MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Topological buildings

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Das von der Studienstiftung des deutschen Volkes geförderte Forschungskolloquium fand unter Leitung von A.E. Schroth (Braunschweig) statt. Der Versuch, eine sinnvolle Definition topologischer Gebäude zu erarbeiten, stand im Mittelpunkt des Interesses. Gleich zu Beginn wurden die verschiedenen Ansätze, topologische Gebäude zu definieren, vorgestellt. Im Laufe der Tagung zeigte sich, daß diese verschiedenen Definitionen doch einen größeren gemeinsamen Kern haben, als zu Beginn vermutet. Weitere Schwerpunkte des Kolloquiums waren Fragen im Umfeld topologischer verallgemeinerter Polygone sowie neue Ergebnisse in der allgemeinen Gebäudetheorie. Neben den unten aufgeführten Vorträgen fanden noch zahlreiche kleinere Gruppensitzungen statt.

Das Treffen, an dem fast alle, die auf dem Gebiet topologische Gebäude und topologische Polygone arbeiten, teilgenommen haben, erwies sich als sehr erfolgreich. Nicht nur, daß ein intersiver Austausch stattgefunden hat, der wahrscheinlich zu der ein oder anderen gemeinsamen Arbeit führen wird, es wurde auch erarbeitet, wie weiter verfahren werden soll. Die verschiedenen Ansätze, topologische Gebäude zu definieren, sollen auch noch weiterhin verfolgt werden, da sie nichts grundverschiedenes ergeben werden, wohl aber durch die feinen Unterschiede reizvolle Erkenntnisse zu erwarten sind.

Da sich gezeigt hat, wie notwendig und erfolgreich dieses Treffen war, wurde beschloßen, sich in ein bis zwei Jahren wieder in ähnlicher Runde zu treffen.

Vortragsauszüge

Theo Grundhöfer:

Topological buildings: the approach of Burns - Spatzier

The concepts and results of a paper by Burns - Spatzier (Publ. IHES 65, 1987, 5 - 34) are discussed. Their definitions appear to be appropriate in the compact case only. Another definition of topological buildings, due to Linus Kramer, requires continuity of projections of the buildings on suitable domains. For projective spaces and generalized polygons, this definition reduces to the usual one, and it is compatible with the definition of Burns - Spatzier in the compact case.

Martina Jäger:

Topological buildings: an approach via projections

Let Δ be a spherical building with vertex set $V = \bigcup_{i=1}^{n} V_i$ such that each V_i consists of all vertices of a fixed type; let each V_i carry a topology. This also provides a topology on Δ.

For each $k \in \{1, \ldots, n\}$, consider the set $D^k = \{(u,v) \in V^2 \setminus \mathrm{I} \mid \mathrm{proj}_u\, v$ has a vertex of type $k\}$ (where I denotes the incidence relation and proj the projection mapping in Δ) and the function $p^k : D^k \to V^k : (u,v) \mapsto$ vertex of type k of $\mathrm{proj}_u\, v$.

Definition: Δ is a **topological building**, if all maps p^k are continuous. Comparing topological buildings with topological projective spaces and topological generalized n-gons, we have the following results:

A projective space of any dimension is a topological building, if and only if it is a topological projective space.

A generalized quadrangle is a topological building, if and only if it is a topological quadrangle. In the case of generalized n-gons with $n > 4$ we only know that a generalized n-gon which is a topological building is a topological n-gon.

Finally, in every topological building links are again topological buildings.

Regina Kühne:

Topological buildings: an approach via convex hulls

I define a topological spherical building by requiring continuity of the map that sends every pair of opposite chambers to the apartment they span, where this apartment is considered as the set of its chambers. In addition, I need the following conditions: the set of pairs of opposite chambers is open and the canonical projections from chambers to vertices are open. For technical reasons the space of chambers obtains its topology as a subspace of the product of the spaces of vertices. As an important consequence it turns out that in the case of topological projective spaces, i.e. buildings of type A_n, this definition is equivalent to the usual one. For buildings of type C_2 my definition implies that of a topological quadrangle, but probably the converse is not true. So my approach seems to be somehow stronger. For buildings of type C_n, with $n > 2$, I deduce several continuous maps between vertices of a certain distance. Here the main result yields a residue that is a topological projective plane, hence by the above a topological building of type A_2.

Linus Kramer:

Coordinates in topological buildings

A spherical building admits a decomposition into Schubert cells, and these cells are products of punctured panels in a rather natural way. If the building carries a good topology (e.g. compact Hausdorff), then this decomposition is very similar to a CW-complex decomposition. This has the following applications:

(i) The underlying field becomes a topological field, and the correspondence between the building topology and the field topology is unique.

(ii) root collineations are automatically continuous.

(iii) the cohomology of the flag space Δ_J of J-flags is $\mathbb{Z}_2{}^{W/W_J}$, if Δ_J is connected and finite dimensional.

Andreas Schroth:

Half regular and regular points in compact polygons

The notion of half regular and regular points in polygons is introduced and treated in the case of compact polygons. It is shown that in a compact polygon with a half regular point the dimension of line pencils is at most the dimension of pointrows. If these two dimensions are the same, then the derivation in a half regular point yields not just a linear space but a topological projective plane and the derivation in a regular point yields a topological projective plane in case of quadrangles and a topological quadrangle in case of hexagons. From this a characterization of the symplectic quadrangle over $\mathbb{R}$ or $\mathbb{C}$ and of the split Cayley hexagon over $\mathbb{R}$ or $\mathbb{C}$ is deduced.

Hendrik Van Maldeghem:

Amalgons, ovoids and unitals

Let $\mathcal{S} = (\mathcal{P}, \mathcal{L}, \mathrm{I})$ be a generalized quadrangle or hexagon having a flag $\{p, L\}$ such that p is a regular point and L is a regular line. The derived geometries are respectively $\mathcal{S}_p$ and $\mathcal{S}_L$. I describe a method to reconstruct the polygon $\mathcal{S}$ from the two polygons $\mathcal{S}_p$ and $\mathcal{S}_L$.

Self polar generalized n-gons contain ovoids resp. unitals when $n = 4$ resp. $n = 6$. We give a construction of the "classical" ovoids and unitals in $Q(4, \mathbb{R})$ and $H(\mathbb{R})^{\text{dual}}$ and raise the question if there are other connected compact ovoids or unitals.

We also give a geometric construction of the Ree-Tits Unital in $G_2(\mathbb{F})$, $\mathbb{F}$ finite and of characteristic three, $|\mathbb{F}| = 3^{2 \text{ or } 1}$.

Michael Joswig:

Slanted symplectic quadrangles

A simple modification of symplectic quadrangles yields examples of non-classical generalized quadrangles. The finite quadrangles of this type are well known.

With the example of the two smallest thick examples, all these quadrangles inherit their group of automorphisms from the classical symplectic quadrangles (as a stabilizer of a point). These slanted quadrangles are not Moufang.

Furthermore, it turns out that all even finitary permutations arise as projectivities of the slanted symplectic quadrangles. As a consequence, they cannot be turned into topological quadrangles.

Linus Kramer:

Polygons with flag transitive automorphism groups

Theorem: Let $\mathcal{P}$ be a compact connected flag homogeneous polygon. If the dimensions of linepencils and pointrows are equal, then $\mathcal{P}$ is classical, i.e. $\mathcal{P}$ arises from the canonical BN-pair of a simple Lie group.

Bernhard Mühlherr:

Geometries fixed by an automorphism of a building

The diagrams used by Tits in order to classify the semi-simple algebraic groups have a natural geometric interpretation. They provide Coxeter complexes which are "weak subcomplexes" of a given one.

These complexes consist of simplices fixed by a group of automorphisms of the Coxeter complex in question. This observation can be easily generalized to spherical buildings, which provides a geometric interpretation of forms of simple algebraic groups.

Cécile Huybrechts:

The commutativity of the ground division ring of a D_n-geometry

If Γ is a thick and residually connected D_n-geometry, it is well known that Γ is defined over a ground division ring which is commutative. I give an elementary proof of the commutativity based on the construction of null polarities in the projective subspaces of Γ, for $n = 4$.

Serge Lehmann:

Line-spaces and buildings

I explained how to make a connection between buildings and line-spaces. In the direction building to line-space I used the concept of shadow spaces, in the other direction a "right" choice of types. The concept of spaces is used to gain insight into other structures. One essential aspect of line-spaces is concerned with axioms; i.e. given a set S of algebraic structures, the goal is to list a set of properties of line-spaces, such that any line-space with those properties corresponds to a member of S. One axiom system in terms of points and lines is well known for the shadow spaces $C_{n,k}$, e.g. polar spaces ($k = 1$) or dual polar spaces ($k = n$). I gave an axiom system in terms of points and lines for cuboctahedric spaces, i.e. the case $C_{3,2}$.

Theo Grundhöfer:

Moduli spaces of compact projective planes

For a compact topological space X, the set of all isomorphism classes of topological projective planes with pointspace X can be endowed with a natural topology. We dare to call this the moduli space $\mathcal{M}_X$ of topological projective planes on X. The planes with large automorphism groups are expected to be singular points of $\mathcal{M}_X$.

Berichterstatter: A.E. Schroth

Liste der Beteiligten

Theo Grundhöfer
Mathematisches Institut der
Universität Tübingen
Auf der Morgenstelle 10
7400 Tübingen
e-mail: mmisa01 at convex.zdv.uni-tuebingen.de

Cécile Huybrechts
Université Libre de Bruxelles
Département de Mathematiques
Campus Plain C.P. 216
Bd du Triomphe
B-1050 Bruxelles
e-mail: huyb at ulb.ac.be

Martina Jäger
Gutenbergstr. 26
2300 Kiel 1

Michael Joswig
Mathematisches Institut der
Universität Tübingen
Auf der Morgenstelle 10
7400 Tübingen
e-mail: mmisa01 at convex.zdv.uni-tuebingen.de

Linus Kramer
Mathematisches Institut der
Universität Tübingen
Auf der Morgenstelle 10
7400 Tübingen
e-mail: mmisa01 at convex.zdv.uni-tuebingen.de

Regina Kühne
Institut für Analysis
Abteilung für Topologie
Pockelsstr. 14
3300 Braunschweig
e-mail: i1011003 at dbstu1.bitnet

Serge Lehmann
Université Libre de Bruxelles
Département de Mathematiques
Campus Plain C.P. 216
Bd du Triomphe
B-1050 Bruxelles
Belgien

Bernhard Mühlherr
Rue Darwin 6
B-1060 Brüssel
Belgien

Andreas Schroth
Franz-Liszt-Str. 37
3300 Braunschweig
e-mail: i1011001 at dbstu1.bitnet

Hendrik van Maldeghem
Seminarie voor Meetkunde en Kombinatoriek
Rijksuniversiteit Gent
Krijgslaan 281
B-9000 Gent
Belgien
e-mail: hvm at cage.rug.ac.be