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This Functional Equations meeting was held just a couple of weeks after the 30th anniversary of the first Functional Equations meeting, also in Oberwolfach. To celebrate the occasion, several survey papers were given which drew from different branches of Functional Equations. These highly successful talks, which presented recent theoretical results and applications, showed the growth and vitality of the discipline.

Professor Rätz, when opening the meeting, looked back over the years and recalled some of the highlights of past meetings and such notable participants as Otto Haupt, Alexander Ostrowski, Georg Aumann, Stanislaw Gołąb, Einar Hille, Børge Jessen, Joseph Kampé de Fériet, Helmut Kneser, Eugene Lukacs, Karl Menger, Olga Taussky, John Todd and William Tutte.

The organizing committee consisted of Professors János Aczél (Waterloo, Ontario), Walter Benz (Hamburg), Jürg Rätz (Bern), and Mark Taylor (Wolfville, Nova Scotia). The secretary of the symposium was Dr. Kazimierz Nikodem.

The 47 participants came from Austria, Canada, Czechoslovakia, Germany, Hungary, Italy, Kuwait, Poland, Russia, Spain, Switzerland, and the USA. There were fewer participants than in recent years, partly because of the Institute's request to restrict the number of participants and partly because of the start of the North American academic year. The quality of presentations, however, continued to be very high. The selection of participants particularly favored promising young members of the different Functional Equations schools.

Right after the opening, Professor Roman Ger paid tribute to the late Marek Kuczma and Zbigniew Gajda.

In addition to the survey talks, 35 regular talks were given. The talks encompassed the following topics: conditional equations; stability, convexity and other functional inequalities; functional equations in one and several variables; the translation equation and iteration; equations on algebraic structures; functional equations describing nowhere differentiable functions and those characterizing the theta function; spectral synthesis; regularization theorems and general methods; applications to information measures, statistics, quantum mechanics, gas dynamics, astrophysics, general relativity, geometry, architecture, and to the behavioral and social sciences.

As to the general emphasis of the discipline, not only is the theory by now highly developed and further developing, but in more and more other fields (e.g. mathematical economics and psychology, theory of measurement, dimensional analysis, physics, geometry, actuarial science) the essential role of Functional Equations is recognized by experts.

The highlights of the symposium included: characterizations of invariants of general relativity theory without differentiability and continuity assumptions, new foundations to dimensional analysis, crucial new developments in Ulam's stability theory which made it more powerful and suitable for applications. The symposium also saw two important results leading to the completion of the solution of the second part of Hilbert's Fifth Problem.

Based on the experiences of the meeting, several important goals became apparent. Among these are:

(1) Characterization of the groups and other structures for which the Cauchy equation is stable.

(2) Satisfactory theories of the inhomogeneous Cauchy equation

$$f(x+y) - f(x) - f(y) = h(x,y)$$

(in particular dependence of f upon the properties of h) and of the very general equation

$$\sum_{j=1}^{m} f_j(\phi_j(x) + \psi_j(y)) = \sum_{i=1}^{n} g_i(x)h_i(y)$$

should be developed.

(3) Describing orthogonally additive functions on more general spaces and without homogeneity assumptions concerning the orthogonality relation.

(4) Characterization, without continuity assumptions, of null-preserving mappings of Lorentz-Minkowskian manifolds in the spirit of A.D. Alexandrov's theorem.

(5) It is now known that the diagram of the general aggregation problem does not commute. In order to meet the needs of applications, an appropriate functional inequality with error estimation should be constructed.

Almost every one of the 18 scientific sessions was followed by a period devoted to remarks and open problems. These turned out to be even more lively than usual. It is remarkable that some problems posed here were solved during the symposium, as well as some problems left open at previous meetings.

A short general meeting discussed the format of future meetings, in particular with regard to survey talks and special sessions.

At the closing session the first ISFE medal for outstanding contribution to the meeting was awarded to Gyula Maksa. Then Professor Benz closed the meeting.

Abstracts of the talks follow in alphabetical order, and we conclude with lists of participants and of e-mail addresses.

ABSTRACTS

JÁNOS ACZÉL : *Some recent applications of functional equations to the social and behavioral sciences. Further problems.* (Survey)

Recent applications of functional equations to the questions of allocation, aggregation, utility, taxation, theories of measurement and dimensional analysis are discussed and open problems formulated.

CLAUDI ALSINA: *The bisectrix transform and the conditional Cauchy equation* $f(x+y) = f(x) + f(y)$ *whenever* $||x|| = ||y||$.

We study the functional equation of the bisectrix transform

$$f\left(\lambda\left(\frac{||x||_1}{||x||_1 + ||y||_1}\, y + \frac{||y||_1}{||x||_1 + ||y||_1}\, x\right)\right)$$
$$= g(\lambda)\left[\frac{||f(x)||_2}{||f(x)||_2 + ||f(y)||_2}\, f(y) + \frac{||f(y)||_2}{||f(x)||_2 + ||f(y)||_2}\, f(x)\right],$$

where $f : E_1 \to E_2$ is a mapping from the real normed space $(E_1, ||\cdot||_1)$ into another real normed space $(E_2, ||\cdot||_2)$, continuous at 0 and $f(x) = 0$ iff $x = 0$, while $g : \{0, \lambda_0, 1\} \to \{0, k_0, 1\}$ satisfies $g(0) = 0 < g(\lambda_0) = k_0 < g(1) = 1$. The general solution f is determined.

This problem has motivated a study (done jointly with J.L. Garcia-Roig) on the conditional Cauchy equation on rhombi: $f(x+y) = f(x) + f(y)$ if $||x|| = ||y||$. We show that, if $f : E \to F$ is a continuous mapping from a real inner product space of dimension greater than 1 into a topological real linear space, then f is a continuous linear transformation.

ROMAN BADORA: *On some generalized invariant means and their application to the stability of Hyers-Ulam type.*

We present some extensions of the concept of an invariant mean to a space of vector-valued functions defined on a semigroup. In particular, we prove that, if a semigroup S is amenable, then there exists a linear operator on a space of all bounded functions mapping S into a normed space having the binary intersection property (see M.M. Day, Normed linear spaces, Springer-Verlag, New York, 1973) which behaves like an invariant mean.

Next, for a function F defined on the Cartesian product of two amenable semigroups into a normed space having the binary intersection property, we study the stability of the following functional equation:

$$C^2C^1F = 0,$$

where C^i denotes the partial Cauchy difference with respect to the i-th variable ($i = 1, 2$). Applying the generalized invariant mean, we show that this equation is stable in the Hyers-Ulam sense.

MARIUSZ BAJGER: *An iterative Pexider equation.*

We consider the Pexider equation $F_{st} = H_s \circ G_t$ for (s,t) belonging to the domain of a binary operation on a groupoid K, where $\{F_t : t \in K\} \subset Z^X, \{G_t : t \in K\} \subset Y^X, \{H_t : t \in K\} \subset Z^Y$ are unknown families of functions. Conditions are established under which the equation may be reduced to the Cauchy equation. The solution of the equation is given in the case where there exists a unit element e in K and H_e is an injection, G_e is a surjection. Finally, using the above result, we solve the following problem: when does it follow from the equality $F_{st} = H_s \circ G_t$ for (s,t) belonging to a set $L \subsetneq R_+^2$ that $F_{st} = H_s \circ G_t$ for $(s,t) \in R_+^2$?

KAROL BARON: *Recent results on functional equations in a single variable, perspectives and open problems.* (Survey)

Two years ago Cambridge University Press published the by now well known monograph "Iterative Functional Equations" by Marek Kuczma, Bogdan Choczewski and Roman Ger. I discuss mainly papers which I consider important but which were published later and/or were not discussed in this book. No completeness is claimed (since 1980 the Mathematical Reviews published about 400 reviews of papers devoted to functional equations in a single variable.)

WALTER BENZ: *Invariants and mappings. Functional equations in geometry.* (Survey)

Let $M \neq \emptyset$ be a set and let G be a group of bijections of M. Then invariants and invariant notions in the sense of *Klein's Erlangen Program* can be defined. Questions like the following lead to functional equations: A) Find all mappings $f : M \to M$ preserving an invariant (an invariant notion), B) find invariants and invariant notions of (M, G). Results in this directions are by Aczél, Gołąb, Kuczma, Lester, Siwek, Beckman, Quarles, A.D. Alexandrov, and others. They concern the solutions of the functional equations involved. We are particularly interested in geometries (M, G) which are solutions of *Einstein's field equations* $R_{ij} = \lambda g_{ij} + \kappa T_{ij}$, like Einstein's cylinder universe, de Sitter's world and others. A very recent result concerns the 2-point-invariants of Einstein's cylinder universe C^n which are additive on C^n-lines and which behave locally Lorentz-Minkowskian. They are given by

$$d(x, y) = \sqrt{|\text{arc}\cos(x_1 y_1 + \cdots + x_n y_n)^2 - (x_{n+1} - y_{n+1})^2|}.$$

MARIO BONK: *The characterization of theta functions by functional equations.*

For every ϑ-function of one complex variable there are functions f_1, f_2, g_1, g_2 such that, for all $x, y \in C$,

$$(*) \qquad \vartheta(x+y)\vartheta(x-y) = f_1(x)g_1(y) + f_2(x)g_2(y).$$

With some obvious modifications, ϑ-functions are the only continuous solutions $\vartheta : R^n \to C$ of (*). This characterization theorem can be used to find the general continuous solution on R^n of the functional equation given by the following addition theorem of Weierstass's σ-function:

$$\begin{aligned} &\sigma(u+x)\sigma(u-x)\sigma(y+z)\sigma(y-z) + \\ &\sigma(u+y)\sigma(u-y)\sigma(z+x)\sigma(z-x) + \\ &\sigma(u+z)\sigma(u-z)\sigma(x+y)\sigma(x-y) \ = 0. \end{aligned}$$

ZOLTÁN BOROS: *Generalizations of the concept of completely additive functions.*

Z. Daróczy, I. Kátai and T. Szabó proved the following theorem: If the sequence $(\lambda_n) : \mathbb{N} \to \mathbb{R}$ with

$$\text{(i)} \ \ 0 < \lambda_{n+1} < \lambda_n \ , \ n \in \mathbb{N}, \ \ \text{and} \ \ \text{(ii)} \ \ \sum_{n=1}^{\infty} \lambda_n < \infty$$

is interval-filling, i.e. the set

$$S := \left\{ \sum_{n=1}^{\infty} \epsilon_n \lambda_n \mid \epsilon_n \in \{0,1\}, n = 1, 2, \ldots \right\}$$

is an interval, then any function $F : S \to \mathbb{R}$ satisfying

$$\text{(iii)} \quad F\left(\sum_{n=1}^{\infty} \epsilon_n \lambda_n \right) = \sum_{n=1}^{\infty} \epsilon_n F(\lambda_n) \quad \text{for} \quad (\epsilon_n) \in \{0,1\}^{\mathbb{N}}$$

is linear.

The author replaces in (i) and (ii) the sequence (λ_n) by $(|\lambda_n|)$, while the set $\{0,1\}$ is also replaced by an arbitrary (non-empty) finite subset of $\mathbb{R}$. Supposed that all tails

$$T^m(\lambda_n) := (\lambda_{m+1}, \lambda_{m+2}, \ldots) \ , \ m = 0, 1, 2 \ldots$$

of the sequence (λ_n) are interval-filling in this sense, any function $F : S \to \mathbb{R}$ satisfying the analogue of (iii) has to be linear.

JANUSZ BRZDĘK: *On some functional equations of Gołąb-Schinzel type.*

We present an application of the well known result of J. Aczél, describing the continuous, cancellative and associative binary operations on a real interval, to the problem of finding continuous solutions of some functional equations of Gołąb-Schinzel type.

JACEK CHMIELIŃSKI: *On the stability of the generalized orthogonality equation.*

This research concerns the generalized orthogonality equation investigated by C. Alsina and J.L. Garcia-Roig. Here we deal with mappings defined on the euclidean space $R^n (n \geq 2)$ satisfying the generalized orthogonality equation with some accuracy. In other words, a class of approximate solutions of the generalized orthogonality equation is defined and investigated. The following theorem contains the main result.

Theorem. *If $T : R^n \to R^n (n \geq 2)$ is an approximate solution of the generalized orthogonality equation, i.e., it satisfies, with $\epsilon \geq 0$, the double inequality*

$$\frac{1}{1+\epsilon} |u \circ v| \leq |T(u) \circ T(v)| \leq (1+\epsilon)|u \circ v| \quad for \quad u, v \in R^n$$

then there exists a mapping $T_* : R^n \to R^n$ satisfying the generalized orthogonality equation, i.e.,

$$|T_*(u) \circ T_*(v)| = |u \circ v| \quad for \quad u, v \in R^n$$

such that

$$||T(u) - T_*(u)|| \leq [(1+\epsilon)^{3/2} - 1] \quad \min\{||T(u)||, ||T_*(u)||\} \text{ for } u \in R^n.$$

ZOLTÁN DARÓCZY: *Functional equations of Abel type.*

Let $I :=]0,1[$ and $x \circ y := (1-x)/(1-xy)$ for every $x, y \in I$. A function $h : I \to \mathbf{R}$ is said to be an Abel type function if it satisfies the functional equation

$$h(x \circ y) + h(y \circ x) = A(x,y) \quad (x, y \in I), \tag{1}$$

where the function $A : I \times I \to \mathbb{R}$ has a given structure. Then (1) is called a functional equation of Abel type. The following cases are investigated:

$$(a)\ A(x,y) = g(xy), \quad (b) A(x,y) = g(x) + g(y), \quad (c) A(x,y) = g(xy) - g(x) - g(y),$$

where $g : I \to \mathbb{R}$ is an unknown function too. Some additional properties of h are also supposed.

WOLFGANG FÖRG-ROB: *The generalized cosine equation* $\sum_{k=0}^{n-1} f(x+\sigma^k y) = nf(x)f(y)$.

This equation was treated in my talk at the 29th ISFE meeting in Canada, 1991: Here we deal with the following questions:

1) Answers to the questions connected to the 1991 talk.

2) Solutions after dropping some of the assumptions.

3) A generalization to an arbitrary finite group of automorphisms.

4) Further results.

GIAN LUIGI FORTI: *Stability of functional equations.* (Survey)

This talk is devoted exclusively to equations in several variables. As by now traditional, the well known problem proposed by S.M. Ulam in 1940 and the result of Hyers in 1941 are taken as starting points.

The main streams of research on stability can be roughly classified as follows: stability of the Cauchy equation in more general settings; stability for other equations or systems; superstability; other definitions of stability and their mutual relations; stability of functional inequalities (convexity, etc.).

In the talk results in the first two fields, about the inequalities

$$||f(x+y)-f(x)-f(y)|| \le \varphi(x,y) \quad \textit{and} \quad ||f(x+y)-f(x)f(y)|| < \epsilon\,,$$

are presented and compared.

For each of them some open problems are stated. Further attention is directed to some relatively new problems about the "perturbations in Banach algebras".

JAIME-LUIS GARCIA-ROIG: *On a system of functional equations in connection with linear normed spaces.*

Following [1] and [2], we study, for a map $f: E_1 \to E_2$ between two real linear normed spaces E_1, E_2, the system of functional equations

(i) $f(tu) = tf(u)$, for all real $t > 0$ (and u in E_1),

(ii) $f(u+v) = f(u) + f(v)$, whenever $||u|| = ||v||$, and

(iii) $||f(u)||_2 = K||u||_1$, for some positive constant K.

The existence of non-trivial solutions imposes severe conditions on both the possible maps f and on the possible norms.

We observe, however, that the solutions of the above system, in the case where both E_1 and E_2 are inner product spaces, are easily obtained (cf. [1] and [2]).

REFERENCES

[1] ALSINA, C., On the functional equation of the bisectrix transform. Manuscript, 1991.

[2] ALSINA, C., GARCIA-ROIG, J-L, On a conditional Cauchy equation on rhombi. Manuscript, 1991.

ROMAN GER: *Multiplicative abstract monomials.*

Multiplicative quadratic selfmappings of the real line were described by C. Hammer and P. Volkmann in their paper "Die multiplikative Lösungen der Parallelogrammgleichung" (Abh. Math. Sem. Univ. Hamburg 61 (1991), 197-201). Because of the existence of discontinuous automorphisms of the complex plane, a nontrivial multiplicative quadratic function on $\mathbb{R}$ need not be a quadratic monomial.

We look for additional conditions eliminating such a phenomenon and we investigate a similar problem for abstract monomials of higher orders.

ROLAND GIRGENSOHN: *Functional equations and nowhere differentiable functions.*

For fixed $b \in \mathbb{N} \setminus \{1\}$, we consider the following system (F) of b functional equations for an unknown function $f : [0,1] \to \mathbb{R}$:

$$f\left(\frac{x+\nu}{b}\right) = a_\nu f(x) + g_\nu(x) \quad \text{for} \quad \nu = 0, \ldots, b-1. \tag{F}$$

Here, $a_0, \ldots, a_{b-1} \in \mathbb{R}$ are given constants with $|a_\nu| < 1$ and $g_0, \ldots, g_{b-1} : [0,1] \to \mathbb{R}$ are given functions.

Some of the most prominent nowhere differentiable functions (such as Weierstrass or van der Waerden functions) can be characterized as solutions of the system (F). Moreover, the non-differentiability of those functions can be shown in a uniform way: It is possible to compute certain Schauder expansions for solutions of (F) and then to infer differentiability conditions from the Schauder coefficients.

This method allows us to state criteria for a system of type (F) to have a non-differentiable solution.

DETLEF GRONAU: *An asymptotic formula for the iterates of a function and related functional equations.*

Let $\mathbb{K}$ be either $\mathbb{R}$ or $\mathbb{C}$ and $D \subseteq \mathbb{K}$ an open set containing 0 and starlike with respect to 0 (thus an open interval containing 0 in the case $\mathbb{K} = \mathbb{R}$). Suppose that the continuous function $f : D \to \mathbb{K}$ with fixed point has the following asymptotic formula for the kn-th iterates of f:

$$f^{(kn)}\left(\frac{x}{n}\right) = \sum_{i=1}^{r} \frac{1}{(nk)^i} f_i(kx) + o\left(\frac{1}{n^r}\right), \tag{1}$$

for $n \to \infty, k, n$ and r being positive integers and x close enough to 0. Then one can derive from (1) some functional equations for the f_i's.

The first "asymptotic approximation" f_1 satisfies the functional equation

$$\frac{1}{(k_m)} f_1((k+m)x) = k\frac{1}{k} f_1\left(\frac{k}{m} f_1(mx)\right),$$

which is equivalent to

$$f_1^{(m)}(x) = \frac{1}{m} f_1(mx). \tag{2}$$

Its general analytic solution is

$$f_1(x) = \frac{x}{1-ax} \ (a \in \mathbb{K}).$$

The function f_2 satisfies

$$\frac{1}{(k+m)^2} f_2((k+m)x) = \frac{1}{k^2} f_2\left(\frac{k}{m} f_1(mx)\right) + \frac{1}{(k+m)^2} \frac{f_1((k+m)x)^2}{f_1(mx)^2} f_2(mx),$$

which can be transformed to a conditional logarithmic functional equation. The solution is given, under certain regularity conditions by

$$f_2(x) = f_1(x)^2 b \ln(1-ax), \ (b \in \mathbb{K}).$$

HEIKO GROSS: *On the characterization of inset information measures by the sum property and by additivity conditions.*

We present a brief survey concerning the characterization of k-dimensional inset information measures on the open domain by generalized sum property and additivity conditions.

Here "inset" means that the considered information measure depends both on the probabilities and on the events.

ANTAL JÁRAI AND LÁSZLÓ SZÉKELYHIDI: *General methods and regularization in the theory of functional equations.* (Survey)

In the talk a summary of recent results on general and regularization methods for functional equations is presented. In the first part polynomial and related equations, quadratic equations and equations of d'Alembert type are considered. Results depending on spectral synthesis are also mentioned. The second part is devoted to the study of regularity properties of solutions of functional equations and to their improvements. Open problems are stated as well.

HANS-HEINRICH KAIRIES: *On the structure of replicative functions.*

Let be $D \subset \mathbb{R}$ and $S \subset \mathbb{N}$. A function $f : D \to \mathbb{R}$ is called replicative of type (D, S, u, v) if there are functions $u, v : S \to \mathbb{R}$ such that, for every $x \in D$ and $k \in S$,

$$\frac{1}{k} \sum_{\nu=0}^{k-1} f\left(\frac{x+\nu}{k}\right) = u(k)f(x) + v(k)$$

holds. In the case $S = \mathbb{N}$ it is sufficient to consider the types $(D, \mathbb{N}, u, 0)$ with a completely multiplicative $u : \mathbb{N} \to \mathbb{R}$ and $(D, \mathbb{N}, 1, v)$ with a completely additive $v : \mathbb{N} \to \mathbb{R}$.

There exist very regular and extremely pathological replicative functions. E.g., for $u(k) = 1/k$ both the Bernoulli polynomial B_1 and every discontinuous additive $f : \mathbb{R} \to \mathbb{R}$ with $f(1) = 0$ are of type $(\mathbb{R}, \mathbb{N}, u, 0)$. In between there are replicative functions in many interesting function spaces. We report on this and give several general existence and uniqueness statements.

PALANIAPPAN KANNAPPAN: *Inner product spaces and functional equations.* (Joint work with B.R. Ebanks and P.K. Sahoo)

We determine the general solutions of the functional equation

$$f_1(x+y) \ + \ f_2(x-y) \ = \ f_3(x) \ + \ f_4(y) \quad (x, y \in G)$$

for $f_i : G \to F$ $(i = 1, 2, 3, 4)$, where G is a 2-divisible group and F is a commutative field of characteristic different from 2. The motivation for studying this equation came from a result due to Drygas who proved a Jordan - von Neumann type characterization theorem for a "quasi-inner" product. This equation is a generalization of the quadratic functional equation investigated by several authors in connection with inner product spaces and their generalizations. Special cases of this equation include the Cauchy equation, the Jensen equation, the Pexider equation and many more. Here we determine the general solution of this equation without any regularity assumptions on f_i.

ALEXANDER KHOLODOV: *Exactly solvable functional equations.*

We determine all regular solutions of the functional equation

$$g(x+y) = g(x)g(y)H(f(x)f(y)),$$

We also determine all regular solutions of the functional equation

$$u(x+y) = (u(x)v(y)+u(y)v(x))H(f(x)f(y)),$$

where f is a solution of the first equation.

KÁROLY LAJKÓ: *A functional equation related to the characterization of theta functions.*

The functional equation

$$f(x)g(y) = h(ax+by)k(cx+dy), \tag{1}$$

where f, g, h, k are real (or complex) valued functions of a real variable or functions defined on $\mathbb{R}^n$ and a, b, c, d are fixed non-zero real numbers or non-singular square matrices of order n, respectively, was considered, among others, by J.A. Baker, I. Ecsedi, K. Lajkó and in special cases by J. Aczél, R.L. Hall and A. Lundberg. Equation (1) has applications to the quantum mechanical three-body problem and to the characterization of normal distributions.

Recently, M. Bonk has obtained an application of a special case of (1) to the characterization of theta functions. In this talk some new results concerning the functional equation (1) are presented.

LÁSZLÓ LOSONCZI: *Structure theorems for sum form functional equations.*

Let $k, \ell \geq 2, N$ be natural numbers and suppose that the functions $f_{ij}, g_{it}, h_{jt} :]0,1[\to \mathbb{C}$ $(i = 1,\ldots,k; j = 1,\ldots,\ell;\ t = 1,\ldots,N)$ satisfy the functional equation

$$\sum_{i=1}^{k}\sum_{j=1}^{\ell}[f_{ij}(x_iy_j) - \sum_{t=1}^{N} g_{it}(x_i)h_{jt}(y_j)] = 0$$

for all $x_i, y_j > 0$ with $x_1 + \cdots + x_k = y_1 + \cdots + y_\ell = 1$.

If the functions $f_{ij}(i = 1,\ldots,k;\ j = 1,\ldots,\ell)$ are measurable on $]0,1[$ then there exist distinct complex numbers $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 2, \ldots, \lambda_M$ and natural numbers $m_1,\ldots,m_M$ such that each f_{ij} is in the linear space spanned by the functions

$$x \mapsto x^{\lambda_p}(\log x)^q \quad (\ x \in]0,1[\ ;\ p = 1,\ldots,M;\ q = 0,\ldots,m_p - 1).$$

An upper bound is given for $m_1 + \cdots + m_M$, the dimension of this linear space. Under suitable conditions also the explicit form of the functions g_{it}, h_{jt} can be given.

GYULA MAKSA: *Results on t-Wright convexity.*
(Joint work with K. Nikodem and Zs. Páles).

Let X be a real linear space, D a convex non-empty subset of X and t a fixed number from $]0,1[$. A function $f : D \to R$ is said to be t-Wright convex if

$$f(tx + (1-t)y) + f((1-t)x + ty) \leq f(x) + f(y) \quad x, y \in D.$$

J. Matkowski posed the following problem: Is every t-Wright convex function, with a fixed t from $]0,1[$, Jensen convex?

In this talk we show that the answer is positive for all rational t from $]0,1[$ and for certain algebraic numbers, however it is negative for all those values of t that are either transcendental or there is an algebraic conjugate s of t such that $|s - 1/2| \geq 1/2$.

JANUSZ MATKOWSKI: *On subadditive functions.*
(Joint work with T. Świątkowski)

The main result is that *every one-to-one subadditive function* $f : (0,\infty) \to (0,\infty)$, *such that*

$$(*) \qquad \lim_{t\to\infty} f(t) = 0,$$

is continuous everywhere. This improves our recent result (cf. [1]) where f was assumed to be bijective. We construct a broad class of discontinuous subadditive bijections of $(0,\infty)$

which are bounded in every vicinity of 0. In the construction we use some new criterions of subadditivity.

Suppose that $f : (0, \infty) \to \mathbb{R}$ is subadditive and satisfies (*). We examine the conditions under which there exists an even subadditive function $F : \mathbb{R} \to \mathbb{R}$ such that $f = F|_{(0,\infty)}$.

Some applications are also mentioned.

REFERENCE

[1] MATKOWSKI, J. and ŚWIĄTKOWSKI, T., Quasi-monotonicity, subadditive bijections of $\mathbb{R}_+$, and characterization of L^p-norm. J. Math. Anal. Appl. 154 (1991), 493-506.

ZENON MOSZNER AND GYÖRGY TARGONSKI: *General theory of the translation equation and a survey of recent results and problems in iteration theory.* (Survey)

We give a survey of results in the general theory of the translation (transformation) equation

$$F(F(\alpha, x), y) = F(\alpha, x \star y)$$

where $F : \Gamma \times G \supset D_F \to \Gamma$ and $\star : G \times G \supset D_\star \to G$, which appeared after 1973 and some open problems. This survey is a continuation of the paper of Z. Moszner, The translation equation and its application, Demonstratio Math. *6* (1973), 309-327.

In the second part of the lecture we survey results in iteration theory during the past decade. The topics range from commuting real functions to cellular automata. A number unsolved problems is discussed.

FRANTIŠEK NEUMAN: *Functional and differential equations.*

In the first part of his 5th problem Hilbert asked: Is every locally euclidean topological group a Lie group, i.e., does it have a parameterization in which the group operations are analytic? The affirmative answer was obtained, after consecutive partial results of several mathematicians, by von Neumann, Pontrjagin, Chevalley, Gleason, Yamabe, Montgomery and Zippin between 1933 and 1953.

Inspired by this, one may ask whether it is possible to transform ("reparametrize") a linear differential equation into another one with smoother (analytic) coefficients. For second order equations, continuous coefficients can always be transformed into analytic ones. This is a correct, however rather misleading result, because for $n \geq 3$ the answer is negative, in general. C^{n-2} is the order that ensures the affirmative answer; $C^{n-2} = C^{0}$ for $n = 2$. Lower orders cannot be improved, they are invariants.

Functional equations play an important role in the proof of this result.

KAZIMIERZ NIKODEM: *Some functional inequalities connected with convex functions.* (Joint work with K. Baron and J. Matkowski).

We consider the functional inequality

$$f(tx + (1-t)y) \leq tg(x) + (1-t)g(y). \tag{1}$$

Theorem 1. *Let $I \subset \mathbb{R}$ be an interval. Functions $f, g : I \to \mathbb{R}$ satisfy the inequality* (1) *for all $t \in (0,1)$ and $x, y \in I$ if, and only if, there exists a convex function $h : I \to \mathbb{R}$ such that*

$$f(x) \leq h(x) \leq g(x)\ ,\ x \in I.$$

As a consequence of the above result we obtain the following.

Theorem 2. *Let λ be a positive number. A function $f : [0, \infty) \to \mathbb{R}$ satisfies the inequality*

$$f(tx + (\lambda - t)y) \leq tf(x) + (\lambda - t)f(y) \tag{2}$$

for all $t \in (0, \lambda)$ and $x, y \in [0, \infty)$ if, and only if, there exists a convex function $g : [0, \infty) \to \mathbb{R}$ such that

$$\hat{g}(x) \leq f(x) \leq g(x)\ ,\ x \in [0, \infty),$$

where $\hat{g}(x) := \lambda^{-1} g(\lambda x)$.

LUIGI PAGANONI AND JŪRG RĀTZ: *Conditional functional equations and orthogonal additivity* (Survey)

Some examples of classes of conditional equations coming from information theory, geometry and from the social and behavioural sciences are presented. Then the classical case of the Cauchy equation on a restricted domain Ω is extensively discussed. Some results concerning the extension of local homomorphisms and the implication "Ω-additivity implies global additivity" are illustrated. Problems concerning the equations

$$[cf(x+y)-af(x)-bf(y)-d][f(x+y)-f(x0-f(y)]=0$$
$$[g(x+y)-g(x)-g(y)][f(x+y)-f(x)-f(y)]=0$$
$$f(x+y)-f(x)-f(y)\in V \quad \text{(a suitable subset of the range)}$$

are presented.

The consideration of the conditional Cauchy equation is subsequently focussed to the case when it makes sense to interpret Ω as a binary relation:

$$f:(X,+,\perp)\to(Y,+)\,;\ f(x+z) \;=\; f(x)+f(z)\ \ (\forall x,z\in Z; x\perp z).$$

A brief sketch on solutions under regularity conditions is given. It is then shown that all regularity conditions can be removed. Finally, several applications (also to physics and to the actuarial sciences) are discussed. In all these cases the attention is focused on open problems and possible extensions of previous results.

ZSOLT PÁLES: *Symmetric functional inequalities.*

In the talk we deal with the functional inequality

$$(*) \qquad F(x_1,\cdots,x_n)\le f(x_1)+\cdots+f(x_n).$$

where $F: I_i\times\cdots\times I_n\to\mathbb{R}$ is a given symmetric C^1 function with the property that

$$x_j\mapsto \partial_i F(x_1,\cdots,x_j,\cdots,x_n)\ \ (x_j\in I_j)$$

is a nondecreasing function for each fixed $x_1,\cdots x_{j-1},x_{j+1},\cdots,x_n$, while the functions $f_i: I_i\to\mathbb{R}$ are unknown ($I_1,\cdots,I_n$ are compact intervals of $\mathbb{R}$). The solutions of (*) are completely described. As application, we obtain the arithmetic-geometric mean inequality and Young's and Jensen's inequalities.

LUDWIG REICH: *Ein Kriterium für die Existenz iterativer Wurzeln von Potenzreihentransformation und Verteilungsfragen.*

Wir setzen unsere Untersuchungen über die Verteilung iterierbarer Potenzreihentransformationen in einer Unbestimmten bezüglich der schwachen und der starken Topologie fort, hier für Transformationen mit iterativen Wurzeln einer gegebenen Ordnung. Wesentlich ist als neues Hilfsmittel ein Kriterium über die Existenz iterativer Wurzeln einer Potenzreihentransformation gegebener Ordnung und mit gegebenem Linearteil, welches verwandt ist mit einem ähnlichen Kriterium von Schwaiger-Scheinberg.

MACIEJ SABLIK: *New results on an equation of Abel.*

We continue investigating the functional equation

$$(E) \qquad \psi[xf(y)+yf(x)] = \varphi(x)+\varphi(y)$$

which was first considered by N.H. Abel in [1]. In the paper [2] we gave a complete list of continuous solutions of (E) in the case where x and y were in a real interval I containing 0. The purpose of the present work is to solve (E) without assuming that 0 is in I. Instead we assume that $0 \in f(I)$.

[1] ABEL, N.H., Sur les fonctions qui satisfont à l'équation $\varphi x + \varphi y = \psi(xfy + yfx)$. Oeuvres Complètes de N.H. Abel rédigées par ordre du roi par B. Holmboe, Christiania, 1839, tome premier, 103-110. Original version in J. Reine Angew. Math. 21 (1827), 386-394.

[2] SABLIK, M., The continuous solution of a functional equation of Abel. Aequationes Math. 39 (1990), 19-39.

WOLFGANG SANDER: *Sum form information measures.*

We give a survey on recent results concerning generalized additive sum form information measures. There are only few open problems in this area and it seems that they could be solved if the general solution of the following functional equation would be determined:

$$f(xy)+f(x(1-y))+f((1-x)y)+f((1-x)(1-y)) = 0 \quad (x,y \in (0,1)^n).$$

JENS SCHWAIGER: *On the stability properties of a system of functional equations for generalized trigonometric functions.*
(Joint work with W. Förg-Rob)

Let G be an abelian group, $n \geq 2$ an integer, σ an automorphism of G with $\sigma^n = \mathrm{id}_G, \omega$ a primitive n-th root of unity in $\mathbb{C}$, and t an integer such that $1 \leq t \leq n$. Let us consider functions $f_j : G \to \mathbb{C}, 0 \le j \le n-1$ and the vector $F :=^t (f_0, f_1, \cdots, f_{n-1})$. Then we define $A := A_F :=^t (A_F^0, A_F^1, \ldots, A_F^{n-1})$ by

$$A_F^j(x,y) := f_j(x + \sigma^t(y)) -$$
$$- \left(\sum_{\ell=0}^{j} \omega^{(j-\ell)^t} f_\ell(x) f_{j-\ell}(y) - \sum_{\ell=j+1}^{n-1} \omega^{(n+j-\ell)^t} f_\ell(x) f_{n+j-\ell}(y)\right)$$

for $0 \le j \le n-1$.

We investigate the solutions of the inequalities

$$||A_F(x,y)|| \le \epsilon \text{ and } ||A_F(x,y)|| \le b(x), \quad (x, y \in G)$$

where ϵ is a nonnegative real number, $||\cdot||$ a norm on $\mathbb{C}^n$, and $b : G \to \mathbb{R}_+$ some given function.

ABE SKLAR: *The generalized translation equation.*
(Joint work with D. Zupnik).

We take the generalized translation equation in the form

$$(*) \qquad F(s, F(t,x)) = F(G(s,t), x)$$

for all s, t in T and x in X, where F is a function from $T \times X$ into X and G is a groupoid on T, i.e. a function from $T \times T$ into T. We consider the following problems: (1) Given G, characterize the F's such that (*) holds. (2) Relate G, which need not be associative, to groupoids that are, e.g., semigroups. (3) Relate F to semigroups, in particular, by characterizing the F's where left-multiplications (the functions f_t on X defined by $f_t(x) = F(t,x)$) are precisely the left-multiplications of semigroups on X.

GYÖRGY SZABÓ: *Orthogonally quadratic mappings on normed spaces.*

Mappings that are quadratic on orthogonal pairs of vectors have been studied by several authors. In [1] a general approach was given to this problem, using sesquilinear orthogonality. Under some weak conditions, such a mapping is unconditionally quadratic. Here we consider this question on a real normed space equipped with the well-known Birkhoff-James orthogonality. Even in this more complicated situation some analogous results are valid.

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JOZEF TABOR: *Quasi-linear mappings.*

Let E_1, E_2 be real normed spaces and $\epsilon \in [0,1)$. A mapping $f : E_1 \to E_2$ is called quasi-linear if it satisfies the following inequalities

(1) $\quad ||f(x+y) - f(x) - f(y)|| \le \epsilon \min\{||f(x+y)||, ||f(x)+f(y)||\}$ for $x, y \in E_1$,

(2) $\quad ||f(\alpha x) - \alpha f(x)|| \le \epsilon \min\{||f(\alpha x)||, |\alpha|\ ||f(x)||$ for $x \in E_1, \alpha \in R$.

Some basic theorems concerning linear mappings also hold for quasi-linear ones.

The problem of the existence of linear approximates for quasi-linear mappings is also considered.

MARK TAYLOR: *Some irreducible balanced equations.*

Let $W_1 = W_2$ be an equation in terms of a single n-ary operation. This equation is said to be Belousov if, for every subterm u_i of w_i, there exists a unique subterm v_j of W_j containing precisely the same variables, $i, j \in \{1,2\}$. Krapež and Taylor determined that the irreducible Belousov equations on binary quasigroups were characterized by certain polynomials over $\mathbb{Z}_2$. Later Taylor showed that every nonempty system of ternary Belousov equations was equivalent to either a single irreducible equation or a pair of specific equations.

In this paper conditions are given which characterize the irreducible Belousov equations for ternary quasigroups by certain polynomials over $\mathbb{Z}_3$. The result is generalized for p-ary (p prime) quasigroup operations in terms of $\mathbb{Z}_p[x]$.

MARIA SANTOS TOMÁS: *On heights and norms and some characterizations of inner product spaces.*
(Joint work with C. Alsina and P. Guijarro).

In a real normed space $(E, ||\ ||)$ we consider the one-sided derivatives $\rho'_\pm$ of the convex functional $\rho(x) = ||x||^2/2$.

The mappings ρ'_+ play a crucial role in characterizing inner product spaces. In fact, when the norm is derivable from an inner product $(E, <>)$, then $\rho'_\pm(x,y) = <x, y>$.

Using these mappings, we have considered several notions of a height of a triangle and we have studied them in relation with characterizations of inner product spaces.

PETER VOLKMANN: *Eine Charakterisierung von polynomialen Funktionen mittels der Dinghasschen Intervall-Derivierten.*
(Gemeinsame Arbeit mit A. Simon).

Es sei $J \subseteq \mathbb{R}$ ein nichtausgeartetes Intervall, $f : J \to \mathbb{R}$ und $\Delta_h f(x) = f(x+h) - f(x)$.

Satz. $\Delta_h^n f(x) = 0 (x, x+nh \in J)$ *ist äquivalent zu*

$$D^n f(x) := \lim_{\substack{\alpha \leq x \leq \beta \\ \beta - \alpha \downarrow 0}} \left(\frac{n}{\beta-\alpha}\right)^n \Delta^n_{(\beta-\alpha)/n} f(\alpha) \;=\; 0 \quad (x \in J).$$

MAREK CEZARY ZDUN: *Commuting functions and simultaneous Abel equations.*
(Joint work with W. Jarczyk and K. Loskot)

Let T be a non-void set and let $-\infty \leq a < b \leq \infty$. We consider the continuous solutions of the system of Abel equations

$$\alpha(f_t(x)) \;=\; \alpha(x) \;+\; \lambda(t)\;,\; t \in T\;,\; x \in (a,b),$$

where $f_t, t \in T$ are homeomorphisms mapping the interval (a,b) onto itself, which have no fixed points and are pairwise commuting, and $\lambda : T \to \mathbb{R}$ is a given function.

In particular, we show that, for every rational flow of homeomorphisms $\{f^t, t \in \mathbb{Q}\}$ (i.e. $f^t \circ f^s \;=\; f^{t+s}, t, s \in \mathbb{Q}$), mapping (a,b) onto itself without fixed points, there exists a continuous solution α, unique up to an additive constant, of the system of Abel equations

$$\alpha(f^t(x)) \;=\; \alpha(x) \;+\; t\;,\; t \in \mathbb{Q}.$$

This solution is monotonic.

Berichterstatter: Kazimierz Nikodem

LIST OF PARTICIPANTS

Prof.Dr. János Aczel
Dept. of Pure Mathematics
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof.Dr. Walter Benz
Mathematisches Seminar
Universität Hamburg
Bundesstr. 55

W-2000 Hamburg 13
GERMANY

Prof.Dr. Claudi Alsina
Departament de Matematicas ETSAB
Universidad Politecnica de Catalunya
Diagonal 649

E-08028 Barcelona

Mario Bonk
Institut für Analysis
Abt. für Topologie und Grundlagen der Analysis, Uni Braunschweig
Pockelsstr. 14

W-3300 Braunschweig
GERMANY

Prof.Dr. Roman Badora
Institute of Mathematics
Silesian University
Bankowa 14

40-007 Katowice
POLAND

Prof.Dr. Zoltan Boros
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. Mariusz Bajger
Inst. of Mathematics
Pedagogical University
Rejtana 16A

35-310 Rzeszow
POLAND

Prof.Dr. Janusz Brzdek
Dept of Math.
Pedagogical University

35-959 Rzeszow
POLAND

Prof.Dr. Karol Baron
Institute of Mathematics
Silesian University
Bankowa 14

40-007 Katowice
POLAND

Prof.Dr. Jacek Chmielinski
Instytut Matematyki
Wyzsza Szkola Pedagogiczna
ul. Podchorazych 2

30-084 Krakow
POLAND

Prof.Dr. Bogdan Choczewski
Institute of Mathematics
University of Mining and Metallurgy
Al. Mickiewicza 30

30059 Kraków
POLAND

Prof.Dr. Jaime-Luis Garcia-Roig
Departament de Matematicas ETSAB
Universidad Politecnica de
Catalunya
Diagonal 649

E-08028 Barcelona

Prof.Dr. Zoltán Daróczy
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. Roman Ger
Institute of Mathematics
Silesian University
Bankowa 14

40-007 Katowice
POLAND

Prof.Dr. Wolfgang Eichhorn
Institut für Wirtschaftstheorie
und Operations Research
Universität Karlsruhe
Kaiserstraße 12

W-7500 Karlsruhe 1
GERMANY

Dr. Roland Girgensohn
Institut für Mathematik
TU Clausthal
Erzstr. 1

W-3392 Clausthal-Zellerfeld 1
GERMANY

Prof.Dr. Wolfgang Förg-Rob
Institut für Mathematik
Universität Innsbruck
Technikerstr. 25/7

A-6020 Innsbruck

Prof.Dr. Detlef Gronau
Institut für Mathematik der
Karl-Franzens-Universität
Heinrichstraße 36

A-8010 Graz

Prof.Dr. Gian Luigi Forti
Dipartimento di Matematica
Universita di Milano
Via C. Saldini, 50

I-20133 Milano

Heiko Groß
Institut für Analysis
TU Braunschweig
Pockelsstr. 14

W-3300 Braunschweig
GERMANY

Prof.Dr. Antal Jarai
Fachbereich Mathematik-Informatik
Universität-GH-Paderborn
Warburger Str. 100

W-4790 Paderborn
GERMANY

Prof.Dr. Laszlo Losonczi
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. Hans-Heinrich Kairies
Institut für Mathematik
TU Clausthal
Erzstr. 1

W-3392 Clausthal-Zellerfeld 1
GERMANY

Prof.Dr. Gyula Maksa
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. P. Kannappan
Dept. of Pure Mathematics
University of Waterloo

Waterloo ON N2L 3G1
CANADA

Prof.Dr. Janusz Matkowski
Dept. of Mathematics
Techn. Univ.
Willowa 2

43 309 Bielsko-Biała
POLAND

Prof.Dr. Alexander Kholodov
Institute Problems in Cybernetics
Vavilova Str. 37

117312 Moscow
RUSSIA

Prof.Dr. Zenon Moszner
Inst. of Math.
Pedagogical Univ.
ul. Podchorazych 2

30-084 Krakow
POLAND

Prof.Dr. Karoly Lajko
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. Frantisek Neuman
Mathematics Institute
Czechoslovak Academy of Sciences
Mendelovo nam. 1

60300 Brno
CZECHOSLOVAKIA

Prof.Dr. Kazimierz Nikodem
Dept.of Mathematics
Techn. Univ.
Willowa 2

43 309 Bielsko-Biala
POLAND

Prof.Dr. Maciej Sablik
Institute of Mathematics
Silesian University
Bankowa 14

40-007 Katowice
POLAND

Prof.Dr. Luigi Paganoni
Dipartimento di Matematica
Universita di Milano
Via C. Saldini, 50

I-20133 Milano

Prof.Dr. Wolfgang Sander
Institut für Analysis
TU Braunschweig
Pockelsstr. 14

W-3300 Braunschweig
GERMANY

Prof.Dr. Zsolt Páles
Fachbereich Mathematik
Universität des Saarlandes
Bau 27

W-6600 Saarbrücken
GERMANY

Prof.Dr. Jens Schwaiger
Institut für Mathematik der
Karl-Franzens-Universität
Heinrichstraße 36

A-8010 Graz

Prof.Dr. Jürg Rätz
Mathematisches Institut
Universität Bern
Sidlerstr. 5

CH-3012 Bern

Prof.Dr. Abe Sklar
Dept. of Mathematics
Illinois Institute of Technology

Chicago , IL 60616
USA

Prof.Dr. Ludwig Reich
Institut für Mathematik der
Karl-Franzens-Universität
Heinrichstraße 36

A-8010 Graz

Prof.Dr. György Szabó
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. Laszlo Szekelyhidi
Institute of Mathematics
Lajos Kossuth University
Pf. 12

H-4010 Debrecen

Prof.Dr. Maria Santos Tomas
Departament de Matematicas ETSAB
Universidad Politecnica de
Catalunya
Diagonal 649

E-08028 Barcelona

Prof.Dr. Jozef Tabor
Inst. of Math.
Pedagogical Univ.
ul. Podchorazych 2

30-084 Kraków
POLAND

Prof.Dr. Peter Volkmann
Mathematisches Institut I
Universität Karlsruhe
Postfach 6980

W-7500 Karlsruhe 1
GERMANY

Prof.Ph.D. György Targonski
Fachbereich Mathematik
Universität Marburg
Hans-Meerwein-Str.
Lahnberge

W-3550 Marburg
GERMANY

Prof.Dr. Marek Cezary Zdun
Inst. of Math.
Pedagogical Univ.
ul. Podchorazych 2

30-084 Krakow
POLAND

Prof.Dr. Mark A. Taylor
Dept of Math
Acadia Univ.

Wolfville NS B0P 1X0
CANADA

E-MAIL ADDRESSES

ACZÉL, János, JDACZEL@MATH.UWATERLOO.CA

ALSINA, Claudi, ALSINA@UPC.ES

BARON, Karol, BARON@PLKTUS11

BONK, Mario, I1011201@DBSTU1

CHOCZEWSKI, Bogdan, GRCHOCZE@PLKRCY11

FÕRG-ROB, Wolfgang, WOLFGANG.FOERG-ROB@UIBK.AC.AT

FORTI, Gian Luigi, FORTI@IMIUCCA.CSI.UNIMI.IT

GARCIA-ROIG, Jaime Luis, ALSINA@UPC.ES

GER, Roman, ROMANGER@PLKTUS11

GIRGENSOHN, Roland RGIRGENS@JEEVES.UWATERLOO.CA

GRONAU, Detlef, GRONAU@EDVZ.UNI-GRAZ.ADA.AT

JÁRAI, Antal, JARAI@UNI-PADERBORN.DE , H3029JAR@ELLA.HU

KANNAPPAN, Palaniappan, PLKN@WATSERV1.UWATERLOO.CA

KHOLODOV, Alexander, DUG@IPK.MSK.SU , subject : to Kholodov

LAJKÓ, Károly, H2494LOS@ELLA.HU

LOSONCZI, László, H2494LOS@ELLA.HU

MAKSA, Gyula, H3983SZA@ELLA.HU

PAGANONI, Luigi, PAGANON@IMIUCCA.CSI.UNIMI.IT

PÁLES, Zsolt, H2913PAL@ELLA.HU

REICH, Ludwig, GRONAU@EDVZ.UNI-GRAZ.ADA.AT

SABLIK, Maciej, MSSABLIK@PLKTUS11

SANDER, Wolfgang, I1010901@DBSTU1

SCHWAIGER, Jens, GRONAU@EDVZ.UNI-GRAZ.ADA.AT

SZABÓ, György, H3983SZA@ELLA.HU

TAYLOR, Mark, TAYLOR@ACADIAU.CA

TOMÁS, Maria Santos, ALSINA@UPC.ES