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Mathematical Finance

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Mathematical finance is a relatively new field, with most observers attributing its birth in 1971 to the development by Black and Scholes of their celebrated formula for the price of a call option. They took the price of a stock to be geometric Brownian motion and used the economic concept of arbitrage to argue that the price of a call option is the solution of a certain partial differential equation. In subsequent years these ideas have been greatly extended and generalized by mathematicians, financial economists, and researchers in the finance industry. In particular, it was soon recognized that many of the powerful tools of stochastic calculus and martingale theory could be employed to derive results of both fundamental and practical importance. Research in this field is now rapidly accelerating as mathematicians interact with financial researchers and as the world's financial markets continue to grow and become more sophisticated.

The first conference devoted to mathematical finance was held at Cornell University during the summer of 1989. Prompted by its success, the Oberwolfach conference was developed and organized. Thirty-six scholars from thirteen countries attended. Most of these were mathematicians, but included were several financial economists plus two researchers from the financial industry. Twenty-nine papers were presented, divided among the following topics:

- fundamentals of arbitrage and martingale measures
- statistical estimation
- consumption, investment, and optimal choice
- options and futures
- term structure models
- insurance, risk and actuarial problems
- miscellaneous financial applications.

Vortragsauszüge

K. K. Aase:

The risk premium puzzle seen in the light of jump dynamics

This paper derives the equilibrium excess returns on risky assets in an exchange economy where the underlying exogenous uncertainty is a combination of a pure multidimensional jump process and a diffusion model.

We derive closed form solutions for the interest rate and the risk premiums on risky assets for a traditional class of separable utility indices. Our analysis demonstrates that when the underlying jumps of the aggregate consumption process are not negligible, then the traditional form of the consumption-based capital asset pricing model need not hold and the asset risk premiums may be larger than predicted by the traditional CCAPM in continuous time, based on pure Itô-diffusion processes.

Our analysis suggests an explanation for the large estimates of the risk premiums reported in empirical tests of the single beta CCAPM.

Ph. Artzner (and F. Delbaen):

Default Risk Insurance

The default on a marketed security A is defined by the process $A^{\sigma-}$, where σ is a stopping time. The compensator of the risk process is the process of continuous, discounted, premium payments of $\alpha(t)(\Delta A_t + \Pi(A)_t)$, Π pricing operator, α predictable intensity of σ. For example, the default on a bond paying floating rate and 1 at maturity, is fairly insured by a nominal premium flow equal to the intensity.

If risk-free loans and risky loans are both marketed, it is shown that the pure insurance contract providing 1 at default date σ, can be reconstructed when the following is satisfied:

- σ is a totally inaccessible stopping time
- the martingale 1 and the martingale related to the risk-free loan generate all continuous martingales
- the martingale related to the risky loan has no discontinuity outside of σ.

H. Bühlmann:

Stochastic Discounting seen by an Actuary

In order to apply models of stochastic interest in actuarial work we need models in discrete time scale (Discretization of continuous models very often does not reflect all aspects of the problem in discrete time).

Therefore: Take a stream of stochastic payments $X \sim (X_1,X_2 \dots,X_N) \in L_N^2$ (for technical reasons).

If we insist that the valuation of X (called $Q[X]$) be a linear functional, we have the following representation

$$Q[X] = E[\sum_{k=1}^{N} \varphi_k X_k] \qquad \varphi = (\varphi_1,\varphi_2,\dots,\varphi_N) \in L_N^2 .$$

Therefore any model describing stochastic interest in discrete time amounts to defining a probability measure of the vector $\varphi = (\varphi_1,\varphi_2,\dots,\varphi_N) \sim$ discount vector.

It is convenient to factorize

$$\varphi_k = Y_1 \cdot Y_2 \cdot \dots \cdot Y_k \qquad (k = 1,2,\dots,N)$$

and model the vector $Y = (Y_1,Y_2,\dots,Y_N)$. As example I have given two models:

Model I: a) $Y_k = \varepsilon(1 - Z_k) + \delta Z_k$, $\varepsilon,\delta \in (0,1]$ with i.i.d.

$Z_k \sim \frac{1}{M}$ Binomial(M,p) and p given

b) $p \sim$ Beta(α,β)

Model II: Y_k follow a "Generalized Ehrenfest Urn" with transition probabilities

$$p_{y,y+1/M} = \tfrac{1}{2} + a(b - y) , \qquad p_{y,y-1/M} = \tfrac{1}{2} - a(b - y)$$

F. Delbaen (and W. Schachermayer):

Absence of No Free Lunch and Existence of Local Martingale Measures are not the Same

If $(X_t)_{0\le t\le 1}$ is a continuous price process, defined on a probability space $(\Omega,\mathfrak{F},\mathbb{P})$, and adapted to a filtration $(\mathfrak{F}_t)_t$ then we define K as the vector space of all possible gains $K = \{ \int_{[0,1]} H_u dX_u \mid H$ a simple bounded predictable process$\}$. No Arbitrage (NA) is defined as $K \cap L_+^0 = \{0\}$ where L_+^0 is the cone of positive functions in the space of all measurable functions L^0. We give an example of a process X_t such that under the original measure $\mathbb{P}$, the process X is a local martingale but NA is not satisfied, i.e. there is a free lunch. The concept of NFLBR (no free lunch with bounded risk) is defined as follows: <u>if</u> f_n is a sequence in K, $f_n \xrightarrow{\mathbb{P}} f : \Omega \longrightarrow [0,\infty]$ and $f_n \geq -1$ <u>then</u> $f = 0$. (Sometimes we also allow for integrands H which are only predictable). We give an example of a process X that satisfies NFLBR (in the stronger sense), $\sup_t \|X_t\|_p < \infty$ for all $p < 3$, $\mathbb{P}$ is the only <u>local</u> martingale measure for X but there is no martingale measure.

R. J. Elliott:

Estimating the Volatility of an Exchange Rate

Suppose S_t, representing the DM/$ exchange rate, is given by the equation $dS_t = S_t\mu_t dt + S_t\sigma_t dW_t$. (Here μ_t is predictable and σ_t may be Markov in S.) Then $1/S_t$ represents the $/DM exchange rate and $1/S_t$ is described by the equation $d(1/S_t) = -1/S_t((\mu_t - \sigma_t^2)dt + \sigma_t dW_t)$. Formally, therefore, $(dS_t/S_t) + (d(1/S_t))/(1/S_t) = \sigma_t^2 dt$. The Milstein approximation for numerical solutions to stochastic differential equations is introduced to justify this expression.

H. Föllmer (and M. Schweizer):

A microeconomic approach to diffusion models for price fluctuation

We present a case study for the derivation of diffusion models for stock prices which combines the microeconomic point of view with a suitable invariance principle. We consider a sequence of temporary equilibria, where individual excess demand involves liquidity demand, price assessments and dynamic hedging strategies. Passage to continuous time leads to a class of generalized geometric Ornstein- Uhlenbeck processes in a random environment. In this model information trading (in the sense of Black (86)) introduces a recurrent component while noise trading adds a transient drift. Extending a result of Brandt (86) from discrete time to the present setting, we derive a bound on noise trading which assures that the price fluctuation remains ergodic. Beyond that bound the process becomes highly transient. In the special Markovian case we obtain explicit formulae for the invariant distributions which arise in the ergodic case.

L. P. Foldes:

Martingale Conditions versus Programming Conditions for Portfolio Optimality

Brief review of a model of optimal saving and portfolio selections with general semimartingale investments over an infinite horizon in continuous time. Discussion of the relationship between conditions of optimality expressed in terms of martingale properties of shadow prices and those expressed as programming conditions. The use of integer-valued random measures and their predictable compensators to obtain programming conditions in the form of integral equations (or inequalities). Illustration of the procedure in the special case of logarithmic utility. Also, a simple proof of the existence of an optimum in this case.

In a longer, written, version, I would consider some additional points: Difficulties in calculating solutions in general, in particular difficulties arising from the interdependence of saving and portfolio decisions. Review of special assumptions which avoid some of the difficulties, distinguishing between cases with continuous processes and those with jumps.

D. Heath (and Ph. Artzner):

Completeness and Non Unique Pricing

We present an example consisting of countably many discontinuous stochastic processes $\{v_i(t)\}$ for $t \in [0,T]$ for which the set of equivalent martingale measures is a segment $[P_1,P_2]$. Moreover, under each P_i the $\mathcal{L}^2$ (or $\mathcal{L}^1$) closure of span $\{v_i(T)\}$ is all of $\mathcal{L}^2$ (resp. $\mathcal{L}^1$), so in financial terms there are two equivalent martingale measures and the market is complete for each. Changing terminology slightly, this also provides an example of a price system (M,Π) with net trades $M = \text{span } \{v_i(T)\}$ and prices $\Pi(m) = E_{P_i}(m)$ in which agents with preferences given by E_{P_1} or E_{P_2} can find optimal affordable net trades (in M): In this setting Π cannot be extended to a positive continuous linear functional Ψ on $X = \mathcal{L}^2(\frac{P_1+P_2}{2})$ so that each agent has an optimal affordable net trade in X.

F. Jamshidian:

Option and Futures Evaluation with Deterministic Volatilities

Several risk-neutral expectation formulae are derived in a general multifactor setting. Specializing to deterministic covariances of returns, they lead to new formulae for forward and futures prices. Formulae for options on forward and futures contracts are provided. The results are applicable to currencies, bonds, commodities with stochastic convenience yield, and stock indices. For currencies, a no-arbitrage relation between domestic and foreign economies is formulated and applied to evaluate quanto futures and options.

J. Jacod:

Adaptive estimation of the diffusion coefficient for diffusion processes

Consider a diffusion process whose diffusion coefficient depends on a parameter ϑ. We assume that the measures corresponding to different ϑ are all mutually singular, when complete observation is provided (i.e. the path of the process is known).

Now, we can only observe the process at n times, to be chosen as best as possible, and we are looking for asymptotic optimality of estimators of ϑ, as n goes to infinity. In particular we show the existence of a "uniformly best" procedure of adaptive type (i.e. choose the i-th observation time as a function of the (i-1) previously observed values).

We also examine the case of a Cauchy process, with a multiplicative parameter ϑ. In this case, an adaptive scheme improves the estimation even in the homogeneous case (contrary to what happens in the homogenous Wiener case).

M. Jeanblanc–Picqué:

Impulse Control Method and Exchange Rate

In a first part of this paper, we study the following impulse control problem: Let

(1) $$K(t) = k + \eta\, t + \sigma W(t)$$

be a diffusion process. A band $[a,b]$ is given.

An admissible pair is a sequence $(T_i,\zeta_i)_i$ where T_i is a stopping time and ζ_i a $\mathfrak{F}_{T_i}$-measurable variable such that the controlled process $Y(t)$ belongs to $[a,b]$, where

(i) $$Y(T_i^+) := Y(T_i) + \zeta_i$$

(ii) $dY(t) = dK(t)\ ;\ T_i < t \leq T_{i+1}$.

We associate to each sequence (T_i, ζ_i) a cost defined by

$$J(k,T,\zeta) = E_k[\Sigma \exp(-\lambda T_i)(\gamma + c|\zeta_i|)] .$$

We prove that there exists an optimal control characterized by a pair (α, β) with $a < \alpha < \beta < b$, which minimizes the cost function under the admissible pairs. This optimal pair is given by

$$T_i = \inf\{t \geq T_{i-1};\ Y(t) = a \text{ or } b\}$$

$$Y(T_i^+) := \alpha \text{ if } Y(T_i) := a$$

$$Y(T_i^+) := \beta \text{ if } Y(T_i) := b .$$

In a second part we apply this methodology to the exchange rate. The logarithm of the exchange rate is denoted by $X(t)$. We suppose that it is equal to $K(t) + \vartheta E[dX(t)/dt|\mathfrak{F}_s]$ where K is the "fundamental" and ϑ a parameter. We suppose that X is a function g of K $[X(t) = g(K(t))$; the function g is supposed to be C^2 and is determined using Ito formula]. Without any control, the fundamental follows (1). If there is a jump for the fundamental the exchange rate must be continuous.

The problem is to control the fundamental in order to keep the exchange rate in a fixed band and to minimize the cost for the interventions. We prove that there exists a solution.

D. Lamberton:

Convergence of the critical price in the approximation of American options

We consider the American put option in the Black-Scholes model. When the value of the option is computed through numerical methods (such as the binomial method and the finite difference method), the approximation yields an approximate critical price. We prove the convergence of this approximate critical price towards the exact critical price. Our proof (in the case of the binomial method) relies on the work of Kushner on the approximation of optimal stopping problems.

A. Morton (and S. Pliska):

Portfolio Management with Fixed Transaction Costs

We study optimal portfolio management policies for an investor who must pay a transaction cost equal to a fixed fraction of his portfolio value each time he trades. We focus on the infinite horizon objective function of maximizing the asymptotic growth rate, so the optimal policies we derive approximate those of an investor with logarithmic utility at a distant horizon. When investment opportunities are modelled as m correlated geometric Brownian motion stocks and a riskless bond, we show that the optimal policy reduces to solving a single stopping time problem. When there is a single risky stock, we give a system of equations whose solution determines the optimal rule. A numerical example illustrates a general qualitative result that even with very low transaction cost levels, the optimal policy entails very infrequent trading.

S. Müller:

Underpricing of initial public offerings: a common value model

This paper presents a theoretical model, where average underpricing of an initial public offering results from the strategic interaction of investors and the seller (the investment bank) in a situation of incomplete information. The model uses a priority pricing set-up as in Parson/Ravier (1985). However, the model has the following specific features:

- All investors attribute the same value to the stock. It is given by the opening price in the secondary market, which is unknown at the time of the sale.
- The distribution of the opening price is common knowledge.
- All market participants have private information about the opening price.
- The value variable and all information variables are affiliated.
- All market participants are risk neutral.

The following results are derived:

1. The expected revenues from the sale in the primary market are smaller than the expected revenues accrueing under the opening price in the secondary market. (Average underpricing)
2. The information policy of complete revelation is optimal for the seller.
3. Average underpricing decreases with an increase in the number of investors.

Ph. Protter:

Anticipating Stochastic Differential Equations

We study anticipating stochastic differential equations of the form

$$X_t = X_0 + \int_0^t \sigma(s,X_s,Y)\circ dW_s + \int_0^t b(s,X_s,Y)ds \tag{1}$$

and

(2) $\qquad Z_t = X_0 + \int_0^t \sigma(s,Z_s,Y) \circ dW_s + \int_0^t b(s,Z_s,Y)ds$

where equation (1) uses the Skorohod-Stratonovich integral for the Brownian motion W, and (2) uses the Itô-Stratonovich integral via an expansion of filtrations. The anticipation comes from X_0 and Y, which are in $\mathfrak{F}_\infty$. Equation (1) is more general than is (2), but when the domains of definition agree, any solution of (1) is also a solution of (2), hence the uniqueness of solutions of (2) - which is easy - gives uniqueness of (1). We also show existence of solutions to (1) (and (2)), and an important Wong-Zakai type of stability. These equations can be used in Finance Theory to model the activity of "insider trading" in the stock market. This talk is based on joint work with H. Ahn, A. Kohatsu, and J. León.

C. Rogers (and S. Zhan):

Bounds on the value of an Asian option

The problem of computing the price of an Asian option is the problem of computing

$$E[(\int_0^T(S_u - K)du)^+]$$

where $S_u = S_0 \exp(\sigma X_u)$, $\quad X_u = B_u + \mu u$, $\quad \sigma\mu = r - \frac{1}{2}\sigma^2$,
B is a standard BM, K is the strike price, S_0 the start price.

This appears to be quite hard to compute in closed form. The methods used here are basically quite primitive: one takes a random variable Z, conditions on Z; and uses

$$E(\int_0^T(S_u - K)du \mid Z)^+ \leq E[(\int_0^T(S_u - K)du)^+ \mid Z] \leq E[\int_0^T(S_u - K)^+ du \mid Z],$$

and simple variants of this.

A variety of Z are considered, for example, Gaussian Z, and $Z = (\sup_{s \leq T} X_s, X_T)$. The numerical bounds obtained are not particularly sharp, so far, but can be computed reasonably quickly and are certainly useful.

W. J. Runggaldier (and F. Mercurio):

Option Pricing: Approximations and their Interpretation

The purpose in this study is twofold:

1) Obtain a computable approximation for the value of an European call option when the price of the underlying risky asset is given by a jump-diffusion model.

This is accomplished by approximating the coefficients in the original model with piecewise constant time functions.

Since a continuous time model with piecewise constant coefficients has associated a discrete time model - we shall call it the approximating model - the second purpose is then

2) Show that the approximating option value can be interpreted as being itself an option value for the approximating model; i.e. we show that the approximating option value in 1) coincides with the initial value of a portfolio corresponding to a mean self financing and risk minimizing hedging strategy for the discrete time problem induced by the approximating piecewise constant coefficients.

If a perfect hedging strategy for the continuous time problem is available, the discrete time hedging strategy can also be obtained by a "projection" of the continuous strategy.

K. Sandmann:

The Pricing of Options with an Uncertain Interest Rate: A Discrete Time Approach

We consider the pricing of European options written over a stock under the assumption of a stochastic interest rate in a discrete time context. Therefore, the well-known binomial approach for the description of the stock price movement is combined with a binomial model for the term structure. The resulting market structure is a four state model. From this it is obvious that the market is incomplete. The idea behind this is that complete markets should not be naturally assumed for the valuation of long-running options. Basically two problems are connected with this approach.

First, it is necessary to formulate a relationship between stock price movements and interest rate movements. It turns out that correlation is not enough. If it is assumed that there are no arbitrage opportunities the description of the stock price process must take into account the stochastic interest rate movement.

Second, as the market is incomplete, it is no longer possible to set up a self-financing strategy to duplicate the option. To solve the latter problem, the idea of risk minimizing strategies of H. Föllmer and D. Sondermann (1986) is used.

W. Schachermayer (and F. Delbaen):

No Free Lunch with Bounded Risk and Equivalent Martingale Measures

A basic problem in mathematical finance is to determine precisely under which conditions a stochastic process $(S_t)_{t\in\mathbb{R}_+}$ on $(\Omega,\mathfrak{F},P)$ is a martingale under a measure R equivalent to P .

We define the notion of "no free lunch with bounded risk". The equivalence of this notion to the existence of an equivalent martingale measure R was shown for continuous bounded processes by F. Delbaen and for processes in discrete, infinite time by the author.

The main result obtained jointly with F. Delbaen presented in this talk extends this result to general bounded stochastic processes. However, for the validity of this theorem it is indispensable to use general stochastic integrals in the definition of "no free lunch with bounded risk" and not just "easy integrands".

R. Schöbel:

Arbitrage Pricing in Commodity Futures Markets: Is it Feasible?

In commodity markets we find situations of abundance as well as situations of shortage. Arbitrage pricing is not possible in periods of relative scarceness. Pricing in equilibrium misses the possibilities of arbitrage in periods with enough redundant stocks. It is shown that only a hybrid approach solves this fundamental asymmetry. As a by-product, this approach for the first time uses an endogeneous, non-negative convenience yield.

M. Schweizer:

Mean-variance hedging

We consider a continuous semimartingale ("stock price") of the form $X = X_0 + M + \int \alpha d\langle M\rangle$ and such that $X \in \mathfrak{S}^2$. For any $H \in \mathcal{L}^2(\mathfrak{F}_T,P)$ ("contingent claim") and $c \in \mathbb{R}$ ("initial capital"), we want to minimize

$$E[(c + \int_0^T \vartheta dX - H)^2]$$

("disutility (expected) of net terminal loss") over the set Θ of all predictable ϑ ("trading strategies") such that $\int \vartheta dX$ is in $\mathfrak{S}^2$. We solve this problem under the assumptions that

(i) H has the form $H = H_0 + \int_0^T \xi^H dX + L_T$ for some $H_0 \in \mathbb{R}$, $\xi^H \in \Theta$ and $L \in \mathfrak{M}_0^2$ with $L \perp M$.

(ii) The process $\int \alpha^2 d\langle M\rangle$ is deterministic.

Under (i) and (ii), we also determine the strategy $\vartheta \in \Theta$ which minimizes

$$\mathrm{Var}[\int_0^T \vartheta dX - H] .$$

The results can be extended to allow for cases where X is $\mathbb{R}^d$-valued and possibly discontinuous.

H. Shirakawa:

Asymptotic Estimation for Markowitz Efficient Portfolios Under Ergodic Security Return Processes

Assume that the vector security return process has a weak Ergodic property with respect to sample mean, variance and covariances. Also suppose that the investor maximizes the prior sharp measure based on the sample estimators. Then we first show that the prior optimal investment strategy is consistent in the sense that it converges to the posterior maximized sharp measure. Next we analyze the bias of the prior optimal sharp measure and that of posterior value under the finite sample size. When the vector security returns are independent and normally distributed, we derive the exact mean of the prior optimal sharp measure. Also we propose the efficient simulation technique to estimate the posterior sharp measure. The relationship between the prior and posterior bias is numerically studied for a simple case. Finally, the theoretical results are empirically tested for the Japanese stock market.

C. Skiadas:

Foundations of choice under uncertainty and asset pricing

Utility functions that are separable across time and states of nature are known to have serious limitations in representing economically important aspects of behaviour. In this talk a general framework will be presented for modelling temporal and inter-state dependencies through the notions of conditional preferences, utilities, and aggregators. Conditions for additive aggregration will be given, with state-dependent expected utility as a special case. This machinery will also be used to provide an axiomatization of recursive utility and to model information affinity or aversion. Finally, we will show that the more general class of preferences discussed leads directly to interesting asset pricing formulas.

D. Sondermann (and K. Sandmann):

On one-factor lognormal term structure models

Binomial models for the short term interest rate can be classified by their continuous time limit

$$dr = \mu(t,r)dt + \sigma(t,r)dW .$$

For $\sigma(t,r) = \sigma(t)\cdot r$ one obtains lognormal distributions of r. Such models have been studied by Black-Derman-Toy (1990), Hull-White (1990), Black-Karasinski (1991) and Sandmann-Sondermann (1988, 1990, 1991). The latter give a general existence theorem, obtain limit results and the distribution of discount factors.

C. Stricker:

Predictable representation property and maximum price of a contingent claim

In a complete financial market the price of a contingent claim B is given by $E^Q[B]$ where Q is the martingale measure of the discounted price process. The following result characterizes the contingent claims which can be represented as a stochastic integral with respect to the vector of discounted security prices.

$$\mathfrak{M} = \{Q: Q \sim P \text{ and } X \text{ is a } Q \text{ local martingale}\}$$

Theorem: Let B be a contingent claim that is integrable for all $R \in \mathfrak{M}$. Then the following conditions are equivalent.

i) B is attainable.

ii) $\exists Q \in \mathfrak{M}$ such that $\forall R \in \mathfrak{M}$ $\quad E^R[B] \leq E^Q[B]$.

R. Viswanathan:

Numerical methods for the pricing of American options

In the Black-Scholes model it is possible to derive analytical representation formulae für American style calls and puts thanks to a theorem by El Karoui and Karatzas. The representation involves in a crucial way the optimal exercise boundary.

The paper first describes a number of results on the optimal exercise boundary with particular emphasis on the asymptotic behaviour of the boundary.

The second part of the paper deals with numerical methods which attempt to calculate the value function using an approximation of the optimal exercise boundary. These techniques differ in spirit from binomial trees, variational inequalities and finite difference methods which are commonly used to price American options. The efficiency of the method is explored together with the related problem of evaluating derivatives of the value function.

W. Willinger (and N. Cutland, E. Kopp):

From Discrete to Continuous to Hyperfinite Financial Markets: New Convergence Results for Option Pricing

In Cutland, Kopp, and Willinger (Math. Finance 1, 1991), methods from Nonstandard Analysis were used to give substance to the claim that the Black-Scholes Model contains a built-in version of the Cox-Ross-Rubinstein binomial models. Here we show that nonstandard methods give standard information about convergence of claims, trading strategies etc. in the discrete Cox-Ross-Rubinstein models to their continuous counterparts in the continuous Black-Scholes model. We develop a standard notion of convergence called D^2-convergence which is stronger than weak convergence, the commonly used mode of convergence in the finance literature. Our results show that D^2-convergence is characterized both by a natural nonstandard lifting condition, and by standard L^2-convergence together with a discretization scheme. The results suggest a procedure for calculating good (i.e. convergent) discrete time trading strategies within a continuous version of the Black-Scholes model.

M. Yor:

The pricing of Asian options

With the help of several different methods, a closed formula is obtained for the laws of the exponential functional of Brownian motion with drift taken at certain random times, particularly exponential times, which are assumed to be independent of the Brownian motion.

This study yields, in particular, an explicit integral formula for the Laplace transform in time of the price of Asian options.

T. Zariphopoulou:

Investment-Consumption models with stochastic income

An optimal investment-consumption model in which the investor is endowed with a stochastic income is treated. This income is driven by a Wiener process which is, in general, correlated with the one associated with the price of the risky asset. Therefore, the market is incomplete since the stochastic income cannot be replicated by trading the available securities. The optimization problem is solved via analytic and in particular viscosity solution techniques. It is first shown that the value function can be characterized as the unique constrained viscosity solution of the associated Hamilton-Jacobi-Bellman (HJB) equation. This property is used to get convergence results for a wide class of numerical schemes for the value function and the optimal policies. In the special case of HARA utility, the problem can be reduced to a one-dimensional consumption-portfolio problem with almost HARA utilities. It is then shown that the value function is actually the unique smooth solution of the (HJB) equation and the optimal policies are given from the first order conditions.

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