MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Komplexe Analysis

30.8.-5.9.1992

Die Tagung fand unter der Leitung von W. Barth (Erlangen), H. Grauert (Göttingen) und R. Remmert (Münster) statt.

Es wurde versucht, die gegenwärtigen Entwicklungen in den verschiedenen Teilgebieten der komplexen Analysis durch Vorträge zu repräsentieren. Es gab u.a. Vorträge zum Problem der Gruppenaktionen, zu den analytischen Äquivalenzrelationen, zu den q-vollständigen Räumen, und es wurden Fragestellungen der komplexen algebraischen Geometrie diskutiert.

Vortragsauszüge

N. Mok:
When is a compact Kähler manifold projective-algebraic?

We look for topological conditions on a compact Kähler manifold X guaranteeing that the algebraic dimension is positive. The principle result is the following

Theorem: *Let X be a compact Kähler manifold, $\pi_1(X) = \Gamma$. Let $\Phi : \Gamma \to \mathrm{SL}(n, \mathbb{R})$ be a discrete semisimple representation such that G:=Zariski closure of $\Phi(\Gamma)$ is noncompact. Then, replacing X by a finite étale covering if necessary (hence replacing Γ by a subgroup of finite index), there exists a modification $\hat{X} \to X$, a non-singular projective variety Z of general type, a surjective holomorphic map $\sigma : \hat{X} \to Z$, and a representation $\Xi : \pi_1(Z) \to \mathrm{SL}(n, \mathbb{R})$ such that $\Phi = \Xi \circ \sigma_*$ on Γ, where $\sigma_* : \pi_1(\hat{X}) \to \pi_1(Z)$ is induced by σ and*

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$\pi_1(\hat{X})$ is canonically isomorphic to $\pi_1(X) \cong \Gamma$.

This theorem and its proof also allow us to give sufficient conditions for the projective-algebraicity of X. For instance, if $\Phi(\Gamma)$ is torsion-free and the induced map $H_{2n}(X, \mathbb{Z}) \to H_{2n}(N, \mathbb{Z})$, $n = \dim_{\mathbb{C}} X$,

$$N = \Phi(\Gamma) \setminus \mathrm{SL}(n, \mathbb{R})/\mathrm{SO}(n),$$

is nontrivial, then X is projective-algebraic. A special case is the following. Suppose $Y = \Omega/\Gamma'$ is a compact quotient of a bounded symmetric domain and X is a compact Kähler manifold homotopic to some complex submanifold S of Y, then X is projective-algebraic.

Our key point consists of studying holomorphic foliations arising from harmonic maps and proving that the leaves are complex-analytic closed subvarieties. (More precisely, the holomorphic foliations are defined outside some singular set and we purse that the closures of the leaves are also complex-analytic.) For the proof we study holomorphic foliations equipped with compatible semi-Kähler metrics and exploit the semisimplicity of Φ.

T. Peternell:

Varieties with trivial canonical sheaf

Consider a normal projective variety X with at most terminal or canonical singularities whose canonical sheaf K_X is numerically trivial (for short: minimal variety with $K \equiv 0$). It is suspected (and true by Mori in dim ≤ 3) that every projective manifold X with $\kappa(X) = 0$ is birational to a minimal variety with $K \equiv 0$. Therefore it would be important to have a kind of Beauville decomposition for those varieties, generalizing the classical case X smooth, $c_1(X) = 0$.

I discussed several approaches to the problem, presented some special results and made a conjecture what should be the final solution of the problem introducing a notion of singular Calabi-Yau resp. symplectic varieties.

J. Schürmann:

Einbettungen Steinscher Räume in affine Räume minimaler Dimension

Für einen (nicht notwendig reduzierten) komplexen Raum X sei

$$X(k) := \{x \in X \mid \operatorname{emdim}_x X \geq k\} \text{ und } n(k) := \dim X(k),\ k \in \mathbb{N}_0.$$

Im Falle $m := \operatorname{emdim} X := \sup\{\operatorname{emdim}_x X \mid x \in X\} < \infty$ gilt für die Größe $b'(X) := \sup\{k + [\frac{1}{2}n(k)] \mid k \in \mathbb{N}_0 \text{ mit } k \leq m\}$, (wobei $[.]$ den ganzzahligen Anteil bezeichne) die Abschätzung:

$$n + [\tfrac{1}{2}n] \leq b'(X) \leq \max\{n + [\tfrac{n}{2}], m + [\tfrac{1}{2}\dim S(X)]\},$$

wobei $S(X)$ der singuläre Ort von X und $n := \dim X$ ist.

Mit diesen Bezeichnungen gilt der

Einbettungssatz: *Für einen n-dimensionalen Steinschen Raum X mit $m < \infty$ gibt es eine holomorphe Einbettung in den affinen Raum $\mathbb{C}^N$ mit $N =$* $\max\{n + 1 + [\frac{n}{2}], b'(X), 3\}$.

Wesentliche Hilfsmittel beim Beweis des Satzes sind:

1) Eine Version des "Okaprinzips"von Gromov, 1989.

2) Die Konstruktion (fast-)eigentlicher holomorpher Abbildungen nach Bishop, 1961.

3) Ein "Lefschetzsatz" von Hamm 1983/86 über den Homotopietyp spezieller Mengen in Steinschen Räumen.

Einfache Beispiele zeigen, daß dieses Ergebnis für $n \geq 2$ oder $m \geq 3$ die im allgemeinen kleinstmögliche Dimension N liefert.

F. Campana:

The class $\mathcal{C}$ is not stable by small deformations

A compact connected (smooth) n-fold is said to be in the class $\mathcal{C}$ if it is bimeromorphic to some compact Kähler manifold. Moishezon manifolds are thus in $\mathcal{C}$. This class $\mathcal{C}$ is very stable under geometric operations.

So it is natural to ask (as A. Fujiki did in 1982) wether it is also stable under small deformations, i.e.:

Question: If $X_0 \in \mathcal{C}$, are all small deformations of X_0 in $\mathcal{C}$?

We answer negatively this question by a counterexample in the smallest possible dimension $n = 3$ with an X_0 which is moreover Moishezon (and even rational). This was also observed by C. Lebrun - Y.S. Poon.

The counterexample X_0 is a twistor space constructed by H. Kurke and C. Lebrun. (Although it is Moishezon, it is not homeomorphic to any projective 3-fold; observed in a joint work with T. Peternell.)

The original proofs heavily depend on the theory of twistor spaces. We present here another (parallel) proof independent of any differential geometric consideration.

P. Heinzner:

Holomorphic extensions of proper actions

Let G be a Lie group which acts properly on a real analytic manifold X.

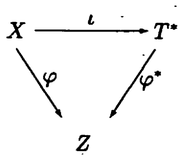
Theorem. *There exists a complex space X^* with a holomorphic action of the complexified group $G^{\mathbb{C}}$ and a G-equivariant real analytic map $\iota : X \to X^*$ such that*

(i) the quotient of X^* with respect to the smallest complex analytic equivalence relation which contains the $G^{\mathbb{C}}$-orbits is a Stein space, denoted by $X^*/\!\!/G^{\mathbb{C}}$.

(ii) the map $\iota : X \to X^*$ induces a map $\tau : X/G \to X^*/\!\!/G^{\mathbb{C}}$ which realizes X/G as a closed semianalytic totally real subspace of $X^*/\!\!/G^{\mathbb{C}}$.

(iii) $\iota(X)$ is a closed semianalytic totally real subspace of X^*.

(iv) to every G-equivariant real semianalytic map from X into a complex space Z where $G^{\mathbb{C}}$ acts holomorphically there exists a $G^{\mathbb{C}}$-invariant neighborhood T^* of X in X^* and a $G^{\mathbb{C}}$-equivariant holomorphic map $\varphi^* : T^* \to Z$ such that the following diagram commutes



If G is a linearly reductive group, then X^* is a Stein manifold and one can proof the following

Theorem. *There exists a closed linear equivariant embedding* $\varphi : X \to \mathbb{R}^N$ *if the G-orbittype of X is finite.*

F. Kutzschebauch:

Äquivariante Vektorraumbündel

Linearisierungsproblem: Sei $K \hookrightarrow \mathrm{Aut}_0(\mathbb{C}^n)$ eine kompakte Liegruppe. Gibt es ein $\phi \in \mathrm{Aut}_0(\mathbb{C}^n)$, so daß nach Konjugation mit ϕ gilt $K \hookrightarrow \mathrm{GL}_n(\mathbb{C})$?

Gegenbeispiele in der algebraischen Kategorie wurden von G. Schwartz (C.R. Acad. Sci. Paris, t. 309, Seite I) konstruiert.

Mit Hilfe folgender K-äquivarianter Verallgemeinerung des Grauertschen Oka-Prinzips wird gezeigt, daß diese Art von Gegenbeispielen in der holomorphen Kategorie nicht möglich sind:

Satz: *Seien $E \to X$, $F \to X$ zwei holomorphe K-Vektorraumbündel über einer Steinschen K-Mannigfaltigkeit X. Dann sind E und F genau dann holomorph K-äquivalent, wenn sie stetig K-äquivalent sind.*

Zum Beweis dieses Satzes wird das Problem zuerst auf Vektorraumbündel mit $K^{\mathbb{C}}$-Wirkung ($K^{\mathbb{C}}$ ist universelle Komplexifizierung von K) zurückgeführt und dann eine äquivariante Version von Theorem principal (s. Cartan: Symposium Int. de Topologica Algebraica, Mexico, 97-121) bewiesen:

Sei X Steinsche $K^{\mathbb{C}}$-Mannigfaltigkeit, M eine Kempf-Ness-Menge in X. Für ein spezielles $K^{\mathbb{C}}$-Gruppenbündel P (allgemeine Faser $GL_n(\mathbb{C})$) wird eine Garbe $\mathcal{F}$ bezüglich kompakter Hausdorffräume $N \subset H \subset C$ auf dem kategorischen Quotienten $X/\!\!/K^{\mathbb{C}}$ wie folgt definiert: Für $U \subset X/\!\!/K^{\mathbb{C}}$ offen sei

$\mathcal{F}(U) := \{f : \pi_M^{-1}(U) \times C \cup \pi^{-1}(U) \times H \to P$, f stetig K-äquivariant, so daß

$f_t : \pi_M^{-1} \to P$ stetiger K-Schnitt für $t \in C \setminus H$,

$f_t : \pi^{-1}(U) \to P$ holomorpher K-Schnitt für $t \in H$,

$f_t \equiv \mathbb{1}$ für $t \in N$ gilt.$\}$

Satz. *Wenn N Deformationsretrakt von C ist, gilt:*

(1) $H^0(X/\!\!/K^{\mathbb{C}}, \mathcal{F})$ ist kurvenzusammenhängend.

(2) Für jedes $\mathcal{O}_{X/\!\!/K^{\mathbb{C}}}$-konvexe $U \subset X/\!\!/K^{\mathbb{C}}$ hat $H^0(X/\!\!/K^{\mathbb{C}}, \mathcal{F}) \to H^0(U, \mathcal{F})$ dichtes Bild.

(3) $H^1(X/\!\!/K^{\mathbb{C}}, \mathcal{F}) = 0$.

M. Zaidenberg:

Estimates for cuspidal plane curves

(A report on a joint work with S. Orevkov.)

One of the most popular methodes to obtain restrictions on the number of singular points on a plane projective curve is applying Miyaoka-Yau-type inequality. The method of computation of the selfintersection numbers, involved in the inequality, is discussed, which allowed to generalize the earlier known inequalities for cuspidal curves (Hirzebruch, Ivinskis, Yoshihara) to the case of arbitrary cusps.

Theorem. *Let $\overline{D}$ be an irreducible curve in $\mathbb{P}^2$ with only irreducible singularities $z_1, \ldots, z_s$ of multiplicities $m_1, \ldots, m_s \leq m$, and $d = \deg \overline{D}$. Let $s \geq 2$ or $g(=$ the genus of $\overline{D}) > 0$. Then*

$$\sum_1^s \mu_i \leq \frac{2m}{2m+1}\left(d^2 - \frac{3}{2}d\right),$$

where μ_i is the Milnor number of z_i.

Corollary. *In the hypothesis above,*

$$g \geq \frac{d^2 - (9m+3)d}{2(2m+1)} + 1.$$

Matsuoka und Sakai (89) proved that $d < 3m$ for a rational cuspidal curve that is better than the bound $d \leq 9m + 2$, given by the Corollary above. In fact, an expression of quantities in MY-inequality in terms of Puiseaux characteristic sequences is obtained in the course of the proof.

D. van Straten:

A Quintic Hypersurface in $\mathbb{P}^4$ with 130 Nodes

The maximal number $N_n(d)$ of ordinary double points (nodes) a hypersurface of degree d in $\mathbb{P}^n$ can have, is not known in general. For hypersurfaces in $\mathbb{P}^4$ one has $N_4(3) = 10$, realised by the Segre Cubic and $N_4(4) = 45$, realised by the Burkhardt Quartic. The value of $N_4(5)$ is unknown, but ≤ 135.

In this lecture I describe some properties of a remarkable Quintic M that has 130 nodes. Consider $\mathbb{P}^5$ with coordinates $X_0, \ldots, X_5$ and let $S_i = S_i(\underline{X})$ be the i-th elementary symmetric function of X_i. Then:

Theorem. *The variety defined by $S_1 = S_5 + S_2S_3 = 0$ has exactly 130 nodes.*

The cohomology of a small resolution $\tilde{M}$ is $\dim H^2(\tilde{M}) = 30$, $\dim H^3(\tilde{M}) = 2$. The cohomology on $H^3 = H^{3,0} \oplus H^{0,3}$ gives rise to a weight 4 form for $\Gamma_0(6)$.

G. Dethloff:

Hyperbolicity of the complements of plane algebraic curves

There exists the following Conjecture of Kobayashi-Zaidenberg:

Let $C(d_1, \ldots, d_k) = \{\Gamma = \Gamma_1 \cdot \ldots \cdot \Gamma_k \mid \Gamma_i$ is a curve in $\mathbb{P}^2$ of degree $d_i \in \mathbb{N}\}$.

Let $\sum_{i=1}^{k} d_i \geq 5$. Let

$$H(d_1, \ldots, d_k) = \{\Gamma \in C(d_1, \ldots, d_k) : \mathbb{P}^2 \setminus \Gamma \textit{ is complete hyperbolic and hyperbolically embedded}\}.$$

Then $H(d_1, \ldots, d_k)$ contains a Zariski open subset of $C(d_1, \ldots, d_k)$.

In joint work with G. Schumacher and P.M. Wong I proved the following

Theorem.

a) *The conjecture is true for $C(d_1, \ldots, d_k)$, $k \geq 4$.*

b) *The conjecture is true for $C(2,2,2)$.*

c) *The conjecture is true "in codimension 1" for $C(2,2,1)$. (That shall mean that there exists a variety of codimension 1 in the space of all configurations on which it is true.)*

The tools used in the proof come mainly from value distribution theory. For the proof of a) it suffices to calculate some defects of images of curves under morphisms of $\mathbb{P}^k$. For the proof of b), the 3 quadrics are "replaced" by 12 lines in general position, on which the SMT of VDT is applied in two different ways. For the proof of c), we proof a generalized Borel's Lemma, where one summand may be vanishing somewhere, but is a square. We finish up the proof by using that in codimension one, the linear system given by two generic quadrics and a generic doubleline contains a doubleline other than the given one.

B. Siebert:

Meromorphic Quotients and Geometric Invariant Theory

We compare Hilbert-Mumford-Quotients with meromorphic quotients.

More precisely, let X be an affine or projective variety with a linear action of a reductive group G. Then the invariant regular functions give rise to the HM-quotient $X/\!\!/G = \operatorname{Spec} \mathbb{C}[X]^G$ (resp. Proj. $\mathbb{C}[X]^G$) which is a quotient for the closed orbits (resp. the stable ones).

On the other handside the closure $\overline{R}_G$ of the orbit equivalence relation $R_G = \{(x, gx) \in X \times X \mid x \in X, g \in G\}$ is an analytic set looking like an analytic equivalence relation outside some nowhere dense analytic set $P \subset X$ (Rosenlicht), i.e. $\overline{R}_G$ is a meromorphic equivalence relation in the sense of Grauert. The main theorem on meromorphic equivalence relation shows that there is a canonical G-equivariant proper modification $\tilde{X} \to X$ such that the proper transform $\tilde{R}$ of $\overline{R}_G$ is a geometrically flat ($\Leftrightarrow$ open if $\tilde{X}$ is normal) analytic equivalence relation, thus having a quotient $\tilde{X}/\tilde{R}$. This is our meromorphic quotient $X/\overline{R}_G$, (cf. Grauert: On meromorphic Equivalence Relations and my thesis).

The connection between $X/\!\!/G$ and $X/\overline{R}_G$ is done using the notion of fibre cycli spaces - a complex space parametrizing limits of generic fibers. In general $X/\overline{R}_G$ is a proper modification of $X/\!\!/G$ and the spaces coincide iff the quotient map $X \to X/\!\!/G$ (resp. $\hat{X} \to X/\!\!/G$, $\hat{X}$ blow up of the nullcone in the projective case) is geometrically flat.

Finally, this suggests to study holomorphic group actions with $\overline{R}_G$ being a meromorphic equivalence relation.

K. Fritzsche:

Proper holomorphic maps onto q-complete spaces

Motivation (by a paper of Ch. Okonek): Let $f : X \to \mathbb{P}^n$ holomorphic, X a compact complex space, dim(fibres) $\leq k$, $H \subset \mathbb{P}^n$ hyperplane. Then $X \setminus f^{-1}(H)$ is $(k+1)$-complete.

Notations: φ "q-convex in x" means: The Leviform $\mathcal{L}_{\varphi,x}$ has $n - q + 1$ positive eigenvalues, where $n = \dim_{\mathbb{C}} T_x(X)$. So 1-complete means Stein. A "differentiable function" on a complex space X is a section in the sheaf $\mathcal{D}_X$, where $\mathcal{D}_X$ is defined as follows:

If $X = (A, \mathcal{O}/(f_1, \ldots, f_r))$ is a local model, then

$$\mathcal{D}_X = \mathcal{A}/(\operatorname{Re} f_1, \ldots, \operatorname{Re} f_r, \operatorname{Im} f_1, \ldots, \operatorname{Im} f_r)$$

with $\mathcal{A}$ = sheaf of C^∞-functions.

Conjecture: Let $f : X \to Y$ be a proper holomorphic map between complex spaces, dim (fibres) $\leq k$, Y q-complete. Then X is $(q+k)$-complete.

Remarks. It is sufficient to consider the case X, Y reduced, f surjective. The conjecture is true for f a finite map (Ancona) and for locally trivial fibre bundles (trivial).

Theorem. *The conjecture is true for the case $q = 1$.*

Proof (joint work with V. Vijitu). Use a filtration $X = X_0 \supset X_1 \supset \ldots \supset X_N \supset X_{N+1} = \emptyset$, where X_j closed complex subspace, $S_j = X_j \setminus X_{j+1}$ regular, $f|S_j$ submersion. For $x \in S_j$ set $V_x := \mathrm{Ker} f_{*,x} \cap T_x(S_j)$. Then $\dim_{\mathbb{C}} V_x \leq k$ always. For $j = 1, 2, \ldots$ find different functions η_j on X with $\eta_j|X_j = 0$, $\eta_j|X \setminus X_j > 0$, $\mathcal{L}_{\eta_j,x} \geq 0$ on X_j and > 0 on $T_x(X) \setminus T_x(X_j)$ for $x \in S_j$. Set $g_j := \eta_j + \varphi \circ f$ for $j = 1, 2, \ldots$ and $g_0 := \varphi \circ f$ (where φ is a 1-convex exhaustion function on Y). Now do descending induction. Set $\psi_N := g_N$, $\psi_j := \chi \circ g_j + \psi_{j+1}$ with a very fast increasing function χ, and show that for $x \in X_j$ there exists a sub-vectorspace $E_x^{(j)} \subset T_x(X) \setminus V_x'$ (with $V_x' = V_x \setminus \{0\}$) such that $\dim_{\mathbb{C}} E_x^{(j)} \geq \mathrm{embdim}_x(X) - (k+q) + 1$, $\mathcal{L}_{\psi_j,x} > 0$ on $E_x^{(j)}$. The case $i = 0$ gives the theorem.

W. Decker:

On abelian and bielliptic surfaces in $\mathbb{P}^4$

(joint work with A. Aure, K. Hulek, S. Popescu, K. Ranestad)

Every smooth algebraic surface X can be embedded in $\mathbb{P}^5$, but only a few of them in $\mathbb{P}^4$. Moreover Ellingsrad and Peskine proved that there are only finitely many families of such surfaces $X \subseteq \mathbb{P}^4$ which are not of general type.

Problem: Classify these surfaces.

The talk is concerned with the known bielliptic and abelian surfaces. Two new examples will be constructed.

1. *The abstract point of view.* We recall how to obtain (minimal) abelian and bielliptic surfaces in $\mathbb{P}^4$ of degree 10 via linear systems. We describe a $\mathbb{P}^2$-bundle $\mathbb{P}^2_E$ over an elliptic curve E. $\mathbb{P}^2_E$ contains a pencil of abelian surfaces and 8 bielliptic surfaces. $\mathbb{P}^2_E$ may be mapped to $\mathbb{P}^4$ embedding the abelian and bielliptic surfaces. The image of $\mathbb{P}^2_E$ is the unique quintic hypersurface containing the bielliptics.

2. *The geometric point of view.*
2.1 Nonminimal abelian surfaces of degree 15 can be obtained by linkage from the minimal ones in degree 10.
2.2 We obtain a new family of nonminimal bielliptic surfaces of degree 15 by applying the quadro-cubic Cremona transformation of Semple to the minimal bielliptic in degree 10.

3. *The syzygy point of view.* We explain how to construct all surfaces mentioned by constructing the finite length graded cohomology modules of their ideal sheaves first. As un upshot we present a new family of nonminimal abelian surfaces of degree 15. This family is different from the one mentioned in 2.1.

M. Schneider:
Special projective varieties

(joint work with R. Braun, G. Ottaviani, F.O. Schreyer)

A projective variety $X \subset \mathbb{P}_N$ is called to be of *log-general type* if $K + H$ is big and nef.

Theorem. *Let* $X \subset \mathbb{P}_5$ *be a smooth 3-fold which is not of log-general type. Then* $\deg X \leq 12$ *and* X *is in the following list:*

$\deg X = D$	
3	cubic Segre scroll
4	complete intersection of 2 quadrics
5	quadric fibration over $\mathbb{P}_1$
6	complete intersection of quadric and cubic
6	Bordiga scroll over $\mathbb{P}_2$
7	Palatini scroll over cubic in $\mathbb{P}_3$
7	blow up in one point of complete intersection of 3 quadrics
7	del Pezzo fibration over $\mathbb{P}_1$
8	del Pezzo fibration over $\mathbb{P}_1$
9	conic bundle over $\mathbb{P}_2$
9	scroll over K3-surface
12	conic bundle over quartic in $\mathbb{P}_3$

The conic bundle of degree 12 was not known classically.

W. Ebeling:

Homology Hopf Surfaces

(joint work with Ch. Okonek)

A rational (or integral) homology Hopf surface is a compact complex surface which has the same rational (or integral) homology as $S^1 \times S^3$. Building on K. Kodaira's results we give a classification of such surfaces. Examples of such surfaces are complex structures on $S^1 \times \Sigma^3$, where Σ^3 is a homology 3-sphere. First examples of complex structures on $S^1 \times \Sigma^3$, where Σ^3 is not homeomorphic to the standard sphere, appeared implicitly in a paper of E. Brieskorn an A. Van de Ven and are related to singularities. We generalize their construction.

We derive the following classification of complex structures on $S^1 \times \Sigma^3$, where Σ^3 is a rational homology 3-sphere: If Σ^3 is irreducible and sufficiently large in the sense of Waldhausen but is not Seifert fibred, then $S^1 \times \Sigma^3$ does not possess any complex structure. If, on the other hand, Σ^3 is Seifert fibred, then there always exists a complex surface X homeomorphic to $S^1 \times \Sigma^3$; any such X is a Hopf surface or an elliptic surface. If, in addition, Σ^3 is a large Seifert manifold, we determine precisely the surfaces homeomorphic to $S^1 \times \Sigma^3$.

S. Bauer:

Diffeomorphism types of elliptic surfaces with $p_g = 1$

Simply connected elliptic surfaces with geometric genus 1 can be constructed from an elliptic K3-surface via logarithmic transformation of two smooth fibres with multiplicities p and q. These numbers should be coprime. It is known from Kodaira that these surfaces are deformation equivalent if and only if the multiplicities of the multiple fibres do match. The main result is:

Theorem. *Two simply connected elliptic surfaces with $p_g = 1$ are diffeomorphic if and only if they are deformation equivalent.*

This result has also been obtained by Morgan - O'Grady. It relies on a partial computation of Donaldson's polynomial invariants, which are known to be polynomials in the intersection form Q and the class $k = \frac{1}{pq}F$, where F denotes the Poincaré dual of a generic fiber.

In particular, for fixed Chern classes $c_2 = 3$, $c_1 = K_{\hat{S}}$, the polynomial can be written

$$q = \sum_{i=0}^{3} a_i Q^{(3-i)} k^{2i}.$$

The leading term a_0 is known by results of Friedman - Morgan to be $a_0 = pq$. The second term a_1 then has the explicit form

$$a_1 = \begin{cases} \frac{pq}{12}(3p^2q^2 - p^2 - q^2 - 1) & \text{if } pq \text{ is odd,} \\ \frac{pq}{12}(3p^2q^2 - p^2 - q^2 - 1 - 3(p^2 - 1) & \text{if } q \text{ is even.} \end{cases}$$

G. Müller:

Hypersurface singularities and infinite dimensional Lie groups

(joint work with H. Hauser, Innsbruck)

Let $X \subseteq (\mathbb{C}^n, 0)$ be a reduced hypersurface. Its group $G_x = \{\varphi \in \mathrm{Aut}(\mathbb{C}^n, 0) \,|\varphi(X) = X\}$ of embedded automorphisms can be provided (in a natural way) with the structure of an infinite dimensional Lie group. For this purpose, a Rank theorem for special analytic maps between special infinite dimensional spaces is proven. Finally, it is shown (in case $n \geq 3$) that X is uniquely determined (up to isomorphism) by its Lie group G_x.

J. Winkelmann:

Subvarieties of complex parallelizable manifolds

(joint work with A.T. Huckleberry)

Subvarieties of complex compact parallelizable manifolds (i.e. manifolds with holomorphically trivial tangent bundle) are classified according to their Kodaira-dimension. For $Z \subset X = G/\Gamma$ (X parallelizable) $k(Z) = 0$ iff Z is a closed orbit of a Liesubgroup of G. If $k(Z) > 0$ then there exists the Iitaka-reduction which has particular nice properties here, i.e. with $H = \mathrm{Aut}_0(Z)^0$ one obtains a holomorphic fiber bundle $Z \to Z/H$, where Z/H is a manifold of general type. Based on this it can be proven that one has strong dimension bounds. If G is semisimple, Γ discrete cocompact and $Z \subset G/\Gamma$ is a subspace of general type or in class $\mathcal{C}$, then $\dim(Z) \leq \frac{1}{3}\dim(G/\Gamma)$. If G is simple and Z an arbitrary subspace of $X = G/\Gamma$, then $\dim(X) - \dim(Z) \geq \sqrt{\dim X}$. If G is non-commutative, then there are always non-trivial subspaces, because there exists a maximal connected abelian subgroup with a closed orbit. For G semisimple the component of the cycle space containing this cycle is always non-compact. The theory of vector bundles over tori may be carried over to parallelizable manifolds to a substantial degree, yielding the following result:

Let M be an arbitrary compact complex manifold. Then there exists a non-trivial holomorphic vector bundle over M.

Berichterstatterin: S. Terveer

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