MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 15/1993

Combinatorial Convexity and Algebraic Geometry

28.03. bis 03.04.1993

The conference was organized by G. Ewald (Bochum), P. McMullen (London) and R. Stanley (Cambridge, MA).

In 37 talks the current progress of the research which relates the fields of Combinatorial Convexity and Algebraic Geometry was shown. One may say that this part of mathematics is very modern and is getting more and more attention. Not only well known mathematicians were present but also many young mathematicians, which seems to be very hopeful for future progress in this kind of mathematics.

The talks addressed many subjects out of the fields of Combinatorics and Algebraic Geometry. Many results in Algebraic Geometry are achieved with the help of Combinatorics but also some new results in Combinatorics and Number Theory are achieved with the help of Algebraic Geometry.

Some of the addressed topics are: The Polytope Algebra, Counting Lattice points, The Ehrhart polynomial, the Cd-Index. Polytopes related to singularities and deformations, Chowgroups of toric varieties, The Picard group of toric varieties, Invariant Theory, h-vectors, Matroids, Betti-numbers, and much more.

After the talks and especially in the evening, there were many fruitful (mathematical) discussions.

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Vortragsauszüge:

Klaus Altmann:

<u>Deformation of affine toric varieties and Minkowsi sums of convex polyhedra</u>

A deformation of an affine toric variety $Y = \mathrm{Spec}\,\mathbf{C}[\check{\sigma}^{\vee} \cap \mathbf{Z}^{k+1}]$ is called toric if the total space together with the embedding of the special fibre Y are contained in the category of toric varieties.
Moreover, it is called homogeneous if the Kodaira-Spencer map maps the whole parameter space into a homogeneous piece of T^1_Y.

In this article we construct a one-one-correspondence between homogeneous toric deformations on the one hand and certain decompositions of affine slices of the cone $\check{\sigma}$ into a Minkowski sum on the other hand.
Finally, we compute the Kodaira-Spencer map and provide some examples.

E.K. Babson:

<u>A flag variety analog to oriented matroids</u>

An (a) oriented matroid projection on $[d]$ is a rank a oriented matroid on the set $\{1, \ldots, d\}$. An $(a_1 \ldots a_r)$ oriented matroid projection on $[d]$ is a rank a_1 oriented matroid each of whose covectors is the basis set for an $(a_2 \ldots a_r)$ oriented matroid projection on $[d]$. These arise from maps of pseudosphere arrangements if the support in $[d]$ of every nonzero basis has order at least $1 + \Sigma(a_i - 1)$.

A. Barvinok:

<u>Exponential valuations on convex lattice polytopes</u>

We introduce a family of valuations defined by certain exponential integrals on convex lattice polytopes. It turns out that such a valuation can be explicitly expressed in terms of volumes of faces and an additive measure on the supporting cones at the faces. We further use an analogue of "Fourier expansion" for a valuation to compute the Ehrhart polynomial and the number of integral points in a polytope. In particular, we show that the computation of an arbitrary fixed numer of the highest coefficients of the Ehrhart polynomial can be reduced in pseudopolynomial time to the computation of the volumes of faces. We also prove some identities for the coefficients of the Ehrhart polynomial.

Marge Bayer:

Subdivisions of polytopes and local h-vectors

Associated with any polyhedral complex is the h vector, defined combinatorially by Stanley. (The definition is motivated by formulas for intersection homology Betti numbers of the boundary complex of a rational convex polytope). We are interested in how the h-vector $h(\Gamma)$ of a polyhedral subdivision of a polytope relates to the h-vector $h(P)$ of the polytope itself. Stanley defined the local h-vector of a polytope relative to a subdivision. The local h-vector of a face of a polytope measures the contribution of the subdivision of that face to the difference $h(\Gamma) - h(P)$. A triangulation of a polytope P is shallow if each simplex in the triangulation is contained in a face of P of dimension at most twice as large. A shallow triangulation of a polytope has the same h-vector as the original polytope, and thus the local h-vector of every face is 0. We discuss how the local h-vectors essentially locate violations of shallowness.

S.N. Bespamyatnykh

Constructing minimum spanning trees in $\Re^K_\infty$ and covering of the k-cube.

We establish a connection between the problem of finding the minimum spanning trees in $\Re^k_\infty$ and the covering of the $(k-1)$-cube. The minimum spanning tree in $\Re^k_\infty$ can be found in $O(2^k k \gamma_{k-1} n (lg\ n)^{k-2} lglg\ n)$ time, where γ_{k-1} is the minimum number of simplices to cover the $(k-1)$-cube (the vertices of simplices are the vertices of the cube). The minimum spanning tree algorithm uses the region approach of Yao A.C. and the minimum value algorithm of Gabow H.N., Bentley J.L. and Tarjan R. E.

Haiman M. proved that $(\tau_{kl}/(kl)!)^{1/kl} \leq (\tau_k/k!)^{1/k}$ where τ_k is the minimum number of simplices to triangulate the k-cube and $k \geq 1$, $l \geq 1$. Sallee J.F. proposed the middle-cut triangulation, which gives $\tau_8 \leq 13248$. Hence $\tau_k = O(p^k k!)$ where $p = (13248/40320)^{1/8} \approx 0.870$. It is clear that $\gamma_k \leq \tau_k$. We propose the method of dimension reduction which gives for instance, $\tau_{120} \leq 0.145302548 \cdot 10^{192}$. Hence $\gamma_k = O(p^k k!)$, where

$$p = \frac{\gamma_{120}}{120}^{\frac{1}{120}} \leq \left(\frac{0.145302548 \cdot 10^{192}}{0.668950291 \cdot 10^{199}}\right)^{\frac{1}{120}} \approx 0.863258 \leq 0.864.$$

Gabow H. N., Bentley J. L. and Tarjan R. E. discovered an $O(2^k k! n (lg\ n)^{k-2} lg\ lg\ n)$-time and $O(2^k n)$-space algorithm for finding the geometric minimum spanning tree in $\Re^k_\infty$. We improve the time bound by a factor $\frac{1}{0.864^k}$ and show how to reduce space to $O(kn)$.

L. J. Billera

Products of Minors

We consider $\Sigma(m, n) \subset \mathbf{R}^{m \cdot n}$, the Newton polytope of the product of *all* minors of an $m \times n$ matrix. By a result of Gel'fand, Kapranov and Zelevinsky, $\Sigma(m, n) = \Sigma(\Delta_{m-1} \times \Delta_{n-1})$, the secondary polytope of the product of an $(m-1)$-simplex and $(n-1)$-simplex. A result of Sturmfels and myself shows this to be the fiber polytope $\int_{\Delta_{m-1} \times \Delta_{n-1}} P(a, b) da db$ where

$$P(a, b) = \{X \in \mathbf{R}^{m \times n} \mid X \geq 0, \; m \times n \text{ matrix with row sums a, column sums b}\}$$

This interpretation of $\sum(m, n)$ is used to obtain some information about facets. This is partially joint work with E. Babson.

A. Björner

Orbit subspace arrangements and the complexity of some decision problems

Let W be a finite reflection group acting in $\mathbf{R}^n$, and let K be a subspace obtained as the intersection of some of the reflecting hyperplanes. Consider the orbit $W(K)$. Based on some evidence from the "type A" case we propose the optimistic conjectures that

(1) $\mathbf{R}^n \setminus U\{w(K) : w \in W\}$ has torsion-free cohomology (same for $\mathbf{C}^n \setminus U\{w(K)\}$), and more strongly that

(2) $S^{n-1} \cap (U\{w(K) : w \in W\})$ has the homotopy type of a wedge of spheres (typically of different dimension).

For $W = S_n$ acting on $\mathbf{R}^n$ by permuting coordinates (the "type A" case) the orbit arrangements $W(K)$ are naturally indexed by number partitions λ, n(*i.e.*, $\lambda = (\lambda_1, \ldots, \lambda_p)$, $\lambda_1 \geq \cdots \geq \lambda_p > 0$, $\lambda_1 + \ldots + \lambda_p = n$). These conjectures have been verified for $\lambda = (k, 1, \ldots, 1)$ in joint work with V. Welker, and also for $\lambda = (k, k, \ldots, k)$. Known results by Goresky and MacPherson and Ziegler and Zivaljevic reduce these questions to essentially combinatorial questions about certain lattices of set partitions.
Finally, we described an application of the $\lambda = (k, 1, \ldots, 1)$ orbit arrangements to a question in computational complexity, relating the sum of Betti numbers of their complement to the size of so-called linear decision trees (joint work with L. Lovasz).

A. Borovik

Combinatorial Convexity and Combinatorics of Flag varieties

The talk will be devoted to some convexity-like structures on Coxeter groups related to WP-matroid introduced by I.M. Gelfand and V.V. Serganova for study of combinatorics of flag varieties.

F. Brenti

Combinatorial properties of Betti numbers of some toric varieties

Let P be a simplicial convex polytope. It is then well known that one can associate to P a toric projective variety $X(P)$ and that the sequence of even dimensional Betti numbers of $X(P)$ equals the h vector of P. In this talk we use this result to study, from a combinatorial and enumerative point of view, (q-analogues of) these Betti numbers in the case that P is a Coxeter complex of type A_n, B_n a and D_n. While for type A_n these Betti numbers are known to coincide with classical Eulerian numbers, and thus have been extensively studied from a combinatorial point of view, only some results of a combinatorial nature are known for type B_n, and little is known for type D_n. In this talk we show that essentially all of the classical results for Eulerian numbers have analogues for these other Betti numbers. Our results generalize and unify previous results of Dolgachev, Lunts, Stembridge and Stanley, and generalize the classical theory of Eulerian numbers and polynomials. As a by-product of our combinatorial analysis we obtain a combinatorial proof of a simple relation between these three types of Betti numbers and we are led to several conjectures, both of a geometric and combinatorial character, about them.

M. Brion

Embeddings of homogeneous spaces (some generalizations of toric varieties)

Spherical varieties are a class of embeddings of homogeneous spaces; they are classified in terms of "colored fans" (a combinatorial object, which extends the well-known notion of a fan).

We study morphisms between spherical varieties, by means of their combinatorial classification, and of some ideas of Mori theory.

W. Bruns

On the computation of a a-invariants

In our lectures we will mainly concentrate on computing the a-invariants of graded algebras with straightening laws on upper semi-modular lattices. Let Π be an upper semi-modular lattice. We may construct a chain in $\Pi \cup \{\infty\}$ as follows: $\xi_1 = \mu_1 \sqcup \ldots \sqcup \mu_n$ are the minimal elements of Π, and inductively $\xi_{i+1} = \nu_1 \sqcup \ldots \sqcup \nu_s$ where $\nu_1, \ldots, \nu_s$ are the covers of ξ_i, provided $\xi_i \neq \infty$. The construction stops after a finite number m of steps. We call $\mathcal{P}(\Pi) = \xi_1, \ldots, \xi_m$ the *principal chain* of Π. (It may of course happen that $\mathcal{P}(\Pi) = \infty$.)

Theorem 1. Let R be a monotonely graded ASL over a field k on an upper semimodular lattice Π with principal chain $\mathcal{P}(\Pi) = \xi_1, \ldots, \xi_m$. Set $\deg \infty = 0$. Then

$$a(R) = -\sum_{i=1}^{m} \deg \xi_i.$$

The proof uses a standard inductive scheme. As a typical application we will easily prove a formula for the a-invariant of a determinantal ring.
My student M. Barile has found the connection between the principal chain and Hibi's fundamental faces:

Theorem 3 Under the hypothesis of Theorem 1, the principal chain of Π is the supremum of the fundamental faces of the order complex Π (in the set of chains of Π under its natural parial order).

Mark McConell

Polytopes, Finite Projective Geometry, and Reduction Theory for the Symplectic Group

In recent years, relations have emerged between two areas: the combinatorics of geometric configurations in projective spaces, and locally symmetric spaces (with their connections to Lie groups, number theory and algebraic geometry). Let $\mathbf{P}^3(p)$ be projective three space over the field of p elements, with extra structure coming from a symplectic form. MacPherson and I define a set $\mathbf{C}$ of configurations in $\mathbf{P}^3(p)$, with a partial order coming from inclusion. The order complex Δ of $\mathbf{C}$ has the homotopy type of X/Γ, where X is the symmetric space for the symplectic group $G = Sp(4, \mathbf{R})$ and $\Gamma \subseteq G$ is an arithmetic subgroup. For instance, $\mathbf{C}$ can be used in principle to compute the intersection cohomology of important algebraic varieties. The configurations in $\mathbf{C}$ are beautiful, and Δ is a polyhedral complex with interesting cells. The proof of our theorem mixes techniques from algebraic group differential geometry, and polytope theory (e.g., we had to shell some large triangulated objects explicitely).

B.V. Dekster

The Jung Theorem for the spherical and the hyperbolic spaces

We extend the Jung Theorem to the spherical and the hyperbolic n-spaces establishing a lower bound of the diameter of a set there in terms of its circumradius R. In the hyperbolic case, the bound is the greatest. In the case of the sphere S^n, it is also the greatest over the segment $R \in [0, \arccos\frac{-1}{n+1}]$. For a greater $R(\leq \pi)$, we estimate the greatest lower bound from above and from below. In the spherical case, the greatest lower bound over the shorter segment $R \in [0, \arccos\frac{-1}{n+1}]$ was obtained be Molnar in 1957.

I. Dolgachev

Flips In Geometric Invariant Theory And Toric Geometry

The notion of the quotient space X/G in Geometric Invariant Theory depends on a choice of a G-linearized line bundle L on X. It turns out that, when we let L vary in a certain closed convex cone, the different quotients are related by a sequence of some special birational transformations which are called flips. We shall announce some general results concerning these variations of quotients obtained in a joint work with Y. Hu. Then we shall give some applications to the wall geometry of fans considered earlier by M. Reid, and T. Oda, H. Park

A. Duval

The exterior face-ring and a combinatorial decomposition of simplicial complexes

We find a decomposition of simplicial complexes that implies and sharpens the Björner-Kalai characterization of the f-vector and Betti numbers of a simplicial complex. The proof uses the *exterior face-ring* of a simplicial complex, the "exterior algebra version" of the face-ring. We also present several open problems on generalizations.

W. Ebeling

Polytope complexes associated to singularities

Let $f : (\mathbf{C}^n, 0) \to (\mathbf{C}, 0)$ define an isolated hypersurface singularity. The intersection matrix S with respect to a strongly distinguished basis of vanishing cycles of f is represented by a graph D, called a Coxeter-Dynkin diagram of f. This is a graph with edges weighted

by integers and with a numbering of the vertices. A closed path in D is called a monotone cycle if in traversing the path the numbering of the vertices is increasing (except for the last step). We denote by $\mathcal{M}(D)$ the set of monotone cycles of D.
We show that for many f there exist Coxeter-Dynkin diagrams D where only the weights $+1$ or -1 occur and where the set $\mathcal{M}(D)$ has the structure of an n-dimensional abstract simplicial or polytope complex. The Euler characteristic of this complex is equal to the trace of the Coxeter transformation. In this way, to each such f there is associated a class of polytope complexes.
We study combinatorial properties of this class of complexes. We discuss the following conjecture: If f has at least one diagram D with $\mathcal{M}(D)$ a polytope complex then the minimum over all diagrams D with $\mathcal{M}(D)$ a polytope complex of the minimal embedding dimension of $\mathcal{M}(D)$ is equal to corank of f.

M. Eikelberg

The Picard group of a compact toric variety

In our lecture we explain our generalization of our method for the calculation of the Picard group we explained at the recent conference in 1989. Now our method applies to all compact toric varieties:
Let X_Σ be a compact toric variety given by a fan $\Sigma T_N CDIV(X_\Sigma)$ the group of all T-invariant Cartier divisors, and $SF(N;\Sigma)$ the group of all 'support functions' as defined by T. Oda. Using a special element of $T_N CDIV(X_\Sigma)$ which always exists, we present a formula for the calculation of the Picard group Pic X_Σ of a compact variety:

$$\text{Pic} X_\Sigma = \mathbf{Z}^{n-d-\dim_Q \Sigma_{\sigma\in\Sigma(d)} AD(\sigma)}$$

This formula provides a formula for the calculation of $T_N CDIV(X_\Sigma)$ and $SF(N;\Sigma)$, as well. We explain how to find the special element of $T_N CDIV(X_\Sigma)$ mentioned above, state our theorem, and give an outline of the proof.
By an example we demonstrate our method of calculation. Furthermore we show what type of results can be obtained using our formula. It is known, that Pic X_Σ may not already be determined by the combinatorial type of Σ but dependend on metrical properties of Σ (cf. our lecture in 1989). Our results are also on the question what can be said about Pic X_Σ if only the combinatorial type of Σ is known.

G. Ewald

On minimal resolutions of singularities in affine toric 3-varieties

We consider an affine toric 3-variety spec$\mathbf{C}[\check{\sigma} \cap \mathbf{Z}^n] = X_{\check{\sigma}} =: X_{\Sigma(\sigma)}$ where σ is a lattice cone generated by 3 linearly independent simple lattice vectors, $\check{\sigma}$ its dual cone, $\Sigma(\sigma)$ the fan composed of σ and its faces. If $S_1, \ldots, S_q$ are stellar subdivisions which turn $\Sigma(\sigma)$ into a regular fan Σ then by

$$\begin{array}{ccccccc} \Sigma & \overset{S_q}{\leftarrow} & \cdots & \overset{S_1}{\leftarrow} & \Sigma(\sigma) \\ \downarrow & & & & \downarrow \\ X_\Sigma & \overset{\varphi_q}{\rightarrow} & \cdots & \overset{\varphi_1}{\rightarrow} & X_{\Sigma(\sigma)} \end{array}$$

a resolution of singularities is given (ϕ_i toric morphisms, $i = 1, \ldots, q$). We assume the resolution to be minimal in the sense that no ϕ_i is a blowdown. Then we can classify for $q \leq 3$ all pairs $X_{\Sigma(\sigma)}$, X_Σ explicitely.

K.H. Fieseler

Chern classes for singular toric varieties

In this talk we prove that for a toric variety X the Schwartz-MacPerson Chern class $c.(X) = \sum_i c_{2i}(X) \in H.^{dd}(X)$ is given by the formula

$$c.(H) = \sum_{O \subset X \ T-orbit} [\bar{O}].$$

That result originates from joint work with G. Barthel and J.P. Brasselet; but as we learnt later on, there existed already an analogous computation by F. Ehlers. We show that $c.(X)$ in general does not come from a cohomology class (via the Poincaré duality homomorphism), but over the rationals, it always lifts (non canonically) to an intersection homology class.

J. Fine

Intersection homology Betti numbers of algebraic varieties and convex polytopes

I wish to present geometric proofs for the Mayer-Vietoris and the ICI equations for convex polytopes.
The rest of the talk will be devoted to the following problems
a) verifying the author's conjecture on the intersection homology Betti numbers of algebraic varieties
b) constructing intersection homology for convex polytopes, without recourse to algebraic varieties
c) defining generalised Betti numbers for convex polytopes, which are to express the Bayer-Billera equations as duality for a variant of intersection homology.

T. Hibi

A lower bound theorem on Erhart polynomials of convex polytopes

Let Γ be an integral polyhedral complex in $\mathbf{R}^n$ and suppose that the underlying space $X = |\Gamma|$ of Γ is homeomorphic to the d-ball. Set $i(X, n) = \#(nX \cap \mathbf{Z}^n)$ for $n = 1, 2, 3, \ldots$. It is known that $i(X, n)$ is a polynomial in n of degree d with $i(X, 0) = 1$. which is called the Erhart polynomial of X. We define the sequence $\delta_0, \delta_1, \delta_2, \ldots$ of integers by

$$(1-\lambda)^{d+1}[1 + \sum_{n=1}^{\infty} i(X, n)\lambda^n] = \sum_{i=0}^{\infty} \delta_i \lambda^i.$$

Then $\delta_i = 0$ for every $i > d$. We say that $\delta(X) = (\delta_0, \delta_1, \ldots, \delta_d)$ in the δ-vector of X.

Theorem 0.1 *Suppose that X is* "star-shaped" with respect to some $\alpha \in (X - \partial X) \cap \mathbf{Z}^n$. Then

(i) $\delta_0 + \delta_1 + \cdots + \delta_i \leq \delta_d + \delta_{d-1} + \cdots + \delta_{d-i}, 0 \leq \forall i \leq [d/2]$;
(ii) $\delta_1 \leq \delta_i, 2 \leq \forall i < d$.

R. Koelman

A criterion for the ideal of a projectively embedded toric surface to be generated by quadrics

We show that the ideal of a projectively embedded toric surface is generated by quadrics if and only if the polygon which corresponds to the embedding has more than 3 lattice points on its boundary.

C. Lee

Convex polytopes, rigidity and stress

We discuss a generalization of classical stress and infinitesimal rigidity to higher-dimension faces of simplicial complexes. In particular, we mention some relationship to

- the Stanley-Reisner ring
- shellability
- bistellar operations
- the g-theorem for simplicial convex polytopes
- the polytope algebra

R. Morelli

Volumes of lattice polytopes, Ehrhart polynomials, and Todd classes of toric varieties

Let $n \approx \mathbf{Z}^d$ be a lattice of rank d. We construct a sequence of messures $\mu_K^{td_k}, K = 0, 1, \ldots, d$ with the following properties:
1. $\mu_K^{td_k}$ is defined on the collection of all n-dimensional rational polyhedal cones in $N_R = N \otimes R$ and takes values in the space of rational functions on the grassmannian of (d-K+1)-planes in N_R.
2. $\mu_K^{td_k}$ is linear with respect to subdivisions of cones

$$\mu_K^{td_k}(\sigma_1 \cup \cdots \cup \sigma_N) = \sum_{i=1}^{N} \mu_K^{td_k}(\sigma_i)$$

if the σ_i have disjoint interiors.
3. $\mu_K^{td_k}$ is natural with respect to isomorphisms of lattices.
4. $\mu_K^{td_k}$ are given by an explicit formula.

Theorem: 1. For a lattice polytope $P \subseteq M \otimes \mathbf{R} = N^\vee \otimes \mathbf{R}$ the Ehrhart polynomial $\#(P;n) = \mathrm{Card}(nP \cap M) = \#_0(P) + \#_1(P)n + \cdots + \#_d(P)n^d$ is given by

$$\#_K(P) = \sum_{F \subseteq P \ \dim F = K} M_{d-K}^{td_k}(\Theta_F(P)^\vee) vol(F)$$

where $\Theta_F(P)$ is the angle cone of P along F.
2. For a completely arbitrary toric variety X_Δ with fan Δ,

$$Td_K = (X_\Delta) = \sum_{\sigma \in \Delta \ \dim \sigma = d-K} \mu_{d-K}^{td_{d-K}}(\sigma)[V(\sigma)].$$

Peter McMullen

Simple polytopes

The conditions characterizing the f-vectors of simple polytopes were proposed by the speaker in 1970. Around 1979-1980, the characterization was established. The sufficiency of the conditions was proved by Billera and Lee, using a direct (and ingenious) construction. However, the necessity, proved by Stanley, employed deep results from algebraic geometry, namely the hard Lefschetz theorem applied to the cohomology ring of the toric variety associated with a rational simple polytope. In this talk, a proof of the necessity entirely within convexity will be presented. However, there are striking parallels with Stanley's proof, and related results on the cohomology ring, which suggest deeper connexions that remain to be explored.

James Pommersheim

Todd classes of toric varieties and counting lattice points

We give a formula for the Todd class of a toric variety, which we use to obtain results about the Ehrhart polynomial of a convex lattice polytope. In particular, the codimension two part of the Todd class is expressed in terms of Dedekind sums. This leads to an expression for the coefficient of the degree $n-2$ term in the Ehrhart polynomial of an n-dimensional polytope given in terms of Dedekind sums. Another consequence of the Todd class formula is new relations among Dedekind sums.

Lauren Rose

Combinatorial conditions for homological dimension of modules of piecewise polynomials

For a polyhedral complex Δ embedded in $\mathbf{R}^d$, we consider modules of piecewise polynomial functions defined on Δ. We describe combinatorial und topological conditions on Δ for these modules to have homological dimension equal to k, $0 \leq k \leq d$. To do this, we make use of a connection between these modules and the face ring of Δ.

E. Shustin

Glueing of Newton polygons, construction of singular curves and deformation of singularities

We solve completely the problem of classification of nodal multisingularities for real plane algebraic curves and for deformation of real plane curve singular points. Our approach is based on the following generalization of Viro's construction: any finite set of real polynomials, whose Newton polygons form a regular subdivision of a convex polygon Δ, determines a real polynomial with Newton polygon Δ and nodal multisingularity equal to the disjoint union of nodal multisingularities of the initial polynomials.

Robert Simon

The Shelling Extension Conjecture

If Δ is a (pure) rank d shellable simplicial complex such that Δ is not a uniform matroid ($\Delta = \{F \mid |F| \leq d\}$) does there exist a set B with $|B| = d$ and $\Delta \cup \bar{B}$ a shellable simplicial complex? A proof of the above for Δ vertex decomposable will be given, and progress on this question will be presented.

Richard Stanley

Flag f-vectors and the cd-index

The flag f-vector of a graded poset P counts the number of chains of P whose elements have specified ranks. If P is an Eulerian poset (e.g., the face-poset of a regular CW-sphere) then the cd-index $\Phi_p(c,d)$ is a noncommutative polynomial in c and d, due to J. Fine, which efficiently encodes the flag f-vector. We conjecture that the cd-index of a Cohen- Macaulay Eulerian poset has nonnegative coefficients, in which case this would give all linear inequalities satisfied by flag f-vectors of Cohen-Macaulay Eulerian posets. We prove the conjecture in several cases, including face-lattices of convex polytopes.

Bernd Sturmfels

Chow cohomology of toric varieties

This lecture deals with ongoing joint work with W. Fulton. We study the Chow homology groups $A_k X$ of k-dimensional algebraic cycles on a toric variety X, modulo rational equivalence.

If X is complete, then $A_k X$ is shown to be dual to the operational Chow cohomology group $A^k X$, as defined by Fulton and MacPherson. Our main result is a combinatorial description, in terms of Minkowski weights, of the ring $A^* X = \oplus_k A^k X$. The subring generated by the Picard group $Pic(X) \simeq A^1 X$ is isomorphic to McMullen's polytope algebra $\Pi(P)$, provided X is projective and P the associated convex polytope.

David Wagner

Toric varieties associated with finite distributive lattices

Associated with a poset P in the polytope P consisting of all order-reversing functions $f : P \longrightarrow \mathbf{R}$ with $O \leq f(p) \leq 1$ for all $p \in P$. The toric variety $V(D)$ associated with P is also defined by the ideal $(X_q X_r - X_{q\cup r} X_{q\cap r} : q, r \in D)$ in $k[X_q : q \in D]$, where D is the distributive lattice of downsets of P. A subtorus of $V(P)$ corresponds to a generous sublattice L of D and its closure is isomorphic to $V(L)$. For a downset $q \in D$ we determine the structure of the associated graded ring of the local ring of $V(D)$ along $V([\emptyset, q])$; as a consequence we show that $V(D)$ is nonsingular along $V([\emptyset, q])$ if and only if $P \setminus q$ is a forest of downward-branching trees. Order dual results also hold so that $V(P)$ is a nonsingular variety if and only if P is a disjoint union of chains. The technique seems likely to yield similar information for all generous sublattices L of D. Our hope is that these varieties will provide some leverage in dealing with purely order-theoretic questions.

Uwe Wessels

Complete fans with $\leq d+2$ generators are strongly polytopal

The close relation between the convex geometry of cone systems and the algebraic-geometric theory of toric varieties has been used to solve geometrical problems applying algebraic tools as well as to provide a geometrical treatment of algebraic objects. Interesting examples of complex analytic spaces have been found this way. We focus on minimal examples of compact, non-projective toric varieties.
Regarding smooth toric varieties, Oda 1988 provided an example of a 3-dimensional non-projective compact toric variety with Picard number 4. Its minimality has been established by Kleinschmidt and Sturmfels, 1989. The minimal known singular example is given by a 3-dimensional fan with 6 1-cones. We establish its minimality, asserting that any non-simplicial complete fan with $d+2$ generators is spanned by a pyramid, whereas the simplicial case is settled by obvious modifications of the Kleinschmidt-Sturmfels proof.

J.M. Wills

Minkowski-type inequalities for the lattice point enumerator

Let $K \subset E^d$ be an 0-symmetric convex body, $L \subset E^d$ a lattice and $G(K, L) = \text{card}\,(K \cap L)$. Further let

$$\lambda_i(K, L) = min\{\lambda > 0 \mid \dim \text{aff}(\lambda K \cap L) \geq i\} \quad i = 1, \ldots, d$$

Minkowski's successive minima. We show

$$G(K, L) \leq \lfloor 2/\lambda_1(K, L) + 1 \rfloor^d$$

and, if K strictly convex

$$G(K, L) \leq 2\lceil 2/\lambda_1(K, L) \rceil^d - 1$$

For $\lambda_1 = 1$ (i.e. $intK \cap L = \{0\}$) one gets Minkowski's results $G(K, L) \leq 3^d$ and $G(K, L) \leq 2^{d+1} - 1$. Further results in this direction are given.

This is a joint paper with U. Betke and M. Henk.

David Yavin

A chain complex for intersection homology of toric varieties

We present a topological description of a (compact) toric variety $X = X_\Sigma$ as a quotient $P \times T^n/_\sim$, where P is a cell complex dual to the (complete) fan Σ which defines X.

$T^n = \mathbf{R}^n/\mathbf{Z}^n$ is the n-torus, and the relation $\sim$ collapses the n-torus over each point in a k-face $F \subset P$ modulo the $(n-k)$-torus determined by the $(n-k)$-cone $\sigma \in \Sigma$ dual to F.

Using this near-product structure of X, we show that in order to compute the intersection homology $IH_*^{\bar{p}}(X; \underline{V})$ (for any perversity $\bar{p}$ and any local system $\underline{V}$), it suffices to consider chains of the form $c \times t/_\sim$, where $c \in sdP$, and t is a chain on the torus. We show that, after suitable geometric interpretation, the bar complex $\bar{W}_*(\mathbf{Z}^n; \underline{V})$ can be used as a complex on the torus.

Andrei Zelevinsky

Maximal minor polytopes

For $2 \leq m \leq n$ the maximal minor polytope $\Pi_{m,n}$ is the Newton polytope of the product of all $m \times m$-minors of an $m \times n$ matrix of indeterminates. The study of $\Pi_{m,n}$ was initiated in a joint work with B. Sturmfels (to appear in "Advances in Math.") and continued in joint works with P. Santhanakrishnan (to appear in "Discrete Computational Geometry"). The polytopes $\Pi_{m,n}$ have several algebraic-geometric applications, including various systems of local coordinates on Grassmannians. This talk concentrates on combinatorial and geometric properties of $\Pi_{m,n}$. We discuss various ways of encoding vertices of $\Pi_{m,n}$, some special facets, and a special class of simple vertices.

G.M. Ziegler

Constructing the Permuto-Associahedron

We construct a family of polytopes KPA_{n-1}, the "Permuto- Associahedra". Here KPA_{n-1} is an $(n-1)$-dimensional polytope, whose vertices correspond to the complete bracketings of permutations of $\{1, 2, \ldots, n\}$, with a natural notion of adjacency. This solves a problem of M.M. Kapranov, who had defined KPA_{n-1} as a combinatorial object and showed that it corresponds to a cellular ball.

Our construction combines the construction of the associahedron as a special "secondary polytope" with a compatible action of the permutation group. It can be generalized to yield "Coxeter- associahedra" for the signed permutation groups B_n and D_n. The proofs yield integral coordinates, with all vertices on a sphere, and include complete descriptions of the facet-defining inequalities.

This is joint work with Victor Reiner (Minneapolis).

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