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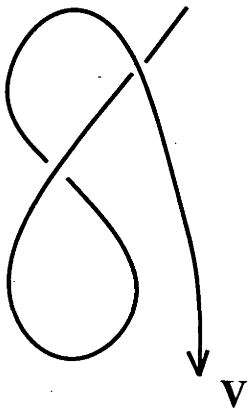
TOPOLOGIE

6.–12. 9. 1992

$\Theta_V^{-2} =$ 

The conference was organized by J.D. Jones (Warwick),I. Madsen (Aarhus) and E. Vogt (Berlin). 48 participants from Europe and the United States attended the conference. The topics of the 19 talks dealt with new developments in algebraic and geometric topology. In particular, the following areas were discussed: knots and 3-manifolds and their connections to topological quantum field theory, algebraic K-theory with applications to group actions and differential geometry, the relationship between analysis and topological invariants,homotopy theory.

Christian Kassel:

Framed tangles and ribbon categories

Ribbon categories as defined by Reshetikhin-Turaev are the right kind of monoidal categories in order to produce isotopy invariants of framed links in $\mathbf{R}^3$. In joint work with Vladimir Turaev we associate a ribbon category $\mathcal{D}(\mathcal{C})$ to any monoidal category $\mathcal{C}$ with (left) duals. In some sense it is the right adjoint of the forgetful functor from ribbon categories to monoidal categories with left duals.

If $\mathcal{C} = A - \mathrm{Mod}_f$ is the monoidal category of finite dimensional representations of a Hopf algebra A, then $\mathcal{D}(\mathcal{C})$ is isomorphic to the ribbon category $\mathcal{D}(A)(\Theta) - \mathrm{Mod}_f$ of finite dimensional representations of the universal ribbon algebra associated to Drinfeld's quantum double $\mathcal{D}(A)$ of A.

Johan Dupont:

Formulas for characteristic classes for flat bundles

Let G be a complex reductive Lie group. The Cheeger-Chern-Simons classes for flat G-bundles give cohomology classes in $H^*(BG^\delta; \mathbf{C}/\mathbf{Q})$, where G^δ is the discrete underlying group for G. We give an explicit formula for these classes in terms of the bar-complex for the discrete group G^δ, given a choice of the following data: In the singular complex with rational coefficients $C_* = C_*^{\mathrm{sing}}(G;\mathbf{Q})$ choose a projection h onto a subspace of representatives for the homology of G and a chain homotopy $s : C_* \longrightarrow C_{*+1}$, such that $s\partial + \partial s = \mathrm{id} - h$. In the case $G = \mathbf{C}^*$, the C-C-S-class is just given by $z \mapsto \frac{1}{2\pi i}\log(z)$, and h corresponds to the choice of $1 \in \mathbf{C}^*$ as a basepoint, whereas h is the choice of an arc α in the formula $\log(z) = \int_\alpha \frac{dz}{z}$. Thus our formula is a first step in getting a "polylogarithmic" formula for the C-C-S-classes.

Erik K. Pedersen (joint work with G. Carlsson):

Continuously controlled algebraic K-theory

Let Γ be a group such that $B\Gamma$ is finite. Suppose $E\Gamma$ has a compactification X ($E\Gamma$ an open dense subset of X) such that

(i) the Γ-action extends to X,

(ii) X is metrizable and contractible,

(iii) compact subsets of $E\Gamma$ become small near $Y = X \setminus E\Gamma$ i.e. $\forall y \in Y\ \forall U_y \subset X$ nbd. of y $\forall K \subset E\Gamma$ compact $\exists V_y \subset X$ so that if $gK \cap V_y \neq \emptyset$ then $gK \subset U_y$.

Theorem: The assembly maps

(i) $B\Gamma^+ \wedge KR \longrightarrow KR\Gamma$

(ii) $B\Gamma^+ \wedge \mathbb{L}^{-\infty}(R) \longrightarrow \mathbb{L}(R\Gamma)$

(iii) $B\Gamma^+ \wedge A(Y) \longrightarrow A(B\Gamma \times Y)$

are split monomorphisms (of spectra). In (i) R is a ring, in (ii) a ring with involution and in (iii) Y is a space.

The proof of (i) is based on continuously controlled algebraic K-theory as developed by Anderson, Conolly, Ferry and Pedersen. The proof of (ii) uses the L-theory of additive categories as developed by Ranicki, and (iii) uses continuously controlled A-theory as developed by Pedersen and Vogell.

Andrew Ranicki:

The algebraic theory of bands

A *band* is a finite CW complex X with a finitely dominated infinite cyclic cover $\bar{X}$. (The terminology is due to Siebenmann.) Let A be a ring with Laurent polynomial extension $A[z, z^{-1}]$.

A *chain complex band* is a finite f.g. free $A[z, z^{-1}]$-module chain complex C which is A-finitely dominated, i.e. is A-module chain equivalent to a finite f.g. projective A-module chain complex.

Let $A((z)), A((z^{-1}))$ be the completions of $A[z, z^{-1}]$.

Theorem: A finite f.g. free $A[z, z^{-1}]$-module chain complex C is a chain complex band if and only if

$$H_*(A((z)) \otimes_{A[z,z^{-1}]} C) = H_*(A((z^{-1})) \otimes_{A[z,z^{-1}]} C) = 0$$

The theorem has applications to the obstruction theory for deciding which compact manifold bands fibre over $\mathbf{S}^1$. In particular, it can be used to relate the surgery-theoretic Farrell-Siebenmann obstruction to the Morse-theoretic Novikov-Pazhitnov obstruction.

Dietrich Notbohm (joint work with J. Aguadé and C. Broto):

Some spaces with interesting cohomology and a conjecture of Cooke

We are interested in the realization and classification of the homotopy types of spaces realizing algebras over the Steenrod algebra of the form $\mathbb{F}_p[x] \otimes E(y)$, where $deg(x) = 2n$, $deg(y) = 2n+1$ and the Bockstein maps x onto y. These algbras are called PE–algebras of type $(2n, 2n+1)$. Examples of spaces realizing PE–algebras were first constructed by Cooke as follows.

Let Y'_k be the homotopy fiber of the degree p^k map $S^3 \longrightarrow K(\mathbb{Z},3)$. The p-adic completion $Y_k := Y'^{\wedge}_{k\,p}$ carries a $\mathbb{Z}/(p-1)$-action. For r dividing $p-1$ and for $k=0$, the homotopy orbit $E\mathbb{Z}/r \times_{\mathbb{Z}/r} Y_k$ realizes a PE-algebra of type $(2pr, 2pr+1)$. For $k=1$ we get a realization of a PE-algebra of type $(2r, 2r+1)$. Cooke conjectured that a PE-algebra of type $(2n, 2n+1)$ is realizable iff n divides $p(p-1)$. For $n = pr$, the action of the Steenrod algebra $\mathcal{A}_p$ is already determined. For $n|p-1$ there are two different actions of $\mathcal{A}_p$, which can be distinguished by the action of P^1 or Sq^2 on the exterior generator.

Now we assume that p is an odd prime. Then the Cooke conjecture is true, but the above constructed spaces are not the only homotopy types realizing PE-algebras.

Theorem: If the cohomology $H^*(X;\mathbb{F}_p)$ of a topological space X is a PE-algebra of type $(2n, 2n+1)$, then n divides $p(p-1)$.

Theorem: Let B be a fixed PE-algebra of type $(2pr, 2pr+1)$, where r divides $p-1$.

(i) For any $s \in \mathbb{N}$ there exists a p-complete space Y_s realizing B, and all of these spaces are pairwise not homotopy equivalent.

(ii) Any p-complete space, Y realizing B, is homotopy equivalent to one of the spaces Y_s.

The proof of this theorem and of the Cooke conjecture involves a lot of 'Lannes-theory'. In particuliar we have to face the problem to calculate the cohomology of a mapping space $map(B\mathbb{Z}/p, Y)_f$, where the application of Lannes' T-functor does not give an algebra which vanishes in degree 1.

Klaus Johannson:

Homotopy of Heegaard strings

Let M be a 3-manifold, orientable and compact. M is Haken, if it is irreducible, ∂-irreducible and sufficiently large in the sense that it contains at least one incompressible surface. M is simple if it contains no essential annulus or torus and if it is Haken. An arc $t \subset M$ is a Heegaard-string, if its complement is a handlebody.

Theorem: Any two Heegaard-strings in a simple 3-manifold are homotopic iff they are ambient isotopic.

It has been indicated how to apply this result to a calculation of the mapping class group of a 1-relator 3-manifold.

As a corollary one also gets that it can be decided whether two given Heegaard strings in a simple 3-manifold are ambient isotopic.

John Rognes:

Filtering the spectrum $K(\mathbb{Z})$ by rank

We approximate the K-theory spectrum of the integers using a spectrum level rank filtration. By means of a certain poset spectral sequence we explicitly compute the first three subquotients of this filtration. Assuming a conjecture about the filtration's rate of convergence, we conclude that $K_4(\mathbb{Z}) = 0$ and $K_5(\mathbb{Z})$ is a copy of $\mathbb{Z}$ (the Borel summand) plus two–torsion of order at most eight.

Michael Weiss:

Pincing and concordances

The sphere theorem of Rauch-Berger-Klingenberg states that a simply-connected complete Riemannian manifold M^n whose sectional curvature satisfies $1/4 < \sec(M) \leq 1$ is homeomorphic to $\mathbf{S}^n$. Question, popular to differential geometers: Is M *diffeomorphic* to $\mathbf{S}^n$? One way to attack the question is by using concordance theory as follows.

Suppose that M is *any* homotopy sphere. Let $W(M)$ be the space of Morse functions f on M having exactly two critical points (necessarily maximum and minimum points). The map $W(M) \longrightarrow M \times M \setminus \text{diagonal}, f \mapsto (\text{min.point of} f, \text{max.point of} f)$ is a $\mathbb{Z}/2$-map; the generator of $\mathbb{Z}/2$ acts by $f \mapsto -f$ and $(x, y) \mapsto (y, x)$.

Passing to the quotients gives a fibre bundle

$$p_M : \frac{W(M)}{\mathbb{Z}/2} \longrightarrow \frac{M \times M \setminus \Delta}{\mathbb{Z}/2} \simeq \mathbf{R}P^n.$$

Theorem: If M has a Riemannian metric with $1/4 < \sec(M) \leq 1$, then p_M has a section.

Calculation: Assume that M (with some orientation) has *even* order in the group of oriented homotopy n-spheres modulo diffeomorphism, and that $M = \partial N$ where N^{4k} is smooth, compact parallellized. Then p_M does *not* have a section.

More precise results are available, relating the order of $[M]$ to the maximal k such that p_M has a partial section over $\mathbf{R}P^k \subset \mathbf{R}P^n \simeq (M \times M \setminus \Delta)/(\mathbb{Z}/2)$.

The calculation uses: (1) The fiber of p_m is homotopy equivalent to the space of smooth concordances of the sphere $\mathbf{S}^{n-1}$;

(2) Waldhausen's theory relating smooth concordance spaces to algebraic K-theory.

The calculation is based on joint work with Bruce Williams.

Jun Murakami:

A formula for the HOMFLY polynomial of satelite links

Let P be the HOMFLY polynomial of links defined by the skein relation. Let $H_{n,m}$ be a quotient by the skein relation of the semigroup ring of $(n+m)$–tangles whose left n–strings (right m–strings) on the edges are oriented downwards (upwards). It is a generalization of the Iwahori–Hecke algebra and its simple modules are parametrized by pairs of Young diagrams (Y, Y'). For a satelite link K_L coming from a knot K in S^3 and a link L in $B^2 \times S^1$, we have $P(K_L) = \sum_{(Y,Y')} f_{(Y,Y')}(L)\, P_{(Y,Y')}(K)$. In this formula, $f_{(Y,Y')}$ is an invariant of links in $B^2 \times S^1$ coming from the character of $H_{n,m}$ parametrized by (Y, Y'), and $P_{(Y,Y')}(K) = P(K_{(Y,Y')})$, where $K_{(Y,Y')}$ is a linear combination of satelite links of K corresponding to a primitive idenpotent of $H_{n-1,m-1}$ parametrized by (Y, Y').

Alexander L. Fel'shtyn (joint work with Richard Hill):

Reidemeister torsion, Nielsen Theory and Dynamical zeta functions

We continue to study the Reidemeister and Nielsen zeta functions. We prove rationality and functional equations of the Reidemeister zeta function of an endomorphism of any finite group and of a self–map of a polyhedron with finite fundamental group. The same results are obtained for eventually commutative endomorphisms of groups , and for eventually commutative self–maps of compact polyhedra. We connect the Reidemeister zeta function of a group endomorphism with the Lefschetz zeta function of the Pontryagin dual endomorphism, and as a consequence obtain a connection of the Reisemeister zeta function with the Reidemeister torsion. We also obtain arithmetical congruences for the Reidemeister and Nielsen numbers similar to those found by Dold for the Lefschetz numbers.

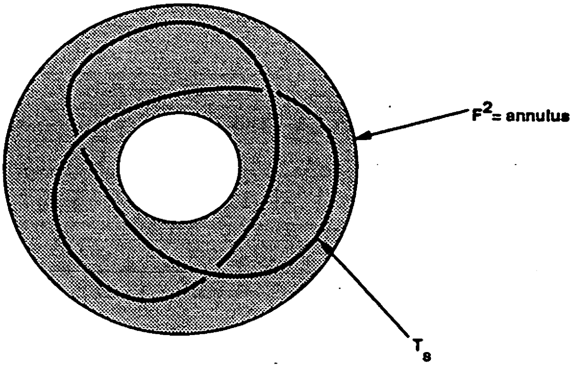
Thomas Fiedler:

Small state sums for knots and their applications

In the first part of the talk we introduced a new invariant for knots in real line bundles over non–simply connected surfaces by means of a very simple state sum. This invariant was used to calculate a relative unknotting number for knots in certain 3–manifolds which fiber over the circle.

In the last part of the talk this invariant was generalized to an isotopy invariant of isotopies of knots. Let F^2 be an oriented smooth non–simply connected surface and let $\arg : F^2 \times \mathbf{C}^* \to S^1$ be the map given by the argument of the second factor.
Definition. An oriented closed smooth embedded surface $T \hookrightarrow F^2 \times \mathbf{C}^*$ is called a *transversal 2-dimensional* braid if $\arg|_T$ has no critical points. (For $T = S^1$ and $F^2 = I$ we obtain the usual closed braids.) We construct an isotopy invariant for 2–braids which lives in the free $\mathbf{Z}$–module generated by ordered pairs of elements from $(H_1(F^2), H_1(F^2)/_{\pm 1})$.

Example:



Let $T_s = \arg^{-1}(s) \cap T, s \in \mathbf{S}^1$ be the standard $\mathbf{Z}/3\mathbf{Z}$–symmetric diagram of the trefoil in $F^2 \times \mathbf{R}$.

If we rotate the annulus by $\frac{2\pi}{3} n$, $n \in \mathbf{Z}$, and glue it to itself we obtain a 2–dimensional braid $T(n)$.

Theorem. *$T(n)$ is not isotopic as a 2–dimensional braid to $T(m)$ for $n \neq m$.*

Remark. It is well known that ordinary closed braids are isotopic as braids iff they are isotopic as links in the solid torus. Does the analogue statement hold for 2–dimensional braids? If "yes" our invariant provides a new isotopy invariant for certain smooth surfaces in certain 4–manifolds.

Oleg Viro:

Triangulations of smooth manifolds

Let X be a smooth manifold of dimension n with a smooth triangulation T. Let σ be a simplex of T of dimension p. Then the normal space of T is equipped with a decomposition into simplicial cones consisting of vectors directed inside to the adjacent simplices of T. A subcomplex of T is called locally convex, if at each point of its boundary the cones corresponding to its simplices constitute a convex cone.

Theorem 1. If Σ is a simplex of T with $lk_T\Sigma$ simplicially isomorphic to the boundary of a q–dimensional simplex and $St_T\Sigma$ is locally convex, then the bistellar transformation of T centered at Σ produces a triangulation smooth with respect to the same smooth structure of X.

Theorem 2. If T is simplicially isomorphic to the boundary of an $(n+1)$–dimensional simplex then X is diffeomorphic to S^n.

Corollary. Any smooth triangulation of an exotic 7–dimensional sphere can not be transformed to a triangulation isomorphic to the triangulation of the boundary of the 8–simplex by bistellar transformations centered at simplexes with locally convex stars.

Pierre Vogel:

The Heisenberg group of a surface and Topological Quantum Field Theory

The Witten–Reshetikhin–Turaev invariants can be modified and generalized into invariants Z_p, associated to every oriented closed 3–manifold M equipped with a banded link and a p_1–structure (i.e. a null–homotopy of its first Pontrjagin class). This invariant takes values in an algebraic extension k_p of the ring $\mathbb{Z}[A, A^{-1}]$ quotiented by the cyclotomic polynomial $\phi_{2p}(A)$. It has nice properties with respect to surgery and change of p_1–structure or orientation and, as function of the link, it satisfies the Kauffmann skein relation (with the variable A).

In a complete formal way, one associates to this invariant a TQFT: for every closed surface Σ (equipped with a p_1–structure) there is a k_p–module $V_p(\Sigma)$, and for every compact 3–manifold M (equipped with a banded link and a p_1–structure) there is a canonical element $Z_p(M)$ in the module $V_p(\partial M)$.

One constructs a Heisenberg type group $\Gamma(\Sigma)$ which is a central extension of $H_1(\Sigma, \mathbb{Z}/2)$ by $\mathbb{Z}/4$. The elements of this group are represented by certain links in $\Sigma \times I$. By naturality, this links induce endomorphisms on $V_{2p}(\Sigma)$ and one gets an action of the group $\Gamma(\Sigma)$ on this module. By looking at the characters of this group one deduces a natural decomposition of the module $V_{2p}(\Sigma)$:

Theorem: *If p is odd, there is a natural decomposition:*

$$V_{2p}(\Sigma) \simeq V_2(\Sigma) \otimes V_p(\Sigma)$$

If p is even, there is a natural decomposition:

$$V_{2p}(\Sigma) \simeq \otimes_u V_p(\Sigma, u)$$

where the direct sum runs over all spin structures of Σ if p is divisible by 4 and all $\bmod 2$ *cohomology class of Σ if p is conguent to* $2 \bmod 4$.

The modules $V_p(\Sigma, u)$ are new modules depending on u, and produce other TQFT for spin manifolds and for manifolds equipped with a $\bmod 2$ cohomology class.

Jürgen Eichhorn:

Index theory on non-compact manifolds

Let (M^n, g) be an open, complete Riemannian manifold, $(\zeta, \nabla) \to M$ a Clifford bundle, D the associated generalized Dirac operator.

Assume $r_{\text{inj}}(M) > 0, |\nabla^i R| \le C_i, |(\nabla^\zeta)^i R^\zeta| \le D_i, 0 \le i \le k, k > n/2$.

Denote by $k_t(x,y)$ the heat kernel of e^{-tD^2}. $k_t(x,y)$ has an asymptotic expansion

$$k_t(x,y) \sim \sum_{k\geq 0} t^{(k-n)/2}\psi_k(x) \qquad (t \to 0+),$$

where $\psi_k \in \Omega(\mathrm{End}\zeta \otimes \Lambda^n T^*)$.

Assume additionally ζ and D graded, i.e. there is an involution η of ζ, $\eta^2 = \mathrm{id}$, $\zeta = \zeta_+ \oplus \zeta_-$, $D\eta + \eta D = 0$. Let $I_D = \mathrm{tr}(\eta\psi_n)$. Assume M has a so called regular exhaustion. This exhaustion defines a fundamental class $m \in ({}^bH^n)^*$ and $I_D \in {}^bH^n=$ bounded cohomology. D is elliptic and has a well defined Index $\mathrm{Ind}D \in K_0(U_{-\infty})$, $U_{-\infty} =$ quasilocal operators of order $-\infty$. An $A \in U_{-\infty}$ has a kernel k_A. Define a trace τ by $\tau(A) = \langle \mathrm{tr}(\eta k_A), m\rangle$ and set $\mathrm{ind}_a D = \tau(\mathrm{Ind}D)$.

Theorem: $\mathrm{ind}_a D = \mathrm{ind}_t D := \langle I_D, m\rangle$.

John Roe proved this for $k = \infty$. We dropped the assumption to $k > n/2$.

Mel Rothenberg:

Survey of recent developments in Reidemeister torsion

Classical Reidemeister torsion associates to a manifold M and a homomorphism $\chi : \pi = \pi_1(M) \to O(n)$ an invariant $\tau(M,\chi) \in R^+$ provided an associated complex is acyclic. This is done in two ways: A combinatorial way yielding τ_{PL} and an analytic–geometric way yielding τ_a.

The equivalence of the two is a famous Theorem of Cheeger and independently of Müller.

Modern work of Carey-Mathai, Lück-Rothenberg, Lott and Lott-Lück has extended this construction to certain families of infinite dimensional representations, most important the regular representation. The invariants are closely linked to L^2 invariants of $\widetilde{M}$. The invariants are closely linked to both geometry and homotopy of M, and yield for example for hyperbolic M, the volume of M.

Peter May:

Rings and modules in stable homotopy theory

An E_∞ ring spectrum is the nearest analog in stable homotopy theory of a commutative and associative ring. We have constructed a good category of module spectra over an E_∞ ring spectrum. The essential point is the definition of smash product modules $M \wedge_R N$ and function spectra modules $F_R(M,N)$. This new chapter in stable homotopy theory allows one to contemplate a serious translation of commutative ring theory into stable homotopy theory. Immediate aplications include new constructions of a variety of spectra usually obtained from MU by the Bass–Sullivan theory of manifolds with singularities and new universal coefficient and Künneth spectral sequences. One defines

$$\pi_q(M \wedge_R N) = \mathrm{Tor}^R_q(M,N) \quad \text{and} \quad \pi_{-q}F_R(M,N) = \mathrm{Ext}^q_R(M,N)\,.$$

On specialization to Eilenberg–MacLane spectra, these ralize the classical homological algebra functors, and in general there are spectral sequences starting in classical homological functors and having these groups as targets. The new theory has potential applications to Quillen's algebraic K–theory of rings, Waldhausen's algebraic K–theory of spaces, Bökstedt's topological Hochschild homology, as well as to stable homotopy theory. Especially interesting is the relation to the recent result of Hopkins and Miller that versions of the spectra $E(n)$ carry E_∞ ring structures. Other examples are S, HR for a commutative ring, kO, kU, MO, MU, KR for a commutative ring, and many others.

Ian Hambleton (joint work with E.K. Pedersen):

Bounded Topology and Non–Linear Similarity

Let G be a finite group. Two finite–dimensional real G–representations are *topologically similar* if there exists an equivariant homeomorphism $h : V_1 \to V_2$ between them. When V_1, V_2 are *not* linearly isomorphic, h is called a non–linear similarity. After choosing a G-invariant metric,the $S(V_i)$ inherit linear G–actions.

Observation (M. Steinberger?): V_1, V_2 are topologically similar ($V_1 \sim_t V_2$) if and only if $S(V_1)$ is $G-h$–cobordant to $S(V_2)$.

In the lecture I reviewed previous work by Cappell, Shaneson, Steinberger, Weinberger, West, Hsinang, Pardon, Madsen, Rothenberg. The aim of the talk was to give a construction of non-linear similarities using bounded toplogy.

If V, W are G-representations, $S(V \oplus W) = S(V) * S(W)$ and there is a natural map $S(V \oplus W) - S(W) \to W$ making the diagram (of G-maps) commute:

$$\begin{array}{ccc} S(V\oplus W) - S(W) & \approx & S(V) \times W \\ & \searrow \quad \swarrow \text{proj}_2 & \\ & W & \end{array}$$

where the last map is the second factor projection. After quotient by G, the pair $(S(V)\times_G W \to W/G)$ is a bounded manifold in the sense introduced by Ferry–Pedersen. Our main result is:

Theorem: Let V_1, V_2 be free representations of G. Then $V_1 \oplus W \sim_t V_2 \oplus W$, for some G-representation W, if and only if $(S(V_1) \times_G W \to W/G)$ and $(S(V_2) \times_G W \to W/G)$ are boundedly h-cobordant.

We then use the bounded surgery theory of Ferry-Pedersen (as extended in our paper in J.A.M.S. 4(1991)) and relate the existence of non-linear similarities to the vanishing of a transfer map:

$$\begin{array}{ccccccc} \to & L_n^h(\mathbb{Z}G) & \longrightarrow & \mathcal{S}(S(V_1)/G) & \longrightarrow & \mathcal{N}(S(V_1)/G) \\ & \Big\downarrow \text{trf}_W & & \Big\downarrow \text{trf}_W & & \Big\| \\ \to & L^h_{n+|W|}(\mathcal{C}_{W,G}(\mathbb{Z})) & \longrightarrow & \mathcal{S}^b\begin{pmatrix} S(V_1)\times_G W \\ \downarrow \\ W/G \end{pmatrix} & \longrightarrow & \mathcal{N}\begin{pmatrix} S(V_1)\times_G W \\ \downarrow \\ W/G \end{pmatrix} \end{array}$$

The upper row is the ordinary surgery exact sequence for structures on $S(V_1)/G$, and the lower row is the new bounded surgery exact sequence. The bounded L-group is the L-group of an additive category, and so has an algebraic description.

This formulation leads to new techniques for finding examples. A new collection of non-linear similarities was given for the quaternionic groups $G = Q(2^k p)$, p odd and $k \geq 4$.

Bob Oliver:

Decompositions of classifying spaces: a survey

For a compact Lie group G and a prime p, there are two decompositions of BG at the prime p which have been very useful in recent years for studying homotopy properties of BG:

A. [Jackowski–McClure] Let $A_p(G)$ be the category whose objects are subgroups $(\mathbb{Z}/p)^r \cong E \subseteq G$ $(r > 0)$, and where $\mathrm{Mor}_{A_p(G)}(E, E') \subseteq \mathrm{Hom}(E, E')$ is the subset of homomorphisms given by inclusions and conjugation. Then

$$BG \simeq_p \underset{E \in A_p(G)}{\underrightarrow{\mathrm{hocolim}}}\ BC_G(E)\,.$$

B. [Jackowski–McClure–Oliver] Let $R_p(G)$ be the category of orbits G/P, where P is p–toral, $N(P)/P$ is finite, and $O_p(N(P)/P) = 1$ (morphisms are G–maps). Then

$$BG \simeq_p \underset{G/P \in R_p(G)}{\underrightarrow{\mathrm{hocolim}}}\ EG \times_G (G/P)\,.$$

(where $EG \times_G G/P \cong EG/P \simeq BP$).

The second decomposition has been useful, for example, when describing maps between classifying spaces, and (in work by Notbohm) when showing the uniqueness of classifying spaces in many cases.

Elmer Rees:

Radon-Hurwitz revisited

Using Clifford algebras, there is a construction of a bilinear, norm-preserving map

$$R^{\rho(n)} \times R^n \to R^n$$

where $\rho(n) = 8b + c$ if $n = 2^a m$ with m odd and $a = 4b + c$, $0 \leq c \leq 3$. In 1923, Hurwitz and Radon, independently, showed that the dimension $\rho(n)$ cannot be increased. The construction gives a $\rho(n)$-dimensional linear subspace V of $\mathrm{Hom}(R^n, R^n)$ such that every non-zero matrix in V is non-singular. In 1962, Adams, Lax and Phillips proved, as a consequence of Adams' solution of the vector fields on spheres problem, that the dimension of any such V cannot be greater than $\rho(n)$. A more direct version of their proof is to consider the map $V \times R^n \to V \times R^n$ defined by $(A, x) \to (A, Ax)$. This map induces an isomorphism $n\lambda \to n\varepsilon$ of vector bundles over the projective space $\mathsf{P}(V)$; the result follows by a simple direct calculation using the fact that the reduced real K-theory of $\mathsf{P}(V)$ is cyclic of known order $2^{\varphi(d)}$ with $\lambda - \varepsilon$ as a generator.

The same method can be used to estimate the dimension of a linear subspace V of $\mathrm{Hom}(R^n, R^n)$ such that every non-zero matrix in V has given rank k. In this case there is an exact sequence

$$0 \to F \to n\lambda \to n\varepsilon \to G \to 0$$

where F and G are both vector bundles of dimensionn $n - k$. Using this one can deduce, when $k = n - 1$, that the dimension of V can be at most

$$\begin{array}{ll} \rho(n) & \text{when } n \text{ is even} \\ \rho(n-1) & n \equiv 1 \bmod 4 \\ \rho(n+1) & n \equiv 3 \bmod 4\,. \end{array}$$

These estimates are best possible except in the last case where spaces only of dimension $\rho(n) - 1$ can be constructed (for the two cases $n = 3, 7$, it can be shown that this lower esimate is the largest possible).

There is an algebraic geometric approach to the closely related problem of estimating the maximum dimension of linear spaces as above but with the condition that every non-zero matrix in V has rank $\geq k$. The method is to study the variety $\mathcal{X}_k$ of $n \times n$ real matrices of rank less than k. A linear space V of dimension complementary to that of $\mathcal{X}_k$ must intersect it if the degree of $\mathcal{X}_k$ is odd. In the simplest case one obtains the following result: Let $\ell = n - k + 1$ and choose s so that $\ell \leq 2^s < 2\ell$, and suppose that V is a linear space whose non-zero elements are $n \times n$ real matrices of rank at least k and if $n \equiv \pm\ell \bmod 2^{s+1}$ then the dimension of V is at most ℓ^2. A construction shows that, in these cases, the estimates cannot be improved.

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