MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 52 / 1992

Theory and Numerical Methods for Initial-Boundary Value Problems

6.12. bis 12.12.1992

Die Tagung fand unter der Leitung von H.-O. Kreiss (Los Angeles) und J. Lorenz (Albuquerque) statt. Im Zentrum des Interesses stand die Theorie und die numerische Behandlung zeitabhängiger partieller Differentialgleichungen. Es war ein Ziel der Tagung, praktische und theoretische Aspekte zusammenzubringen. Die Anwendungen reichten von technisch wichtigen Strömungen bis zur Medizin. Es zeigte sich, daß die Analysis von großer Bedeutung ist für die numerische Behandlung und für die Interpretation von Ergebnissen.

Vortragsauszüge

G. Bader:

Numerical Simulation of Laminar Flame Problems on Parallel Computers with Distributed Memory

We present a new data decomposition algorithm for the solution of stationary 2D-Flame Models. In particular, we discuss the efficient implementation of Block-ILU preconditioning for the iterative solution of highly unsymmetric type of linear systems. Numerical results for the so called flame-sheet approximation are presented. These show, both, the high efficiency and the robustness of the approach taken.

G. Baker:

Numerical Methods for Free-Surface Flows

Many free-surface flow problems (fingering in Hele-Shaw cells or ground water, Rayleigh-Taylor instabilities, Kelvin-Helmholtz instabilities) are ill-posed in the sense of Hadamard. In particular, the smallest scales grow the fastest unless some physical regularization is present. In the limit of small physical regularization, the motion can be extremely difficult to compute numerically by standard methods. However, the analytic continuation of the equations into the complex physical plane presents a new formulation that is well-posed for numerical calculation. We describe a spectral method capable of advancing the finger in a Hele-Shaw cell for long times and for studying the competition of fingers in a Hele-Shaw cell with high accuracy. Our methods generalize to other free-surface flows that are ill-posed.

Ch. Bernadi:

Finite Element Discretization of Navier-Stokes Equations with Varying Density

In the incompressible Navier-Stokes equations, when the Boussinesq approximation is not valid, the variations of the density must be taken into account. We study two variational formulations of a model where the density is assumed to be given but non-constant. We compare finite discretizations which rely on these formulations. New extensions are presented.

W.-J. Beyn:

Numerical Approximation of Connecting Orbits

We consider the numerical computation of orbits which connect steady states or periodic orbits in a parameter dependent dynamical system. Such problems typically arise when determining the shape and speed of travelling waves in parabolic systems. Connecting orbits satisfy a boundary value problem on the real line. We analyze the error caused by truncation to a finite interval and by the choice of boundary conditions. More specifically, we consider orbits connecting a steady state to a periodic orbit. It will be shown that a crucial role is played by the property of "asymptotic phase" and by the corresponding foliations of stable and unstable manifolds.

F. Bornemann:

Adaptive Rothe's Method for Time-Dependent PDEs

An adaptive approach for IBVPs stemming from diffusions-dominated or Schrödinger-type equations is presented. It consists in adaptively discretizing the evolution operator in time first and viewing the spatial discretization as a perturbation. This allows time-steps which belong to the dynamics of the problem and gives an easy matching of time- and space-accuracies. The use of a multigrid-type algorithm on highly non-uniform triangulations in space provids an elliptic subproblem solver which is optimal. This allows easily a new triangulation at each time-step. Application to the 2D and 3D simulation of the heating of tumors (hyperthermia) in cancer-therapy is given.

Y. Brenier:

Existence and Uniqueness of a Pressure Field for the Incompressible Euler Equations

We consider the motion of an ideal incompressible fluid in a compact domain $X \subset \mathbb{R}^d$, d=2 or 3. Instead of the initial value problem, we deal with the following two point boundary value problem in time: Given T>0 and a diffeomorphism h of X with Jacobian determinant equal to 1, find a time-dependent family $t \to g(t)$ of diffeomorphisms of X that minimizes the "action"

$$\frac{1}{2}\int_0^T \int_X |\partial_t g(t,x)|^2 \, dx \, dt \ , \qquad \textit{subject to} \ \begin{cases} g(0,x) = x \\ g(T,x) = h(x) \\ \det Dg \equiv 1 \end{cases}$$

Local existence results (when h is close to the identity map in some strong Sobolev norm) were obtained by Ebin & Marsden. In the large, uniqueness can break down (which is an easy observation) but, more seriously, existence can fail when d = 3. (This was shown by Shnirelman in 1987). We recall some previous result (YB, 1989) showing global existence of generalized solutions (obtained in the spirit of Young measure theory). In the present talk, we establish the existence and uniqueness of the dual solution of the minimization problem, which turns out to be the pressure field.

D. L. Brown:

Adaptive Composite Overlapping Grids for Gas Dynamics

A method under development for the numerical solution of the compressible Euler equations of gas dynamics in regions of complex geometry is presented. Regions of complex geometry in two and three space dimensions are represented by the method of composite overlapping grids, as developed by Chesshire and Henshaw (J. Comp. Physics **90**, p.1). A composite overlapping grid consists of a set of logically rectangular or hexahedral non-orthogonal curvilinear grids that overlap where they meet and completely cover the computational region. PDEs are solved using standard finite difference techniques, but with additional boundary conditions that interpolate the solutions between component grids. The Adaptive Mesh Refinement (AMR) method developed by M. Berger is combined with the overlapping grid method to give adaptive resolution of complex structure in the flows. Since the AMR method is also based on logically rectangular grids, very few changes are required in the algorithm in order to use it with overlapping grids. The Euler equations are solved on the grids using a class of high-order Godunov methods of the type developed by P. Colella. A discretization for the Euler equations must have the basic properties that shock singularities propagate at the correct speed, and that the error committed in the shock is damped out extremely rapidly as it moves away from the shock. The high-order nature ot the Godunov method guarantees that the smooth parts of the solution are computed accurately, and thus the end-states for the shock and hence its speed will be correct. The upstream nature of the method ensures that shock errors are damped out quickly. Numerical examples are presented demonstrating the high-order Godunov method on overlapping grids.

M. Giles:

Non-Reflecting Boundary Conditions for the Euler Equations

This talk discusses the variety of far-field boundary conditions used for the solution of the Euler equations in the context of two-dimensional flows in turbomachinery and external aerodynamics. Starting from an assumption of linear perturbations to a uniform steady flow it is possible to construct exact non-local non-reflecting b.c.'s. These then lead to a number of different approaches

* 1D b.c.'s.: These assume waves leaving normal to the boundary. An improved well-posed version assumes known non-zero angle.
* Steady-state: Taking the limit as frequency approaches zero gives spatially non-local b.c.'s which are easily applied in turbomachinery. Similar b.c.'s can also be used when there is a single known non-zero frequency.
* 2D approximate: Using the ideas of Engquist and Majda produces approximate local non-reflecting b.c.'s. Using the theory of Kreiss it can be shown that the outflow b.c. is well-posed but not the inflow. A modfication to the inflow b.c. leads to it being well-posed and fourth-order non-reflecting.

M. Goldberg:

Stable Difference Schemes for Parabolic Systems

The main purpose of this talk is to discuss the employment of generalized numerical radii in order to investigate stability of implicit difference schemes for the initial-value problem associated with a general, well-posed, multi-space-dimensional, parabolic system of the form

$$\frac{\partial u}{\partial t} = \sum_{1\le p\le q\le d} A_{pq}\frac{\partial^2 u}{\partial x_p \partial x_q} + \sum_{1\le p\le d} B_p \frac{\partial u}{\partial x_p} + Cu\ , \quad x_p \in \mathbb{R},\quad t\ge 0,$$

where A_{pq}, B_p, and C are fixed matrices.

T. Hagstrom:

Boundary Conditions at Artificial Boundaries with Applications to Fluid Flow Simulations

We describe our work on the development, analysis and testing of boundary conditions at artificial boundaries for the Navier-Stokes and Euler equations. The approach is to study in detail boundary condition construction for the equations linearized about simple base flows, and experiment numerically on the full equations. We display conditions based on the Orr-Sommerfeld equations for parallel flows, small viscosity and Mach number expansions of the exact boundary operators for linearizations about constant flows, and conditions for the Euler equations derived from asymptotic expansions of the Riemann variables. A negative result for the long time error behavior for solutions of the two-dimensional wave equation and standard boundary operators is also given.

T. Y. Hou:

Stabilizing Effect of Surface Tension and Formation of Pinching Singularities in Fluid Interfaces

In this talk, I present our recent results on the stabilizing effect of surface tension for inviscid, incompressible fluid interfaces. We show that the fluid interface is linearly well-posed when linearized around any prescribed, time-dependent smooth solution. Using this result, we prove stability and convergence of a spectrally accurate boundary integral method. We then use our spectrally accurate method to study the nonlinear stability effect of surface tension numerically. An efficient implicit scheme is designed to relax the severe time step constraint due to the presence of surface tension. We found that if surface tension is above certain value, the interface problem has a global smooth solution. However, if surface tension is below certain critical value, the interfaces can form a pinching singularity in a finite time. And the type of singularity is different from that in vortex sheets.

H. Jarausch:

Computing Periodic Solutions of Parabolic Systems by Using a Singular Subspace as "Reduced Basis"

The search for a periodic solution is reduced to a fixed point problem for the Poincaré map: $\mathcal{P}(u)=u$. Let Z and Y be orthogonally complementary subspaces built up from singular vectors of $\mathcal{P}'(u)$-I. These subspaces are mapped into the corresponding left singular subspaces $\bar{Z}$, $\bar{Y}$. To get back to a fixed point problem one constructs an orthogonal mapping J which maps Z, Y to $\bar{Z}$, $\bar{Y}$. By applying J^T to $0=\mathcal{P}(u)-u$ this FPeq splits into two locally decoupled FPeqs. By seeking the "right" singular subspace $Z(\bar{Z})$ the FPeq projected onto Y gets contravtive and the (usually small) FPeq projected onto Z shows many features of the full system. One can show that derivatives up to the 2nd order coincide with the correspon-ding Ljapunov-Schmidt reduction. Since in many applications the dimension of the "reduced basis" Z is very low one has a useful technique to sudy (branches of) periodic solutions with and of a tiny system. The computation of Z and $\bar{Z}$ leads to a Riccati equation which can be solved using bordering techniques which are well known.

R. Jeltsch:

On a Boundary Layer in Hypersonic Reacting Euler Flow

We consider hypersonic flow around a blunt body of a mixture of gases which are chemically not in an equilibrium. It is shown that the chemical reactions induce an extremely thin, unphysical boundary layer. A modification of the Van Leer flux vector splitting is presented which is able to indicate the presence of the boundary layer. This is joint work with M. Fey, ETH Zürich and S. Müller, RWTH Aachen.

C. Johnson:

Adaptive Finite Element Methods for Conservation Laws.

We present adaptive finite element methods for systems of conservation laws in one dimension based on a posteriori error estimates.

K. Kirchgässner:

Long-Time-Asymptotics of Perturbed Gravity Waves

This describes joint work with Mariana Haragus from Nice. It is well known that solitary waves exist under the influence of gravity on the surface of an inviscid fluid, when the Froude number is greater than one. However, the description of the large-time behavior of local perturbations of this wave as solutions of the full 2d Euler equations is an open problem. In this lecture we discuss a method for its resolution. First it is observed that the slowest transport happens in a four-dimensional subspace of the phase space. There we construct a so-called Eichform via a Floquet-transformation. The Eichform isolates the given solitary wave in a one parameter family of these waves, and it reveals the space-asymptotics which a local perturbation has to obey. An asymptotic Eichform is derived then, such that the full equations contain in addition only fast decaying terms in space. These terms yield decay of the solution which is at least O(1/t) faster than the slowest decay, which is determined by the asymptotic Eichform. The final result then shows that the long-time behavior is given by two modulated waves moving in opposite directions and decaying like $0\ (t^{-3/2})$.

C. Klingenberg:

On Two-Dimensional Hyperbolic Equations: Stability of Difference Schemes with Shock Tracking

When computing compressible inviscid gas flow there are cases when it is of particular importance to preserve the identity of a surface representing a particular shock wave. An example is the bow shock of a reentry vehicle. We consider here the case in two space dimensions where a curve representing a shock wave evolves for a certain time, always separating two smooth flow regions. In particular, we consider a shock wave moving into a gas at rest. The numerical scheme we consider for computing this flow is based on a moving underlying grid where the shock position always determines one grid line. We shall consider the linear stability of a class of finite difference schemes in the back of this shock wave coupled with a scheme explicitely tracking this interfaces. The main ingredient in this work is a stability theorem for a two-dimensional initial boundary value problem using energy methods. This fixed boundary then becomes a moving boundary to obtain the desired result.

P. Knabner:

Error Estimates for Finite Element Approximation of Degenerate Parabolic Systems

We consider the following model problem for reactive solute transport in porous media, with an adsorption reaction, either in equilibrium ($k=\infty$) or in non-equilibrium ($k<\infty$):

$$\left.\begin{aligned} \partial_t u + \partial_t v - \Delta u &= f \\ \partial_t v &= k(\phi(u) - v) \end{aligned}\right\} \quad in\ \Omega\subset\mathbb{R}^N, \quad t\in[0,T]$$

together with boundary and initial conditions, where $k\leq\infty$, i.e. for $k=\infty$ the problem reduces to the scalar equation $\partial_t(u+\phi(u))-\Delta u=f$. The problems are degenerate, as typical non-linearities have the form $\phi(u)=\alpha u^p$, $\alpha>0$, $p\in(0,1)$. We study error estimates for the semi-discrete Galerkin approximation with linear finite elements, consistent or with quadrature, and the fully discrete versions based on the backward Euler method. We derive error estimates in energy norms, which partially exhibit the full approximation power of the trial space despite of the degeneration. It turns out that regularization is a useful technical tool in general and in case of non-degeneracy conditions, i.e. if there is a minimal growth of the solution away from the front ∂supp u, the regularization improves on the results. (Joint work with J. W. Barrett (London)).

H. C. Kuhlmann:

Hydrodynamic Instabilities in Systems Driven by Surface Forces

The linear stability of 2-dimensional toroidal flows in a cylindrical liquid bridge driven by thermocapillary forces is investigated by the application of spectral methods. The 2-dimensional basic flow and temperature fields are calculated by a Galerkin tau method. The neutral modes giving rise to 3-dimensional instabilities are obtained by Galerkin-collocation-tau. Although the applicability of this method is limited to moderate Marangoni numbers, the threshold value and the space-time structure of the neutral disturbances can be obtained with reasonable good accuracy. The instability mechanisms and the physical properties of the supercritical flows are discussed.

J. Lorenz:

Continuation of Invariant Tori

We consider a dynamical system depending on a parameter λ. Assume that for $\lambda=\lambda_0$ an invariant 2-torus M_0 is known in terms of a parametrization w_0. Under suitable assumptions, there is a branch of invariant tori $M(\lambda)$ for λ near λ_0, which we try to follow computationally. The main idea is to use a coordinate system determined by w_0 and to update the coordinates as the computation proceeds. In practice, the parametrizations can only be determined on a grid. We give an error analysis as the gridsize tends to zero.

G. Lube:

Stabilized Galerkin Methods for Solving the Incompressible, Nonisothermal, Nonstationary Navier-Stokes Equations

We consider the finite element discretization of the incompressible Navier-Stokes problem with Boussinesq approximation. Spurious numerical solutions of the standard Galerkin finite element methods may be caused by dominating convective terms and/or inappropriate pairs of velocity and pressure interpolation functions which do not pass the Babuska-Brezzi condition. As a remedy we add least-squares formulations of the basic equations. It turns out that the resulting Galerkin/least-squares method stabilizes both instabilities arising from dominating convective terms and inappropriate velocity/pressure pairs. First we analyze the stability and convergence of the time-integration procedure. Secondly we consider the stability and convergence of the method for a linearized problem arising from the simple iteration procedure. In particular, we consider the parameter design problem for the given method. We conclude with some numerical results.

M. Luskin:

The Dynamics of Crystalline Microstructure and Phase Boundaries

The deformation $y(x,t): \Omega\times\mathbb{R}^{+} \to \mathbb{R}^{3}$ of solid crystals where the spatial domain $\Omega\subset\mathbb{R}^{3}$ can be modelled by

$$y_{tt}(x,t) = div\,\sigma(\nabla y(x,t))$$

with appropriate boundary and initial conditions. Equilibrium solutions to these models often have highly oscillatory deformation gradients when the stress tensor $\sigma(F) = \partial\phi(F)/\partial F$ is derived from a non-convex energy density $\phi(F)$ where $F = \nabla y(x,t)$. We can describe these highly oscillatory solutions by a mathematical definition of material microstructure using the Young measure. We will present numerical methods and results for the dynamical development of material microstructure and for the propagation of phase boundaries in the presence of material microstructure.

Y. Maday:

Adaptivity Using Wavelets Basis for the Approximation of PDE

The aim of this work is to use the localization properties of the wavelet basis for the simulation of PDE. This property is used in an adaptive framework in order to minimize, at most, the number of degrees of freedom to represent accurately the solution, especially when one can infer that it developes local singularities of sharp gradients. In opposition to other methods that require error estimators that are added to the numerical tools, our strategy uses the existing discretization parameters as error estimators. The adaptivity can then be done at each time step resulting in a very cheap representation of the solution. Numerical and theoretical evidences of the possibility of the method will be presented not only for 1D problems.

H. Mittelmann:

Computing Stability Bounds for the Thermocapillary Convection under Zero-Gravity

In the float-zone process of crystal growth temperature-gradient induced surface tension gradients along the outer free surface drive convection rolls in the float-zone. These are present even under zero-gravity. For increasing temperature differences measured by the Marangoni number this convection becomes unstable leading to poor crystal quality. It is thus desirable to determine bounds for the stability limit. Both energy and linear theory results are obtained for a model problem. In addition to the solution of the underlying Boussinesq equations generalized large and sparse eigenvalue problems have to be solved. The numerical approach is outlined and results are compared to those from experiments.

K. W. Morton:

Evolution Galerkin Methods in One and Two Dimensions

In discretizing evolutionary problems a combination of three principle ideas has proved to be particularly useful: approximating the evolution operator; Galerkin or Petrov Galerkin projection onto the finite element trial space; and a recovery procedure to obtain higher order accuracy in an adaptive manner. Several examples of the first will be given, of which that based on tracing the characteristics will be considered in more detail. In the form of Brenier's transport collapse operator, using L_2 projection onto piecewise constants and with recovery by piecewise linears, it gives a family of schemes of the form $U^{n+1}=PE_{\Delta}RU^{n}$. They are explicit, unconditionally stable, TVD or TVB and converge to the entropy-satisfying solution of a scalar conservation law. Using a Riemann-Stieltjes parametrization, several equivalent forms are given in 1D; and the importance of certain corner terms in the the 2D case are pointed out. Systems of equations are dealt with by wave decomposition to approximate the evolution operator.

N. A. Petersson:

Computing the Oscillations of a Free Surface Jet

A numerical method for computing the motion of an inviscid and irrotational fluid jet issuing from an elliptical orifice is described. The differential equation for the evolution of the potential on the boundary, and of the shape of the boundary is discretized by 4'th order accurate centered differences in space. The the resulting system of ODEs is integrated in time by a 4'th order accurate four stage Runge-Kutta method. To evaluate the time derivatives of the potential and the shape of the boundary, it is necessary to solve Laplace's equation with Dirichlet data. The problem is transformed onto a fixed computational domain where the elliptical equation is solved by a 4'th order accurate finite difference method on a composite overlapping grid. One advantage of this approach is that the computational domain only needs to be gridded once. Instead, the transformed Laplace equation will get coefficients that vary both in time and space. By studying the spectrum of the discrete linearized operator, it is found that the spatial discretization is not completely satisfactory because one complex conjugated pair of eigenvalues of the linearized operator has a small positive real part. However, numerical examples show that the equations can still be integrated successfully, at least until time of the order O(1), if the initial cross-section is sufficiently close to a circle. For initial cross-sections with large aspect ratio, the break down time decreases when the number of grid points increases.

R. Rannacher:

On the Approximate Inertial Manifold Approach to the Navier-Stokes Equations

The concept of "inertial manifold" was recently introduced by Foias, Sell and Temam in an effort to reduce the study of the long time dynamics of the Navier-Stokes equations to that of a finite system of ordinary differential equations. Following that Foias, Manley and Temam introduced the concept of "approximate inertial manifold" in an effort to make the theory practical, ultimately for large scale computation of turbulent flow. Identifying the large and small scales of motion with the low and high modes of a spectral representation they proposed a method (the "AIM") for determining approximate values of the high modes directly as functions of the low modes, rather than as solutions of evolutionary equations. This has subsequently developed into a computational scheme, the associated "nonlinear Galerkin method" of Marion and Temam, that consists of inserting these values for the high modes into the Galerkin equations for the approximation of the low modes. Because of the apparent deoth of this inside, seemingly reaching to the very physics of turbulence, these ideas have been received with a sense of exitement and followed upon in the research papers of many authors. This lecture will contribute to the discussion about the ability of the AIM method to model turbulent flow and about the theoretical potential of the AIM/NGM to provide a computational basis for the calculation of turbulent flow.

H.-J. Reinhardt:

On Approximation Methods for Illposed Parabolic Equations

Cauchy problems for parabolic initial value problems will be considered with the Inverse Heat Conduction Problem (IHCP) as a model example. Such problems may be formulated, e.g. as integral equations of the first kind. Due to the illposedness of the problems, large errors in the approximating solutions may occur in the presence of small errors in the data. Approximation methods for such problems should therefore stabilize - or regularize - this behaviour. For several methods, rules for stabilizing are well-known, however, a rigorous analysis is often not available. In this contribution, a short overview of available approximation methods will be presented. In more detail, a sequential approximation scheme for the IHCP will be discussed including a stability and error analysis.

P. Seifert:

Numerische Behandlung von Anfangswertaufgaben der chemischen Kinetik mit Hilfe der Linienmethode

Eine Reihe von chemischen Aufgabenstellungen befaßt sich mit den Reaktionen und der gleichzeitigen Diffusion von Substanzen in Mischreaktoren. Ein typisches Beispiel dieser Diffusions-Reaktions-Prozesse, das näher untersucht werden soll, ist die radikalische Copolymerisation von zwei chemischen Stoffen unter Anwesenheit eines Initiators. Es werden dabei in mehreren Teilschritten Ketten von Polymeren gebildet, während gleichzeitig Diffusionsprozesse ablaufen. Das zugehörige mathematische Modell besteht aus einem System von nichtlinearen parabolischen Differentialgleichungen mit vorgegebenen Anfangswerten und Neumannschen Randbedingungen. Dieses Anfangs-Randwertproblem wird mit der numerischen Linienmethode behandelt, wobei nach der Teildiskretisierung in Ortsrichtung verschiedene bekannte "Solver" zur Lösung der entstehenden Anfangswertaufgaben gewöhnlicher Differentialgleichungen (steife Systeme) benutzt werden. Es werden Resultate der numerischen Rechnungen angegeben und Vergleiche der verwendeten "Anfangswert-Solver" angestellt.

B. Sjögreen:

The Practical Use of High Order Difference Methods

We consider the implementation of centered high order difference methods to solve the 2D compressible Navier-Stokes equations on a curvilinear grid. To obtain a stable method we first develop a computer program to perform stability analysis according to the theory by Gustafsson, Kreiss, and Sundström. For the equation $u_t + au_x = 0$ on $x>0$ it turns out that fourth order centered differencing with one sided fourth order differences at the boundaries are stable. Eighth order centered differencing with eighth order one sided boundary operators can be made stable by adding a twelfth order artificial dissipation term. We test the method on the Mach 3 flow past a disk. For the centered differences, the shock is fitted to the grid boundary . The results are compared with results from using a second order TVD shock capturing method. The fourth order method can resolve the boundary layer on the used grid. This was not possible using a second order method due to limited computer power. The TVD method on a coarse grid could be made to agree with the fully resolved solution by streching the grid towards the wall, thereby producing cells of very high aspect ratio.

M. Slodička:

On a Numerical Approach to Nonlinear Degenerate Parabolic Problems

This contribution deals with a fully discrete linear scheme for solving nonlinear singular parabolic problems like the Stefan problem or porous medium equations. The main idea is the approximation of nonlinear P.D.E. by a linear elliptic equation at each time step. This is solved using piecewise linear finite elements. The numerical integration is taken into account. The convergence of the method is proved and some error estimates are derived.

T. Sonar:

Adaptive Techniques in the Computation of Inviscid Compressible Flow

A finite volume scheme is used for the computation of steady and unsteady inviscid compressible flow fields in complex geometries. The method works on general conforming triangulations and is an upwind TVD-MUSCL type of scheme. Two adaptive techniques are used to insert/remove points in the triangulation. The finite-element residual is used as an error indicator. The question of norms in which this residual can be measured is addressed.

M. Stynes:

Pointwise Error Estimates for a Streamline Diffusion Method on a Shishkin Mesh for a Time-Dependent Convection-Diffusion Problem

We analyze the streamline diffusion method on a special piecewise uniform mesh for a model time-dependent convection-diffusion problem in one space-dimension. The mesh (due to Shishkin) is not locally quasiuniform; it resolves part but not all of the boundary layer. Using piecewise linear finite elements, we show that our method is convergent, uniformly in the diffusion parameter, of almost order 5/4 outside the boundary layer and almost order 3/4 inside the boundary layer.

A. Szepessy:

Adaptivity and Error Control for Hyperbolic Problems

The discrete kinetic Broadwell modell

$$\begin{aligned} f_{+t} + f_{+x} &= f_0^2 - f f_- \\ f_{0t} &= -\frac{1}{2}(f_0^2 - f f_-) \\ f_{-t} - f_{-x} &= f_0^2 - f f_- \end{aligned}$$

describes the evolution of the distribution (f_+, f_0, f_-) of the three velocities (1, 0, -1). In the fluid dynamical variables $\rho = f_+ + 4f_0 + f_-$, $m = f_+ - f_-$, $z = f_+ + f_-$ the Broadwell model takes the form

$$\begin{aligned} \rho_t + m_x &= 0 \\ m_t + z_x &= 0 \\ z_t + m_x &= \frac{1}{8}((\rho - z)^2 - 4(z^2 - m^2)) \end{aligned} \tag{1}$$

For this model, Broadwell found explicit expressions for travelling shock waves connecting equilibrium states satisfying the Rankine-Hugoniot condition. In this talk I presented joint work with Zhonping Xin on asymptotic stability of Broadwell shocks. We have proved that Broadwell shocks, which initially are locally perturbed, converge time asymptotically to a superposition of a translated shock wave, a diffusion wave, and a linear coupled diffusion wave (with zero mass). The sum of the diffusion wave and the linear wave solves a Navier-Stokes type approximation of (1), with slightly modified viscosity in the shock region compared to the traditional Navier-Stokes eaquation obtained from the Chapman-Enskog expansion.

L. Tobiska:

Finite Element Methods for Solving the Boussinesq-Approximation of the Navier-Stokes Equations

We consider stability and convergence of finite element discretizations for the incompressible Navier-Stokes equation. There are different reasons for spurious numerical oscillations of standard Galerkin finite element methods, e.g. dominance of convective terms, inappropriate pairs of finite elements for approximating the velocity and pressure field, etc. We propose to combine the stable nonconforming Crouzeix/Raviart element with an upstream technique for handling the influence of the convective terms. In order to solve the nonlinear system of equations in each time step a multigrid method is used. Finally, we give some results on numerical test examples.

G. Warnecke:

Zur Entropiekonsistenz von Verfahren mit großem Zeitschritt

Zur numerischen Approximation von Lösungen hyperbolischer Erhaltungsgleichungen gibt es eine Klasse von Verfahren, bei denen die Anfangsdaten durch stückweise konstante Funktionen ersetzt werden und dann zwischen diesen Werten jeweils das Riemannsche Anfangswertproblem gelöst wird, das aus elementaren Wellen (Stößen, Kontaktunstetigkeiten, Verdünnungswellen) besteht. Die resultierende Lösung verwendet man bis zu der Courant-Zahl, bei der sich benachbarte Wellen schneiden können. Zu diesem Zeitpunkt wandelt man die Daten wieder in stückweise konstante Funktionen um und wiederholt das Verfahren. Man hat dann in jedem Zeitstreifen eine exakte Lösung der Erhaltungsgleichungen, die der Entropiebedingung genügt. Beispiele solcher Verfahren sind das Glimm-Verfahren, das Godunov-Verfahren und die MUSCL-Verfahren. Von Le Veque wurden 1983 die Verfahren mit großem Zeitschritt eingeführt, bei denen man größere Courant-Zahlen verwendet und bei sich schneidenden benachbarten Wellen ihre Interaktion als linear annimmt, um den numerischen Aufwand nicht zu erhöhen. Sowie sich diese Wellen schneiden, liegt in dem Zeitstreifen keine exakte Lösung der Erhaltungsgleichungen mehr vor. Trotzdem sind diese Verfahren erstaunlicherweise für beliebige Courant-Zahlen stabil und konvergent. Außerdem erhält man die besten Approximationen für große, aber nicht zu große, Zeitschritte. Der Beweis der Entropiekonsistenz dieser Verfahren, d.h. daß die approximierenden Lösungen gegen exakte Lösungen konvergieren, die einer Entropiebedingung genügen, erwies sich als schwierig und blieb offen. In dem Vortrag werden erste Resultate, die in Zusammenarbeit mit Wang Jinghua erzielt wurden, vorgestellt.

Berichterstatter: H. C. Kuhlmann

Tagungsteilnehmer

Dr. Georg Bader
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Yann Brenier
Analyse Numérique, Tour 55-65
Université Pierre et Marie Curie
4, place Jussieu

F-75230 Paris Cedex 05

Prof.Dr. Greg Baker
Department of Mathematics
Ohio State University
231 West 18th Avenue

Columbus , OH 43210-1174
USA

Dr. David L. Brown
Los Alamos National Laboratory
Computer Research Dept.
Mail Stop B 265

Los Alamos , NM 87545
USA

Prof.Dr. Christine Bernardi
Laboratoire d'Analyse Numérique,
Tour 55-65
Université P. et M. Curie(Paris VI)
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Evangelos A. Coutsias
Dept. of Mathematics and Statistics
University of New Mexico

Albuquerque , NM 87131
USA

Prof.Dr. Wolf-Jürgen Beyn
Fakultät für Mathematik
Universität Bielefeld
Postfach 10 01 31

W-4800 Bielefeld 1
GERMANY

Prof.Dr. Michael B. Giles
Numerical Analysis Group
Computing Laboratory
Oxford University
Keble Road

GB- Oxford OX1 3QD

Dr. Folkmar Bornemann
Konrad-Zuse-Zentrum für
Informationstechnik Berlin
Heilbronner Str. 10

W-1000 Berlin 31
GERMANY

Prof.Dr. Moshe Goldberg
Department of Mathematics
Technion
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof.Dr. Thomas Hagstrom
Dept. of Mathematics and Statistics
University of New Mexico

Albuquerque , NM 87131
USA

Prof.Dr. Herbert B. Keller
Applied Mathematics 217-50
California Institute of Technology

Pasadena , CA 91125
USA

Prof.Dr. Thomas Y. Hou
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street

New York NY 10012-1110
USA

Prof.Dr. Klaus Kirchgässner
Mathematisches Institut A
Universität Stuttgart
Postfach 80 11 40

W-7000 Stuttgart 80
GERMANY

Dr. Helmut Jarausch
Institut für Geometrie und
Praktische Mathematik
RWTH Aachen
Templergraben 55

W-5100 Aachen
GERMANY

Dr. Christian Klingenberg
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Rolf Jeltsch
Seminar für Angewandte Mathematik
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Peter E. Kloeden
Depart.of Computing & Mathematics
Deakin University

Geelong,Victoria 3217
AUSTRALIA

Prof.Dr. Claes Johnson
Dept. of Mathematics
Chalmers University of Technology
and University of Göteborg
Sven Hultins gata 6

S-412 96 Göteborg

Dr. Peter Knabner
Institut für Angewandte Analysis
und Stochastik
Hausvogteiplatz 5-7

O-1086 Berlin
GERMANY

Prof.Dr. Heinz Otto Kreiss
Dept. of Mathematics
University of California
405 Hilgard Avenue

Los Angeles , CA 90024-1555
USA

Prof.Dr. Mitchell B. Luskin
School of Mathematics
University of Minnesota

Minneapolis MN 55455
USA

Prof.Dr. Gunilla Kreiss
NADA
The Royal Institute of Technology

S-10044 Stockholm

Prof.Dr. Yvon Maday
Laboratoire d'Analyse Numerique,
Tour 55-65
Universite P. et M. Curie(Paris VI)
4, Place Jussieu

F-75252 Paris Cedex 05

Dr. Hendrik C. Kuhlmann
ZARM
Universität Bremen
Fachbereich 4
Postfach 330440

W-2800 Bremen 33
GERMANY

Prof.Dr. Hans D. Mittelmann
Department of Mathematics
Arizona State University

Tempe , AZ 85287-1804
USA

Prof.Dr. Jens Lorenz
Dept. of Mathematics and Statistics
University of New Mexico

Albuquerque , NM 87131
USA

Prof.Dr. Keith W. Morton
Computing Laboratory
Oxford University
8 - 11, Keble Road

GB- Oxford OX1 3QD

Dr. Gert Lube
Institut für Analysis
Fakultät für Mathematik
Technische Universität Magdeburg
Universitätsplatz 2, PSF 4120

O-3040 Magdeburg
GERMANY

Prof.Dr. Anders Petersson
CNLS, MS B258
Los Alamos National Laboratory

Los Alamos , NM 87545
USA

Prof.Dr. Rolf Rannacher
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

W-6900 Heidelberg 1
GERMANY

Marian Slodicka
Institute of Applied Mathematics
Comenius University
Mylinska dolina

84215 Bratislava
CZECHOSLOVAKIA

Prof.Dr. Hans-Jürgen Reinhardt
Fachbereich 6 Mathematik
Universität/Gesamthochschule Siegen
Hölderlinstr. 3

W-5900 Siegen
GERMANY

Thomas Sonar
Dt.Versuchs- und Forschungsanstalt
für Luft- und Raumfahrt
- DLR -
Bunsenstr. 10

W-3400 Göttingen
GERMANY

Dr. Wolfgang Schröder
MBB AG, RTT 32
Postfach 801169

W-8000 München 80
GERMANY

Dr. Martin Stynes
Dept. of Mathematics
University College

Cork
IRELAND

Dr. Peter Seifert
Institut für Numerische Mathematik
Technische Universität Dresden
Mommsenstr. 13

O-8027 Dresden
GERMANY

Dr. Anders Szepessy
NADA
The Royal Institute of Technology

S-10044 Stockholm

Prof.Dr. Björn Sjögreen
Department of Scientific Computing
Box 120

S-75104 Uppsala

Prof.Dr. Lutz Tobiska
Institut für Analysis
Fakultät für Mathematik
Technische Universität Magdeburg
Universitätsplatz 2, PSF 4120

O-3040 Magdeburg
GERMANY

Prof.Dr. Eric Van de Velde
Applied Mathematics 217-50
California Institute of Technology

Pasadena , CA 91125
USA

Dr. Gerald Warnecke
Mathematisches Institut A
Universität Stuttgart
Postfach 80 11 40

W-7000 Stuttgart 80
GERMANY

Prof.Dr. Bodo Werner
Institut für Angewandte Mathematik
Universität Hamburg
Bundesstr. 55

W-2000 Hamburg 13
GERMANY