

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 26/1993

Analysis auf kompakten Varietäten

6.6.-12.6.1993

Die Tagung fand unter der Leitung von T. Peternell (Bayreuth) und Y.-T. Siu (Harvard) statt. Thema der Tagung war das Zusammenwirken analytischer und algebraischer Methoden beim Studium komplex-algebraischer Mannigfaltigkeiten. Schwerpunkte bildeten: spezielle Kählermetriken (Kähler-Einstein, etc.), Calabi-Yau-Mannigfaltigkeiten, Verschwindungssätze, Hyperbolizität, Mannigfaltigkeiten allgemeinen Typs und "Ströme" (currents), sowie Effektivitätsprobleme für globale Erzeugung.

Vortragsauszüge

T. Bandman:

Holomorphic mappings between projective varieties of general type

Joint work with D. Markushevich.

We sketch the proof of the following

**Theorem.** *There exists a function  $\delta = \delta(n, r, k)$  such that for any pair of varieties  $X, Y$  with at most canonical singularities and nef and big canonical line bundles  $K_X$  resp.  $K_Y$  the number  $R(X, Y)$  of rational dominant maps  $f : X \dashrightarrow Y$  is bounded by  $\delta(n, r, k)$ , where  $n = \dim(X)$ ,  $k = (K_X)^n$  and  $r = r_X \cdot r_Y$  is the product of indices of the varieties  $X$  and  $Y$ .*

**Corollary 1.** *For any two surfaces  $X, Y$  of general type we have  $R(X, Y) \leq \delta(h^1, h^2)$ , where  $h^1 = \dim H^1(X, \mathbb{Z})$ ,  $h^2 = \dim H^2(X, \mathbb{Z})$ , and  $\delta$  is a function depending only on 2 variables.*

**Corollary 2.** *There is a function  $\delta$ , such that for arbitrary threefolds  $(X, Y)$  of general type  $R(X, Y) \leq \delta(r, k_m)$ , where  $r = r_X \cdot r_Y$  and  $k_m$  is the cube of the canonical class of the minimal model of  $X$ .*

V. Batyrev :

Variations of Hodge structures on Calabi-Yau manifolds

The talk is devoted to some relations between two different variations of Hodge structures on Calabi-Yau manifolds and theoretical physics. The two variations are connected by Mirror Symmetry discovered by physicists.

T. Bouche:

Asymptotics of certain determinants

Let  $X$  be a compact complex manifold of dimension  $n$  endowed with an arbitrary metric  $\omega$  and  $L$  a positive hermitian line bundle over  $X$  with a global section  $s \in H^0(X, L)$ . We consider the asymptotic behaviour of the quadratic form  $Q_k(\sigma) = \|s \otimes \sigma\|_2^2$ , where  $\|\cdot\|_2$  is the global  $L_2$ -norm on  $H^0(X, L^k)$ . If  $\lambda_0^k \leq \dots \leq \lambda_{N_k}^k$  denote the eigenvalues of  $Q_k$  w.r.t. the  $L^2$ -norm and  $c_1(L, h)$  the first Chern form associated to the metric  $h$  of  $L$ , we prove

**Theorem.**

- (a)  $\lambda_0^k \rightarrow 0$  and  $\lambda_{N_k}^k \rightarrow \|s\|_\infty^2$
- (b)  $\sum_{i=0}^{N_k} \lambda_i = \frac{k^n}{n!} \int_X |s|^2 c_1(L, h)^n + o(k^n)$
- (c)  $\text{Log}\left(\prod_{i=0}^{N_k} \lambda_i\right) = \frac{k^n}{n!} \int_X \text{Log}|s|^2 c_1(L, h)^n + o(k^n)$

The proof relies heavily on heat kernel estimates.

F. Catanese:

Representation of fundamental groups of some open surfaces and applications

If  $F$  is a  $C^\infty$  4-manifold, and  $P$  an  $SO(3)$  topological bundle, we consider the case where the virtual dimension  $V - \dim = -2p_1 - 3(b^+ - b^1 + 1)$  is 0. For a generic metric  $g$ , the moduli space  $\mathcal{M}(P, g)$  has a number of points  $q_{\mathcal{M}}(P)$  counted with multiplicity, which yields a  $C^\infty$  invariant of  $M$ . To calculate  $q_{\mathcal{M}}(P)$ , Kronheimer introduced a theory of orbifold bundles on orbifold space  $X$ , in particular considering the case where  $P$  arises from a resolution of  $X$ , by representations  $\rho: \pi_1(X^\#) \rightarrow SO(3)$ ,  $X^\#$  being the nonsingular part of  $X$ . When  $M = S$  a smooth algebraic surface, we want  $X$  to be its canonical model, and with a lot of singular points. We study the example where  $X_{n,m}$  is a double cover of  $\mathbb{P}^1 \times \mathbb{P}^1$  branched over  $2n$  horizontal and  $2m$  vertical lines, and describe explicitly all these representations, showing that in all cases but one, they are determined by the Stiefel-Whitney class  $w_2$ . Modulo some Ansatz we prove

**Theorem.** Consider a Horikawa surface ( $K^2 = 2\chi - 6$ ) with  $16|K^2$ . If  $p_1 = -6\chi$  and  $w^2 \equiv 1(4)$  for all  $w (= w_2(P))$ , then there is a number  $\alpha$  such that  $q_S(P) = 0, 1$ , or  $\alpha$ .

We need to study the other family of these Horikawa surfaces, and to calculate the  $q_5(P)$ 's there. For these we have some examples by U. Persson for which we have calculated  $\pi_1(X^\#)$ , but not yet the representation  $\rho$ .

K. Cho:

Several characterizations of projective space and hyperquadrics

Joint work with Y. Miyaoka.

The following characterizations of projective space and hyperquadrics are well-known: Siu and Yau gave a characterization of projective space in terms of the positivity of the bisectional curvature in complex differential geometry. Later Siu gave a characterization of hyperquadrics in a similar method. Finally Mok gave a characterization of hermitian symmetric spaces of compact type.

In algebraic geometry, Mori gave a characterization of projective space in terms of ampleness of the tangent bundle. Before this there was a characterization of projective space and hyperquadrics by Kobayashi-Ochiai. In this talk I give a new characterization which generalizes Mori's and Kobayashi-Ochiai's results.

G. Dethloff:

Hyperbolicity of complements of 3 curves in  $\mathbb{P}^2$

Joint work with G. Schumacher and P. M. Wong.

**Conjecture (Kobayashi, Zaidenberg).** Let  $l(d_1, \dots, d_k) = \{\bigcup_{i=1}^k \Gamma_i : \Gamma_i \text{ is a hyper-surface of degree } d_i \text{ in } \mathbb{P}^n\}$  and  $\mathcal{H}(d_1, \dots, d_k) = \{\Gamma \in l(d_1, \dots, d_k) : \mathbb{P}^n \setminus \Gamma \text{ is complete hyperbolic, hyperbolically embedded}\}$ . Let  $\sum_{i=1}^k d_i \geq 2n + 1$ . Then  $\mathcal{H}(d_1, \dots, d_k)$  contains a nonempty Zariski open subset of  $l(d_1, \dots, d_k)$ .

We deal with the surface case ( $n = 2$ ). The case  $k \geq 4$  of the conjecture is true, for  $k = 1, 2$  it is unknown. In the case  $k = 3$  we have the following results:

**Theorem.** (a) The conjecture is true for 3 quadrics  $l(2, 2, 2)$ .

(b) The conjecture is true for 2 quadrics and a line  $l(2, 2, 1)$ .

(c) The conjecture is true for  $d_1 + d_2 + d_3 \geq 7$ , except possibly in the cases  $l(1, 1, d)$ ,  $l(1, 2, d)$ ,  $l(1, 3, 3)$ .

In the talk ideas of the proofs of (a) and (b) are given. P. M. Wong proves (c) in a separate talk.

L. Ein:

Pluricanonical and adjoint linear systems on smooth 3-folds

Joint work with R. Lazarsfeld.

Using cohomological techniques developed by Kawamata, Shokurov and others, we find effective results of Reider-type on freeness of linear systems on smooth projective 3-folds.

In particular we show that if  $X$  is a smooth minimal 3-fold of general type, then  $|mK_X|$  is free for  $m \geq 7$ .

A. Fujiki:

On the holomorphic symplectic structure of moduli spaces of vector bundles on certain open surfaces

Motivated by the work of Nakajima and Kronheimer we look for a natural formulation for the existence of a holomorphic symplectic form on the moduli space of holomorphic vector bundles on certain open algebraic surfaces. In fact, this leads to an analogue of Mukai's theorem in the open case, where the moduli space  $\mathcal{M}$  in question is that of simple sheaves framed along the boundary.

Our main result is: Let  $S$  be a compact smooth surface and  $D$  a divisor on  $S$  such that  $K_S + 2D$  is trivial, where  $K_S$  is the canonical bundle. Then  $\mathcal{M}$  is smooth and admits a natural holomorphic symplectic structure.

Here, the result can actually be formulated in the category of  $V$ -surfaces, and then it is applicable to the case of Nakajima and Kronheimer above.

H. Gillet:

Chow groups of degenerations

Joint work with S. Bloch and C. Soulé.

If  $X$  is a smooth projective variety over a discretely valued field  $K$ , we define analogs of various groups of forms and currents on varieties over  $\mathbb{C}$  (which has an archimedean valuation). These are defined as direct/inverse limits of Chow homology/cohomology groups of special fibres of models of  $X$  over the valuation ring.

For these groups we prove an analog of regularity of  $dd^c$ , and also that the kernel and cokernel of  $dd^c$  can be computed on any single model with special fibre a divisor with normal crossings.

If  $\mathcal{X}$  is semistable and we replace Chow groups by homology then we show  $\text{Ker}(dd^c) \simeq \text{coker}(dd^c)$ .

R. Kobayashi:

Ricci-flat Kähler metrics on affine algebraic manifolds

The talk is about the existence of Ricci-flat Kähler metrics on affine algebraic manifolds of the form  $X - D$ , where  $X$  is a Fano manifold and  $D$  is a divisor with simple normal crossings. Examples of classes of  $(X, D)$  such that  $X - D$  admits a complete Ricci-flat Kähler metric are given and the idea of the proofs of existence are indicated. Finally it is shown that Van de Ven's conjecture can be solved using Ricci-flat Kähler metrics on affine algebraic manifolds.

T. Mabuchi:

Uniqueness and periodicity of extremal Kähler vector fields

Let  $X$  be a compact connected complex manifold endowed with a Kähler class  $\gamma$ . If  $\gamma$  admits an extremal Kähler metric  $\omega$  in the sense of Calabi, then we have a holomorphic vector field  $V_\omega = \frac{1}{\sqrt{-1}} \sum g^{\bar{\beta}\alpha} \partial_{\bar{\beta}} \sigma_\omega \partial_\alpha$  on  $X$ , where  $\sigma_\omega$  denotes the scalar curvature of  $\omega$ . Then we have

**Theorem A.** *Let  $\omega_1, \omega_2$  be extremal Kähler metrics in the class  $\gamma$ . Then there exists an element  $g \in \text{Aut}^0(X)$  such that  $g \cdot V_{\omega_1} = V_{\omega_2}$ .*

**Theorem B.** *Assume that  $\gamma = c_1(X)$ . Then the real vector field  $(V_\omega)_\mathbb{R} := V_\omega + \bar{V}_\omega$  associated to  $V_\omega$  (where  $\omega$  is an extremal Kähler metric in the class  $\gamma$ ) is periodic, i.e., generates an  $S^1$ -action.*

The proof depends on a detailed analysis of moment maps associated to  $(G_m)^r$  actions on  $X$ . We actually use some  $\mathbb{Q}$ -structures on moment maps and define a  $\mathbb{Q}$ -bilinear form on the Lie algebra associated to a maximal algebraic torus of  $\text{Aut}^0(X)$ .

L. Manivel:

Vanishing theorems for ample vector bundles

Let  $E$  be an ample vector bundle of rank  $r$  on a smooth projective variety  $X$  of dimension  $n$ . On the relative flag manifold  $Y = G_r(E) \xrightarrow{\pi} X$  live line bundles  $L^\rho$  indexed by sequences  $\rho \in \mathbb{N}^r$ . If  $\rho$  is decreasing and non-negative,  $\pi_* L^\rho = S^\rho E$  is the vector bundle on  $X$  associated to  $E$  and the irreducible representations of  $GL(r, \mathbb{C})$  with dominant weight  $\rho$ . When  $E$  is ample,  $L^\rho$  also is and we get vanishing theorems using the Borel-Le Potier spectral sequence and applying Kodaira-Nakano vanishing to  $L^\rho$ .

If  $\rho$  is some partition, let  $l(\rho)$  (resp.  $|\rho|$ ) be the number of its non-zero (resp. the sum of its) components. Let  $\chi$  be the permutation of  $\{1, \dots, r\}$  reversing the order.

**Theorem.** (A) *Let  $\mu$  and  $\nu$  be partitions,  $E$  ample of rank  $r \geq l(\mu) + l(\nu)$  on  $X$ . Then*

$$H^{n,q}(X, S^{\mu-\chi(\nu)} E \otimes (\det E)^l) = 0 \text{ if } q > |\nu|, l \geq l(\mu) - \nu_1.$$

(B) *Suppose  $E$  is nef and that for some partition  $\pi$  of  $n$ ,  $\int_X s_\pi(E) > 0$ ,  $s_\pi$  being the characteristic class of  $E$  defined by the Schur symmetric function of weight  $\pi$ . Then for every partition  $\rho$  such that  $l(\rho) \leq l(\pi)$ ,*

$$H^{n,q}(X, S^\rho(E) \otimes (\det E)^l) = 0 \text{ if } q > 0, l \geq l(\pi).$$

Y. Miyaoka:

On characterizations of projective space

Joint work with K. Cho.

This is a continuation of the talk given by K. Cho. We give a new numerical characterization of complex projective space and smooth hyperquadrics. Let  $X$  be a Fano manifold of dimension  $n$  over the complex numbers (i.e.  $(-K_X)$  ample or equivalent  $c_1(X) > 0$ ). Define the length  $l(X)$  of  $X$  as  $l(X) := \min\{(-K_X \cdot C) \mid C \text{ is a rational curve } \subset X\}$ . It is well-known that  $0 < l(X) \leq n + 1$ . Then our theorem can be stated as:

**Theorem.** *Let  $X$  be a Fano manifold of dimension  $n$  over  $\mathbb{C}$ .*

*If  $l(X) = n + 1$ , then  $X$  is isomorphic to  $\mathbb{P}^n$ .*

*If  $l(X) = n$ , then  $X$  is isomorphic to a hyperquadric.*

N. Mok:

The gap phenomenon on hermitian locally symmetric spaces

Let  $\Omega/\Gamma$  be a hermitian locally symmetric space of the non-compact type. Conjecturally there exists a constant  $\epsilon > 0$  depending only on  $\Omega$  for which the following holds:

Let  $S \subset X$  be a compact complex submanifold such that the second fundamental form  $\sigma_{S|X}$  is everywhere of norm  $< \epsilon$ . Then  $S$  is a totally geodesic complex submanifold when  $X$  is not exceptional.

A proof of this conjecture is presented in the special case of curves  $S$ .

K. Oguiso:

Fibre space structure of Calabi-Yau 3-folds and related topics

The purpose of the talk is to study an algebraic fibre space structure on a minimal Calabi-Yau 3-fold  $X$  via the numerical structure on  $X$ . More precisely we construct an algebraic fibre space structure on  $X$  from a pair  $(X, D)$  with  $D$  a nef effective divisor on  $X$  and classify all of them by numerical invariants  $\nu(X, D)$  and  $D \cdot c_2(X)$  into 6 types, each of which actually exists.

T. Ohsawa:

$L^2$ -cohomology vanishing and stability of harmonic forms

Let  $X$  be a reduced compact complex space of pure dimension  $n$  and  $H_{(2)}(X)$  the  $L^2$ -cohomology of  $X$  in the sense of Cheeger.

**Theorem 1.**  $H_{(2)}(X) \simeq IH(X)$ , where  $IH(X)$  denotes the (middle) intersection cohomology. In particular,  $H_{(2)}(X)$  is a topological invariant of  $X$ .

**Theorem 2.** If  $X$  admits a Kähler metric,  $H_{(2)}(X) = \bigoplus H_{(2)}^{p,q}(X)$ . Here  $H_{(2)}^{p,q} := \{[u] \in H_{(2)}(X) \mid u \text{ is of type } (p, q)\}$

Theorem 1 is a consequence of an  $L^2$ -cohomology vanishing theorem and Theorem 2 is obtained by generalizing Theorem 1 to certain complete metrics on  $X_{reg}$  and establishing a stability theorem for harmonic forms.

B. Shiffman:

Algebraic approximation of holomorphic maps and the Kobayashi metric on projective manifolds

Joint work with J. P. Demailly and L. Lempert.

It is shown that every holomorphic map from a Runge open set  $\Omega$  in an affine algebraic variety into a projective algebraic manifold  $X$  can be approximated uniformly by Nash algebraic maps from any relatively compact domain  $\Omega_0 \subset\subset \Omega$ . In the case when  $\Omega$  is the unit disc, this result can be used to show that the Kobayashi pseudo-distance on  $X$  can be computed solely in terms of the closed algebraic curves on  $X$ . More generally, when  $\Omega$  is the unit ball in  $\mathbb{C}^p$ , the approximation theorem shows that the  $p$ -dimensional Eisenman metric of a quasi-projective algebraic manifold can be computed in terms of the Eisenman volume of its  $p$ -dimensional algebraic subvarieties. As another application it is shown that if  $\Omega_0 \subset\subset \Omega \subset S$ , where  $S$  is an affine algebraic manifold and  $\Omega$  is Runge, and if  $E$  is a holomorphic vector bundle on  $\Omega$ , then there exists a Nash algebraic embedding  $\Omega \hookrightarrow Z$  into an algebraic manifold  $Z$  and an algebraic vector bundle  $\tilde{E}$  over  $Z$  such that  $\tilde{E}|_{\Omega} \simeq E|_{\Omega_0}$ .

Y. T. Siu:

Effective Matsusaka Big Theorem

In the talk the following theorem on the effective bound for the Matsusaka theorem is presented with a sketch of its proof:

**Theorem.** *Let  $X$  be a compact complex manifold of complex dimension  $n$  and  $L$  be an ample line bundle over  $X$  and  $B$  be a numerically effective holomorphic line bundle over  $X$ . Let  $K_X$  be the canonical line bundle of  $X$  and  $C$  be the Chern number  $((n+2)L + B + K_X)L^{n-1}$ . Then the line bundle  $mL - B$  is very ample over  $X$  when  $m$  is not less than  $(24n^n C(1+C)^n)^{n(6n^3)^n}$ .*

The proof uses the following two new techniques: (1) a numerical criterion for the existence of global holomorphic sections of a multiple of the difference of two ample line bundles; (2) the stratification of an unreduced subspace defined by multiplier ideal sheaves instead of by finite neighbourhoods of its reduction. The precise statement of the first new technique is as follows. Let  $F$  and  $G$  be ample line bundles over  $X$ . Suppose

$$F^n - \sum_{q=0}^{\lfloor n-1/2 \rfloor} \binom{n}{2q+1} F^{n-2q-1} G^{2q+1} > 0.$$

Then the dimension of global sections of  $k(F-G)$  over  $X$  grows as a positive multiple of  $k^n$  as  $k$  goes to infinity. Here the symbol  $[(n-1)/2]$  means the largest integer not exceeding  $(n-1)/2$ . The proof of this statement uses the strong Morse inequality of Demailly. The use of the stratification by subspaces defined by multiplier ideal sheaves depends on the vanishing theorem of Nadel.

C. Soulé:

Green currents

Joint work with H. Gillet and J.-B. Bost

Given  $Y \subset X$ , a closed irreducible subvariety  $Y$  of codimension  $p$  in a smooth projective variety  $X$  over the complex numbers, a Green current  $g$  for  $Y$  is a current of type  $(p-1, p-1)$  such that, if  $\delta_Y$  is the current given by integration over  $Y$ ,  $dd^c g + \delta_Y$  is smooth on  $X$ .

We prove that these exist, discuss their unicity, and we show that we can choose  $g = [\eta]$ , where  $\eta$  is a smooth form on  $X - Y$  which is  $L^1$  on  $X$  and has log growth along  $Y$ . We discuss the possibility of finding  $g = [\eta]$  with  $\eta$  positive on  $X - Y$ . Finally we give an example of a curve  $Y$  in a 3-fold  $X$  such that such positive  $g$  do not exist.

M. Teicher:

New examples of surfaces of general type with zero signature and finite  $\Pi_1$

In this talk we give some new examples of surfaces of general type with zero signature and finite fundamental group which are spin manifolds.

A. Teleman:

Stable vector bundles over non-Kähler surfaces and geometric structures

We give a complete proof with differential geometric methods for the following Theorem of Bogomolov:

**Theorem.** *A class VII<sub>0</sub> surface with trivial second Betti number is isomorphic to either a Hopf or an Inoue surface.*

Using Lübke's inequality we reduce the problem to the following

**Proposition.** *A projectively flat hermitian metric on a surface is locally conformally flat Kähler.*

In the second part of the talk we announce the following

**Theorem.** *Let  $(X, g)$  be a hermitian compact complex manifold and  $(E, h)$  a  $C^\infty$  hermitian bundle on it. Then there is a natural real analytic open embedding of moduli spaces*

$$\mathcal{M}_{\text{irr}}^{HE}(E, h) \hookrightarrow \mathcal{M}^{\text{simple}}(E).$$



If  $g$  is Gauduchon, the image is precisely the set of  $g$ -stable bundles in  $\mathcal{M}^{\text{simple}}(E)$ .

H. Tsuji :

### Analytic Zariski decomposition

Let  $X$  be a smooth algebraic variety over  $\mathbb{C}$  and  $L$  a line bundle over  $X$ . It is a fundamental problem to study  $R(X, L) = \bigoplus_r H^0(X, L^{\otimes r})$ . If  $L$  is ample this problem is not so difficult, but if  $L$  is not, it is hard to study  $R(X, L)$ . The aim of this talk is to reduce the problem to the case of "positive sheaves". Namely we have:

**Theorem.** Let  $X$  be a smooth projective manifold and  $L$  a big line bundle on  $X$ . Then there exists a singular hermitian metric  $h$  on  $L$  such that

- (1)  $\text{curv}(h) \geq 0$  (positivity)
- (2) If  $\mathcal{F}(i)$  is the sheaf of germs of local  $L^2$ -holomorphic sections of  $(L^{\otimes i}, h^{\otimes i})$ , then  $H^0(X, \mathcal{F}(i)) \simeq H^0(L^{\otimes i})$  holds for all  $i \geq 0$ . (maximality)
- (3) For all modifications  $f : Y \rightarrow X$ , the pair  $(f^*L, f^*h)$  has the properties (1) and (2). (naturality)

We call  $T = \text{curv}(h)$  analytic Zariski decomposition of  $L$ .

J. Winkelmann:

### Complex geometry on parallelizable manifolds

Joint work with A. T. Huckleberry.

Let  $X$  be a compact complex parallelizable manifold. By a result of Wang,  $X = G/\Gamma$ , with a complex Lie group  $G$  and  $\Gamma$  a discrete subgroup. There always exist abelian Lie subgroups  $A$  of  $G$  with closed orbits in  $X$ , hence submanifolds which are tori. On the other hand, up to translation, there are only countably many parallelizable submanifolds. A submanifold  $Z \subset X = G/\Gamma$  has Kodaira dimension zero iff it is parallelizable. In general, if  $Z \subset X = G/\Gamma$ , the pluricanonical fibration may be realized as a quotient by an action of a Lie subgroup  $H$  of  $G$ :  $Z \rightarrow Y = Z/H$ . Therefore the base  $Y$  is of general type. Furthermore this implies restrictions on the dimension, e.g. if  $G$  is simple, then  $\text{codim}(Z) \geq \sqrt{\dim G}$  for all  $Z \subset X = G/\Gamma$ .

Other results: Let  $f : \mathbb{C} \rightarrow X = G/\Gamma$  holomorphic. Then the Zariski closure of the image of  $f$  is parallelizable. For every compact complex manifold  $M$  there exists a holomorphic vector bundle  $E$  which is not holomorphically trivial.

P. M. Wong:

### Complement of curves in $\mathbb{P}^2$

Joint work with G. Dethloff and G. Schumacher.

This talk is a continuation of the one given by G. Dethloff where an outline of the proof of the case of 3 quadrics was given. In general we have:

**Theorem.** Let  $C = C_1 + C_2 + C_3$  be a curve in  $\mathbb{P}^2$  where  $C_i$ ,  $i = 1, 2, 3$ , are irreducible components. Assume that each  $C_i$  is smooth and in general position. Assume further that the logarithmic Chern numbers satisfy the condition  $\bar{c}_1^2 - \bar{c}_2 > 0$ , and  $C + K_{\mathbb{P}^2} > 0$ . Then every entire holomorphic curve  $f : \mathbb{C} \rightarrow \mathbb{P}^2 \setminus C$  is algebraic degenerate.

Note that the assumptions of the theorem are satisfied if  $\deg C_i = d_i = d \geq 3$ ,  $i = 1, 2, 3$ .

**Corollary.** Let the assumptions on  $C$  be as in the theorem and assume moreover that none of the curves passes through the points of tangency of a common tangent of the other two curves. Then  $\mathbb{P}^2 \setminus C$  is Kobayashi hyperbolic and hyperbolically embedded in  $\mathbb{P}^2$ .

P. Yang:

Extremal metrics for the determinant

On a compact Riemannian 4-manifold  $(M, g_0)$  the conformal anomaly formula for  $\log \frac{\det L_\omega}{\det L_0}$  for the conformal Laplacian operators  $L_\omega$  and  $L_0$  of the pair of conformally related metrics  $g_\omega = e^{2\omega} g_0$  splits naturally into two parts. The first of which contains the quadratic form of the conformally invariant operator  $P = \Delta^2 + \operatorname{div}(\frac{2}{3}R - R_{ij})d$ . We define conformal invariants  $k_D = 16\pi^2\chi - \frac{3}{2} \int |Weyl|^2$  and  $k_P = 16\pi^2\chi - \frac{1}{2} \int |Weyl|^2$  and prove that if  $k_D < 32\pi^2$ , the log determinant is extremized at a conformal metric  $g_D$  which satisfies a fourth order equation. If  $k_P < 32\pi^2$ , the main part of the log determinant is extremized at a conformal metric  $g_P$  which satisfies a simpler fourth order equation. In several important instances we can identify these extremal metrics  $g_D$  or  $g_P$  with natural canonical metrics. This is done through uniqueness assertions for the extremals.

Berichterstatter: O. Kühle

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