MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 6/1993

Asymptotics and Adaptivity in Computational Mechanics

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The conference was organized by D. Braess (Bochum), Ph. Ciarlet (Paris) and E. Stein (Hannover). The participants came from numerical engineering, mechanics, and from numerical or applied analysis. The following interacting topics were discussed from various theoretical and applied points of view.

The asymptotic properties of two dimensional plate models were one of the main topics of the conference. Various derivations started from a three dimensional model and led to new (e.g. geometrically exact) plate models or to the von Karman equations. In addition, existence, regularity and asymptotics of the solutions of shells and membranes were discussed as well as related aspects of their numerical approximation. The Reissner–Mindlin plate model gave rise to the discussion of mixed finite element methods and the problem of locking. Appropriate (e.g. stabilized) element formulations as well as robust multigrid or preconditioning methods were presented for the solution of the discrete linear system of equations. The number of iteration steps is small and independent of some crucial parameter (e.g. the thickness of the plate).

The second important topic of the conference was the development and application of adaptive methods which admit an easy computation of solutions with possible singularities. Their algorithmic aspects — mainly concerning the numerical treatment of the discrete equations (e.g. via a multigrid method) — as well as their theoretical foundation were discussed. In particular, the comparability of different error indicators in the case of a structured meshes and its relation to superconvergence phenomenons became important. Adaptive methods were presented not only for linear elliptic problems but applied also to flow problems, nonlinear shell buckling, localization in plasto mechanics, obstacle problems, and in dimensional adaptivity.

D.N. ARNOLD:
$P^2 - P^1$ Stokes Elements
We investigate the finite element approximation of the Stokes equations using continuous piecewise quadratic elements for the velocity and discontinuous piecewise linear elements for the pressure, both computationally and theoretically. This seemingly simple choice of elements exhibits a surprisingly rich variety of stability and convergence behavior depending on the mesh configuration. According to the mesh family there may or may not exist local spurious pressure modes (usually associated with singular vertices) and/or global spurious pressure modes. Of course such pressure modes cause the method to fail the inf–sup condition and in this sense to be unstable. When the pressure modes are removed from the finite element space, a filtering process which leaves the velocity unchanged, the resulting reduced system may or may not be stable. For a number of irregular and regular mesh families we show it is stable, while for a number of other families, including the simplest uniform triangular mesh generated by three lines, it is not. However the evidence seems to indicate that the unstable situations are quite special and not very robust and this element delivers optimal order velocity approximation in most cases. We conjecture that the velocity approximation is at worst one order suboptimal in any case.

E. BÄNSCH:
Algorithmic Aspects of Selfadaptive FE–Techniques for Instationary Problems
Adaptive strategies for instationary problems are presented. These strategies apply to a quite general class of problems. Our emphasis is on 3D problems. The following points are outlined: local grid refinement/coarsening, criteria for grid modification, interpolation between different grids, overall strategies.
Numerical examples are presented:
- Transient flow simulation in 3D
- Solidification in undercooled media/dendrite growth in 2D and 3D.

J. BEY:
Adaptive Multigrid Methods with Local Smoothers for 3D Elliptic Problems
We consider the solution of convection diffusion equations in three space dimensions. These are discretised by a finite volume method applied to a nested sequence of tetrahedral meshes. Upwinding is used to stabilize the discretization in the case of dominating convection. The resulting linear system is solved using an adaptive multigrid algorithm with a local Gauss-Seidel smoother. Therefore

the order of unknowns of each level is properly aligned to the direction of convective flux. This ordering turns out to be essential in order to guarantee good convergence rates independent of the proportion of diffusion and convection terms.

H. BLUM:

Finite Element Error Analysis on Structured Meshes

The consistency error of finite element discretizations allows for a more detailed analysis and for better estimates on structured meshes composed of uniformly refined blocks than on arbitrary regular meshes. These results provide the basis for the derivation of various superconvergence properties which cannot hold true in general. Further, the accuracy can be improved by means of defect correction techniques. Using certain natural local filtering and smoothing procedures, these techniques carry over to mixed and nonconforming schemes.

F. BOURAUIN:

Substructuring Methods for Fluid-Structure Interaction

We consider the problem of elastoacoustic vibrations of a fluid-structure system in the vicinity of an equilibrium state. The pure displacement variational formulation exhibits a small parameter, ratio of the fluid density to the density of the solid body. The asymptotic expansion of the solutions with respect to ϵ is proved to be rigorous in Sanchez–Hubert and Sanchez Palencia, and leads to a natural substructuring algorithm that allows us to compute the elastic eigenmodes as simple static perturbations of the cavity modes in vacua and of the acoustic modes in the rigid cavity. This algorithm can take advantage of a hybrid diplacement–pressure–diplacement potential formulation and requires a fast procedure to compute the forced response of a linear system.

D. BRAESS:

On Finite Elements for Mindlin Plates

When locking free elements for Mindlin–Reissner plates are developed, it is appropriate to consider this plate model as a perturbation of the Kirchhoff model. Some folklore on saddle point problems (mixed problems) with penalty is misleading. A regular perturbation is always stabilizing while a singular one is only if the quadratic form for the variational functional is elliptic on the whole space. The Helmholtz decomposition used by Brezzi and Fortin admits a splitting such that the singular part refers to a subspace with this property. We show that the MITC–elements (or more generally elements which satisfy the axioms of Brezzi, Bathe, Fortin) admit a discrete Helmholtz decomposition. To this end a discrete version of the curl operator is defined as a distributional derivative with test functions from the finite element space for the strain. With this, almost optimal error estimates can be derived.

S.C. Brenner:

Mulitigrid Methods for Parameter Dependent Problems

Multigrid methods for parameter dependent problems will be discussed. The contraction numbers of the algorithm are bounded away from one, independent of the parameter and the mesh levels. Examples include the pure displacement and pure traction boundary value problems in planar linear elasticity, the Timoshenko beam problem, and the Reissner–Mindlin plate problem.

C. Carstensen:

On Adaptive BEM and BEM/FEM Coupling

We state some a posteriori error estimates for Symm's integral equation, for an integral equation with the hypersingular operator, for some transmission problem and the symmetric coupling of boundary elements and finite elements. We sketch the main idea for a proof of these estimates following the approach for finite elements due to Erikson and Johnson. Then we present an adaptive feedback algorithm and discuss some illustrating numerical examples.

H. Cramer:

Adaptive Analysis of Elastic Structures Using Recursive Substructuring

In this lecture an adaptive treatment of problems in plane elasticity using recursive substructuring is described. The applied procedures in the different steps of the adaptive process are discussed. In the first part some frequently used error indicators and estimators are considered. Their performance is illustrated by the results of a model problem. This will be followed by a section dealing with mesh refinement, where the h–process of remeshing is shortly presented. Finally the solution of the algebraic equations using direct solvers is considered. It is pointed out that recursive substructuring is a very well suited solution technique for an adaptive process. Possibilities of parallelization of the calculation are discussed.

R.S. Falk:

Derivation, Asymptotic Properties and Numerical Approximation of some Two Dimensional Plate Models

Two dimensional plate models derived from Galerkin approximations to two forms of the Hellinger–Reissner mixed variational principle are considered. These include the family of minimum energy models, Reissner's model, and a new family of models, similar to Reissner's model, which may also be viewed as complementary energy models. Comparisons are made regarding the boundary layer behavior and regularity of the solutions of these models as a function of the plate thickness t. Uniform in t error estimates are presented for a finite element approximation scheme for the lowest order minimum energy model.

M. FORTIN:

Finite Element Methods, Stabilized Formulations and Error Estimation

Error estimation and adaptivity are essential for advanced computation, specially for Computational Fluid Dynamics. We are considering a strategy in which a stabilized formulation, such as a Galerkin least squares method, can provide some form of error estimation.

R. KORNHUBER:

On Adaptive Multilevel–Methods for Elliptic Problems

We will give a brief introduction to the basic concepts for multilevel preconditioning and a posteriori error estimates which are then applied to some elliptic problems. In particular we will consider the BPX preconditioner in three space dimensions. Using active set strategies, the solution of obstacle problems can be reduced to the solution of a sequence of reduced linear sub–problems. It turns out that the basic concepts of multilevel methods can be extended to the preconditioning of these reduced problems. To allow for local mesh refinement we derive semi–local and local a posterori error estimates, providing lower and upper bounds for the global error. The theoretical results are illustrated by numerical computations.

A. MIELKE:

A Derivation of a Geometrically Exact Plate Model

We consider an infinitely extended thick plate and show that all solutions having uniformly small strains can be described by a two–dimensional PDE. It consists of 9 scalars associated to each fiber. The method is based on a Fourier decomposition of the linearized problem. This shows that at least a 9–director–model is needed. The nonlinear problem is treated by a generalized Lyapunov–Schmidt reduction. In particular, the deformation energy functional of the plate can be deduced rigorously from the energy functional of the three–dimensional continuum.

E. SANCHEZ PALENCIA:

Membrane Approximation for Thin Hyperbolic Shells

Membrane approximation is the limit behavior of shells as the thickness tends to zero provided that the surface, along with the kinetic boundary conditions is geometrically rigid, i.e. displacements with $\gamma_{\alpha\beta} = 0$ ($\gamma_{\alpha\beta}$= variation of the coefficients of the first fundamental form of the surface produced by the displacement u) are only trivial displacements. In this case, the membrane energy form defines a norm on the space of displacements and we may construct the space $\bar{\bar{V}}$ of displacements with finite membrane energy. Functions of this space are not smooth, their non–smoothness depending highly on the geometry of the surface. The asymptotic curves of the surface are the characteristics of the system of the membrane approximation, and singularities may propagate along them. The corresponding space $\bar{\bar{V}}$ is an anisotropic space of functions not having traces along the characteristics curves. It is possible to eliminate the normal component of the displacement to get a closer form of the space in terms of the tangential components. It turns out that the membrane approximation in the hyperbolic case is such that the strain–stress relation is positive but not definite–positive: the corresponding energy vanishes for shear (covariant shear corresponding to the directions of the asymptotic curves. The corresponding behavior is rather of a net than of a membrane, but the net is holded by the boundary conditions.

J.-C. PAUMIER:

On the Locking Phenomenon for a Linearly Elastic Clamped Plate

1. A Finite Element method is applied to the variational displacement formulation of a three dimensional clamped plate. The purpose of this talk is to study the locking phenomenon which might occur when the thickness approaches zero. In this phenomenon, the error in the approximation is not necessarily small and can become very large if the thickness decreases. In this talk we give conditions on the Finite Element Method to avoid this difficulty. Over these conditions it is shown that the convergence of the approximation is uniform as the thickness diminishes to zero.

2. To sum up the above results in an abstract formulation: $u_\epsilon \in V$; $\frac{1}{\epsilon}a_0(u_\epsilon, v) + a_1(u_\epsilon, v) = b(v)$ for all $v \in V$, we give a sufficient condition to have an approximate solution u_ϵ^h which converges, uniformly in ϵ, as $h \to 0$. This is a condition on the limit problem: $u_0 \in G$; $a_1(u_0, w) = b(w)$, for all $w \in G$ where G is the kernel of a_0 (not reduce to $\{0\}$).

3. As an application of our main result we consider the validity of models like "Mindlin—Reissner" where the displacement u is a polynomial in the variable x_3. We give a condition to get models which are "asymptotic–convergent" (as the thickness goes to zero the solution converges to the Kirchhoff–Love solution). This condition consists in taking displacement like polynomial in x_3, degree one for horizontal components and degree two for vertical component.

J. PITKÄRANTA:

Asymptotics Vs. Numerics in Shell Problems

The main characteristics of shell asymptotics are 1) strong dependence of the geometry of the shell (elliptic/parabolic/hyperbolic) and 2) (in fixed geometry) strong dependence of kinematical constraints imposed, in some cases even of the type of loading. Another characteristic feature of thin shell problems is the presence of very different length scales, e.g. in boundary layers. If the largest scale is chosen as the length unit, and t= thickness of the shell, then length scales like t, $\sqrt{t}$ and even $\sqrt[4]{t}$ are to be expected in boundary layers. In standard finite element approximations, the error in the energy norm for p–th degree elements of size h behaves typically like $\|u - u^{h,p}\|/\|u\| \leq C(t)(h/L)^p$ where L is the length scale to be resolved. In favorable cases $C(t)$ is bounded, but often $C(t) \to \infty$ as $t \to 0$. In the worst cases $C(t) \sim t^{-1}$. To have error $< tol$ thus requires that $h < (tol)^{1/p} L/(C(t))^{1/p}$. If $C(t)$ is large, this suggests the use of high-order methods.

E. RANK:

A Hierarchical HP–Version

Recently a variant of the hp–version as a combination of a high order approximation with a domain decomposition method has been suggested. The basic idea can be explained as follows. In a first step of the analysis a pure p–version approximation is performed on a coarse finite element mesh. The coarse mesh is then covered partially by a geometrically independent fine mesh. On the second mesh a lower order approximation is performed and the global approximation is defined as the *hierarchical sum* of the p–approximation on the coarse mesh and the h–approximation on the fine mesh. Global continuity of the finite element solution can be guaranteed by imposing homogeneous conditions at the fine mesh boundary. The hierarchical nature of the approximation also reflects in the structure of the arising linear equation system and can be used in an efficient solution algorithm. The paper discusses algorithmic details and shows how the hierarchical approach offers the possibility of a consistent modeling of local-global solution behavior. In numerical examples the efficiency and accuracy is demonstrated.

RAO BOPENG:

Marguerre - von Kármán Equations and Membrane Model

As shown by Ciarlet-Paumier (1986), the Marguerre von Kármán Equations are a good approximation of a nonlinear shallow shell model. These equations are characterized by the specific nonlinear terms of derivatives of order two.

In this work, we first give a general existence result and some smoothness results. Next, in the case of pure traction, we study the behavior of the shell. We show that the solutions of the shallow shell converge to the solution of the membrane model as the intensity of the traction converges to infinity.

A. RAOULT:

Asymptotic Derivation of Plate Models

1) Asymptotic derivation when the thickness goes to zero of both the linear plate model and the von Kármán model has long been known. Although widely used, these models are restricted to the small displacement range. Moreover, they are not frame indifferent.

By applying an asymptotic procedure to the fully nonlinear system of 3D-elasticity for large loads, we exhibit the nonlinear membrane equations as a limit problem. The energy depends only in the first fundamental form of the deformed mid-surface. In this approach, the constitutive law is a result. The proper coefficients are not stated a priori. When the orders of magnitude of the loads are decreased and for a zero leading resultant, we obtain a bending

problem. The energy depends only on the second fundamental form. If we go on with the process, we recover the previous results concerning the von Kármán and the linear model.
2) When using the classical way of deriving the Mindlin-Reissner model by projection on the space of displacements whose horizontal components are affine with respect to x_3 and whose vertical component is independent of x_3, one has to modify a priori the constitutive coefficients. By comparing this (1,1,0) solution with the exact solution in a special case, Babuška and Schwab find two optimal values for k depending on the comparison criterion. We make the following remark: if an approximation of $u(\epsilon)$ is chosen to be $w = u^0 + \epsilon^2 u^2$ in the case where u^2 can be computed (specific boundary conditions), then the restriction to the mid-surface of the vertical component w_3 or its mean-value lead by comparison to the Mindlin-Reissner model to the same optimal values. The coefficient u^2 is a polynomial of degree 2 in x_3. This is an indication that the Mindlin-Reissner model draws information from such an approximation space (see the talk by R. Falk).

E. STEIN:

Dimensional Adaptivity in Linear Elasticity

The goal is the including of disturbed solutions in subdomains of beams, plates and shells - such as at boundary layers, jumps of thickness or areas of concentrated loading - into an integrated h-d-adaptive finite element analysis in order to get an overall reliable and efficient approximation process. Different strategies are investigated, such as an expansion method 2D to 3D subdomains and a reduction method using h-p-adaptivity in the whole 3D domains with reduction to 2D subdomains where the kinematic hypothesis are valid. Up to now, the h-d- and the h-p_t-d-adaptivity is realized where the latter describes $2\frac{1}{2}$D refinements with p-adaptivity in the thickness direction. Crucial problems are a flexible 1D-2D-3D mesh generator as well as the a priori and a posteriori error indicators. D-adaptivity is initialized if the regularity check in 2D - using residual error indicators - and the a priori estimated decay lengths of perturbations both become active.
Some investigated problems like a plate on simple columns show the properties and the efficiency.

A.-M. SÄNDIG:

Calculation of 2D and 3D-Singularities for Inclusions with Conical Points

Let Ω_2 be a two- or three-dimensional domain with an inclusion Ω_1 in the interior. The domain Ω_1 is polygonal in the two-dimensional case or has a rotational symmetric conical boundary point in the three-dimensional case. It follows from the general theory that the solutions of elliptic differential equations in Ω_i, $i = 1, 2$, which satisfy certain transmission conditions on the common boundary $\partial\Omega_1$, have an asymptotic expansion consisting of singular and regular terms. The method, how to calculate the singular terms is demonstrated for different examples, namely for 2D Poisson equations, biharmonic equations and for the 2D and 3D Lamé equation and for the 2D and 3D Lamé equation systems. The singular terms can have the following form: $r^{\alpha} s_i(\alpha, \phi, \vartheta)$, $i = 1, 2$, where (r, α, ϑ), are the spherical coordinates and r is the distance to a conical point. The real parts of the exponents determine the regularity of the solutions. They are calculated numerically for some materials for all openings of the conical points and the corresponding graphs show, when oscillating singularities (α is complex) and when instabilities in the asymptotics appear (branching and crossing points).

B. Seifert:

Mesh-Adaptation in the Nonlinear Finite-Element-Analysis of Shells, Especially Buckling

h–Adaptivity with a-posteriori error indicators for incremental stress states as well as bounds for the incremental geometrical non-linearity provides reliable and effective non-linear equilibrium paths for shells with moderate or even finite rotations. The computation of bifurcation points and branch-switching is realized by error control. Adaptive refinement into buckle states is necessary and realized at branch switching points.

P. Steinmann:

Localization Problems in Plasto Mechanics

The accumulation of inelastic deformations into narrow failure bands is a frequently observed phenomenon in many different materials. Within classical continuum theory localization is described as a local bifurcation problem at the constitutive level. Thereby discontinuity surfaces across which the (spatial) velocity gradient obeys a jump are assumed. The statical admissibility requirement renders the localization condition in terms of the acoustic tensor. Investigations of the acoustic tensor in the large strain regime are presented and the implications of the acoustic tensor analysis for the numerical computation of BVP are highlighted. To this end, h–adaptive studies of a plane tension problem are presented. Finally, a regularisation method invoking a micropolar continuum approach is sketched.

R. Stenberg:

On some Bilinear Finite Elements for Reissner-Mindlin Plates

We consider three methods using quadrilateral bilinear approximations for the deflection and the solution vector: a classical method in which the skew energy is computed inexactly with the one point integration rule (reduced integration), the "MITC4"-element (Mixed Interpolated Tensorial Components) by Bathe and Dvorkin and a new "stabilized" modification of the MITC4. Of the three methods only the last one can be shown to be uniformly optimal accurate with respect to the plate thickness. In our calculations we try to numerically verify the results of the theoretical error analysis. For regular rectangular meshes the expected loss of accuracy of the unstable method cannot be seen. However, if we disturb the mesh, the instabilties show up. It is most clearly seen in the shear force which for the unstable method can be extremely inaccurate (the error can be greater than 100% !). For the stable method, the mesh distribution have no significant influence on the accuracy.

R. VERFÜRTH:

A Posteriori Error Estimates

There are various ways to obtain reliable a posteriori error estimates for finite element approximations of elliptic pdes. There are in particular three variants which are most popular:
1: evaluate the residual of the finite element solution with respect to the strong form of the pde (originally proposed by Babuska) 2: solve local problems of the same type with Dirichlet boundary conditions (originally proposed by Babuska and Rheinboldt) 3: solve local problems of the same type with Neumann boundary conditions (originally proposed by Bank and Weiser). We prove that all these estimators are equivalent in the sense that they yield — up to multiplicative constants which only depend on the polynomial degree of the finite element functions and on the shape regularity of the mesh — the same global upper and local lower bounds on the error of the finite element solution.

W.L. WENDLAND, U. GÖHNER, G. WARNECKE:

Adaptive Finite Element Methods for Transonic Flows

By using elliptic type error indicators, a mesh refinement with h–adaption is driven for the Glowinski conjugate gradient method for solving the Neumann problem of the transonic full potential equation for two-dimensional compressible flows. These we based on localized residuals and flux jumps across the finite element edges. Numerical experiments showed satisfactory performance except at shocks. Based on the condition for selecting the physically correct solution, an additional shock indicator is used for moving the nodes in order to improve the alignment of the mesh with shock curves. It can be shown that this shock indicator is only active if the shock is within the element or the next neighbor. Furthermore, for a piecewise smooth solution, optimal order convergence for the velocity and the family of adapted refinements can be shown. The lecture presents results of U. Göhner's PhD thesis, a joint paper by U. Göhner and G. Warnecke and a joint paper of the three authors.

References: 1. U. Göhner: Adaptive Finite-Element-Methoden für transsonische Strömungen. Dissertation, Universität Stuttgart 1991.
2. U. Göhner and G. Warnecke: A shock indicator for adaptive transonic flow computations. Preprint 92-9, Math. Inst. A, University Stuttgart.
3. U. Göhner, G. Warnecke and W. Wendland: Error indicators for adaptive transonic flow computations; in preparation.

N.-E. WIBERG:

Patch Recovery Based on Superconvergent Derivatives and Equilibrium

In this paper a postprocessing technique is developed for determining first order derivatives (fluxes, stresses) at nodal points based on derivatives in superconvergent points. It is an extension of the superconvergent patch recovery technique presented by Zienkiewicz and Zhu. In contradiction to that technique all flux or stress components are interpolated at the same time, coupled by equilibrium equations at the superconvergent points. The equilibrium equations and use of one order higher degree of the interpolation polynomials of stresses give a dramatic decrease of the error of recovered derivatives even at boundaries.

G. WITTUM:

Adaptivity and Robustness

Robust solvers are crucial for the fast and efficient solution of singularly perturbed problems. On the other hand, singularly perturbed problems typically show local phenomena like boundary layers. Thus they require adaptive local refinement. In the present lecture we discuss a concept how to combine robustness and adaptivity in multi-grid methods and show the success of this concept applied to special problems. Further we discuss a diffusion problem modelling the drug diffusion through skin and show robustness of the local mg solver.

Berichterstatter: C. Carstensen

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