

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 43/1992

Darstellungstheorie endlicher Gruppen

27.09. bis 03.10.1992

The conference was organized by Bertram Huppert (Fachbereich Mathematik, Universität Mainz, Saarstraße 21, 6500 Mainz, Germany) and Gerhard O. Michler (Institute for Experimental Mathematics, Ellernstraße 29, 4300 Essen 12, Germany). It was attended by 48 mathematicians, 23 of whom came from abroad: Australia (1), Denmark (1), Great Britain (3), France (1), Israel (2), Japan (2), Russia (2), and the USA (11). 30 lectures were given on recent developments in representation theory.

The main topics of the meeting were:

1. Reductions of Brauer's and Alperin's conjectures in modular representation theory.
2. The representation theory of groups of Lie type and their Hecke algebras.
3. Isotypies, Morita equivalences and other correspondences between blocks, especially principal blocks of finite groups with abelian Sylow  $p$ -subgroups.
4. The symmetric group, especially decomposition matrices of spin blocks and related developments in combinatorics.

A special lecture was organized on the Thursday evening in which the participants could present some important open problems arising in their research.

The participants did not only benefit from the original and interesting lectures, but also from the intensive private discussions among each other during the whole week. Especially the young mathematicians from East Germany were able to meet and learn in the subject from famous experts they had never been able to talk to before. The meeting proved again that the representation theory of finite groups is an old and difficult area of research in algebra which is still very active.

## Abstracts of the talks

### E.C. Dade: Counting Characters in Blocks of Finite Groups

If  $B$  is a  $p$ -block of a finite group  $G$ , it is conjectured that the number  $k(B)$  of ordinary irreducible characters of  $B$  with a fixed defect  $d$ , lying over a fixed linear character  $\varphi$  of  $O_p(G)$  (assumed to be central in  $G$ ) is a sum of the form

$$(1) \quad k(B) = \sum_{\substack{C \in R/G \\ |C| > 0}} (-1)^{|C|+1} \sum_{\substack{b \in B \\ b \in B | k(N_G(C))}} k(b)$$

where  $R/G$  is a family of representatives for the  $G$ -conjugacy classes of radical  $p$ -chains of  $G$ . This formula holds only when  $p^{d(B)} > |O_p(G)|$ . We can show that, under the same assumptions ( $O_p(G) \leq Z(G)$  and  $p^{d(B)} > |O_p(G)|$ ), this is equivalent to the conjecture that

$$(2) \quad k(B) = \sum_{\substack{C \in R/G \\ |C| > 0}} (-1)^{|C|+1} \sum_{\substack{b \in B \\ b \in B | k(N_G(C))}} k_w(b)$$

where  $k_w(B)$  counts the number of ordinary irreducible characters  $\psi$  of  $b$  with fixed defect  $d$ , lying over  $\varphi$  and satisfying  $d(\psi) = d(\xi)$ , for any  $\xi \in \text{Irr}(P_1)$  lying under  $\psi$ . Here  $C : P_0 = O_p(G) < P_1 < \dots < P_n$ .

### G. R. Robinson: A Note on Alperin's Conjecture

In this talk, I will discuss a "Möbius Inversion Formula" for the "Lefschetz Conjugation Module" introduced by R. Knörr and myself in our work on Alperin's Conjecture. This is an equation in the Green ring which shows that the Lefschetz Conjugation Module of a block  $B$  is closely related to the module  $\bigoplus_{S \in B} \text{Hom}_F(S, P(S))$  where  $S$  runs through simple modules over  $B$ .

It gives an equality of virtual modules which in a sense generalizes the statement of Alperin's Conjecture.

### M. Weidner: The 3<sup>rd</sup> Loewy Layer of $P_1(G)$

Let  $p$  denote a prime,  $G$  a  $p$ -solvable group,  $\mathbb{F}G$  the groupring over the field with  $p$  elements,  $J$  the radical of  $\mathbb{F}G$  and  $\mathbb{P}_1$  the projective cover of the trivial  $G$ -module.

Then (W. Gaschütz (1977))  $\mathbb{P}_1 J / J^2$  is the direct sum of the complemented  $p$  chief factors of a chief series of  $G$ . We show:  $\mathbb{P}_1 J^2 / J^3$  is a direct sum of some (non complemented)  $p$  chief factors and the head of an accessible module  $W$ .

A consequence of this description is: If  $\dim \mathbb{P}_1 = p^n$  then  $\dim W \leq n(n+1)/2 \geq \dim \mathbb{P}_1 J^2 / J^3$ . If  $l_1$  is the composition length of  $\mathbb{P}_1 J / J^2$ , then the head of  $W$  has composition length at least  $l_1(l_1 - 1)/2$ .

#### D. Chillag: Characters, Classes and Eigenvalues of Regular Representations of Semi Simple Algebras

Analog results on conjugacy classes and characters (ordinary and Brauer characters) can be proved using the fact that the values of characters (ordinary, Brauer and central characters) are eigenvalues of the regular representation of a corresponding algebra.

#### U. Stammbach: Vertices of certain indecomposable modules

Let  $S$  be a  $p$ -Sylow subgroup of  $G$ , and let  $k$  be a field of characteristic  $p$ . We consider the exact sequence

$$0 \rightarrow R \rightarrow kG \otimes_S k \rightarrow k \rightarrow 0.$$

Let  $R = \bigoplus C_i$  with  $C_i$  indecomposable. For each  $C_i$  we choose a vertex group  $V_i \subseteq S$ . Then, for  $A$  a  $kG$ -module and  $n \geq 0$ , the following sequence is exact

$$(*) \quad 0 \rightarrow H^n(G, A) \rightarrow H^n(S, A) \rightarrow \bigoplus H^n(V_i, A) / (H^n(V_i, A))^{N_G(V_i)}.$$

This result was motivated by the result of Robinson, that  $\{(V_i, N_G(V_i))\} \cup (S, N_G S)$  is a weak conjugation family. An application of (\*) yields a simple proof of the following result of Benson, Carlson, Robinson: If  $H$  is a weakly  $p$ -embedded subgroup of  $G$ , and if  $M$  is a module in the principal block of  $kG$ , then  $H^n(G, M) \xrightarrow{\sim} H^n(H, M)$ ,  $n \geq 0$ .

#### R. Gow: The Steinberg Character of a Group of Lie Type

Let  $G$  be a group of Lie type (finite). A scaled version of the Killing form defines a non-degenerated symmetric or alternating bilinear form on  $L/Z \times L/Z$ , where  $L$  is the adjoint module of  $G$  and  $Z$  its centre. Using the associated geometry on  $L/Z$ , we obtain a generalized character of  $G$  which, when slightly modified, gives the Steinberg character. Irreducibility of the character is proved by re-proving Steinberg's theorem that the number of unipotent elements is the square of the order of the Sylow  $p$ -subgroup.

#### C. W. Curtis: On the Decomposition of Gelfand-Graev Representations of Reductive Groups over Finite Fields

Let  $G$  be a connected, reductive algebraic group, defined over a finite field  $F_0$ , with Frobenius map  $F$ , assumed to be of split type in this abstract. Let  $U_0$  denote the unipotent radical of an  $F$ -stable Borel subgroup  $B_0$ , containing a maximal  $F$ -stable torus  $T_0$ . Let  $\Phi$  be the root system of  $G$  with respect to  $T_0$ ,  $\Phi^+$  the set of positive roots corresponding to  $B_0$ , and  $\Pi$  the set of simple roots in  $\Phi^+$ . A linear representation  $\psi$  of  $U_0^F$  is called nondegenerate provided that its restriction to a positive root subgroup  $U_\alpha^F$  is nontrivial if and only if  $\alpha \in \Pi$ . The Gelfand-Graev representations of the finite group  $G^F$  are the induced representations  $\psi^{G^F}$ , for the nondegenerate linear representations  $\psi$  of  $U_0^F$ . In case the center  $Z(G)$  is connected, there is only one Gelfand-Graev representation (up

to equivalence), and the irreducible components of its character  $\Gamma$  were constructed by Deligne and Lusztig as linear combinations of the virtual characters  $R_{T,\theta}$ . In general, the nondegenerate characters  $\psi$ , and the Gelfand-Graev characters associated with them, are parametrized by the elements  $z \in H^1(F, Z(G))$ , and are denoted by  $\psi_z$  and  $\gamma_z$ , respectively. Each Gelfand-Graev representation  $\gamma_z$  is known to be multiplicity free, so its Hecke algebra, or endomorphism algebra  $H_z = e_z K G^F e_z$  is commutative. Here  $K$  is an algebraically closed field of characteristic zero, and  $e_z$  is the primitive idempotent in  $KU_0^F$  corresponding to  $\psi_z$ .

The first result asserts that for each Gelfand-Graev character  $\Gamma_z$ , and each pair  $(T, \theta)$  consisting of a maximal  $F$ -stable torus  $T$  and an irreducible character  $\theta$  of  $T^F$ , there exists a unique irreducible character  $\chi_{T,\theta,z}$  of  $G^F$  such that  $(\chi_{T,\theta,z}, \Gamma_z) \neq 0$  and  $(\chi_{T,\theta,z}, R_{T,\theta}) \neq 0$ . The irreducible representations of  $H_z$  correspond to the irreducible components  $\{\chi_{T,\theta,z}\}$  of  $\Gamma_z$ , and are denoted by  $\{f_{T,\theta,z}\}$ .

Let  $z \in H^1(F, Z(G))$ , and let  $(T, \theta)$  be a pair as above. The main result can be stated as follows. There exists a unique homomorphism of algebras  $f_{T,z} : H_z \rightarrow KT^F$ , independent of  $\theta$ , with the property that each homomorphism  $f_{T,\theta,z}$  can be factored,  $f_{T,\theta,z} = \tilde{\theta} \circ f_{T,z}$ , where  $\tilde{\theta} : KT^F \rightarrow K$  is the representation of the group algebra  $KT^F$  extending  $\theta$ . Explicit formulas are obtained for the values  $\{f_{T,z}(c_i)\}$  on the set of standard basis elements  $\{c_i\}$  of  $H_z$ . These are given in terms of the Green functions  $\{Q_i^{G(T)^\circ}\}$  for semisimple elements  $t \in G^F$ , and the values of the character  $\psi_z$ . In case  $G = SL_2$ , these formulas were obtained by Gelfand and Graev in 1962, and were called Bessel functions over finite fields.

## R. Dipper: Harish-Chandra Series of Irreducible Representations of Finite General Linear Groups

As in characteristic zero the irreducible representations of finite groups of Lie type in non-describing characteristic are divided into Harish-Chandra series. This was first proved by Hiss in terms of character theory. Those series are given by so-called semisimple Harish-Chandra vertices and sources, a generalisation of Green's vertex theory.

For finite general linear groups we determine the semisimple Harish-Chandra vertices for the irreducible representations in non-describing characteristic. They are given by  $l$ - $p$ -adic decomposition of weights. The proof involves a version of Steinberg's tensor product theorem in non-describing characteristic.

## P. Fleischmann: On Conjugacy Classes of Chevalley Groups

This is joint work with I. Janiszczak.

Recently we were able to finish the computations of generic class numbers for adjoint exceptional Chevalley groups of type  $E_6$ ,  $E_7$  and  $E_8$ . In case  $E_8$  our result is an independent check (with some corrections) of earlier work of Mizuno.

We use Lusztig's Jordan decomposition of characters, and determine the numbers of semisimple classes in the (simply connected) dual group. This method can be generalized to groups of classical type. It involves combinations of subsystems of roots and Moebius

functions of partition lattices. As an application of our results we obtained (together with W. Lempken) a theorem on centralizers of finite groups  $G$ : If  $p \mid |G|$  then there is  $g \in G$  such that  $g_p \in C_G(g) \setminus C_G(g)'$ . The proof uses classification of finite simple groups.

### J. F. Carlson: Quotient Categories of Modules

The lecture is a report on joint work with Peter Donovan and Wayne Wheeler. Let  $G$  be a finite group and  $k$  a field of characteristic  $p > 0$ . Let  $\text{Stmod} - kG$  be the stable category of finitely generated  $kG$ -modules modulo projectives. For any integer  $c \geq 0$ , let  $\mathcal{M}_c$  denote the full subcategory of modules of complexity at most  $c$ . If  $r$  is the  $p$ -rank of  $G$  then  $\text{Stmod} - kG = \mathcal{M}_r$ . Each  $\mathcal{M}_c$  is a thick subcategory of  $\mathcal{M}_r$ , and so the quotients  $\mathcal{M}_c/\mathcal{M}_{c-1}$  are triangulated. Let  $Q_c = \mathcal{M}_c/\mathcal{M}_{c-1}$ . In general the objects of  $Q_c$  do not satisfy a Krull-Schmidt theorem. For objects  $M$  and  $N$  in  $\mathcal{M}_c$ , the group of morphisms in  $Q_c$  from  $M$  to  $N$  can be described as the zero grading of the localization of  $\text{Ext}_{kG}^*(M, N)$  at a multiplicative set  $S_{M,N}$  of  $H^*(G, k) \cong \text{Ext}_{kG}^*(k, k)$  determined by a condition on the intersection of varieties. In particular,  $\text{Hom}_{Q_c}(k, k)$  is a direct sum of  $l$  local rings where  $l$  is the number of components of the maximal ideal spectrum of  $H^*(G, k)$  of maximal dimension. The lecture will attempt to make clear the representation theoretic nature of these results.

### B. Külshammer: Finiteness Questions in Representation Theory

Donovan's conjecture says that, for a given prime number  $p$  and a given finite  $p$ -group  $D$ , there are only finitely many Morita equivalence classes of  $p$ -blocks with defect group  $D$ . Our aim is a reduction theorem for this conjecture. Using Dade's theory of block extensions, we show that any  $p$ -block with defect group  $D$  is Morita equivalent to a crossed product  $Y = \bigoplus_{x \in X} Y_x$  satisfying the following conditions:

1.  $X$  is a finite  $p'$ -group with  $|X| \leq |\text{Out}(D)|^2$ ;
2. The identity component  $Y_1$  of  $Y$  is a basic subalgebra of a  $p$ -block with defect group  $D$  in a finite group  $H$  generated by the conjugates of  $D$ .

Since, for a given  $D$ , there are only finitely many possibilities for  $X$ , this leads to the question whether there are, for a given finite  $p'$ -group  $G$  and a given finite-dimensional algebra  $R$  over an algebraically closed field  $F$  of characteristic  $p$ , only finitely many isomorphism types of crossed products  $A = \bigoplus_{g \in G} A_g$  with  $A_1 = R$ . We show that this is indeed the case, using the following result proved independently by T.A. Springer and S. Donkin. For a given finite  $p'$ -group  $G$  and a linear algebraic group  $H$  over  $F$ , there are only finitely many equivalence classes of representations of  $G$  in  $H$ . I had stated this result in form of a question before and proved a number of special cases. Thus we have now reduced Donovan's conjecture to blocks with defect group  $D$  in groups generated by conjugates of  $D$ . For stronger versions of this reduction one would like to have a stronger form of the result by Springer and Donkin above.

### S. Koshitani: Projective Modules of Finite Groups in Characteristic 3

Let  $G$  be any finite group with elementary abelian Sylow 3-subgroups of order 9, and let  $F$  be any field of characteristic 3. Then the Loewy length of the projective cover of the trivial  $FG$ -module is at least 5, where  $FG$  is the associated group algebra. This lower bound is the best possible (e.g. take just  $G$  an elementary abelian group of order 9).

### M. Broué: Isotypies between blocks of finite groups

The notion of type of blocks was introduced by R. Brauer in 1970. Inspired by the analysis of the Deligne-Lusztig operations and the study of blocks of finite reductive groups, a modification-precision of the definition was given by the author in 1988. An "isotypie" between two blocks must be understood as a composable collection of perfect isometries defined at all the local levels of the groups. Since a perfect isometry must in turn be viewed as the "shadow" (at the characters level) of a derived equivalence, we shall try to present a reasonable guess for a module-theoretic explanation of what an isotypie may come from. We conjecture that any block with abelian defect group is isotypic to its Brauer correspondent. We list the known results in this direction.

### P. Fong: Isotypies of Principal Blocks

Broué's conjecture that the 1-blocks of  $G$  and  $H$  are isotypic, where  $G$  is a finite group with an abelian Sylow  $p$ -subgroup  $D$  and  $H = N_G(D)$ , can be reduced to the question of whether "compatible" isometries exist between the 1-blocks of  $X$  and  $Y = N_G(D \cap X)$ , where  $X$  runs over the components of  $G$ . In particular, this is the case for  $p = 2$ .

### G. Malle: Generic Blocks of Finite Reductive Groups

Gemeinsame Arbeit mit M. Broué und J. Michel.

Ausgehend von einer generischen Sylowtheorie endlicher Gruppen vom Lie-Typ wird eine generische Theorie der unipotenten Charaktere entwickelt. Hauptergebnis ist die Beschreibung der Zerlegung des Lusztig-Funktors  $R_L^G$  durch Induktion in relativen Weylgruppen. Dies führt zu einer Verallgemeinerung der Harish-Chandra-Theorie, erlaubt die Blockeinteilung und den Nachweis der Existenz perfekter Isometrien und damit den Beweis der Alperin-Vermutung in gewissen Fällen.

### P. H. Tiep: The Automorphism Groups of Some Integral Euclidean Lattices

In this talk we will consider some integral Euclidean lattices satisfying certain hypotheses of J. G. Thompson and B. H. Gross.

J. G. Thompson suggested the study all pairs  $(G, \Lambda)$ , where  $G$  is a finite group and  $\Lambda$  is a torsion-free  $\mathbb{Z}G$ -module of finite rank with the following property:  $\Lambda/p\Lambda$  is an irreducible  $\mathbb{F}_p G$ -module for all primes  $p$ .

There are known only a few examples of such a pair.

1.  $(W(E_8), E_8)$ , ( $E_8$  is the root lattice);
2.  $(2C_{01}, \Lambda_{24})$ , ( $\Lambda_{24}$  is the Leech lattice);
3.  $(F_3, \Lambda_{248})$ , ( $\Lambda_{248}$  is the Thompson-Smith lattice);
4. Some examples related to the basic spin representations of  $2A_n$  and  $2S_n$  and discovered by R. Gow.

B. H. Gross introduced the following generalization of this situation. Let  $V$  be a  $\mathbb{Q}G$ -module,  $\Lambda$  full  $\mathbb{Z}G$ -module in  $V$ ,  $K = \text{End}_G(V) = \{\varphi \in \text{End}_{\mathbb{Q}}(V) \mid \forall g \in G, \varphi g = g\varphi\}$ .  $V$  is said to be global irreducible, if

- $V \otimes_{\mathbb{Q}} \mathbb{R}$  is irreducible ( $\Rightarrow K$  is a division ring);
- Let  $R$  be a maximal order in  $K$ . Then  $\Delta/\varphi\Lambda$  is an irreducible  $(F/\varphi R)G$ -module for all maximal two-sided ideal  $\varphi$  of  $R$ .

#### Examples.

5.  $G = Sp_{2n}(p)$ ,  $V$  is irreducible of the Weil representation;
6.  $G = PSU_3(q)$ ,  $V$  is related to the irreducible complex representation of  $G$  of minimal degree.

As it is shown by Gross, the two last examples arise also as Mordell-Weil lattices of some elliptic curves considered by N. Elkies. Here we compute the automorphism groups of lattices  $\Lambda$  arising in cases 4-6. We demonstrate also a connection between these lattices and well-known lattices such as the Todd-Coxeter lattice  $K_{12}$ , the Leech lattice  $\Lambda_{24}$ , and the Barnes-Wall lattices  $BW_{24}$ .

Another subject of this talk is to realize some exceptions in the Cohen-Liebeck-Saxl-Seitz list of maximal subgroups for exceptional finite groups of Lie type as the automorphism groups of Euclidean lattices in corresponding Lie algebras.

In all cases the group  $\text{Aut}(\Lambda)$  is a maximal finite subgroup of  $GL_n(\mathbb{Q})$ ,  $n = \dim \Lambda$ .

#### J. L. Alperin: Partial Steinberg Representations

(Joint work with Geoffrey Mason)

Let  $G$  be a universal Chevalley group over a field with  $q = p^f$  elements of type  $A$ ,  $D$ , or  $E$ . A necessary and sufficient condition, in terms of determinants, is given for a subgroup of a root subgroup to be free on a simple module  $V$  for  $G$ .

#### A. Kerber: SYMMETRICA, a computer algebra system for finite symmetric groups and for related classes of groups

In particular, the following items will be discussed:

- The data: Orbits, double cosets, matrices with prescribed row and column sums, tableaux, bi-determinants and their relationship.

- Frobenius' Isometry which allows to formulate everything (in ordinary theory) in terms of multivariate polynomials.
- Schubert polynomials, an important generalization of Schur polynomials.
- Matrix representations, an important advantage of SYMMETRICA, and their use for applications: symmetry adapted bases.

#### B. Srinivasan: A Geometrical Approach to the Littlewood-Richardson Formula

The Littlewood-Richardson formula gives a rule for computing induced characters from characters of Young subgroups of symmetric groups. It is a complicated formula and the existing proofs are, in the opinion of the author, difficult to understand conceptually. An alternative formula and a combinatorial proof of it were given by Remmel and Whitney in 1984. We give an interpretation of this new formula in terms of unipotent classes in  $GL(n, q)$ , and a conceptual proof using a theorem of Steinberg.

#### C. Bessenrödt: Decomposition Matrices of Spin Blocks of Symmetric Groups

In recent joint work with Morris and Olsson, we proved a counterpart to a theorem of James resp. Farahat-Müller-Peel on linear representations of the symmetric groups  $S_n$ , namely that also in the case of spin representations at characteristic  $p = 3$  the upper part of the decomposition matrix is "essentially" a lower triangular matrix (i.e. disregarding the complications arising from associate characters) with respect to a suitable ordering of the characters. Crucial combinatorial ingredients in the proof were a partition identity due to Schur and the new notion of ladders in the  $\bar{p}$ -residue diagram. The partitions labelling the (double-) columns in the decomposition matrix are obtained by a "top node" algorithm on the  $\bar{p}$ -residue diagram. For  $p = 5$ , an internal description of these partitions and a crucial partition identity, which turned out to be equivalent to a conjecture by Andrews from 1974, were recently proved in a joint paper with Andrews and Olsson. With this at hand, a result on the shape of the decomposition matrix at characteristic  $p = 5$  of the type above was then obtained by a more delicate consideration of ladders in the  $\bar{5}$ -residue diagram.

#### G. Pazderski: On the Chief Factors of Solvable Linear Groups

Let  $G$  be a solvable irreducible linear group of degree  $n$  over a field. As is well known the maximum  $r(G)$  among the ranks of all chief factors of  $G$ , the so called chief rank of  $G$ , satisfies  $r(G) \leq n$ . We are interested in the groups  $G$  with  $r(G) = n$ . In this case  $n$  is even and each chief factor of rank  $n$  is a 2-group. If  $G$  is primitive then the only possible values of  $n$  are 2 and 4. Groups of this kind are treated in more detail. They admit an explicit description to a great extent. Finally some generalizations of two results by R. Baer are given concerning the chief rank.



### G. Hiß: Modular representations of Unitary Groups

Let  $G = GU_n(q)$  denote the finite unitary group, and let  $l$  be a prime not dividing  $q$ . In the first part of my talk I shall give a summary on the recent results obtained in joint work with M. Geck and G. Malle on the  $l$ -modular Harish-Chandra series of  $G$ . The modular Steinberg character is introduced and those primes  $l$  are characterized for which it is cuspidal. I shall sketch a proof of this result.

Let  $d$  denote the order of  $-q$  modulo  $l$ . For even  $d$  and  $l > n$  the  $l$ -modular Harish-Chandra series are described. These results lead to a complete description of the decomposition matrices for even  $d$ .  $l > n$ ,  $n \leq 10$ .

If  $d$  is odd, the situation is much more complicated. The smallest unsolved case occurs for  $n = 3$ ,  $l|q + 1$ . The second part of my talk reports on joint work with M. Geck and B. H. Matzat. It is shown how this problem in the representation theory of  $GU_3(q)$  can be transformed first into a combinatorial problem and then into a question on the function field of the Fermat curve.

### M. Geck: Cuspidal Unipotent Characters and Hecke Algebras

There is an  $l$ -modular version of the usual Harish-Chandra theory for finite groups of Lie type  $G^F$  where  $l \neq$  defining characteristic of  $G$ . Among others, the following two problems arise:

1. Is it true that the  $l$ -modular reduction of a cuspidal unipotent character of  $G^F$  is irreducible as a Brauer character?
2. Study characters and decomposition maps for Hecke algebras.

In a joint work with G. Hiß and G. Malle we proved that 1) has an affirmative answer for the finite unitary groups  $U_n(q^2)$ . This is based on results about Harish-Chandra induction of generalized Gelfand-Graev representations.

As a contribution to 2), in a joint work with G. Pfeiffer we introduced the concept of the character table of a generic Hecke algebra  $H$  associated with a finite Weyl group  $W$  with index parameters  $u, s \in S$ .

As an application, we computed the character tables of Hecke algebras of type  $F_4, E_6, E_7$ , and determined the number of irreducible characters of corresponding specialized algebras.

### W. Kimmerle: On a Conjecture of Zassenhaus for $\mathbb{Z}G$

Zassenhaus conjectured that group bases of  $\mathbb{Z}G$  are conjugate by a unit of  $\mathbb{Q}G$  ( $G$  denotes a finite group). The talk deals with this conjecture and certain variations of it (which still imply a positive answer to the isomorphism problem of integral group rings) for the following types of finite groups.

1. Simple groups
2. Groups with abelian Sylow subgroup

The method used with respect to 1. is the use of Brauer trees, with respect to 2. properties of conjugacy class preserving group automorphisms.

#### A. Turull: Brauer Equivalence of $G$ -Algebras

The study of Schur indices for characters of finite groups  $H$  that have some distinguished subgroup  $N$  with  $H/N = G$  abelian leads to the study of certain  $G$ -algebras over number fields in characteristic zero. I introduce a notion of central simple  $G$ -algebra and of Brauer equivalence of such  $G$ -algebras, and I characterize the resulting equivalence classes in certain cases. These results should help in the effective computation of the Schur indices for certain classical groups.

#### W. Plesken: Finite Rational Matrix Groups

One of the more interesting and also difficult aspects of the classification of irreducible maximal finite subgroups of  $GL_n(\mathbb{Q})$  for  $n \leq 23$  with G. Nebe turned out to be the interrelation of these groups by common subgroups fixing the same quadratic forms. A result helping to prove the non-existence of such subgroups is this:

Let  $G \leq GL_n(\mathbb{Z})$  be finite uniform fixing the primitive integral symmetric bilinear form represented by  $A \in \mathbb{Z}^{n \times n}$ . If a prime  $p$  divides  $\det(A)$ , then  $p \mid |G|$ .

This result is wrong for non-uniform groups to the extent that any such  $A$  has prime divisors for its determinant not dividing  $|G|$ .

Often it is even more instructive to investigate common irreducible subgroups having bigger spaces of invariant forms. Examples of three-parametric Bravais groups are given following the classification of irreducible Bravais groups of degree 8 by B. Souvignier.

#### K. Erdmann: Symmetric Groups and Schur Algebras

Let  $\Sigma_r$  be the symmetric group, and let  $K$  be a field of characteristic  $p$ .  $E$  is a fixed  $n$ -dimensional vector space. Then  $E^{\otimes r}$  is a permutation module for  $K\Sigma_r$ , where  $\Sigma_r$  acts on the right (by place permutations). Then the Schur algebra may be taken as  $S = S(n, r) = \text{End}(E_{K\Sigma_r}^{\otimes r})$ . There is a canonical ring homomorphism  $\rho: K\Sigma_r \rightarrow \text{End}({}_S E^{\otimes r})$ ; it has been proved that this is surjective. Let  $\Lambda^+(n, r) = \{\lambda \vdash r: \lambda \text{ has } \leq n \text{ parts}\}$ . The main result is:

**Theorem.** Suppose all  $\lambda$  in  $\Lambda^+(n, r)$  are  $p$ -regular. Then  $K\Sigma_r/\text{Ker } \rho$  is a quasi-hereditary algebra, Morita equivalent to the "Ringel dual" of  $S$  where the Specht modules  $S^\lambda$  are the "Weyl objects", and the Young modules  $Y^\lambda$  are the indecomposable summands of the canonical (tilting) module.

There is also a block version.

This can be used to obtain information on decomposition numbers for  $\Sigma_r$  and also on modules with Specht filtration.

### K. Uno: Auslander-Reiten Sequences and Clifford Theory

Let  $N$  be a normal subgroup of a finite group  $G$ , and  $V$  a  $G$ -invariant  $kN$ -module ( $k$  a field). Then  $E = \text{End}_{kG}(V^G)$  is a  $G/N$ -graded algebra over  $E_1 = \text{End}_{kN}(V)$ . The stable Clifford theory gives us an equivalence of categories between  $\text{Mod}(kG|V)$  ( $kG$ -modules whose restrictions to  $N$  are direct summands of  $V^m$  for some  $m$ ) and  $\text{Mod}(E|E_1)$  ( $E$ -modules whose restrictions to  $E_1$  are projective  $E_1$ -modules). Suppose that  $V$  is indecomposable and  $k$  is algebraically closed and let  $\bar{E} = E/(\text{rad} E_1)E_1$ . Then  $\bar{E}$  is a twisted group algebra over  $k$ . We define a certain category  $C(G)$  by using Auslander-Reiten sequences and prove that there is an equivalence between  $C(G)$  and  $\text{Mod}(\bar{E})$ . This equivalence has interesting properties on compoundings of modules in the Auslander-Reiten quiver. This is in part joint work with T. Okuyama.

### M. Herzog: A Graph Related to Conjugacy Classes in Groups

If  $G$  is a finite group or an infinite  $FC$ -group, we define the following graph  $\Gamma = \Gamma(G)$ : its vertices are the non-central conjugacy classes of  $G$  and two vertices are connected if their cardinalities are not co-prime. The properties of the graph will be described, and the proof of the fact that the diameter of  $\Gamma$  is less than or equal to 3 will be sketched.

### D. Gluck: Sharper Character Value Estimates for Groups of Lie Type

Let  $G$  be a group of Lie type over the field of  $q$  elements. Let  $\chi$  be a nonlinear irreducible character of  $G$  and let  $x$  be a noncentral element of  $G$ . Except when  $x$  is a transvection in an odd characteristic symplectic group, we obtain  $O(1/q)$  bounds for  $|\chi(x)/\chi(1)|$  which give nontrivial information for all  $q \geq 7$ . For all  $q, G, \chi$ , and  $x$ , we show that  $|\chi(x)/\chi(1)| \leq 19/20$ . For unipotent  $x$ , we show that  $|\chi(x)/\chi(1)| \leq 1/(\sqrt{q}-1)$ . The last bound is achieved in  $SL(2, q)$ ,  $q \equiv 1 \pmod{4}$ .

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