MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 49/1992

Numerische Integration

8.11. - 14.11.1992

Die Tagung fand unter der Leitung von H. Braß (TU Braunschweig) und G. Hämmerlin (U München) statt. 46 Mathematiker aus 15 Ländern nahmen daran teil; 33 Vorträge wurden gehalten.

In der Breite der Themen, die auf der Tagung behandelt wurden, spiegelt sich die Tatsache wider, daß die Fragestellungen, die sich im Zusammenhang mit der numerischen Integration ergeben, von der Analysis und Algebra bis zu handfesten Anwendungen reichen. Um nur einige der zentralen Themen zu nennen, bildete wie schon auf früheren Tagungen wieder die Theorie der Gaußschen Integrationsformeln, die zahlreiche auch praktisch interessante Spielarten aufweisen, einen Schwerpunkt. Vor allem für die Berechnung hochdimensionaler Integrale wurden in den vergangenen Jahren Gitterformeln (lattice rules) entwickelt, deren Theorie und praktischer Nutzen in mehreren Vorträgen zur Spache kamen. Daneben wurde über Kubaturformeln für n-dimensionale Integration berichtet, bei denen der Blickpunkt der Optimalität im Vordergrund steht. Fortschritte wurden auch in Untersuchungen von Genauigkeitsfragen unter schwachen Annahmen an den Integranden erzielt, die im Hinblick auf die Anforderungen der Praxis sehr wünschenswert erscheinen. Mehrere Zugänge zur numerischen Behandlung singulärer Integrale und eine Reihe von Beiträgen zu Einzelproblemen vervollständigten das Programm.

Es war der allgemeine Wunsch der Tagungsteilnehmer, in einer Diskussion offene Probleme darzustellen. Dabei kamen zahlreiche ungelöste Fragen zur Sprache, die die Lebendigkeit und Aktualität des Gebiets deutlich machten. Hinzu kamen natürlich die vielen Diskussionen im kleinen Kreis, die durch die gesamte Anlage der Oberwolfacher Tagungen so hervorragend gefördert werden. Zu der Atmosphäre, die dies alles ermöglicht, tragen auch sehr wesentlich die Angehörigen des Instituts bei, die unseren herzlichen Dank verdienen.

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Vortragsauszüge

H. BRASS

Error bounds using total variation

Let R denote the error of a positive quadrature rule for a weighted integral. We are interested in bounds of the type

$$|R[f]| \leq \beta_s \operatorname{Var} f^{(s-1)}$$

that is in the determination of the constants

$$\beta_s(R) : \sup\{|R[f]| : \operatorname{Var} f^{(s-1)} \leq 1\}.$$

Under some assumptions on the weight function the asymptotics of

$$\sup\{\beta_s(R) : R[\tau_n] = 0\} \qquad (\tau_n : \text{Polynomials of degree n})$$

for $n \to \infty$ is determined and it is proved that the supremum is (asymptotically) attained for the Gaussian rule. As a by-product pointwise bounds for the Peano kernels are obtained.

R. COOLS

The construction of cubature formulae of trigonometric degree

In the recent past, several articles on cubature formulae of trigonometric degree appeared in the Russian literature. In the first part of this talk, we give an overview of the results we know about. These include a lower bound for the number of points and some minimal formulae.

In the second part of the talk we will generalise an earlier result on cubature formulae of algebraic degree, to prove that minimal formulae of trigonometric degree must have equal weights. This and the fact that all formulae of trigonometric degree in the literature are lattice rules, motivates us to have a closer look at the relation between "classical" lattice rules and formulae of trigonometric degree.

In the third part of the talk we construct minimal cubature formulae of arbitrary odd trigonometric degree. We compare these with Fibonacci lattice rules, which are optimal w.r.t. the Zaremba index. The comparison is made by the 2 construction criteria used in this talk: the trigonometric degree and the Zaremba index.

F.-J. DELVOS

A trivariate Boolean midpoint rule

Boolean sums of bivariate product trapezoidal rules have been used to generate cubature formulas which are comparable with good lattice rules in two dimensions. These formulas have been extended to bivariate product midpoint rules. These formulas are of similar efficiency and greater simplicity (Math. Comput. *55* (1990)).

The objective of the lecture is to use formulas of trivariate Boolean interpolation to construct trivariate Boolean midpoint rules which are again comparable with good lattice rules in three dimensions.

E. DE DONCKER AND I. VAKALIS

Convergence Properties of Adaptive Integration Processes

We derive convergence results of local adaptive and global region–size adaptive integration algorithms. An adaptive integration process is associated with a tree of regions where each node corresponds to a region and its children nodes to the subregions obtained by one subdivision. The depth of subtrees at distinguished nodes is delimited according to convergence characteristics of the adaptive strategy for the considered function class.

The approach by Rice (1974, 1975) is extended to handle multivariate integration over the N–dimensional hypercube and simplex, where the integrand function may have a vertex singularity. Extensions to cover other types of singular behaviour are possible.

As an application, speedup results can be derived for parallel versions of the algorithms, assuming a model for the parallel processing of the region trees.

S. EHRICH

On the Construction of Gaussian Quadrature Formulae containing preassigned Nodes

A general iterative method is presented for the computation of Gaussian quadrature formulae $Q^G_{n,m}$ that contain m preassigned nodes. The method is locally convergent of the order two. It can be implemented in a way that every iteration step involves $O((2n+m)^2)$ arithmetical operations, and it provides simple and reliable a posteriori error estimates. A comparison is made to methods that are already in use. Numerical examples show the suitability of the method for some of the possible applications.

T.O. ESPELID

Integrating Singularities using Non–uniform Subdivision and Extrapolation

A new approach to the computation of approximations to multi– dimensional integrals over an n–dimensional hyper–rectangular region, when the integrand is singular, is described. This new approach is based on a non–uniform subdivision of the region of integration and the technique fits well to the subdivision strategy used in many adaptive algorithms. The new strategy can be applied to vertex singularities, line singularities and more general subregion singularities. The technique turns out to have good numerical stability properties.

K.-J. FÖRSTER

Variance in Quadrature — a Survey

If the function values $f(x_\nu)$ have random error, it is of practical interest that the quadrature formula Q_n has small variance $\mathrm{Var}(Q_n)$,

$$\mathrm{Var}(Q_n) = \sum_{\nu=1}^{n} (a_\nu)^2,$$

where the a_ν denote the weights of Q_n. The question of small variance was first considered by Chebyshev in 1874. Since that time several investigations on this subject can be found in literature. The purpose of this lecture is to give a systematical survey of the results and open problems in this field.

W. GAUTSCHI

Gauss-Type Quadrature Rules for Rational Functions

When integrating functions that have poles outside the interval of integration but are regular otherwise, it is suggested that the quadrature rule in question ought to integrate exactly not only polynomials (if any), but also suitable rational functions. The latter are to be chosen so as to match the most important poles of the integrand. We describe two methods for generating such quadrature rules numerically and report on computational experience with them, in particular on the evaluation of Fermi–Dirac integrals to high accuracy.

A. GENZ

Adaptive Numerical Integration over HyperSpherical Regions

This talk will begin with a review of numerical methods for integration over an n–dimensional hypersphere, including rule based methods and Monte–Carlo methods. Then the talk will describe an adaptive algorithm. This algorithm subdivides the hypersphere in a sequence of stages, into a collection of subregions each of which is a product of a radial interval and a spherical simplex. The algorithm can then use appropriately transformed integration rules from simpler regions, for approximate integration and error calculation. The algorithm uses a subdivision strategy that chooses for subdivison at each stage the subregion (of the input hypersphere) with the largest estimated error. This subregion is divided in half along a direction that is chosen either from a set of $n(n-1)/2$ possible edge directions or the radial direction, by using information about the smoothness of the integrand. The talk will focus on some mathematical problems associated with the efficient implementation of the algorithm.

A. GUESSAB

Cubature formulae with minimal number of lines

We will consider the problem of approximating a double integral on a convex compact set K as a minimal linear combination of integrals on the real line. We obtain classical cubature formulae, with minimal number of knots, and which are exact on the space $\psi_{2k+1}(K)$ of all polynomials of degree $2k+1$ with respect to each variable $x_i, i = 1, 2$.

S.-Å. GUSTAFSON

Quadrature rules derived from linear convergence acceleration schemes.

We consider the task of numerically evaluating integrals over the entire real line or over the positive axis using functional values at equidistant points. In both cases application of the trapezoidal rule yields a representation of the integral in the form of an infinite series. In the first case the theory of analytic functions may be envoked to prove that the discretization error decreases rapidly when the step–size becomes small even if the resulting series is slowly convergent. In this situation a large number of functional values could be required to achieve an accurate estimate of the sum. Rapidly converging acceleration schemes may now be used to approximate the sum as a linear combination of a small number of terms and this will be exploited for the derivation of new quadrature rules. Their accuracy will be demonstrated on numerical examples like the evaluation of slowly converging Fourier and Laplace integrals.

A. HAEGEMANS AND P. VERLINDEN

Construction of fully symmetrical Cubature Rules of very high Degree for the Square

A new method for the construction of fully symmetrical cubature formulae for the square is proposed. Using a transformation of variables and choosing an appropriate basis for the invariant polynomials of the square, the nonlinear system that one has to solve with classical methods breaks down in three systems. Each of these systems can be solved easily. Even for high degrees the formulae are surprisingly easy to compute: the hardest computation to perform is the solution of a system of two polynomial equations of low degree in two unknowns.

T. HASEGAWA

Numerical Integration of Nearly Singular Functions

We propose an automatic integration for approximating the integral $\int_{-1}^{1} f(x)/(x-c)dx$ for a given smooth function $f(x)$, where c is outside, but close to the integration interval $[-1,1]$. Approximating the integral is a more difficult problem, if c is closer to either end of the interval $[-1,1]$, although the integrand is not a singular function.
The present scheme is an extension of the Clenshaw–Curtis method. The function $f(x)$ is approximated by the finite sum of the Chebyshev polynomials and some extrapolation method is made use of to evaluate the integral. Numerical examples are also included.

D.B. HUNTER AND H.V. SMITH

Some problems involving orthogonal polynomials

Let a weight–function w be positive and continuous over $(-1,1)$, and suppose that

$$\int_{-1}^{1} w(x)dx$$

exists. Also let p_n be the polynomial of degree n in the corresponding orthogonal sequence. Some properties of the coefficients in the expansions

$$1/p_n(z) = \sum_{j=0}^{\infty} b_{nj} z^{-n-j}$$

and

$$q_n(z) = \int_{-1}^{1} \frac{w(x)p_n(x)}{z-x}\, dx = \sum_{j=0}^{\infty} c_{nj} z^{-n-j-1}$$

were reviewed, and an application to the error analysis of Gaussian quadrature was described. The rest of the talk was concerned with the function U_n for which

$$U^{(r)}(1) = U_n^{(r)}(-1) = 0, \qquad (r = 0, 1, \ldots, n-1)$$

$$U_n^{(n)}(x) = w(x)p_n(x), \qquad (-1 < x < 1).$$

A generalization of a theorem of Markoff (1886) on the variation of the zeros of p_n when w involves a parameter was conjectured, and the conjecture was applied to determine the form of U_n when $w(x)/w(-x)$ is strictly increasing.

P. KÖHLER

Intermediate Error Estimates for Quadrature Formulas

Let R be a functional which admits estimates of the form $|R[f)| \leq e_i \|f^{(i)}\|$ for $i = 0, \ldots, r$. If e_0 and e_r are known, then estimates for the intermediate error constants $e_1, \ldots, e_{r-1}$ can be obtained in terms of e_0 and e_r. This was proven by A.A. Ligun for 2π–periodic functions. The estimate holds for nonperiodic functions, too, and various generalizations and specializations of this estimate are derived, with special emphasis on the estimation of quadrature errors.

F. LOCHER

A stability test for linear difference forms

The stability of a linear multistep method or of a LTJ–system (e.g. digital filter) depends on the fact whether the characteristic polynomial $p_n, p_n(z) = \sum_{\nu=0}^{n} a_\nu z^\nu$, has all its zeros in the unit circle. We consider the polynomials φ_n, φ_{n-1} with $\varphi_n := \sum_{\nu=0}^{n} a_\nu T_\nu$, $\varphi_{n-1} := \sum_{\nu=0}^{n-1} a_{\nu+1} U_\nu$. Then the unimodular zeros of p_n are determined by the common zeros of φ_n and φ_{n-1} in $(-1, 1)$. All zeros of p_n lie in the interior of the unit circle iff by Euclidean division, started with φ_n and φ_{n-1}, one can build up a Sturm sequence of maximal length n+1 and there occur n sign changes at -1.

D.S. LUBINSKY

Distribution of Interpolation Points for Convergent Interpolatory Integration Rules

Suppose that

$$I_n[f] = \sum_{j=1}^{n} w_{jn} f(x_{jn})$$

is an interpolatory rule, that is

$$I_n[P] = \int_{-1}^{1} P(x)dx, \; degree(P) \leq n-1,$$

and $\{I_n\}_{n=1}^{\infty}$ are convergent, i.e.

$$\lim_{n\to\infty} I_n[f] = \int_{-1}^{1} f(x)dx$$

for each continuous $f : [-1,1] \to \mathbb{R}$. What can we say about the *distribution* of the points $\{x_{jn}\}$ as $n \to \infty$. For the classical (e.g. Gauss) quadratures, the points have arcsin distribution. We show that in the general case the distribution is

$$\frac{1}{2} * \text{arcsin} + \frac{1}{3} * \text{arbitrary probability measure.}$$

Generalizations to integrals with weights, or on $(-\infty, \infty)$, are also presented.

J.N. LYNESS

The Canonical Forms of a Lattice Rule

Much of the elementary theory of lattice rules may be presented as an elegant application of classical results. These include the Kronecker group representation theorem and the Hermite and Smith normal forms of integer matrices. The theory of the canonical form is a case in point. In this paper, some of this theory is treated in a constructive raher than abstract manner. A step–by–step approach that parallels the group theory is described, leading to an algorithm to obtain a canonical form of a rule of prime power order. The number of possible distinct canonical forms of a particular rule is derived, and this is used to determine the number of integration lattices having specified invariants.

G. MASTROIANNI AND G. CRISCUOLO

The error of positive quadrature formulas

Let

$$u(x) = (1-x)^{\alpha}(1+x)^{\beta} \log^{\Gamma_0} \frac{e}{1-x^2} \prod_{k=1}^{r} |x-t_k|^{\gamma_k} \log^{\Gamma_k} \frac{e}{|x-t_k|}$$

where $\alpha, \beta, \gamma_k > -1$, $-1 < t_1 < \ldots < t_r < 1$ and Γ_k nonnegative integers. Let further $e_m(f)_u = \int_{-1}^{1} f(x)u(x)dx - \sum_{k=1}^{m} w_k f(x_k), w_k > 0, f^{(r-1)} \in AC_{\mathrm{LOC}}$ be the error of a p.q.f. with algebraic degree $\geq m-1$. We prove

$$|e_{2m-1}(f)_u| \leq \frac{c\|f^{(r)}\varphi^r u_m\|_{L_1([x_1,x_m])}}{m(m-1)\quad(m-r+1)} + c\|f^{(r)}\varphi^{2r}u\|_{L_1(I_m)},$$

where

$$I_m = [-1,x_1]\bigcup[x_m,1], u_m(x) = (1-x)^{\alpha}(1+x)^{\beta}\log^{\Gamma_0}\frac{e}{1-x^2}\prod_{k=1}^{r}(|x-t_k|+m^{-1})^{\gamma_k} \cdot \log^{\Gamma_k}\frac{e}{|x-t_k|}$$

and e is a positive constant independent of m, f and t_k. We consider some special cases and, as an application, we estimate the coefficients w_k of the formula.

H. NIEDERREITER

Lattice rules for nonperiodic integrands I

Lattice rules were originally designed for the numerical integration of periodic functions over their period interval, which is assumed to be the s–dimensional unit cube $[0,1]^s$. We introduce a slightly modified integration rule T_L associated with an integration lattice L and show that for nonperiodic functions it performs better than the standard lattice rule Q_L. For instance, T_L integrates all linear functions exactly, but this is not true for Q_L. Furthermore, if $s = 2$ and L has a square unit cell, then T_L integrates all polynomials $a + bx + cy + dxy$ exactly. Again for $s = 2$, it is shown that the Fibonacci lattice L_k obtained from the kth Fibonacci number F_k has a square unit cell if and only if k is odd. Thus, for $s = 2$ there exist excellent integration lattices with square unit cell. These results were obtained jointly with Ian Sloan.

G. NIKOLOV

Gaussian Quadrature Formulae for Splines

The explicit quadrature formulae of Gauss, Radau and Lobatto type are found for the spaces of polynomial splines of degree 1 (arbitrary knots) and degree 2 (the case of equidistant knots). It turns out that in the first case the formulae obtained are almost optimal for the Sobolev space

$$W^2_\infty = \{f \in C^1[0,1],\ f' \in AC[0,1],\ \operatorname*{vraisup}_{t \in (0,1)} |f''(t)| \le M\},$$

being in the same time as simple as the classical compound midpoint and trapezoidal rules. The investigation of the Gauss type quadratures for quadratic splines with equidistant knots suggests that they might be near to the optimal rule for W^3_∞.

E. NOVAK

Quadrature formulas for convex classes of functions

We study the problem of optimal recovery in the case of a nonsymmetric convex class of functions. We begin with the problem of optimal integration of convex functions. We prove that adaption cannot help in the worst case but considerably helps in the case of Monte Carlo methods.

Then we study more generally certain Gelfand widths that are useful for nonsymmetric classes. We give examples for linear problems on a convex set where adaptive methods are much better than nonadaptive ones.

F. PEHERSTORFER

Characterization of positive quadrature formulas

We say that an interpolatory quadrature formula

$$\int_{-1}^{+1} f(x)w(x)dx = \sum_{j=1}^{n} \beta_j f(x_j) + R_n(f),$$

$-1 < x_1 < x_2 < \ldots < x_n < 1$, is a positive $(2n-1-m, n, w)$ q.f., if $\beta_j > 0$ for $j = 1, \ldots, n$ and $R_n(f) = 0$ for $f \in \mathbb{P}_{2n-1-m}$. Various characterizations of positive $(2n-1-m, n, w)q.f.$, old and new ones, are given. So it is demonstrated that the positivity of a $(2n-1-m, n, w)$ q.f. is equivalent to the fact that there exists a sequence of positive definite numbers $c_0, c_1, \ldots, c_{2n-1-m}$ such that $c_j = \int_{-1}^{+1} x^j w(x)dx$ for $j = 0, \ldots, 2n-1-m$. Then it is shown that the last statement is equivalent to the fact that $\prod_{j=1}^{n}(x - x_j)$ can be generated by a recurrence relation the recurrence coefficients of which coincide up to $j = 0, \ldots, n - [\frac{m}{2}]$. Furthermore, as consequences of the above characterizations, a simple characterization of positive quadrature formulas with respect to the Chebyshev weight $(1 - x^2)^{\pm\frac{1}{2}}$ and an extension of a theorem of Bernstein on the distribution of the nodes of a positive quadrature formula is given.

K. PETRAS

Quadrature theory of convex functions

First, we give a short overview of some known results on the quadrature of convex functions. Midpoint and trapezoidal formulae have been investigated frequently, but it was shown that they cannot compete in many situations. However, in those classes of convex functions which have caused some interest, the Gaussian rule is not far behind the respective optimal rule. We therefore investigate the Gaussian error for convex functions more detailed, in particular under additional assumptions (for instance, continuous differentiability in the interior of the basic interval) on the integrand. Similarities to the size of Fourier coefficients are remarkable.

G. SCHMEISSER

Positivity and Monotonicity in Quadrature

Given a quadrature method, it is not only of theoretical interest but also of practical importance to know conditions on the integrand which guarantee

(a) a one-sided approximation of the integral (POSITIVITY),

(b) monotone convergence of the remainders (MONOTONICITY).

For most of the familiar quadrature methods these problems have been considered and sign restrictions on certain higher order derivatives have been given as sufficient conditions. Examples indicate, however, that these conditions seem to be far from being necessary. Using the concept of positive definite functions, we specify in the case of the trapezoidal method wider classes of functions that guarantee positivity or monotonicity. Extensions to related quadrature methods are also discussed.

H.J. SCHMID

Gaussian Cubature Rules

This is a joint work with Yuan Xu. Gaussian quadrature rules are characterized by the fact that there are infinitely many even degree formulae, and one of odd degree having a minimal number of nodes. The number of nodes attains the classical lower bound.
For two classes of integrals analogous results will be derived in two dimensions.

C. SCHNEIDER

Rational Hermite Interpolation and Quadrature

After a short introduction to rational functions interpolating f and f' at given nodes we apply these interpolants to numerical integration. Formulas for the weights and the remainder are derived, different choices of the parameters are discussed. Finally, the strong connections between quadrature and rational Hermite interpolation are exhibited by Engels' dual quadratures.

A. SIDI

A new variable transformation for numerical integration

Presently, variable transformations are used to enhance the performance of lattice rules for multidimensional integration. The transformations that are in the literature so far are of either polynomial or exponential nature.
We propose yet a new transformation that is trigonometric in nature. We analyze its effect within the framework of one–dimensional integration and show that it has some very interesting mathematical properties. We demonstrate its numerical efficiency by applying it to various one–dimensional integrals of smooth and singular functions. Present results indicate that the new transformation is more advantageous than the known polynomial transformations, and has less underflow and overflow problems than the exponential transformations.

I.H. SLOAN

Lattice rules for non–periodic integrands II

This paper describes further joint work with H. Niederreiter on the application of lattice rules for integrals over $[0,1]^s$ in the case in which the integrand f is continuous on $[0,1]^s$ but does not have a continous periodic extension. Here s is allowed to be arbitrary. The main focus is a quadrature rule Bf which redistributes the weight $1/N$ (where N is the order of the original rule) which originally is associated with the vertex of the cube at the origin over all 2^s vertices of the cube, with the weights chosen so that Bf integrates exactly all multilinear functions. It is shown that Bf is optimal among modified–vertex–weight rules, in that it minimizes the L_2 discrepancy error bound. Numerical experiments and further work are discussed.

F. STENGER

Collocation and Approximation of Indefinite Convolutions

We present a new collocation–type procedure for approximating either one of the integrals

$$p(x) = \int_a^x f(x-t)g(t)dt, \text{ or } p(x) = \int_x^b f(t-x)g(t)dt, \qquad (0.1)$$

for $x \in (a,b)$, where (a,b) may be a finite interval, a semi – infinite interval, or all of $\mathcal{R}$. We give an explicit construction of a vector $(p_{-N},\ldots,p_N)^T$, and a choice of h, corresponding to $p(x)$ being analytic on (a,b), although $p(x)$ may have certain types of singularities at a and b, (as long as, e.g., $p \in \mathrm{Lip}_\alpha[a,b]$ in the case when (a,b) is a finite interval), and then we show that there exists a constant C_1 which is independent of N, such that

$$\sup_{x\in(a,b)} \left| p(x) - \frac{p_{-N}+\rho(x)p_N}{1+\rho(x)} - \sum_{k=-N}^{N} \left(p_k - \frac{p_{-N}+e^{kh}p_N}{1+e^{kh}} \right) S(k,h)\circ\phi(x) \right|$$

$$\leq C_1 N^{1/2} e^{-(\pi d\alpha N)^{1/2}} \qquad (0.2)$$

In $(0.2), \phi : (a,b) \to \mathcal{R}$ and $\rho = e^\phi$ are explicitly defined functions, and $S(k,h)\circ(x) = \sin\{\pi(x-kh)/h\}/\{\pi(x-kh)/h\}$.

G.W. WASILKOWSKI

Integration of piece–wise smooth functions in probabilistic setting

We discuss integration of scalar functions that are regular everywhere but at some singular points.

In the worst case setting, due to unknown location of singularities, any quadrature requires a substantial number of function values in order to approximate the integral with small error. The situation is drastically different in a probabilistic setting. Indeed, under some stochastic assumptions, we provide an adaptive quadrature that, modulo a small probability, behaves as well as an optimal quadrature for functions without singularities.

H. WOŹNIAKOWSKI (joint work with K. Ritter & G.W. Wasilkowski)

Multivariate Integration in Various Settings

We begin with multivariate integration in the average case setting. We show estimates on the nth minimal average error in terms of the smoothness of a covariance function of stochastic process. The proof technique is based on relations between the average and worst case settings. We illustrate these estimates for isotropic Wiener and Wiener sheet stochastic processes.

We briefly indicate extensions of our estimates for nonlinear quadrature formulas, adaptive information, different error criteria and for the probabilistic setting.

K. ZELLER (with W. Haußmann)

Error in DE: Grid and BOGS

Plum '90 treated differential operators by a numerical homotopy method. He used Simpson's quadrature in cases $f = pq$ (where p fixed, q : computed polynomials), estimating the error via Leibniz' formula and grid bounds for polynomials (Ehlich). We discuss other approaches, based on BOGS (biorthogonal systems; Chebyshev expansions): General estimates, decomposition $f = pq$, Fourier differentiation. Further we mention variants and extensions.

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