in terms of functions of "restricted bounded mean oscillation", the problem of bounded analytic extensions to the polydisk of functions defined in a zero variety, with control of the norm, is given a solution.

P. SJÖLIN

Radial functions and maximal estimates for solutions to the Schrödinger equation

For $f \in \mathcal{S}(\mathbf{R}^n)$ set

$$S_t f(x) = u(x,t) = (2\pi)^{-n} \int_{\mathbf{R}^n} e^{ix\cdot\xi} e^{it|\xi|^a} \hat{f}(\xi)\, d\xi, \quad x \in \mathbf{R}^n, \quad t \in \mathbf{R},$$

where a > 1. We then have u(x,o) = f(x) and in the case a = 2 u is a solution to the Schrödinger equation $\Delta u = i\partial u/\partial t$. We set $S^* f(x) = \sup_{0<t<1} |S_t f(x)|, \quad x \in \mathbf{R}^n,$

and let H_s denote Sobolev spaces. We shall study estimates of the type

$$\left(\int_{B(O;R)} |S^*f(x)|^q dx\right)^{1/q} \le C_R \|f\|_{H_s},$$

where $B(O;R) = \{ x \in \mathbb{R}^n ;\ |x| \le R \}$ and f is a radial function in $\mathbb{R}^n$, $n \ge 2$.

L. VEGA

Local regularity properties of Schrödinger equations

We present the following theorem.

Theorem: (with A. Ruiz) Let $e^{it(\Delta+V)}u_0$ be the solution of the Schrödinger equation

$$\begin{cases} i\partial_t u + \Delta_x u + v(x)u = 0 & (x,t) \in \mathbb{R}^{n+1} \\ \qquad u(x,0) = u_0(x) . \end{cases}$$

Then

$$\left(\sup_{x_0,R} \frac{1}{R} \int_{B(x_0,R)} \int_{-T}^{T} |D^{1/2} e^{it(\Delta+V)} u_0|^2 dt dx\right)^{1/2} \le C(T) \|u_0\|_{L^2(\mathbb{R}^n)}$$

where $D^{1/2} = (-\Delta)^{1/4}$ and $V = V_1 + V_2$ with $\|V_1\|_{L^{n/2}(\mathbb{R}^n)} < \varepsilon(n)$ and $\|V_2\|_{L^\infty(\mathbb{R}^n)} < +\infty$.

$\varepsilon(n)$ is a constant which depends on the dimension.

J. PIPHER

Boundary value problems for 2nd order elliptic operators

We consider the solvability, with data in L^p, of the Dirichlet, Neumann and regularity problems for operators $L = div\, A\nabla$ where A is a symmetric, elliptic matrix with bounded measurable coefficients. In particular, what additional smoothness must be imposed on the coefficients to guarantee solvability of these problems for some p. Results of R. Fefferman - C. Kenig -

J. Pipher and of N. Lim on the Dirichlet problem and results of C. Kenig - J. Pipher on the Neumann and regularity problems were presented.

C. KENIG

The Poisson Equation on Lipschitz and C^1 domains

In joint work with D. Jerisson we study the Poisson equation $\Delta u = f$, $u|_{\partial\Omega} = 0$ on bounded Lipschitz and C^1 domains in $\mathbb{R}^n$. We assume that the data f belongs to Sobolev spaces $W_\alpha^P(\Omega)$ or Besov spaces $B_\alpha^P(\Omega)$. We find optimal conditions on p and α such that for any such domain Ω and any such f, the solution u belongs to $W_{2+\alpha}^P(\Omega)$ or $B_{2+\alpha}^P(\Omega)$.

N. GAROFALO

Unique Continuation for Sub-elliptic Operators

This is recent joint work with Zhongwei Shen. We consider in $\mathbb{R}^3$ the model Grushin operator

$$\mathcal{L} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (x^2+y^2)\frac{\partial^2}{\partial t^2}.$$

$\mathcal{L}$ constitutes a basic example of sub-elliptic operator and is closely related to the sub-Laplacian on the nilpotent Heisenberg group of real dimension 3. The homogeneous dimension of $\mathcal{L}$ at the origin is Q = 4. The action of the nonisotropic delations leads to conjecture that the differential inequality $|\mathcal{L}u| \le |Vu|$ has the strong unique continuation property at points of the manifold $\{(0,0)\} \times \mathbb{R}$, provided that $V \in L^P_{loc}(\mathbb{R}^3)$ with $p \ge \frac{Q}{2}$. Our approach relies on the establishment of an $\left(L^{\frac{4}{3}}, L^4\right)$ Carleman estimate which generalizes to the present context a result of D. Jerison and C. Kenig for the Euclidean Laplacian. The main tool is a discrete version of restriction theorem for the projection operators associated with the spherical harmonics of the operator $\mathcal{L}$.

W. TREBELS

On necessary conditions for Laguerre multipliers

This is a joint work with G. Gasper.
We study multipliers for expansions into Laguerre polynomials on $L^p_{w(\alpha)}$-spaces

$$T_m f(x) \sim \frac{1}{\Gamma(\alpha+1)} \sum_{k=0}^{\infty} m_k \hat{f}_\alpha(k) L_k^\alpha(x), \quad \alpha > -1$$

where

$$\|f\|_{L^p_{w(\alpha)}} = \left(\int_0^\infty |f(x) e^{-x/2}|^p x^\alpha dx\right)^{1/p}, \quad 1 \le p < \infty,$$

and

$$\hat{f}_\alpha(k) = \int_0^\infty f(x) L_k^\alpha(x)/L_k^\alpha(0) e^{-x} x^\alpha dx.$$

Theorem: Let $m \in M^p_{w(\alpha)}$, $1 \le p < 2$, $\frac{1}{p}+\frac{1}{q}=1$, and let $\alpha > -1$ be such that $\max\left\{\frac{1}{3p}, \frac{1}{4}\right\} < (\alpha+1)\left(\frac{1}{p}-\frac{1}{2}\right)$. Then

$$\|m\|_\infty + \sup_N \left(\sum_{k=N/2}^{N} |(k+1)^\lambda \Delta_2 \Delta^{\lambda-1} m_k|^q \frac{1}{k+1}\right)^{1/q} \le C\|m\|_{M^p_{w(\alpha)}},$$

where $\lambda := (2\alpha+1)\left(\frac{1}{p}-\frac{1}{2}\right)$, $\Delta_2 m_k = m_k - m_{k+2}$, $\Delta^\delta m_k = \sum_{j=0}^{\infty} A_j^{-\delta-1} m_{k+j}$.

From this follows immediately for finite sequences $m = \{m_k\}_{k=0}^{n}$ the Cohen-type inequality

$$(n+1)^{(2\alpha+2)\left(\frac{1}{p}-\frac{1}{2}\right)-\frac{1}{2}} \; |m_n| \leq C \, \|m\|_{M^p_{v(\alpha)}}, \quad 1 \leq p < \frac{4\alpha+4}{2\alpha+3}$$

essentially due to C. Markett 1983. The necessary conditions are compared with sufficient ones.

D.R. ADAMS

NxN second order elliptic systems under constraints

Let Ω be a bounded domain in $\mathbf{R}^n$, $n \geq 1$, $L = A\Delta - B$ where A and B NxN real constant matrices, Δ = Laplacian on $\mathbf{R}^n$. Set

$$\mathbf{K} = \{ v = (v^1, \dots, v^N) \in H_0^1(\Omega)^N : v^1 \geq \psi \text{ on } \Omega \}$$

where $\psi \in C^2(\Omega) \cap C^0(\bar{\Omega})$ with $\psi < 0$ on $\partial\Omega$; $f = f^1, \dots, f^N) \in L^\infty(\Omega)^N$. We consider the variational inequality

(VI) Find $u \in \mathbf{K}$ s.t. $\langle Lu, v-u \rangle_{H_0^1, H^{-1}} \geq \langle f, v-u \rangle$ for all $v \in \mathbf{K}$.

Question: What are the conditions on A and B that give existence, uniqueness, and regularity?

The ellipticity assumption is: $\det(A) \neq 0$.

We consider two cases: $(A^{-1})_{11} < 0$ and $(A^{-1})_{11} = 0$, where the last case can be split into N-1 subcases according to the value of $k \in \{1, \dots, N-1\}$, where $k =_{def} \deg(\tilde{p}) + 1$, and $\tilde{p}(M;\lambda)$ is the monic polynomial obtained from $\det[(M-\lambda I)^{\#}]$, $M = SA^{-1}BS^{-1}$, for some especially choosen invertible S. Here the symbol # denotes the (N-1)x(N-1) "upper right hand corner" of the matrix. From this we get canonical forms for the reduction of M via the group

$\mathfrak{C}_N$ = invertible NxN matrices C s.t.

$C_{11} = 1$; $C_{1j} = 0$, $2 \leq j \leq N$; $C_{iN} = 0$, $1 \leq i \leq N-1$. From this we get solution classes and eventually their associated a priori estimate. That eventually lead to a partial answer to our questions.

T. RUNST

Mapping properties of nonlinear operators in function spaces

This is a survey on some recent results of a joint work with W. Sickel, Jena.

We study the boundedness of the nonlinear superposition operator: $\mathcal{G}: F^{s_0}_{p,q}(\mathbb{R}_n) \to F^{s_1}_{p,q}(\mathbb{R}_n)$, $s_0 \geq s_1$, given by $f \to G(f)$, where $G(t)=t^m$, $m=2,3,\ldots$ and G(t) is a sufficiently smooth real-valued function with G(0)=0, respectively, and f belongs to the scale of spaces of Triebel-Lizorkin type $F^s_{p,q}(\mathbb{R}_n)$. Results concerning with the loss of smoothness s_1-s_0 in dependence on the parameter n, s_0 and p are also given.

Theorem: Let

$0 < p < \infty$, $0 < q \leq \infty$, *and* $G \in C^{\mu}(\mathbb{R}_n)$, $\mu > \max(1,s)$ *with* $G(0) = 0$.

(i) Let $s > \sigma_p := n \cdot \max(0, \frac{1}{p} - 1)$. Then there exists a constant c_G

such that

$$\|G(f)\,|F^s_{p,q}\| \leq c_G(\|f|F^s_{p,q}\| + \|f|F^s_{p,q}\|\,\|f|L_\infty\|^{\max(0,s-1)})$$

holds for all $f \in F^s_{p,q}(\mathbb{R}_n) \cap L_\infty(\mathbb{R}_n)$.

(ii) Let $\sigma_p + 1 < s < \frac{n}{p}$ and $\varrho = \dfrac{\frac{n}{p}}{\frac{n}{p} - s + 1}$. Then there exists a constant c_G

such that

$$\|G(f)\,|F^{\varrho}_{p,q}\| \leq c_G(\|f|F^s_{p,q}\| + \|f|F^s_{p,q}\|^{\varrho})$$

holds for all $f \in F^s_{p,q}(\mathbb{R}_n)$.

(iii) Let $n \cdot \max(0, \frac{1}{p}-1; \frac{1}{q}-1) < s < 1$. Then there exists a constant c_G such that

$$\|G(f)|F^s_{p,q}\| \leq c_G \|f|F^s_{p,q}\|$$

holds for all $f \in F^s_{p,q}(\mathbb{R}_n)$.

V.D. STEPANOV

A recent progress of the Hardy-type weighted norm inequalities and related topics

Let $p > 0$ and $\|f\|_p$ denote the usual (quasi)norm of a function in Lebesgue space $L^p(\mathbb{R}^+)$. The weighted space $L^p_v(\mathbb{R}^+)$ is generated by the weighted (quasi)norm $\|f\|_{p,v} = \|fv\|_p$. We consider the weighted inequality

$$\|(Kf)v\|_q \leq C\|fv\|_p, \text{ for all } f, \qquad (1)$$

with the Volterra integral operator K given by

$$Kf(x) = \int_0^x k(x,y)f(y)\,dy \qquad (2)$$

where the kernel k satisfies the following condition

(i) $k(x,y) \geq 0$ if $x > y > 0$ and is non-decreasing in x or non-increasing in y,

(ii) $D^{-1}(k(x,y)+k(y,z)) \leq k(x,z) \leq D(k(x,y)+k(y,z))$ if $x > y > z > 0$.

We prove necessary and sufficient conditions for the inequality (1) to hold and for the operator (2) to be compact.
Similar results are valid in general (non-Volterra) case. Also the dual and weak-type criteria are given.
In case $k(x,y) = (1/\Gamma(n))(x-y)^{n-1}$, $n > 1$ we find out applications to the spectral theory of the ordinary differential operators such that the estimate of the first eigenvalue from above and below,

criterion of discreteness of the spectrum, trace formula and etc. The inequality (1) restricted to the cone of all non-negative monotone functions is a second part of the work. We obtain the similar criteria for this case and then consider the mapping properties of a number of classical operators of harmonic analysis such as maximal function, Hilbert transform, Riesz potential, Fourier operator, etc. in the Lorentz spaces given by

$$\Lambda_p(v) = \{f : \|f\|^{*}_{p,v} = \left(\int_0^\infty f^*(t)^p \, v(t)\, dt\right)^{1/p} < \infty\},$$

$$\Gamma_p(v) = \{f : \|f\|_{p,v} = \left(\int_0^\infty f^{**}(t)^p \, v(t)\, dt\right)^{1/p} < \infty\}.$$

In particular, an analog of recent result of Ariso and Muckenhoupt about the acting of the maximal function in $\Lambda_p(v)$ for the normed Lorentz space $\Gamma_p(v)$ is given.

C.D. SOGGE

Null forms and local smoothing of Fourier integral operators

We consider the Cauchy problem for the wave operator in $\mathbb{R}^{n+1}$, $n \geq 2$:

$$\Box u = 0$$

$$u|_{t=0} = f, \quad \partial_t u|_{t=0} = 0$$

Theorem (I.Peral 1980): If $\alpha_p = (n-1)\left|\frac{1}{p} - \frac{1}{2}\right|$ and $1 < p < \infty$.

Then $\|u(\cdot,t)\|_{L_p(\mathbb{R}^n)} \leq C_{p,t} \|f\|_{L^p_{\alpha_p}(\mathbb{R}^n)}$. This result is sharp.

We present a proof based on joint work with A. Seeger and E.M. Stein which works for variable coefficients.

Finally, we prove local smoothing estimates and show how they imply Bourgain's circular maximal theorem. In particular, we have the following joint result with G.Mockenhaupt and A. Seeger.

Theorem: If $2 < p < \infty$, $\exists\, \varepsilon_p > 0$ such that $u \in L^p_{loc}(\mathbb{R}^{n+1})$ if $f \in L^p_{\alpha_p - \varepsilon_p}(\mathbb{R}^n)$.

No such result holds if $p \leq 2$.

The proof we present relies on a bilinear estimate of S.Klainerman and M.Machedon involving null forms for the wave operator.

H. SMITH

L^p - regularity for diffractive wave equations

Let Ω be a smoothly bounded, strictly convex, compact subset of $\mathbb{R}^n$. Let Δ denote the self adjoint Laplacian acting on L^2 function on $\mathbb{R}^n\backslash\Omega$, the complement of Ω, which satisfy Dirichlet conditions on $\partial\Omega$. In recent joint work with C.Sogge, we show that the diffracted wave operator $e^{it\sqrt{-\Delta}}$ has the same mapping properties on the L^p Sobolev spaces, $1< p < \infty$, as for the unobstructed wave operator:

Theorem: $(1-\Delta)^{-\alpha_p/2}\, e^{it\sqrt{-\Delta}}$ is a bounded operator on $L^p(\mathbb{R}^n\backslash\Omega)$, with operator norm $C_p(t)$ uniformly bounded for t in any compact interval.

This establishes the sharp fixed time regularity of solutions to the mixed wave equation:

$$\begin{aligned}
\partial_t^2 u(x,t) &= \Delta_x u(x,t) \qquad x\in\mathbb{R}^n\backslash\Omega \qquad t\in\mathbb{R}\\
u(x,0) &= f(x)\\
\partial_t u(x,0) &= g(x)\\
u(x,t) &= 0 \qquad\qquad if \;\; x\in\partial\Omega
\end{aligned}$$

The proof uses a nonisotropic decomposition of phase space to decompose the Melrose-Taylor oscillatory integral representation of the Schwartz kernel of $e^{it\sqrt{-\Delta}}$. Although not discussed in this talk, these techniques also are used to establish:

Theorem: Let A be a Fourier integral operator of order m associated to a folding canonical relation. Then $A:L^p_{comp}(\mathbb{R}^n)\to L^p_{loc}(\mathbb{R}^n)$ if:

(1) $m \le -(n-1)\left|\frac{1}{p}-\frac{1}{2}\right|$ and $p \notin [\frac{3}{2},3]$

(2) $m < -\frac{1}{6}-(n-2)\left|\frac{1}{p}-\frac{1}{2}\right|$ and $p \in (\frac{3}{2},3)$

For n = 2, the result (2) is also valid for $m = -\frac{1}{6}$.

D. GELLER

Transversally elliptic operators on strictly pseudoconvex CR manifolds

Let M be a real analytic, strictly pseudoconvex CR manifold. The Kohn Laplacian $\Box_b$ and its natural generalizations, the transversally elliptic operators, play an important role in analysis on M, just as the Laplacian and elliptic operators do on Riemannian manifolds. However, the theory of transversally elliptic operators is still incomplete. Consider, for instance, the "$\Box_b$-Schrödinger operator", $\Box_b + V$, where V is a (real) analytic function. If V is nowhere vanishing, then this operator has an analytic parametrix, if V is identically zero, M compact, it has an analytic parametrix; only relative to a projection onto a large nullspace. For general V, the situation is still unclear. We shall discuss this example and several other recent results.

R. FELIX

Randon transforms associated with certain group actions

Das Radonsche Inversionsproblem besteht darin, eine gegebene Funktion f auf einem Raum $\mathcal{P}$ von "Punkten" zu rekonstruieren aus den Integralen von f längs der "Ebenen" in $\mathcal{P}$.
Wir gehen hier von einer Abelschen Gruppe $\mathcal{P} = U \times E$ aus, auf der eine Gruppe G durch Automorphismen wirkt. (Die Standard-Situation ist natürlich der Fall $\mathcal{P} = \mathbb{R}^n$.) Da das "Ebenensystem"
$\mathcal{E} := \{E(g,u) \mid E(g,u) := g\cdot(u+E),\ g \in G,\ u \in U\}$
im allgemeinen zu groß und damit das Radonsche Problem überbestimmt ist, beschränken wir uns auf ein "reduziertes System"
$\mathcal{E} := \{E(m,u) \mid m \in M, u \in U\}$, wobei M eine geeignet zu wählende Teilmenge von G ist. Die Radon-Transformierte $\mathfrak{R}f$ einer Funktion f auf $\mathcal{P}$ kann dann als Funktion auf M x U angesehen werden. Sei m* die zu $m \in M$ kontragrediente Wirkung auf der dualen Gruppe $\hat{\mathcal{P}}$; sei dm ein Maß auf M und bezeichne $d\eta$ bzw. $d\zeta$ das Haar-Maß auf $\hat{U}$ bzw. $\hat{\mathcal{P}}$. Um zu einer Inversionsformel zu gelangen, treffen wir nun die folgende entscheidende

<u>Voraussetzung:</u> Das Bildmaß des Maßes $dmd\eta$ auf M x $\hat{U}$ unter der Abbildung $F: M \times \hat{U} \to \mathfrak{P}$, $(m,\eta) \to m^{\bullet}\cdot\eta$, existiere und sei äquivalent zu $d\mathcal{E}$. (Diese Voraussetzung stellt sicher, daß $\mathcal{E}$ zwar reichhaltig genug, aber nicht zu groß ist.)
Ist jetzt $w(m,\eta)$ die zugehörige Dichtefunktion und L der durch w bestimmte Fourier-Multiplikator (bzgl. der Fourier-Transformation auf U) so lautet die Inversionsformel

$$f = \mathfrak{R}^{*} L \mathfrak{R} f,$$

wobei $\mathfrak{R}^{*}$ die "duale Radon-Transformation" ist. Ferner gilt die "Plancherel-Formel", welche besagt, daß $L^{\frac{1}{2}}\mathfrak{R}: L^2(\mathfrak{P}) \to L^2(M \times U)$ eine Isometrie ist.
Das Resultat ist anwendbar sowohl auf den klassischen Fall als auch auf die natürliche Wirkung einer jeden zusammenhängenden Gruppe unipotenter Matrizen, so daß die gegebene Konstruktion insbesondere zu einer Radon-Transformation auf beliebigen nilpotenten Lie-Gruppen führt. Hier erweist sich L als ein Differentialoperator.

S. MEDA

<u>On the best semigroup on non-compact symmetric spaces</u>

Let $X=G/K$ be a symmetric space of the noncompact type. Let $\mathcal{L}$ be (minus) the Laplace-Beltrami operator associated to a canonical G-invariant metric on X. Define for all $0 \le \theta \le 1$ the operator $\mathcal{L}_\theta = \mathcal{L} - \theta b$.
In a joint work with Michael Cowhing and Sarains Giulini we proved sharp estimates for the L^p-L^q operator norm of the heat semigroup $(\exp(-t\mathcal{L}_0))_{t>0}$ and we found exactly when the resolvent operator $\mathcal{L}_\theta^{-\alpha/2}$ (here α is a complex number with nonnegative real part) is bounded from L^p to L^q. This is done by using some new recent criteria which give necessary or sufficient condition for a function f to be in $L^p(G)$.

P. LEVY-BRUHL

Coherent states, spectral theory and nilpotent Lie groups

This is a joint work with J.Nourrigat. We prove the following abstract theorem about coherent states:

Theorem 1: Let H an Hilbert space, Z a space with positive measure dz, $z \to \psi_z$ measurable function from Z to H s.t.:

1) $\|\Psi_z\| = K$, 2) $\forall f \in H,\ f = \int (f, \Psi_z)\Psi_z\, dz$ (weak sense).

Let P be a self-adjoint positive operator in H with domain D(P) s.t. $\Psi_z \in D(P)$ for all $z \in Z$. Assume P to have compact resolvent, and let $N(\lambda)$ be the counting function of eigenvalues of P. Then:

i) Assume there exists a positive function s on Z s.t.:

(*) $\forall r > 0$ mes $\{z \mid s(z) \le r\} < \infty$ and:

(1) $\forall f \in D(P)$ (on a curve of P): $\int s(z)\,|(f,\Psi_z)|^2 dz \le (Pf,f) + \lambda\|f\|^2$.

Then $N(\lambda) \le 2K^2$ mes $\{z \mid s(z) \le 4\lambda\}$.

ii) Assume there exists a positive function S on Z, satisfiying (*), and:

(2) $\forall f \in D(P)$ (on a curve of P): $(P(f), f) \le \int S(z)\,|(f,\Psi_z)|^2 dz$

(3) $\exists\, \alpha \ge 0\ \exists\, c > 0$: $\int \left[\dfrac{\sup[S(z),S(w)]}{\inf[S(z),S(w)]}\right]^{\alpha+\frac{1}{2}} |(\Psi_z,\Psi_w)|\,dw \le C\left(\dfrac{S(z)}{\lambda}\right)^{\alpha}$

then for every r > 0, there exists R > 0 and

$N(R\lambda) \ge \dfrac{K^2}{2}$ mes $\{z \mid S(z) \le r\lambda\}$

In a second part, we construct coherent states associated to an induced representation Π of a nilpotent Lie algebra $\mathfrak{g}$.
When the algebra is stratified, theorem 1 and the coherent states associated to $\Pi \circ \delta_{\lambda^{-1}}$ (δ_λ is the usual dilations] can be used to give an estimate of the image $\Pi(D)$ of the sublaplacian of $\mathfrak{g}$ under Π. (This result was obtained before by the authors by a much more complicated technic).
The coherent states constructed here also give an "approximate diagonalization" of $\Pi(P)$, from which we can improve results of J.Nourrigat about Gårding inequality and limit sets of representations.

G.A. KALYABIN

On the exact values and bilateral estimates of certain capacities

The quantities of combinatorical nature introduced by Yu.V.Netrusov (1989) are considered which are equivalent to the capacities of sets with respect to the generalized Besov spaces $B_{p,p}^{(\alpha,N)}(\mathbb{R}^n)$.
The algorithm is constructed which allows to calculate the exact values of these modified capacities for the arbitrary finite union of T "elementary bricks" using only O(T) arithmetic operations.
As a corollary the effective bilateral estiamtes are obtained for capacities of regular lattices.
The results are new even for the ordinary classes $B_p^r(\mathbb{R}^n)$.

L.P. ROTHSCHILD

Analytic discs in real surfaces in $\mathbb{C}^N$

This is a report of joint work with S.Baovendi. A real hypersurface M in $\mathbb{C}^N$ is minimal at p_0 if there is no germ of a complex analytic hypersurface $\tilde{M}$ with $p_0 \in \tilde{M} \subset M$.
Theorem 1: If M is minimal at p_0 and H: M → M is a C^∞ mapping, $H(p_0) = p_0$, $\det dH \neq 0$ and H extending holomorphically above M near p_0, then H is not flat at p_0. That is, some derivative of H is nonzero at p_0.
Theorem 2: If $M \subset \mathbb{C}^2$ is a hypersurface of finite type at p_0 (in the sense of Kohn) and H: M → M is a mapping as in Theorem 1, then H is a local diffeomorphism near p_0.
The proofs of Theorems 1 and 2 use the method of analytic discs.

H.M. REIMANN

Quasiconformal deformations on the Heisenberg group

This is a joint work with A. Koranyi, CUNY.
Quasiconformal mappings $f: H^n \to H^n$ on the Heisenberg group are homeomorphisms with certain regularity properties which preserve the contact structure and distort the conformal structure by a bounded amount at most. They satify a system of differential equations of Beltrami type:

$$Z_j f_k = \sum_{l=1}^{n} \mu_{lj} Z_l f_k \qquad k = 1,\dots,n+1.$$

with μ symmetric and $\frac{1+\|\mu\|}{1-\|\mu\|}$ uniformly bounded.

__Theorem:__ A vectorfield of the form

$$v = \frac{i}{2}\sum_{j=1}^{n} [(\overline{Z}_j p) Z_j - (Z_j p)\overline{Z}_j] + pT \quad \text{with } p \in C_c^1(H^n, \mathbb{R}) \text{ generates a flow}$$

of quasiconformal mappings provided that $\sum_{l,k} \|Z_j Z_k p\|_\infty$ is bounded.

The problem of extending quasiconformal mappings on $H^n = \partial D$ onto $D = \{ z \in \mathbb{C}^{n+1}: Im\, z_{n+1} - \sum_{j=1}^{n} |z_j|^2 > 0 \}$ is discussed.

D. MÜLLER

Spectral multipliers for Heisenberg groups

The following Hörmander-Mikhlin type theorem has been obtained in collaboration with E.M.Stein:
Suppose that $G = G_1 \times \dots \times G_k$ is a product of Heisenberg respectively Euclidean groups, and let $\mathcal{L} = \mathcal{L}_1 + \dots + \mathcal{L}_k$ be a sub-Laplacian on G, where each $\mathcal{L}_j$ is a sub-Laplacian on G_j. Let $\chi \neq 0$ be a bump function supported in [1/2, 1], and assume that $m \in L^\infty(\mathbb{R}^+)$ satisfies

$$\sup_{t>0} \| \chi\, m(t\cdot) \|_{L^2_\alpha} < \infty$$

for some $\alpha > \frac{1}{2} \dim G$. Then the operator $m(\mathcal{L})$ is bounded on $L^p(G)$, for $1 < p < \infty$ and of weak type (1,1). This result is essentially sharp with respect to the critical index $\frac{1}{2} dim\, G$.

The best previously known results held only under the condition $\alpha > Q/2$, where Q is the homogeneous dimension of G.

N. JACOB

Pseudo differential equations in Dirichlet spaces

Let $p:\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}$ be continuous function such that $p(x,.):\mathbb{R}^n \to \mathbb{R}$ is negative definite (in the sense of Beurling-Deny). Further suppose that for large $|\xi|$ we have with a fixed continuous negative definite function $a^2:\mathbb{R}^n \to \mathbb{R}$ the estimate $c_1 a^2(\xi) \le p(x,\xi) \le c_2 a^2(\xi)$. In addition suppose that $p(\cdot,\xi)$ fulfills certain smoothness conditions and that $osc_x\, p(x,\xi)$ is controlled. First it is shown that the pseudo differential operator -p(x,D) defined on $C_0^\infty(\mathbb{R}^n)$ extends to a generator of a Feller semigroup. If further -p(x,D) is symmetric it generates a Dirichlet form B with domain $H^{a^2,1/2}(\mathbb{R}^n)$, $H^{a^2,s}(\mathbb{R}^n) = \{ u \in L^2(\mathbb{R}^n), \int (1+a^2(\xi))^{2s}|\hat{u}(\xi)|^2 d\xi < \infty \}$. If this Dirichlet space is transient, for any bounded open set $\Omega \subset \mathbb{R}^n$ one has the decomposition $D_e(B) = D_\Omega^O(B) \oplus H_B(\Omega)$, where $D_\Omega^O(B) = \overline{C_0(\Omega) \cap H^{a^2,1/2}(\mathbb{R}^n)}^B$ and $H_B(\Omega) = \{ u \in D_e(B);\ B(u,v) = 0,\ \forall\ v \in D_\Omega^O(B)\}$ where $D_e(B)$ is the extended Dirichlet space associated with B. It is shown that locally elements of $H_B(\Omega)$ belong to $H^{a^2,t}(\mathbb{R}^n)$ for any $t \in \mathbb{R}$. This is locally for our case the anologous result to Weyl's decomposition of the classical Sobolev space $H^1(\Omega)$.

A. CARBERY

Hilbert Transforms along flat curves in $\mathbb{R}^n$

For a curve $\Gamma:\mathbb{R} \to \mathbb{R}^n$, $\Gamma \in C^1$, $\Gamma(0) = 0$, let

$H_\Gamma f(x) = \int_{-\infty}^{\infty} f(x-\Gamma(t))\frac{dt}{t}$. Write $\Gamma = (\gamma_1,\dots,\gamma_n)$ we considered the L^p boundedness if this operator in $\mathbb{R}^n$, with particular attention given to

- local geometry of Γ
- global geometry - Calderón Zygmund theory
- regularity of Γ
- extensions to noneuclidian situation

We focussed on flat curves, gave a brief summary of the two-dimensional situation and then presented three theorems in $\mathbb{R}^n$, one for each of the $T_-(n,\mathbb{R})$, $GL(n,\mathbb{R})$ and $diag(n,\mathbb{R})$ cases.
The third one was as follows:
Theorem: (C. and S.Ziesler) If Γ is "balanced" and for t > 0 (similarly for t < 0) satisfies γ_1', γ_{j+1}'/γ_j' increasing $\gamma_1' > 0$, $(\gamma_{j+1}'/\gamma_j')(0) = 0$, and if moreover $(\gamma_{j+1}'/\gamma_j')(ct) \geq 2\cdot(\gamma_{j+1}'/\gamma_j')(t)$ (all j,t, some C) then H_Γ: $L^p \to L^p$ (provided also a technical condition holds).
A sketch of the proofs was given, and in particular a sketch of the proof that if $h'(t) \geq h(t)/t$ for a convex curve $(t,\gamma(t)) \subseteq \mathbb{R}^2$ $(h(t) = t\gamma' - \gamma)$ then H_Γ bounded in L^p was given. This did not use the Fourier-Transform.

W. HEBISCH

Scattering for generalized BBM equation

This is a joint work with P.Biler and J.Dziubanski.
We study solutions with small initial data to multidimensional generalized Benjamin-Bona-Mahony equation:
$u_t - \Delta u_t = (a, \nabla u) + (b, \nabla u) u^p$ in $\mathbb{R}^n$.
We give sufficient conditions on n,s,p to have scattering.
We also study traveling wave solutions to BBM equation (in the case of parallel a and b).
The study of nonlinear problem is based on linearized equation. Our estimates of L^∞ norm in linear problem are sharp, but there is huge gap between p in our positive results and counterexamples given by traveling waves.
It is work on progress to find better norms in order to improve nonlinear estimates.

H. TRIEBEL:

Entropy numbers, compact embeddings in function spaces, eigenvalue distributions

1. Entropy numbers: $T : A \to B$ (quasi-Banach spaces) lin. compact

$$\underset{(k\text{-th entropy number})}{e_k} = \inf\left\{\varepsilon : \underset{\text{unit ball in } A}{TU_A} \text{ can be covered by at most } 2^{k-1} \text{ balls of radius } \varepsilon \text{ in } B\right\}$$

Prop. (Carl 82): $T{:}A \to A$, μ_k eigenvalues of T, then $|\mu_k| \le \sqrt{2}\, e_k$.

2. Non-limiting embedding

$H_p^s(\Omega)$, $0 < p < \infty$, $s \in \mathbb{R}$, fractional Sobolev (Hardy space) Ω bounded, smooth domain in $\mathbb{R}^n$

Theorem 1: If $s_0 - n/p_0 < s_1 - n/p_1$ and $s_0 < s_1$, then

id: $H_{p_1}^{s_1} \to H_{p_0}^{s_0}$ is compact, $e_k \sim k^{-\frac{s_1-s_0}{n}}$.

Application:

$B = b(x)\,(id-\Delta)_D^{-1} b(x)$, $b(x) \in L_v(\Omega)$, $u < v \le \infty$ acting in $L_2(\Omega)$ with Dirichlet condition,

then $|\mu_k| \le c\, k^{-2/n}$.

Example: $0 \in \Omega$, $b(x) = |x|^{-\lambda}$, $A = B^{-1} = |x|^{\lambda}\ (\mathrm{id}-\Delta)_D\ |x|^{\lambda}$

If $0 < \lambda < 1$ then $|\lambda_k| \ge c\, k^{\frac{2}{n}}$ for some c>0, λ_k-k-th eigenvalue of A.

Counterexample: $A = |x|\ (id-\Delta)_D\ |x|$ is not operator of pure point spectrum.

3. Limiting embedding

$\mathrm{H}_p(\Omega) = H_p^{\frac{n}{p}}(\Omega)$, $0 < p < \infty$ $\mathrm{H}_p(\Omega) \to E_v(\Omega)$ (Orlicz-Trudinger)

$E_v(\Omega) = \{ f: \int_\Omega \Phi_v\left(\frac{|f(x)|}{\lambda}\right)dx \le 1$ for some $\lambda > 0\}$ $\Phi_v(t) = t \exp t^v$.

Theorem 2: id: $\mathrm{H}_p(\Omega) \to E_v(\Omega)$ continuous iff $0 < v \le p'$

$(1 < p < \infty,\ \frac{1}{p}+\frac{1}{p'}=1)$, in the case "<" compact

and

$e_k \sim k^{-\frac{1}{p}}$ if $1+\frac{2}{p} < \frac{1}{v}$

Application: Limiting differential operator.

G. MOCKENHAUPT

Some L^p-L^q estimates for oscillatory integrals

Let $E \subseteq \mathbb{R}^n$ be a compact set and $d\mu$ a positive measure supported on E such that $\hat{d\mu} \in L^p(\mathbb{R}^n)$ for $p > \frac{2n}{\dim E}$, here dim E denotes the Hausdorff dimension of E. We are interested in the question for which (p,q)-range the restriction inequality for the Fourier transform

$$\int |\hat{f}|^q \, d\mu \le c \, \|f\|^q_{L^p(\mathbb{R}^n)} \qquad (*)$$

holds. We consider the case where E is a k-dimensional quadratic submanifold, i.e. E is graph of second order polynomials, $d\mu = \varphi d\sigma$, $\varphi \in C_c^\infty$, $d\sigma$ surface measure. Let $G_O(H)$ be the Gaussian curvature of E at O with respect to the unit normal vector $H \in S^{n-k-1} \subseteq N_O(E)$, the normal plane at $O \in E$.

Theorem 1: If $G_O(H)^{-1} \in L^p(S^{n-k-1})$ for $p \ge \frac{\text{codim } E}{\dim E}$ then (*) holds

for $1 \le p < \frac{2(n+\text{codim } E)}{n+3\cdot\text{codim } E}$ and q = 2.

We give also an endpoint result for the optimal restriction inequality for $E = S^1 \subseteq \mathbb{R}^2$, $d\mu$ arclength measure and using a result of R.Salem we show an analog of the restriction phenomenon in one dimension

Theorem 2: There are subset $E \subseteq \mathbb{R}$ of Hausdorff dimension $\alpha > \frac{1}{2}$ and positive measures μ on E s.t. (*) holds for $1 \le p < \frac{4}{3}$ and $p' \ge 3q$.

G. MAUCERI

Spectral multipliers for groups of exponential growth

This is a joint work with S.Guilini and A.Hulanicki.
We prove the existence of a nonholomorphic L^p-functional calculus for a distinguished Laplacean Δ on a class of solvable Lie groups of exponential growth. Let G be a connected, noncompact, semisimple Lie group with Owasawa decomposition G = NAK. Let S = NA. Then S is a solvable Lie group of exponential growth, whose Lie algebra has an orthogonal decomposition $\mathfrak{s} = \bigoplus_{\alpha\in\Sigma^+} \mathfrak{g}^\alpha \oplus \mathfrak{a}$, where the $\mathfrak{g}^\alpha$ are the root spaces corresponding to a choice of positive restricted roots Σ^+. Fix an orthonormal basis $\{H_1,\dots,H_l,X_1,\dots,X_m\}$ of $\mathfrak{s}$

adapted to the above decomposition, where $\{H_1,\ldots,H_l\}$ is an orthonormal basis of $\mathfrak{a}$, and regard its elements as right-invariant vector fields on S. Define the right-invariant Laplacean Δ on S by

$$\Delta = -\left(\sum_{j=1}^{l} H_j^2 + \frac{1}{2}\sum_{k=1}^{m} X_k^2\right).$$

Then Δ is a nonnegative self-adjoint operator on $L^2(S)$ with respect to the left Haar measure.

Let $\Delta = \int_0^\infty \lambda dE(\lambda)$ be its spectral resolution.

For every measurable function m on $[0,+\infty)$ define $m(\Delta) = \int_0^\infty m(\lambda)\, dE(\lambda)$.

Let n be the dimension of S. γ the cardinality of the set Σ_0^+ of short positive roots.

<u>Theorem:</u> Suppose that there exists a positive number a such that:

i) m is in the Sobolev space $H^s([0,a])$ for some $s > \frac{\gamma+1}{2}$;

ii) $m \in C^k((a/2,+\infty))$, $k > \max\left(\frac{\gamma+1}{2}, \frac{n+1}{2}\right)$

and $\sup_{\lambda \geq a/2} |(\lambda D_\lambda)^i m(\lambda)| \leq C$, $i = 0,\ldots,k$.

Then the operator $m(\Delta)$ is bounded on $L^p(S)$, $1 < p < \infty$ and is of weak type 1-1.

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