MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 28/1993

Konvexgeometrie

20.06. bis 25.06.1993

Die Tagung fand unter der Leitung von R. Schneider (Freiburg i. Br.) und J. M. Wills (Siegen) statt. Sie hatte 45 Teilnehmer, von denen 40 Vorträge hielten. Angesichts des unverändert starken Andrangs zu dieser Tagung und der vom Institut vorgegebenen zahlenmäßigen Begrenzungen waren das vertretbare Obergrenzen.
Wie bei der vorherigen Tagung zum selben Thema war man bestrebt, die Beziehungen zwischen der eher klassischen Konvexgeometrie und der Geometrie der Banachräume weiterhin zu verstärken. Aus beiden Gebieten gab es wesentliche Beiträge. Hier sei vor allem auf die Lösung des Busemann-Petty-Problems hingewiesen sowie auf die Fortschritte in der Theorie der gemischten Volumina, Approximation konvexer Körper und Zonoide. Der Anteil an Teilnehmern und Vorträgen aus der Kombinatorischen Konvexgeometrie, der Diskreten Geometrie und Computational Geometry ist, bedingt durch die entsprechenden Oberwolfacher Tagungen, leicht zurückgegangen. Dennoch gab es auch hier bemerkenswerte Fortschritte, insbesondere die Weiterentwicklung der Polytop-Algebra (Beweis des g-Theorems – Notwendigkeit der Bedingungen – in diesem Rahmen), aber auch eine Verbesserung von Minkowskis Schranke für dichteste gitterförmige Kugelpackungen und den Beweis von L. Fejes Tóths Wurstvermutung in hohen Dimensionen.
Weitere Beiträge betrafen geometrische Ungleichungen, geometrische Tomographie, Polytope, orientierte Matroide, Helly-Typ-Sätze, Arrangements, topologische Methoden, jeweils mit Bezügen zur Konvexgeometrie, sowie eine neue Konvexitäts-Struktur auf affinen Grassmann-Mannigfaltigkeiten.
Insgesamt waren die Beiträge sehr vielseitig und zeugten von der ungebrochenen mathematischen Vitalität der Konvexgeometrie.

Vortragsauszüge

K. BALL:

Lattice sphere-packing

It is shown that for each n, there is a lattice in $\mathbb{R}^n$ that packs Euclidean balls with volume density $2^{1-n}(n-1)\zeta(n)$. This estimate slightly improves the bound of Davenport and Rogers. More interesting is that the method is very different from the existing ones and is technically very simple.

I. BÁRÁNY:

The limit shape theorem for convex lattice polygons

Let $\mathcal{P}_n$ denote the set of all convex polygons with vertices from the lattice $\frac{1}{n}\mathbb{Z}^2$ that lie in the unit square. This is the same (up to a homothety) as the set of all convex lattice polygons in the square $[0,n]^2$. A. M. Vershik posed the following beautiful question: Is there a "limit shape" of the polygons in $\mathcal{P}_n$? The answer is yes: let $\chi_P(x)$ stand for the characteristic function of $P \in \mathcal{P}_n$, i.e., $\chi_P(x) = 1$ or 0 according to $x \in P$ or $x \notin P$.
Theorem. There is a convex set C in the unit square such that

$$\lim_{n\to\infty} \frac{1}{|\mathcal{P}_n|} \sum_{P\in\mathcal{P}_n} \chi_P(x) = \begin{cases} 1 \text{ if } x \in \operatorname{int} C \\ 0 \text{ if } x \notin C \end{cases}$$

Actually, the boundary of C consists of four pieces of parabolas, each touching two consecutive edges of the square at their midpoint.
Further, let $\mathcal{C}_n$ denote the set of all convex polygonal paths with vertices from $\frac{1}{n}\mathbb{Z}^2$ that connect $(0,1)$ to $(1,0)$ within the triangle $(0,0)$, $(1,0)$, $(0,1)$. Set $\Gamma = \{(x,y) : \sqrt{x} + \sqrt{y} = 1\}$. There is a limit shape theorem for $\mathcal{C}_n$ as well.
Theorem. For every $\epsilon > 0$ there is $n(\epsilon)$ such that for $n > n(\epsilon)$

$$\frac{\sharp\{C \in \mathcal{C}_n \text{ not in the } \epsilon\text{-neighbourhood of } \Gamma\}}{|\mathcal{C}_n|} < \epsilon.$$

K. BEZDEK:

On affine subspaces that illuminate a convex set

Let K be a convex body in $\mathbb{E}^d$, i.e., a compact convex set with non-empty interior in $\mathbb{E}^d$. We say that a point $P \in \mathbb{E}^d \setminus K$ illuminates the boundary point Q of K if the open ray emanating from Q having direction vector $\vec{PQ}$ meets the interior of K. Let $L \subset \mathbb{E}^d \setminus K$ be an affine subspace in $\mathbb{E}^d$ of dimension ℓ, $0 \leq \ell \leq d-1$. Then L illuminates the boundary point Q of K if there exists a point P of L that illuminates Q. Moreover, we say that the affine subspaces $L_1, L_2, \ldots, L_n \subset \mathbb{E}^d \setminus K$ illuminate K if every boundary point of K is illuminated by at least one of the affine subspaces $L_1, L_2, \ldots, L_n$. Finally, let $I_\ell(K)$ be the smallest number of affine subspaces of dimension ℓ lying in $\mathbb{E}^d \setminus K$ that illuminate K, where $0 \leq \ell \leq d-1$. It is easy to see that $I_0(K) \geq I_1(K) \geq \ldots \geq I_{d-1}(K) = 2$, moreover, $I_\ell(K) \geq \left\lceil \frac{d+1}{\ell+1} \right\rceil$.

Theorem 1. Let K be a convex body in $\mathbb{E}^d$, $d \geq 3$. Then K can be illuminated by two $(d-2)$-dimensional affine subspaces, i.e., $I_{d-2}(K) = 2$.

Theorem 2. Let P be a convex polytope in $\mathbb{E}^d$ such that the inner cone of any boundary point of P contains some orthant of $\mathbb{E}^d$, $d \geq 3$. Then P can be illuminated by two $\left\lfloor \frac{d}{2} \right\rfloor$-dimensional affine subspaces, i.e.,

$$I_{\lfloor \frac{d}{2} \rfloor}(P) = I_{\lfloor \frac{d}{2} \rfloor + 1}(P) = \ldots = I_{d-1}(P) = 2.$$

T. BISZTRICZKY:

Ordinary 3-polytopes

We consider 3-polytopes which are generalizations of cyclic 3-polytopes in the sense that there is a vertex array (a total ordering of vertices) which is instrumental in determining their facial structure.

More precisely, a 3-polytope P with $V = \operatorname{ext} P = \{x_0, \ldots, x_n\}$, $n \geq 3$, is ordinary if there is a vertex array, say, $x_0 < x_1 < \ldots < x_{n-1} < x_n$ such that for any facet F of P : (01) any two points of $V \setminus F$ are separated by an even number of points in $F \cap V$ and (02) if $F \cap V = \{y_1, \ldots, y_m\}$ where $y_1 < y_2 < \ldots < y_m$, then F is an m-gon with edges $[y_1, y_2]$, $[y_{m-1}, y_m]$ and $[y_i, y_{i+2}]$, $i = 1, \ldots, m-2$.

If P is ordinary then there is a k, $3 \leq k \leq n$, such that $[x_0, x_i]$ ($[x_n, x_{n-i}]$) is an edge of P if and only if $1 \leq i \leq k$. We call k the characteristic of P. In addition to being able to describe the facets of P, we remark that $k + \left\lceil \frac{n}{2} \right\rceil \leq f_2(P) \leq n + k - 2$. If $k = n$ then P is cyclic; if $k = 3$ and $f_2(P) = 3 + \left\lceil \frac{n}{2} \right\rceil$ then P has the least number of facets for any 3-polytope with $n+1$ vertices.

J. BOKOWSKI:

Altshuler's sphere M^9_{963} revisited

The smallest uniform non-polytopal matroid polytope (determined by the author in 1978) has played an important role in the theory of oriented matroids. A certain one point contraction of it was used as an example throughout the recent research monograph on oriented matroids. The face lattice of the determining decisive cell in the Falkman Lawrence representation of this oriented matroid is polar to a sphere in Altshuler's list of 3-spheres, the sphere no. 963 with 9 vertices. A new property can now be added: this matroid polytope has no polar (joint work with P. Schuchert), thus answering a question posed by Billera. This provides a minimal rank example for the fact that matroid polytopes have no polar. In addition, a new algorithm for realizing oriented matroids will be mentioned, and interesting aspects from investigations about neighborly polyhedra (joint work with A. Altshuler and P. Schuchert) will be given. No neighborly oriented 2-manifold with 12 vertices can be embedded in the 2-skeleton of any 4-polytope. Thus they cannot be found as a Schlegel diagram of such a polytope in 3-space.

G. FEJES TÓTH:

Packing and covering properties of typical convex discs

The packing density $\delta(C)$ of a convex disc C in the Euclidean plane is defined as the supremum of the densities of all packings of congruent copies of C. Similarly, the covering density $\vartheta(C)$ of C is the infimum of the densities of all coverings with congruent copies of C. If we restrict ourselves to lattice arrangements, we obtain the similarly defined quantities $\delta_L(C)$ and $\vartheta_L(C)$. Let $\mathcal{C}$ denote the space of all convex discs in the plane equipped with the Hausdorff metric. Let $\mathcal{N}_p$ and $\mathcal{N}_c$ be the subsets of $\mathcal{C}$ consisting of those discs C for which $\delta(C) \neq \delta_L(C)$ and $\vartheta(C) \neq \vartheta_L(C)$, respectively. It is shown that $\mathcal{N}_p$ and $\mathcal{N}_c$ are open dense sets in $\mathcal{C}$. The result concerning covering is joint work with Tudor Zamfirescu.

R. J. GARDNER:

Intersection bodies and the Busemann-Petty problem

The Busemann-Petty problem, concerning central sections of centrally symmetric convex bodies, is now solved in each dimension. The talk outlines the interesting history of the problem and some of the recent work on it. In particular, the speaker's positive answer in three dimensions and negative answer for five or more dimensions are sketched. (The negative answer for four dimensions was obtained by Zhang Gaoyong.) The important ingredients in the solution, the concept of an intersection body and the theory of dual mixed volumes, are also discussed, as well as the role of the problem as part of geometric tomography.

P. GOODEY (joint work with Gaoyong Zhang):

Zonoids and mixed volumes

We give a characterization of zonoids by means of inequalities between mixed volumes. This follows from various denseness properties of surface area measures. The results are connected with earlier work of Schneider (1967) and Weil (1976).

J. E. GOODMAN (joint work with R. Pollack):

Convexity on affine Grassmann manifolds

We introduce a natural convexity structure on the "affine Grassmannian" $\mathfrak{G}'_{k,d}$ which extends the standard convexity structure on $\mathbb{R}^d$ $(= \mathfrak{G}'_{0,d})$, and which is invariant under the natural action of the affine group $\mathbf{A}(d, \mathbb{R})$ on $\mathfrak{G}'_{k,d}$. This convexity structure, which is defined by means of a duality operator between subsets of $\mathfrak{G}'_{k,d}$ and families of convex point sets in $\mathbb{R}^d$, turns out to share a number of properties enjoyed by the standard convexity structure on $\mathbb{R}^d$. Examples of convex sets of k-flats for $k > 0$ include the k-flats meeting a convex point set in $\mathbb{R}^d$, the rulings of and lines inside a one-sheeted hyperboloid in $\mathbb{R}^3$, and affine Schubert varieties in $\mathfrak{G}'_{k,d}$.

P. GRITZMANN (joint work with V. Klee):

Closedness of cone + subspace

When X and Y are convex subsets of a topological vector space E, an *external tangent of* the ordered pair (X, Y) is defined as an open ray T that issues from a point of $X \cap Y$, is disjoint from $X \cup Y$, and is such that X intersects each open halfspace containing T. We show that if E is a separable normed space, C is a closed convex cone in E, and L is a line through the origin in E, then the vector sum $C + L = \{c + \ell : c \in C,\ \ell \in L\}$ is closed if and only if the pair (C, L) does not admit any external tangent. When S is a subspace of finite dimension greater than 1, closedness of $C + S$ is shown to be equivalent to the nonexistence of external tangents of a certain pair (C', L), where L is a line through the origin and C' is a second closed convex cone constructed from (C, S).
Questions about the closedness of sets of the form cone + subspace arise from various optimization problems, from problems concerning the extension of positive linear functionals, and from certain problems in matrix theory.

P. M. GRUBER:

Tiling space with smooth tiles

How well can $\mathbb{E}^d$ be packed with smooth bodies? If the bodies are convex and of class C^1, then the set not covered has Hausdorff dimension at least $d-2$. It has Hausdorff dimension at least $d-1$ if the bodies are convex and of class C^ω, i.e. analytic. These estimates are best possible. Related results hold for strictly convex bodies.
The situation changes drastically, if non-convex bodies are admitted: there is a tiling of $\mathbb{E}^d$ with topological d-cells of class C^∞. In addition, for $d = 2, 3$ one may assume that the diameters of the tiles are uniformly bounded, but the differentiability assumption may not be improved much: for any packing of $\mathbb{E}^d$ with bodies having connected boundaries of class C^ω the set not covered has Hausdorff dimension at least $d-1$. There are packings for which equality holds.

M. HENK (joint work with U. Betke and J. M. Wills):

The sausage-conjecture for high dimensions

Let B^d be the d–dimensional unit ball. For $n \in \mathbb{N}$ let $\mathcal{P}_n = \{C_n : C_n = \{x^1, \ldots, x^n\}, |x^i - x^j| \geq 2, i \neq j\}$, i.e. $C_n + B^d$ is a packing of n translates of B^d. Further let $S_n = \{2u, 4u, \ldots, 2nu\}$ for a unit vector u. The sausage-conjecture of László Fejes Tóth claims for $n \in \mathbb{N}$, $\rho = 1$, $d \geq 5$:

$$\min\{V((\operatorname{conv} C_n) + \rho B^d)\} = V((\operatorname{conv} S_n) + \rho B^d),$$

where V denotes the volume.
We show that the conjecture is true for high dimensions, and we even prove that for every $\rho < 2/\sqrt{3}$ there exists a 'sausage'-dimension d_ρ such that for $d \geq d_\rho$ the volume of $(\operatorname{conv} S_n) + \rho B^d$ is minimal. Further we show that a similar result holds for arbitrary centrally symmetric convex bodies.

D. G. LARMAN (joint work with I. Bárány, J. Pach and H. Bunting):

Rich cells in arrangements of hyperplanes

Let $\mathcal{H} = \{H_1, \ldots, H_n\}$ be an arrangement of hyperplanes in $\mathbb{E}^d$. Then $\mathcal{H}$ splits $\mathbb{E}^d$ into regions called cells. We say that a cell is *rich* if each hyperplane of $\mathcal{H}$ meets the boundary of the cell. The arrangement $\mathcal{H}$ is called *convex* if it possesses at least one rich cell. Any arrangement of 4 lines in general position in $\mathbb{E}^2$ is convex. There exist arrangements of 5 lines in general position in $\mathbb{E}^2$ which are not convex. What is the analogous result in $\mathbb{E}^d$? I conjecture that every $2d$ hyperplanes in general position in $\mathbb{E}^d$ form a convex arrangement but there exist some $2d+1$ which do not. Is there a Helly theorem for convex arrangements, i.e., does there exist $k(d)$ such that if any

$k(d)$ subset of $\mathcal{H}$ is convex then so is $\mathcal{H}$? $k(2) = 5$ but no finite $k(d)$ is known for $d \geq 3$. I conjecture $k(d) = 2d+1$. It is shown that the maximum possible number of rich cells in $\mathcal{H}$ is $\sim \frac{n^{d-2}}{d-2!}$ (n large).

M. LASSAK:

Approximation of convex bodies by rectangles

Theorem. Let C be a convex body in the plane. We can inscribe a rectangle R in C such that a homothetic copy S of R is circumscribed about C. The positive homothety ratio is at most 2 and $\frac{1}{2}|S| \leq |C| \leq 2|R|$.
The symbol $|C|$ denotes the area of C. We use the following
Lemma. Let P be a parallelogram inscribed in a plane convex body C. Denote by s_i the ratio of the width of C to the width of P for the direction perpendicular to the i-th pair of parallel sides of P, where $i = 1, 2$. Then $s_1^{-1} + s_2^{-1} \geq 1$. We have $\frac{1}{2}(s_1 + s_2)|P| \leq |C|$. On the other hand, $|C| \leq (2s_1 + 2s_2 - s_1 s_2 - 2)|P|$ for $s_1 \leq 2$ and $s_2 \leq 2$, and $|C| \leq [s_a s_b - \frac{1}{2}s_a^2(s_b - 1)]|P|$ for $s_a \geq 2$ and $s_b \leq 2$ (where $\{a, b\} = \{1, 2\}$).
Conjecture. If P is a parallelotope inscribed in a convex body C in $\mathbb{E}^d$, then $\sum_{i=1}^{d} s_i^{-1} \geq 1$, where s_i denotes the ratio of the width of C to the width of P for the direction perpendicular to the i-th pair of parallel facets of P, where $i = 1, \ldots, d$.

C. LEE:

Squeezed spheres are shellable

Kalai (1988) showed for fixed d and n that there are "many more" triangulated $(d-1)$-spheres with n vertices than simplicial d-polytopes with n vertices. This was done by constructing a large collection of piecewise linear spheres, which he called "squeezed spheres". Though not all piecewise linear spheres are shellable, we show that all squeezed spheres are.

K. LEICHTWEISS:

On inner parallel bodies and evolutions in the equiaffine geometry

In 1941 T. Kaluza and G. Bol investigated inner parallel bodies of convex bodies in the plane in order to get a simple proof of the isoperimetric inequality. In analogy to this the definition of equiaffine inner parallel bodies as floating bodies of a given body is given and several properties of these bodies are pointed out in a theoretic manner and illustrated with the help of computer drawings. Iterated infinitesimal transitions to the equiaffine inner parallel body motivate the notion of an affine

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evolution given by the parabolic PDE $\frac{\partial x}{\partial t} = y$ (x resp. y = position resp. affine normal vector). The suitably rescaled solutions of this PDE converge to an ellipsoid, which yields a new proof of the affine isoperimetric inequality.

J. LINDENSTRAUSS:

Approximating the Euclidean ball by zonotopes with summands of a fixed length

The talk discussed the problem of approximating the Euclidean ball B_n up to ϵ (say in the Hausdorff metric) by a zonotope $Z = \sum_{j=1}^{N} I_j$ where $\{I_j\}_{j=1}^{N}$ are segments and $N = N(\epsilon, n)$ is as small as possible. In a joint paper with Bourgain and Milman it was shown in 1988 that

$$(1) \qquad N(n,\epsilon) \geq c_1(n)\epsilon^{-2(n-1)/(n+2)}.$$

The proof of (1) uses spherical harmonic functions. By using a combination of deterministic and random methods it was proved in a subsequent joint paper with Bourgain that (1) is best possible (up to a logarithmic factor). Namely

$$(2) \qquad N(n,\epsilon) \leq c_2(n)(\epsilon^{-2}|\log\epsilon|)^{(n-1)/(n+2)}.$$

One may further ask what happens if we require that all the segments I_j in the representation of Z have equal length. As noted by Betke and McMullen who first posed this problem it is equivalent to the following question. Find the minimal $N = N(\epsilon, n)$ so that there are $\{u_j\}_{j=1}^{N}$ in S_{n-1} for which

$$\left| s(K) - \frac{c(n)}{N}\sum_{j=1}^{N} V_{n-1}(PK, u_j)\right| \leq \epsilon s(K)$$

holds for every convex body K in $\mathbb{R}^n$ (here $s(K)$ denotes the surface area of K, $V_{n-1}(PK, u)$ the volume of the orthogonal projection of K in the direction u, and $c(n)$ a suitable constant).
It turns out that (2) is also valid in this case. For $n \leq 6$ this was proved by the late G. Wagner while for general n this is proved in a recent joint paper with Bourgain (both papers appeared in Discrete and Combinatorial Geometry vol. 9 no. 2, 1993). The proof requires, besides the tools used earlier, a quadrature formula with equal weights for general continuous functions defined on an arc in the plane.

E. LUTWAK:

The Brunn-Minkowski-Firey theory

In the early '60's, Wm. J. Firey showed how for each real $p \geq 1$ one could define an analog of a Minkowski combination of convex bodies. It is shown that, when Minkowski combinations (in the classical Brunn-Minkowski theory) are replaced by these Firey combinations, an extension of the classical theory is possible.

H. MARTINI:

Cross-section bodies and related topics

Let $\underline{V}_{d-1}(K,u)$ denote the maximal $(d-1)$-volume of a hyperplane section of a convex body $K \subset \mathbb{R}^d$ in direction u. The cross-section body CK of K is the star set having $\underline{V}_{d-1}(K,u)$ as its radius function with respect to the origin. The inclusions $IK \subset CK \subset \Pi K$ (where IK is the intersection body and ΠK the projection body of K) are known. We shall give estimates for the ratios of contrary inclusions, namely for c_1 and c_2 in $CK \subset c_1 \mathrm{I}(K-x)$ and $\Pi K \subset c_2 CK$. (Here $\mathrm{I}(K-x)$ denotes the intersection body of K generated by hyperplanes through some $x \in \mathrm{int}\, K$.) Further results in that direction (e.g. with regard to k-dimensional sections of full-dimensional convex bodies in $\mathbb{R}^d$) shall be presented, too. Also an example is given which shows the difficulty for determining the diameter of CK if K is a convex d-polytope (in view of computational approaches). Finally the question is posed whether the cross-section body CK is convex for an arbitrary convex body $K \subset \mathbb{R}^d$.

P. McMULLEN:

Applications of the polytope algebra

In the polytope algebra, the subalgebra $\Pi(P)$ generated by the classes of summands of a simple d-polytope P reflects many of the combinatorial properties of P. The existence of a Lefschetz decomposition of $\Pi(P)$ implies the necessity part of the g-theorem on f-vectors of simple polytopes. In turn, that existence follows from quadratic mixed-volume inequalities, generalizing those of Minkowski and Aleksandrov-Fenchel. These are proved quasi-combinatorially, without using the Brunn-Minkowski theorem.

M. MEYER:

Convex bodies and concave functions

Let $\tilde{A}$ be the Steiner symmetral of a convex body A in $\mathbb{R}^d$ with respect to some hyperplane H; given a convex body $C \subset \tilde{A}$, symmetric with respect to H, does there exist a convex body $B \subset A$ in $\mathbb{R}^d$ such that its Steiner symmetral $\tilde{B}$ with respect to H satisfies $\tilde{B} = C$? The answer is yes if $d = 2$ and generally no, if

$d \geq 3$. We generalize here this question, and find properties that a class $\mathfrak{C}$ of closed bounded convex subsets of a Banach space E and a mapping $p : \mathfrak{C} \to \mathbb{R}_+$ should satisfy in order to obtain the following result:

Theorem. Let $\mathfrak{C}$ and $p : \mathfrak{C} \to \mathbb{R}_+$ satisfy these properties, and let K be a closed convex subset of $[0,1] \times E$, such that, for every $t \in [0,1]$ the set $K(t) = \{z \in E;\ (t,z) \in K\}$ is an element of $\mathfrak{C}$. Suppose that a concave continuous function $f : [0,1] \to \mathbb{R}$ is given such that

$$0 \leq f(t) \leq p(K(t)), \text{ for every } t \in [0,1].$$

Then, there exists a closed convex subset L of $[0,1] \times E$ such that $L \subset K$,

$$L(t) = \{z \in E;\ (t,z) \in L\} \in \mathfrak{C} \text{ and } f(t) = p(L(t)) \text{ for every } t \in [0,1].$$

Some examples and applications are given. For instance if $K \subset \mathbb{R}^d$, $E = \mathbb{R}^{d-1}$ and all the $K(t)$ are homothetical to some fixed convex body G of $\mathbb{R}^{d-1}$, then for any concave continuous function f such that

$$0 \leq f(t) \leq \left(\mathrm{vol}\,(K(t))\right)^{1/d-1},$$

there exists a convex body $L \subset K$ such that the $L(t)$ are also homothetical to G and satisfy:

$$f(t) = \left(\mathrm{vol}\,(L(t))\right)^{1/d-1}.$$

V. MILMAN:

Inverse Brunn-Minkowski theorem for volumes and mixed volumes

Below K and T are convex compact bodies in $\mathbb{R}^n$, symmetric with respect to the origin, D is the unit euclidean ball and $V_i(K) = V(\underbrace{K,\ldots,K}_{i},D,\ldots,D)$ is the i-th mixed volume of K. We discuss "isomorphic" geometric inequalities as in the following problem:

Question: Is it true that there exists an absolute constant C such that for any K and T there are linear transformations $u_T, u_K \in SL_n$ and for all $1 < i \leq n$

$$(V_i(u_K K + u_T T))^{1/i} \leq C\left(V_i(u_K K)^{1/i} + V_i(u_T T)^{1/i}\right)?$$

This was proved a few years ago for the case $i = n$ (so called "Inverse Brunn-Minkowski inequality"). We discuss an idea of a geometric proof of this fact and some preliminary results for arbitrary i: under some conditions on K and T the above inequality is true.

Another (similar) question, which also was answered for the case $i = n$, is the following: does an absolute constant C exist such that for every $K \subset \mathbb{R}^n$ there exists an ellipsoid $\mathcal{E}_K$ such that for any other body T and any $1 \leq i \leq n$,

$$\frac{1}{C} V_i(r_i \mathcal{E}_K + T)^{1/i} \leq V_i(K + T)^{1/i} \leq C V_i(r_i \mathcal{E}_K + T)^{1/i}$$

where $r_i > 0$ such that $V_i(K) = V_i(r_i \mathcal{E}_K)$?

L. MONTEJANO:

Applications of topology to geometric convexity

We shall review how topology - cohomology, duality, fiber bundles, Stiefel–Whitney classes - has been used to prove some results in geometric convexity. Essentially, all these applications of topology are related with the problem of recognizing properties of a convex body by means of properties of some of its sections. In this sense, we shall review results of Stein, Kosinski, Mani, Burton, Schneider, Gromov, Zelinsky, Scepin, Montejano and some others.

J. PACH:

Partitions of families of convex sets

Let $\mathcal{F}$ be a family of plane convex sets containing no $k+1$ pairwise disjoint members. Then $\mathcal{F}$ can be partitioned into k^4 subfamilies, each consisting of pairwise intersecting members. This can be used to settle an old conjecture of Avital-Hanani, Kupitz, Erdős.

Theorem. (Pach-Töröcsik) Let G be a graph drawn in the plane with (possibly crossing) line segments, and assume that G has no $k+1$ pairwise disjoint edges. Then $|E(G)| \leq k^4 |V(G)|$. (Here $V(G)$ and $E(G)$ denote the vertex set and the edge set of G.)

Let $\mathcal{F}$ be an n-member family of plane convex sets, no k of which have a point in common, and no ℓ can be chosen with empty 3-wise intersections. Katchalski and I proved that then $n \leq 3\binom{k}{3}\ell$. This implies among others

Theorem. (Katchalski-Pach) Let $\mathcal{F}$ and $\mathcal{G}$ be two n-member families of plane convex sets such that every member of $\mathcal{F}$ touches every member of $\mathcal{G}$. Then either $\mathcal{F}$ or $\mathcal{G}$ contains at least ϵn members that have a point in common (for some absolute constant $\epsilon > 0$).

A. PAJOR (joint work with E. Gluskin and M. Meyer):

Zeros of analytic functions and norms of inverse matrices

Let $K \subset \mathbb{R}^n$ be a symmetric convex body and T a linear mapping such that $T(K) \subset K$ and $|\det T| = \theta > 0$, we look for an estimate of λ such that $K \subset \lambda T(K)$. Let k_n be the smallest constant such that for any n-dimensional normed space X and any invertible linear operator $T \in \mathcal{L}(x)$, we have $|\det T| \, \|T^{-1}\| \leq k_n \|T\|^{n-1}$. Let A_+ be the Banach space of all analytic functions $f(z) = \sum_{k \geq 0} a_k z^k$ on the unit disk D with absolutely convergent Taylor series, and let $\|f\|_{A_+} = \sum_{k \geq 0} |a_k|$; define φ_n on $\bar{D}^n$ by $\varphi_n(\lambda_1, \ldots, \lambda_n) = \inf\{\|f\|_{A_+} - |f(0)|; f(z) = g(z) \prod_{i=1}^n (\lambda_i - z), g \in A_+, g(0) = 1\}$. We show that $k_n = \sup\{\varphi(\lambda_1, \ldots, \lambda_n); \lambda_1, \ldots, \lambda_n \in \bar{D}\}$. J. J. Schäffer (1970) proved that $k_n \leq \sqrt{en}$. We show that up to a logarithmic factor, k_n is of the order of $\sqrt{n}$ as $n \to +\infty$. This logarithmic factor was removed recently by Hervé Queffelec.

C. PERI:

On the minimal convex shell of a convex body

Let C be a fixed closed smooth and strictly convex surface in $\mathbb{E}^d$ with the origin in its interior. By a *shell* with centre c we mean the set of all points between surfaces of the form $\rho C + c$ and $\sigma C + c$ where $\rho, \sigma \geq 0$. A *minimal-convex shell* of a convex body K is a shell containing the boundary of K and for which $|\rho - \sigma|$ is minimal.
We generalize results of T. Bonnesen, N. Kritikos and I. Bárány on "spherical" shells to the more general case considered here. Then we extend a result of A. Zucco on spherical shells by showing that a typical convex body touches the boundary of its minimal convex shell in precisely $d+2$ points.
Finally, we improve an extension of Bonnesen's inequality due to Blaschke.

M. A. PERLES:

A Helly type theorem for almost intersecting families

Let $\mathcal{A} = \{A_i : i \in N\}$ denote a finite family of convex sets in $\mathbb{R}^d$, $|N| = n$. We say that $\mathcal{A}$ is k-almost intersecting ($= k$-a.i.) if there is a point z common to at least $n - k$ members of $\mathcal{A}$.
Denote by $h_k(d)$ the corresponding Helly number, i.e., the smallest number h for which the following statement is true:
If $|\mathcal{A}| = n \geq h$, and if every subfamily of size h is k-a.i., then $\mathcal{A}$ itself is k-a.i.
$h_0(d) = d + 1$ is just Helly's theorem. Here we show that $h_1(d) = [(d+1)^2/4] = \max\{k\ell : k, \ell \text{ integers}, k + \ell = d + 3\}$.
For larger k we have so far only the weak upper bound $h_k(d) \leq \frac{1}{4}(d+1)^{k+1}$.

S. REISNER (joint work with M. Meyer and M. Schmuckenschläger)

Volume of intersection of convex bodies

It is proved that for a symmetric convex body K in $\mathbb{R}^n$, if for some $\tau > 0$, $|K \cap (x + \tau K)|$ depends on $\|x\|_K$ only, then K is an ellipsoid. As a part of the proof, smoothness properties of convolution bodies are studied. Some refinements of the above statement are proved as well.

J. R. SANGWINE-YAGER:

A generalization of outer parallel sets

Let K be a d-dimensional convex body, and $f : S^{d-1} \to \mathbb{R}$ be a non-negative function. If we extend each outer normal to the boundary of K in direction $u \in S^{d-1}$ distance $tf(u), t > 0$, then we have a new family K_{tf} of generalized outer parallel sets of K. The volume of this family is represented by a Steiner-type polynomial whose coefficients are integrals of powers of f with respect to the area measures. We show that for all K and L, $K_{\rho(L,u)} \subset K + L \subset K_{h(L,u)}$. It follows that

$$\int \rho(L,u)^{d-1} dS_1(K,u) \leq dV(K, L, \ldots, L)$$

and

$$dV(K, \ldots, K, L, L) \leq \int h(L,u)^2 dS_{d-2}(K,u).$$

These outer parallel sets are also used to give a geometric interpretation of dual quermassintegrals and the Wulff functional.

R. SCHNEIDER:

On the intermediate area measures of convex bodies

Let $S_i(K,\cdot)$ be the i-th area measure of the convex body K in $\mathbb{E}^n$, thus

$$S_{n-1}(K + \epsilon B^n, \cdot) = \sum_{i=0}^{n-1} \epsilon^{n-1-i} \binom{n-1}{i} S_i(K,\cdot)$$

for $\epsilon > 0$, where B^n is the unit ball and S_{n-1} is the area measure of K. For $1 < i < n-1$, the set of all i-th area measures of convex bodies is not yet well understood. Results like the following ones (where 'most' refers to Baire category) add to the impression that intermediate area measures are of a very special nature.
Theorem 1. Let $1 < i < n-1$. For most pairs K, L of convex bodies in $\mathbb{E}^n$, the sum $S_i(K,\cdot) + S_i(L,\cdot)$ is not an i-th area measure.
Theorem 2. Let $1 \leq i < n-1$. For most convex bodies K in $\mathbb{E}^n$, the inequality $S_i(L,\cdot) \leq S_i(K,\cdot)$ for a convex body L of dimension at least $i+1$ implies that L is homothetic to K.

C. SCHÜTT (joint work with E. Werner):

Homothetic floating bodies

Let K be a convex body in $\mathbb{R}^d$. The convex floating body K_δ of K is the intersection of all halfspaces where defining hyperplanes cut off a set of volume δ from K. We discuss the proof of the following result.
Theorem. Let K be a convex body in $\mathbb{R}^d$. Suppose there is a sequence $\delta_k, k \in \mathbb{N}$, of positive numbers that converge to 0 so that $K_{\delta_k} (k \in \mathbb{N})$ are homothetic to K with respect to the same center of homothety. Then K is an ellipsoid.

G. C. SHEPHARD:

Napoleon Bonaparte

Many people claim that Napoleon's Theorem (about equilateral triangles drawn on the sides of an arbitrary triangle) could not possibly have been discovered or proved by Napoleon. But this is based on a misapprehension. Napoleon knew a considerable amount of geometry and the Italian mathematician Mascheroni (1750 - 1800) dedicated his geometry book to him.
This talk was concerned with variants and extensions of Napoleon's Theorem. The two most important are:
Gerber's Theorem (1990). If regular (n/d)-gons are drawn on the sides of an affinely regular (n/d)-gon, then their centres are the vertices of a regular (n/d)-gon.
Theorem (1993). If $c_1, c_2, \ldots, c_n$ are the lengths of parallel diagonals, taken in order, of a regular (n/d)-gon and $P = \{V_1, \ldots, V_n\}$ is any n-gon, then the n points

$$W_i = c_1 V_i + c_2 V_{i+1} + \ldots + c_n V_{i+n} \qquad (i = 1, \ldots, n),$$

where the subscripts are taken mod n, are the vertices of an affinely regular (n/d)-gon.
This remains true even if the initial polygon P is skew in any number of dimensions.

V. SOLTAN:

On Grünbaum's problem about inner illumination of convex bodies

A set $F \subset \operatorname{bd} K$ is called inner illuminating for a convex body $K \subset \mathbb{E}^d$ if for every point $x \in \operatorname{bd} K$ there is a point $y \in F$ such that $]x, y[\subset \operatorname{int} K$. An inner illuminating set F for K is called primitive if no proper subset of F illuminates K.
B. Grünbaum (1964) posed the following
Problem. Prove that any inner primitive illuminating set of a convex body $K \subset \mathbb{E}^d$ has at most 2^d points.

One can prove the following result.

Theorem 1. Any inner primitive illuminating set of a convex body in $\mathbb{E}^3$ has at most 8 points, and only a convex polytope combinatorially equivalent to the 3-cube has an inner illuminating primitive set of 8 points (placed at its vertices).

The proof of Theorem 1 is based on the following

Theorem 2. If for a convex body $K \subset \mathbb{E}^3$ there is an inner primitive illuminating set of at least 8 points, then K is a convex polytope combinatorially equivalent to the 3-cube.

A. VOLČIČ (joint work with R. J. Gardner):

Tomography of convex and star bodies

The i-chord function of a star body $L \subset \mathbb{E}^n$ is defined as $r_{i,L}(u) = r_L^i(u) + r_L^i(-u)$ when $\underline{o} \in \operatorname{int} L$ and $r_{i,L}(u) = ||r_L(u)|^i - |r_L(-u)|^i|$ when $\underline{o} \notin L$. Here $r_L(u) = \max\{c; c \in \mathbb{R}, c \cdot u \in L\}$ is the radial function of L. For $i \in \mathbb{R}\backslash\{o\}$ and $1 \leq k \leq n-1$, define for every $S \in \mathcal{G}(n,k)$

$$\tilde{V}_{i,k}(L \cap S) = \frac{1}{k} \int\limits_{L\cap S} r_L^i(u) d\lambda_{k-1}(u).$$

Theorem 1. If L_1 and L_2 are two star bodies, $\underline{o} \notin \partial L_m$, $m = 1, 2$, $i \in \mathbb{R}\backslash\{o\}$, then $\tilde{V}_{i,k}(L_1 \cap S) = \tilde{V}_{i,k}(L_2 \cap S)\ \forall\ S \in \mathcal{G}(n,k)$ iff $r_{i,L_1} = r_{i,L_2}$.

Theorem 2. If L_1 and L_2 are two star bodies, $\underline{o} \notin \partial L_m$, $m = 1, 2$, $i \neq j$ real, then $r_{i,L_1} = r_{i,L_2}$ and $r_{j,L_1} = r_{j,L_2}$ iff $L_1 \cap t = \pm L_2 \cap t$ for every line t through $\underline{o}$.

A corollary of this result generalizes a theorem due to Süss characterizing the ball.

W. WEIL (joint work with P. Goodey and E. Lutwak):

Functional analytic characterization of classes of convex bodies

Let C be a closed convex cone with a compact base B in a locally convex Hausdorff space X, then the following characterizations of C and of the vector space $C - C \subset X$ in terms of the dual X' hold.

Theorem 1. Let $\mathcal{A} \subset X'$ be dense and $x \in X$. Then $x \in C \Leftrightarrow x'(x) \geq 0\ \forall x' \in \mathcal{A}$ with $x'(y) \geq 0$ for $y \in \operatorname{ext} B$.

Theorem 2. Let $\mathcal{A} \subset X'$ be dense and $x \in X$. Then $x \in C - C \Leftrightarrow \exists c(x) > 0$ such that $x'(x) \leq c(x) \sup_{y \in \operatorname{ext} B} |x'(y)|\ \forall x' \in \mathcal{A}$.

Theorem 1 is applied to zonoids and intersection bodies and gives characterizations of these in terms of mixed volumes resp. dual mixed volumes, if $\mathcal{A}$ is taken to be a suitable set of differences of surface area measures (resp. powers of radial functions). By Theorem 2, corresponding characterizations of generalized zonoids (resp. generalized intersection bodies) are obtained.

B. WEISSBACH:

Zur Approximation konvexer Körper in der Radon-Nikodym Metrik

Betrachtet wird die Approximation eines konvexen Körpers $K_1 \subset \mathbb{E}^n$ durch Angehörige der Familie $\{\lambda K_2 + x : \lambda > 0,\ x \in \mathbb{E}^n\}$ bezüglich der durch das Volumen der symmetrischen Differenz gegebenen Metrik ρ. Angegeben wird eine notwendige Bedingung für beste Annäherung. Für das durch $|\sum_{i=1}^{n} \xi_i| \leq 1$ gegebene reguläre Kreuzpolytop K_1 und den durch $|\xi_i| \leq 1$, $i = 1, \ldots, n$, beschriebenen Würfel K_2 gilt

$$\rho(K_1, \lambda K_2 + x) \geq 2^n \left(\frac{1}{n!} - \frac{2}{\sqrt{\pi}} \left(\frac{2}{n+1} \right)^{n+1} \int_0^\infty \left(\frac{\sin x}{x} \right)^{n+1} dx \right)$$

mit Gleichheit für $x = 0$ und $\lambda = \frac{2}{n+1}$.

J. M. WILLS (joint work with U. Betke and M. Henk):

Mixed volumes – a new approach to packing

Let $\mathcal{K}_0^d$, $d \geq 2$, denote the set of convex bodies $K \subset \mathbb{E}^d$ with $K = -K$ and volume $V(K) > 0$. For $K \in \mathcal{K}_0^d$ and $n \in \mathbb{N}$ let $K_i = K + c_i$, $i = 1, \ldots, n$, and $C_n = \operatorname{conv}(c_1, \ldots, c_n)$. If $\operatorname{int}(K_i \cap K_j) = \emptyset$ for all $i \neq j$, C_n is called admissible and $\{K_1, \ldots, K_n\}$ a finite packing by translates of K. For $\rho \geq 0$ let

$$\Delta(K, n, \rho) = \min\{V(C_n + \rho K)/n \mid C_n \text{admissible}\}.$$

So ρ controls the influence of the boundary region via the mixed volumes of K and the polytope C_n. For $K \subset \mathcal{K}_0^d$ and $\rho \geq 2$ holds $\Delta(K, n, \rho) \geq \Delta(K)$, where $\Delta(K)$ is the volume of an average Dirichlet-Voronoi cell of the densest infinite packing. So we get a new approach to classical packings. Further for $K \in \mathcal{K}_0^d$, $n \in \mathbb{N}$ and $\rho \leq (24d^{3/2})^{-1}$, $\Delta(K, n, \rho)$ is attained if C_n is a segment, i.e., we have a sausage theorem for arbitrary K. This bound can be improved for special K, in particular for the unit ball B^d (cf. M. Henk's abstract).

J. ZAKS:

On curves contained in rectangles

Let Γ be any closed curve in the plane, having length 2π, and let $R(\theta)$ be the minimal rectangle in the plane, containing Γ and having a side in the direction θ.
Leo Moser asked (Problem M50, see Proc. East Lansing Conf. 1974) if the area of $R(\theta)$, for some θ, is at most 4. An affirmative answer was given by E. Lutwak (Amer. Math. Monthly, 1979).

Using reflections, we show the following.

1) The area of $R(\theta)$, for all θ, is at most $\pi^2/2$, and this bound is best.
2) If $R(\theta)$ is a square, then $R(\theta)$ or $R(\theta+\pi/4)$ have area which is at most $(\frac{2+\sqrt{2}}{8})\pi^2 = 4.21\ldots$, and this bound is best.

T. ZAMFIRESCU:

On the dimension of convex curves

Most convex curves (in the sense of Baire categories) are smooth and strictly convex like the circle, but not C^2 and with vanishing curvature a.e. like the quare. So, which is more typical, the circle or the square? An answer via dimension seems absurd, both having dimension 1. Thanks to C. A. Rogers, who introduced in 1988 the dimension print, it is not. The dimension print of the circle is triangular, while that of the square is a line segment.
Theorem. Most convex curves have the dimension print of the square.
More generally, in $\mathbb{R}^n$:
Theorem. Most convex surfaces have the dimension print of a hyperplane.
These results have been obtained jointly with Gh. Crăciun.
Another result in $\mathbb{R}^3$:
Theorem. Most convex bodies cannot escape from some circle. The circle can be chosen to have larger diameter than the body.

G. M. ZIEGLER:

Two problems on polytopes

We present solutions to two problems about convex polytopes.
1. *Polytopes are not extendably shellable.*
Bruggesser & Mani (1971) showed that polytopes can be shelled. However, it was an open problem whether polytopes are *extendably shellable*: whether one cannot get stuck while shelling a polytope? It is easy to see that 3-polytopes are extendably shellable. Here we present a simple construction that shows that 4-polytopes are not extendably shellable. For this we embed a non-shellable ball, as given by Frankl (1931) and Bing (1964), into the boundary of a 4-polytope.
2. *Two-faces cannot be arbitrarily prescribed.*
An extension of Steinitz's theorem, due to Barnette & Grünbaum (1970), says that the shape of one facet of a 3-polytope can be arbitrarily prescribed. This is known to be false for 4-polytopes, but one can try to prescribe a 2-face. It is open whether this is possible, but we construct a 5-polytope with 12 vertices and 10 facets for which

the shape of a hexagon 2-face cannot be prescribed. This is done by constructing the affine Gale diagram of the polar polytope.

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