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Algebraische K–Theorie

June 27–July 3, 1993

The conference was chaired by Prof. Dr. R. Keith Dennis (Ithaca) and Prof. Dr. Ulf Rehmann (Bielefeld).

The program consisted of thirty–two talks.

The following were among the subjects covered:

> Applications of K–theory to number theory, algebraic cycles, Chow groups and intersection theory, algebraic K–theory of special varieties, topological Hochschild homology, cyclic homology and Loday homology, motives.

Also a project to distribute mathematical books electronically was described.

The history of the Novikov conjecture

Andrew Ranicki (Edinburgh)

The talk was only a partial history of the Novikov conjecture, with the aim of providing some background for the talk by Erik Pedersen.

The conjecture originates from an early triumph of algebraic K-theory in topology: the application of $Wh(\mathbf{Z}^j) = 0$ in Novikov's 1967 proof of the topological invariance of the rational Pontrjagin classes.

The Novikov conjecture is that the signatures of certain types of submanifolds are homotopy invariant. The conjecture relates the geometry of manifolds to their underlying homotopy type. Over the years, both the statements and methods of proof of the conjecture have been refined in various directions. The progress of high-dimensional topology in the last 25 years is largely measured in the extent to which the conjecture has been verified.

The history starts with the theorem of Hirzebruch (1952) expressing the signature of an oriented differentiable $4k$-dimensional manifold M

$$\sigma(M) \;=\; \text{signature(intersection form on } H^{2k}(M;\mathbf{Q})) \in \mathbf{Z}$$

in terms of the $\mathcal{L}$-genus

$$\mathcal{L}(M) \;=\; \mathcal{L}(p_1, p_2, \ldots, p_k) \in H^{4*}(M;\mathbf{Q})$$

of the Pontrjagin classes $p_i = p_i(M) \in H^{4i}(M)$, namely

$$\sigma(M) \;=\; \langle \mathcal{L}(M), [M] \rangle \in \mathbf{Z}$$

with $[M] \in H_{4k}(M)$ the fundamental class.

Thom (1956) used the Hirzebruch signature theorem to define the $\mathcal{L}$-genus $\mathcal{L}(M) \in H^{4*}(M;\mathbf{Q})$ of an oriented combinatorial m-dimensional manifold M in terms of the signatures of $4k$-dimensional submanifolds $N^{4k} \subset M^m$ with trivial normal bundle $N \times \mathbf{R}^j \subset M$ $(j = m - 4k)$. If $x \in H^j(M)$ is the cohomology class Poincaré dual to $[N] \in H_{4k}(M)$ then

$$\langle \mathcal{L}(M) \cup x, [M] \rangle \;=\; \langle \mathcal{L}(N), [N] \rangle \;=\; \sigma(N) \in \mathbf{Z} \subset \mathbf{Q}\ .$$

The rational Pontrjagin classes $p_*(M) \in H^{4*}(M;\mathbf{Q})$ are determined algebraically by the $\mathcal{L}$-genus $\mathcal{L}(M)$.

A combinatorial $(= PL)$ equivalence $h : M' \to M$ of combinatorial manifolds preserves the submanifold structure, so that $\mathcal{L}(M)$ and $p_*(M)$ are combinatorial invariants. A homeomorphism of manifolds cannot in general be approximated by a combinatorial equivalence (by the failure of the Hauptvermutung), so it is not at all obvious that $\mathcal{L}(M)$ and $p_*(M)$ are topological invariants of the manifold M.

The $\mathcal{L}$-genus and the rational Pontrjagin classes are not homotopy invariant. A map $h : M' \to M$ of combinatorial m-dimensional manifolds can be made transverse regular at an arbitrary submanifold $N^n \subset M^m$ with the restriction $f = h| : N' = h^{-1}(N) \to N$ a degree 1 normal map of n-dimensional manifolds. In general, if h is a homotopy equivalence the restriction f need not be a homotopy equivalence. In the case $n = 4k$, $\pi_1(N) = \{1\}$ the difference of the signatures $\sigma(N') - \sigma(N) \in \mathbf{Z}$ is the surgery obstruction to having f a homotopy equivalence. Novikov (1967) applied algebraic K-theory to show that if h is a homeomorphism then $\sigma(N) = \sigma(N') \in \mathbf{Z}$, thus proving the topological invariance of $\mathcal{L}(M)$ and $p_*(M)$. The Bass-Heller-Swan computation $Wh(\mathbf{Z}^j) = 0$ was used to show that if $N^{4k} \subset M^m$ has a trivial normal bundle $U = N \times \mathbf{R}^j$ $(j = m - 4k)$ then the open submanifold $U' = h^{-1}(U) \subset M'$ is of the form $U' = N' \times \mathbf{R}^j$ with N' h-cobordant to N, and $\sigma(N) = \sigma(N')$. In connection with his proof Novikov formulated the following generalization of the $\mathcal{L}$-genus.

Definition *Let M be an oriented m-dimensional manifold with fundamental group $\pi_1(M) = \pi$, and let $f : M \to B\pi$ be the classifying map of the universal cover $\widetilde{M}$. The* **higher signatures** *of M are the rational numbers*

$$\sigma_x(M) \;=\; \langle \mathcal{L}(M) \cup f^*(x), [M] \rangle \in \mathbf{Q} \;,$$

one for each cohomology class $x \in H^*(B\pi; \mathbf{Q})$.

Novikov conjecture *The higher signature are homotopy invariant: if $h : M' \to M$ is a homotopy equivalence of manifolds then $\sigma_x(M) = \sigma_x(M') \in \mathbf{Q}$ for any $x \in H^*(B\pi; \mathbf{Q})$, with $\pi = \pi_1(M) = \pi_1(M')$.*

In fact, it suffices to only consider cohomology classes $x \in H^j(B\pi; \mathbf{Q})$ ($j = m-4k$) represented by maps $x : B\pi \to S^j$, in which case the higher signature is the signature of the submanifold $N^{4k} = (xf)^{-1}(pt.) \subset M^m$ with trivial normal bundle $N \times \mathbf{R}^j \subset M$ and $[N] = [M] \cap f^*x \in H_{4k}(M)$

$$\begin{aligned} \sigma_x(M) \;&=\; \langle \mathcal{L}(M) \cup f^*(x), [M] \rangle \\ &=\; \langle \mathcal{L}(N), [N] \rangle \;=\; \sigma(N) \in \mathbf{Z} \subset \mathbf{Q} \,. \end{aligned}$$

The signatures of all submanifolds are homeomorphism invariant. The conjecture predicts which submanifolds have homotopy invariant signatures.

Many special cases of the Novikov conjecture have now been proved, using a wide variety of topological, algebraic and analytic methods. The methods all factor through the algebraic L-theory assembly map

$$A \;:\; H_*(X; \mathbf{L}.(\mathbf{Z})) \;\to\; L_*(\mathbf{Z}[\pi_1(X)])$$

constructed in [1] for any space X, with $L_*(\mathbf{Z}[\pi_1(X)])$ the Wall surgery obstruction groups and $\mathbf{L}.(\mathbf{Z})$ the simply-connected surgery spectrum with $\pi_*(\mathbf{L}.(\mathbf{Z})) = L_*(\mathbf{Z})$.

Theorem [1] *The Novikov conjecture holds for a group π if and only if the algebraic L-theory assembly map $A_\pi : H_*(B\pi; \mathbf{L}.(\mathbf{Z})) \to L_*(\mathbf{Z}[\pi])$ for the classifying space $B\pi$ is a rational injection.*

The algebraic proof of the Novikov conjecture by Carlsson and Pedersen for a reasonably large class of groups π makes use of bounded algebra and topology to prove that A_π is a split injection. The bounded algebra verifies the conjecture by providing an organizational principle for the signatures of submanifolds of manifolds with fundamental group π.

Chapter 24 of [1] contains a more detailed account of the history of the Novikov conjecture. Appendix C of [1] includes an account of the topological invariance of rational Pontrjagin classes using lower K and L-theory, and can be read as an introduction to the forthcoming paper of Carlsson and Pedersen.

[1] A.Ranicki *Algebraic L-theory and topological manifolds* Cambridge Tracts in Mathematics 102, Cambridge University Press (1992)

E.K.Pedersen (Binghamton): On the Novikov conjecture
joint work with Gunnar Carlsson

Let Γ be a group such that $B\Gamma$ is finite and $E\Gamma$ admits a compactification X such that

(i) X is metrizable.

(ii) X is contractible.

(iii) The Γ-action extends to X.

(iv) If $K \subset E\Gamma$ is compact and $y \in Y = X - E\Gamma$, then for every neighborhood U of y, there is a smaller neighborhood V so that if $g \in \Gamma$ and $g \cdot K \cap U \neq$ then $g \cdot K \subset V$.

These conditions are satisfied for many groups e.g word hyperbolic groups. Our main result is that for these groups the assembly maps

$$B\Gamma^{+} \wedge L(\mathbb{Z}) \rightarrow L(\mathbb{Z}\Gamma)$$

$$B\Gamma^{+} \wedge K(\mathbb{Z}) \rightarrow K(\mathbb{Z}\Gamma)$$

are split monomorphisms of spectra, thus confirming the Novikov conjecture and the K-theory analogue of the Novikov conjecture for these groups. Since the Novikov conjecture is a rational statement, we actually get a stronger integral result.

In joint work with Wolrad Vogell we have confirmed the analogous result for A-theory.

Condition (i) is not necessary, it suffices that X is compact Hausdorff. The proof requires using a generalized Čech theory.

Condition (iv) can be replaced by a condition expressing that a compact subset converges to a set which is homotopically trivial in some technical sense.

Condition (ii) can be replaced by the statement that the generalized Čech theory applied to X is trivial

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Normality of Elementary Subgroup Functors and Applications

A. Bak (Bielefeld)
(joint work with N. Vavilov)

We define a notion of group functor G, on categories of graded modules, which unifies all previous concepts of a group functor G possessing a notion of elementary subfunctor E. We show under a general condition, which is easily checked in practice, that the elementary subgroup $E(M)$ of $G(M)$ is normal for all Noetherian and quasi-Noetherian objects M in the source category of G. This result includes all previous ones on Chevalley and classical groups G of rank ≥ 3 over a commutative or module finite ring M and settles positively the still unanswered cases of normality for these group functors, even in some rank 2 cases.

Whereas the concepts of Chevalley and classical group functor are formed in a ridged way in the setting of linear algebra, our concept of a group functor is simply an arbitrary group functor G on a category Λ of graded modules M, e.g. a category of rings or form rings, such that $G(M)$ contains homogeneous components of M in a functorial way. The elementary subgroup $E(M)$ of $G(M)$ is defined as the subgroup of $G(M)$ generated by the embedded homogeneous components of M. This construction obviously defines a subfunctor $E \hookrightarrow G$. In the case of Chevalley or classical-like groups G, it is clear that the elementary matrices define embeddings into G of the ring and form parameter over which G is defined and that the elementary subgroup of G is indeed the subgroup generated by these embedded entities. On the other hand, any Chevalley or classical-like group is itself embedded set theoretically, but still functorially, into a direct sum of copies of the ring over which it is defined. So, we assume in the general setting that our groups $G(M)$ embed set theoretically and functorically into a direct sum $\oplus M$ of copies of M. This extra structure coupled with a simple, but very important observation is enough to proof our main result concerning the normality of $E(M)$ in $G(M)$ in just a few words.

The Adams–Riemann–Roch theorem in the higher K–theory of group scheme actions

B. Köck (Karlsruhe)

Let G/S be a group scheme. For any G–scheme X/S the q–th equivariant K–group $K_q(G,X)$ is defined to be the q–th K–group of the exact category consisting of all locally free G–modules on X.
For any G–projective G–morphism $f : X \to Y$ of complete intersection we can define the generalized Euler characteristic

$$f_* : K(G,X) \to K(G,Y)$$

Conjecturally $K(G,X)$, equipped with the λ–operations constructed by D. Grayson, is a λ–ring.
Let $\hat{K}(G,X)$ be the completion of $K(G,X)$ wrt. the Grothendieck filtration.

Theorem: a) (Adams–Riemann–Roch)
Bott's j–th cannibalistic class $\Theta^j(T_f^\vee)$ is invertible in $\hat{K}_0(G,X)[j^{-1}]$. For any $x \in K(G,X)$ and for any Adams operation ψ^j, we have

$$\hat{f}_*(\Theta^j(T_f^\vee)^{-1} \cdot \psi^j(x)) = \psi^j(f_*(x))$$

in $\hat{K}(G,Y)[j^{-1}]$.
b) (Grothendieck–Riemann–Roch)
If in addition f_* is continuous wrt. the Grothendieck filtrations and if the above conjecture holds, then f_* has a graded degree, and for any $x \in K(G,X)$, we have

$$\hat{Gr} f_*(Td(T_f^\vee) \cdot ch(x)) = ch(f_*(x))$$

in $\hat{Gr}K(G,Y)_{\mathbb{Q}}$.

References:

1) D.R. Grayson: "Exterior Power Operations in Higher K–Theory", K–Theory 3 (1989).
2) B. Köck, „Das Adams–Riemann–Roch–Theorem in der höheren äquivarianten K–Theorie", Jour. reine u. angew. Math. (1991).
3) G. Tamme, "The theorem of Riemann–Roch", in M. Rapoport et. al., "Beilinson's conjectures on special values of L–functions", Academic press (1988).

Improved stability for SK_1 of an affine algebra

W. van der Kallen (Utrecht)
(joint work with R.A. Rao)

Let A be a non–singular affine algebra of Krull dimension $d \geq 2$ over a C_1–field k. If char$(k) \leq d$, assume k is perfect. We show that
1) the map $\mathrm{SL}_r(A)/E_r(A) \to SK_1(A)$ is an isomorphism for $r \geq d+1$. The proof is based on a similar result of Suslin concerning rows.
2) if $d = 3$ then the Vaserstein symbol $\mathrm{Uni}_3(A)/E_3(A) \to W_E(A)$ is bijective. Here the target is a Witt group based on alternating forms under "elementary" equivalence.
3) if $d \geq 3$ then $\mathrm{SL}_{d-1}(A)E_d(A)$ is normal in $\mathrm{SL}_d(A)$. This uses 2) and bijectivity of a universal weak Mennicke symbol.

References:

R.A. Rao, W. van der Kallen, "Improved stability for SK_1 and WMS_d of a non–singular affine algebra", Utrecht preprint 757.
For "failure of 3) over $\mathbb{R}$" see
W. van der Kallen, "A module structure on certain orbit sets of unimodular rows", Jour. of Pure and Appl. Algebra 57 (1989), pp. 281–316.
This paper also introduces the universal weak Mennicke symbol
$\mathrm{Uni}_n(A)/E_n(A) \to WMS_n(A), n \geq 3$. It is universal for the sympletic Mennicke relations of Lemma 5.4 in

L.N. Vaserstein, "The structure of classical arithmetic groups of rank greater than 1", Math. USSR Sbornik 20 (1973) No. 3, pp. 465–492.

K_2 and Gross's p-adic conjecture

Nguyen Quang Do (Besançon)

By a combination of K-theory, Galois cohomology and Spiegelung in Iwasawa theory, we prove the non-vanishing of Gross's p-adic regulator which dictates, in the case of CM-fields, the behaviour at $s = 0$ of p-adic L-functions. Notations: F = a number field, p = an odd prime, F_∞ = the cyclotomic $\mathbb{Z}_p$-extension of F, $\Gamma = \mathrm{Gal}(F_\infty/F)$, O'_∞ = the ring of p-integers of F_∞, $O'^\times_\infty$ = its group of p-units, A'_∞ = the p-group of p-classes of F_∞. We may suppose that F contains a primitive p-th root of unity. The main step is the following

Theorem:

1) We have a "twisted Kummer" exact sequence of Γ-modules:

$$0 \to K_3 F_\infty \otimes \mathbb{Q}_p/\mathbb{Z}_p \to O'^\times_\infty \otimes \mathbb{Q}_p/\mathbb{Z}_p(1) \to K_2 O'_\infty \otimes \mathbb{Z}_p \xrightarrow{K_\infty} A'_\infty \otimes \mathbb{Z}_p(1) \to 0$$

2) The "twisted Kummer" morphism K_∞ induces a pseudo-isomorphism $K_2^w O'_\infty \otimes \mathbb{Z}_p \sim A'_\infty(1) \otimes \mathbb{Z}_p(1)$, where K_2^w denotes the wild kernel.

Via a result on Brauer groups, this theorem allows us to show that $(A'_\infty)_\Gamma$ is finite, which is one of many equivalent formulations of Gross's conjecture in Iwasawa theory.

References:

B.H. Gross, "p-adic L-series at $s = 0$", Jour. Fac. Sci. Univ. Tokyo 1 A 28, 979–994 (1981).

L.J. Federer, B.H. Gross, "Regulators and Iwasawa modules", Invent. Math. 62, 443–457 (1981).

Higher class number formulas

M. Kolster (Hamilton)

In the framework of the Lichtenbaum Conjectures, the following two results can be proved:

Theorem 1: Let F/F^+ be a CM-extension of number fields, χ the non-trivial character of $\mathrm{Gal}(F/F^+)$, $n \in \mathbb{N}$ odd. Then, for any odd prime p

$$L(\chi, 1-n) \underset{p}{\sim} \frac{\# K^{\acute{e}t}_{2n-2}(o_F)^-}{\# K^{\acute{e}t}_{2n-1}(o_F)_{\mathrm{tors}}}.$$

(Here, $\underset{p}{\sim}$ means p-equivalence of rational numbers, and $K^{\acute{e}t}$ is the étale K-theory of Dwyer–Friedlander.)

As an application this yields an easy way of computing $K_4^{ét}$ for rings of integers in imaginary quadratic number fields.

Now let F be totally real abelian, $\Delta = \mathrm{Gal}(F(\zeta_p)/F)$, p odd. Let $n \geq 2$, $\chi_{ét}$ a character of Δ with the same parity as $n-1$.

Theorem 2: $[K_{2n-1}^{ét}(o_{F(\zeta_p)})^{\chi} : (C'_\infty)^{\chi \cdot w^{(1-n)}}(n-1)_{G_\infty}] = \sharp K_{2n-2}^{ét}(o_{F(\zeta_p)})^{\chi}$. (Here, C'_∞ is a group of cyclotomic p–units, w is the Teichmüller character, and $G_\infty := \mathrm{Gal}(F(\zeta_{p^\infty})/F)$.)

A combination of Theorem 2 with either Theorem 1 or the analogous result (Wiles) for n even essentially shows that the Lichtenbaum Conjecture for F (complex abelian) reduces to a precise relation between the regulator maps on cyclotomic elements (Beilinson, Bloch) and the leading terms in the ζ–functions.

Specialization maps on the K–theory of number fields

B. Kahn (Paris)

Let F be a number field, l an odd rational prime. Choosing an embedding of F into $\overline{\mathbb{Q}}$, we get maps on K–theory:

$$K_{2i-1}(F) \to H^0(F, K_{2i-1}(\overline{\mathbb{Q}})),$$

hence on l–primary torsion

(1) $\quad K_{2i-1}(F)\{l\} \to H^0(F, K_{2i-1}(\overline{\mathbb{Q}})\{l\}) = H^0(F, \mathbb{Q}_l/\mathbb{Z}_l(i))$ (Suslin).

By [1], there are canonical maps

(2) $$H^0(F, \mathbb{Q}_l/\mathbb{Z}_l(i)) \to K_{2i-1}(F)\{l\}$$

whose composite with (1) is the identity. (For $i = 1, 2$, these maps are isomorphisms.)

Using appropriate primes of F one can extend (1) to the whole of $K_{2i-1}(F)$. Let S be a set of places of F containing the infinite ones and the ones dividing l, and let $\wp \notin S$. The index

$$m(\wp) := [H^0(\kappa(\wp), \mathbb{Q}_l/\mathbb{Z}_l(i)) : H^0(F, \mathbb{Q}_l/\mathbb{Z}_l(i))]$$

for $i \neq 0$ st. $H^0(F, \mathbb{Q}_l/\mathbb{Z}_l(i)) \neq 0$, is independent of i.

One defines specialization maps

$$\alpha_\wp^i : K_{2i-1}(o_F^S) \to H^0(F, \mathbb{Q}_l/\mathbb{Z}_l(i))$$

whose composite with (2) is multiplication by $m(\wp)$ on $H^0(F, \mathbb{Q}_l/\mathbb{Z}_l(i))$. By Čebotarev there are infinitely many primes $\wp$ st. $m(\wp) = 1$. It is not yet known how $\alpha_\wp^i$ varies with $\wp$ but the speaker has a result which can be interpreted as saying that in the average under Galois action, $\alpha_\wp^i$ only depends on $m(\wp)$.

This result prompts the following question.
There is a spectrum $j(F)$ such that

$$\pi_m(j(F)) = \begin{cases} H^0(F, \mathbb{Q}_l/\mathbb{Z}_l(i)) & , \quad m = 2i-1 \geq 1 \\ 0 & , \quad \text{otherwise} \end{cases}.$$

The $\alpha^i_\wp$ are induced by maps of spectra

$$\alpha_\wp : K(o^s_F) \to j(F)\,.$$

Question: Does the above result already hold on the level of spectra? Namely, is the map of spectra

$$\alpha : K(o^s_F) \to j(F)$$

st. for all $\wp \notin S$, one has $\sum_{\wp'\sim\wp} \alpha_{\wp'} = g \cdot m(\wp)\alpha$?
($\sim$ is conjugation by $\mathrm{Gal}\,(F/\mathbb{Q})$, and $g := \sharp\{\wp' \mid \wp' \sim \wp\}$.)

Reference: [1] B. Kahn, "Bott elements in algebraic K–theory", preprint, Paris 7

Stable norm groups of p–units, and K_2

C. Greither (München)

For a number field F, we consider the stable norm groups in the projective system $\tilde{U}'_n$ (where U'_n is the group of p–units in the n–th layer F_n of the cyclotomic $\mathbb{Z}_p$–extension of F, and $\sim$ indicates tensoring with $\mathbb{Z}_p$ and factoring out torsion). These stable norm groups turn out to be free over the group ring $\mathbb{Z}_pG_n$, with $G_n = \mathrm{Gal}\,(F_n/F)$. (In principle, this follows already from work of Kuz'min (1972).) This leads to a description of $K_2(O_F)$ involving the stable capitulation kernel H; to wit, we reprove part of a result of Keune (obtained by "taking coinvariants"), this time "by taking invariants".

Next we replace p–units by K_3 and show a freeness result for stable norms, and a structural description, in analogy with the case of p–units.

Finally we discuss noncyclotomic $\mathbb{Z}_p$–extensions. In this setting, the stable norm groups of p–units don't have such a direct relation with K_2. Nevertheless, it is perhaps of interest that Galois module theoretic results still can be proved in this generality.

We use results of Levine, Merkurjev–Suslin, Kahn, and Nguyen Quang Do.

J.Rognes (Oslo): $K_4(\mathbb{Z}) = 0$.

The fourth higher algebraic K-group of the rational integers is trivial. We prove this by using a spectrum level rank filtration of the K-theory spectrum $K(\mathbb{Z})$, defined in [1]:

$$* \simeq F_0K(\mathbb{Z}) \longrightarrow F_1K(\mathbb{Z}) \longrightarrow \dots \longrightarrow F_kK(\mathbb{Z}) \longrightarrow \dots \longrightarrow K(\mathbb{Z}).$$

Here the kth stage $F_kK(\mathbb{Z})$ is roughly the subspectrum of $K(\mathbb{Z})$ built from the category of $\mathbb{Z}$-modules of rank k or less. The spectrum homology of the third stage $F_3K(\mathbb{Z})$ is computed in [2] to be

$$H_*^{spec}(F_3K(\mathbb{Z})) \cong (\mathbb{Z}, 0, 0, \mathbb{Z}/2, 0, \mathbb{Z} \oplus (\text{two-torsion}), \dots).$$

This is found using Soulé's calculation of the group homology of $GL_3(\mathbb{Z})$ given in [3]. Next we prove that the inclusion map $F_3K(\mathbb{Z}) \to K(\mathbb{Z})$ is four-connected. This is the more recent addition to these calculations, and is proved using connectivity results for a certain spectrum called the stable building, which can be thought of as a spectrum level version of the Tits buildings. Hence the calculation above determines the spectrum homology of the K-theory spectrum through homological degree four, and the result then follows from the Atiyah–Hirzebruch spectral sequence for stable homotopy theory, which appears below.

6	$\mathbb{Z}/2$	0	0	$\mathbb{Z}/2$	$\mathbb{Z}/2$	?
5	0	0	0	0	0	0
4	0	0	0	0	0	0
3	$\mathbb{Z}/24$	0	0	$\mathbb{Z}/2$	$\mathbb{Z}/2$	?
2	$\mathbb{Z}/2$	0	0	$\mathbb{Z}/2$	$\mathbb{Z}/2$	?
1	$\mathbb{Z}/2$	0	0	$\mathbb{Z}/2$	$\mathbb{Z}/2$	?
0	$\mathbb{Z}$	0	0	$\mathbb{Z}/2$	0	$\mathbb{Z}\oplus ?$
	0	1	2	3	4	5

There is a nontrivial extension in total degree three, and a nonzero differential ending in bidegree $(3, 1)$, from which the result follows. These observations about the spectral sequence follow from a comparison with Bökstedt's spectrum $JK(\mathbb{Z})$, using his two-complete splitting [4] of $\Omega JK(\mathbb{Z})$ off from $\Omega K(\mathbb{Z})$.

References

[1] J. Rognes, *A spectrum level rank filtration in algebraic K-theory*, Topology **31** (1992), 813–845.
[2] ———, *Approximating $K_*(\mathbb{Z})$ through degree five*, K-Theory (to appear).
[3] C. Soulé, *The cohomology of $SL_3(\mathbb{Z})$*, Topology **17** (1978), 1–22.
[4] M. Bökstedt, *The rational homotopy type of $\Omega Wh^{Diff}(*)$*, Algebraic Topology, Aarhus 1982, Proceedings, (I. Madsen and B. Oliver, eds.), Lecture Notes in Math., vol. 1051, Springer Verlag, 1984, pp. 25–37.

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Linear groups: Abelianizations and imperfect groups

A.J. Berrick (Singapore)

This talk presents joint work with D.J.S. Robinson: "Imperfect groups", J. Pure Appl. Alg. (1993), to appear [B–R] and with the late Pere Menal (two articles in preparation) [B–M].

For a group G, define $\mu(G)$ to be the least cardinality of a subset of G whose normal closure is G. In general, $\mu(G)$ is very difficult to calculate (hence the Kervaire Conjecture); however:

Theorem 1 [B–R]: $\mu(G) = \mu(G_{ab})$, the rank of G_{ab}, when G has an imperfect normal subgroup N with quotient G/N of finite composition length.

Here, N is defined to be *imperfect* if it has no nontrivial perfect quotient.

Theorem 2 [B–M]: $\mu(E_2R) = \mu((E_2R)_{ab})$.

This is an interesting numerical invariant of a ring R (= 1 for a field, infinite for $\mathbb{F}_2[t]$). [B–R] calculates the imperfect radical and residual of $\mathrm{GL}_n\mathbb{Z}$. For other rings:

Theorem 3 [B–R]: (a) For a field k, $\mathrm{GL}_n k$ is imperfect iff $(n, |k|) = (2, 2)$ or $\{x^n \mid x \in k\} \neq k$.
(b) For R commutative semilocal with $R \not\twoheadrightarrow \mathbb{F}_2$, we have:

$$\begin{aligned} \mathrm{GL}_n R \text{ imperfect} \quad &\Leftrightarrow \quad \forall \text{ quotient } k \text{ of } R, \{x^n \mid x \in k\} \neq k \\ &\Rightarrow \quad \mu(\mathrm{GL}_n R) = \mu((\mathrm{GL}_n R)_{ab}) \underset{n\geq 3}{=} \mu(K_1 R). \end{aligned}$$

Calculation of $(E_2R)_{ab}$ and $(\mathrm{GL}_2R)_{ab}$ has been made in [B–M] (including relative versions).

Theorem 4: Suppose R is commutative with $srR = 1$.

$$(E_2R)_{ab} \cong (E_2(R/T))_{ab} \cong (R^+ \times \mathrm{GL}_1R)/\{(t,v) \mid t - 3(1-v) \in L\}$$

where T = ideal of all elements of R of torsion coprime to G,
and $L = \mathrm{level}\,([E_2R, E_2R]) = \sum_{r,s\in R}(r^2 - r)(s^3 - s)$. Moreover, $SK_1(R,T) = 0$.

The following has not yet been completely checked by the speaker.

Theorem 5: If $srR = 1$, then

$$(\mathrm{GL}_2R)_{ab} \cong K_1R \oplus (E_2R)/[\mathrm{GL}_2R, \mathrm{GL}_2R]$$

where $(E_2R)/[\mathrm{GL}_2R, \mathrm{GL}_2R] \cong R/\sum_{r\in R} R(r^2 - r)R$.

Bivariant cycle theory
M. Rost (Bonn)

The topic of the talk was the discussion of a theory of correspondences between varieties (including singular ones) over a field. In particular, this theory yields for any variety X a complex $C^*(X)$ which in a way is dual to the complex

$$C_*(X):\ldots \xrightarrow{d} \bigoplus_{x\in X_{(p)}} K_*^M(\kappa(x)) \xrightarrow{d} \bigoplus_{x\in X_{(p-1)}} K_*^M(\kappa(x)) \longrightarrow \ldots$$

There are pairings

$$C^*(X)\otimes C^*(X)\longrightarrow C^*(X)$$

which turn the cohomology $H^*(X)$ into an anticommutative ring. In case of a smooth variety the complexes $C^*(X)$ and $C_*(X)$ are homology equivalent and the product on $H^*(X)$ coincides with the usual intersection product for cycles.

Intersection theory via perfect complexes
R.W. Thomason (Paris)

Motivated by modern techniques in local intersection theory, the known connections between classical intersection theory and algebraic K–theory, and the superiority of Cartier divisors over Weil divisors, we seek a global intersection theory on general schemes, possibly singular or not over a field, where perfect complexes play the role of p–cocycles.

Henceforth, we consider only noetherian schemes with an ample line bundle. These include all quasi–projective algebraic varieties.

We recall a perfect complex on X is a complex of O_X–modules quasi–isomorphic to a bounded complex of algebraic vector bundles ([SGA6]). We seek a filtration of the category of perfect complexes on all X

$$\cdots C^{p+1}(X)\subseteq C^p(X)\subseteq\cdots$$

such that $C^p(X)$ is the full subcategory of all perfect complexes in some sub-triangulated category of the derived category $D(X)_{\mathrm{perf}}$ of perfect complexes. Given such a filtration C^*, we define Chow groups considering $C^p(X)$ as the "p–cocycles":
$CH^p_C(X)$ = free abelian group on objects of $C^p(X)$ modulo relations:
$[E]+[F]\sim[E\oplus F]$,
$[E]\sim[F]$ if E is quasi–isomorphic to F,
$[E]\sim 0$ if $E\in C^{p+1}(X)\subseteq C^p(X)$, and
"rational equivalence": $[E_0]\sim[E_1]$ if $\exists\, H\in C^p(X\times\mathbb{P}^1)$ s.t. $Li_k^*H\cong E_k$ for $i_k: X\to X\times\mathbb{P}^1$ the sections at 0 and 1.

Lemma 1: a) There is a surjection $K_0(C^p(X))\longrightarrow CH^p_C(X)$.
b) There is a well–defined map

$$CH^p_C(X)\to K_0(X)/\,\mathrm{image}\ K_0(C^{p+1}(X)).$$

(Both homomorphisms send the class $[E]$ of E to $[E]$, its class in the other group.)

The first naive guess for a C is to take C^p_{codim} = perfect complexes acyclic off codimension $\geq p$. This gives $CH^{\bullet}_{\text{codim}}(X)$ which are obviously isomorphic to the classical Chow groups for X regular of finite type over a field. However, this $CH^{\bullet}_{\text{codim}}(\ )$ is not functorial on singular X.

Better guess for C: $C^p_{sm}(X) = \{E^{\cdot}$ in the smallest triangulated subcategory of $D(X)_{\text{perf}}$ that contains all "basic p–cocycles", i.e. those of the form

$$R\pi_*(\mathcal{K} \otimes (\mathcal{L} \xrightarrow{s_1} O) \otimes \ldots \otimes (\mathcal{L}_m \xrightarrow{s_m} O_z)$$

where $\pi : Z \to X$ is a smooth projective morphism,

$$\mathcal{K}, \mathcal{L}_1, \ldots, \mathcal{L}_m$$

are line bundles on Z, and $m \geq p$ + fibre dimension of $\pi\}$.

Theorem 1: $CH^{\bullet}_{sm}(\ )$ is a contravariant graded commutative ring-valued functor on the category of noetherian schemes with an ample line bundle.

$CH^{\bullet}_{sm}(\ )$ is a covariant functor for smooth projective maps, where $f_* : CH^{\bullet}_{sm}(X) \to CH^{\bullet-d}_{sm}(Y)$ for $f : X \to Y$ of fibre dimension d shifts degree by d.

Theorem 2: There is a natural surjection

$$\varphi : \mathrm{Pic}(X) \twoheadrightarrow CH^1_{sm}(X)$$

which is an isomorphism if X is either normal, or Cohen–Macanlay, or satisfies the condition S_2 of Serre.

Theorem 3: If $\mathcal{E}$ is a vector bundle of rank $r+1$ on X, then $CH^{\bullet}_{sm}(\mathbb{P}\mathcal{E}_X)$ is a free graded $CH^{\bullet}_{sm}(X)$ module of rank $r+1$, on the basis $\{1, h, h^2, \ldots, h^r\}$ where h is the class of $O_{\mathbb{P}\mathcal{E}}(1) \in \mathrm{Pic}(\mathbb{P}\mathcal{E}_X)$.

Theorem 4: (Moving lemma)
Each $x \in CH^p_{sm}(X)$ is $[E^{\cdot}]$ of some perfect complex E in $C^p_{sm}(X)$ which in addition is acyclic off codimension $\geq p$.

Theorem 5: (Comparison)
$CH^{\bullet}_{sm}(X) \otimes \mathbb{Q}$ is isomorphic as a graded ring to the associated graded of the γ–filtration on $K_0(X) \otimes \mathbb{Q}$.

In particular, for X regular quasi–projetive over a field

$$\oplus CH^p_{sm}(X) \otimes \mathbb{Q} \cong \oplus H^p_{\text{Zar}}(X, K_p) \otimes \mathbb{Q} \cong \oplus CH^p_{\text{classical}}(X) \otimes \mathbb{Q}\,,$$

so our theory agrees rationally with the classical one.

However, for $X/\bar{k}$ a semisimple algebraic group of type G_2 over an algebraically closed field, it is known (Grothendieck, R. Marlin) that $CH^3_{\text{classical}}(X) = \mathbb{Z}/2$, whereas $CH^3_{sm}(X) = 0$. So we still have to find a better filtration $C^\bullet$ than $C^\bullet_{sm}$, as this latter works fairly well, but still leaves to be desired.

Weith Filtrations via commuting Automorphisms

D.R. Grayson (Urbana)

We consider a filtration of the K–theory space $K(R)$ for a regular neotherian ring R which was proposed by Goodwillie and Lichtenbaum. The filtration has the t–th stage defined as

$$W^t = \Omega^t |d \mapsto K(R\mathbb{A}^d, \mathbb{G}_m^{\wedge t})|$$

where $R\mathbb{A}$ is the standard simplicial polynomial ring over R, and $K(R, \mathbb{G}_m^{\wedge t})$ denotes the K–theory space of the category of projective finitely generated R–modules equipped with t commuting automorphisms $\Theta_1, \ldots, \Theta_t$. The notation $\mathbb{G}_m^{\wedge t}$ indicates that mod out by the subspaces where $\Theta_i = 1, i = 1, \ldots, n$.

We prove that the quotient W^t/W^{t+1} (delooping of the homotopy fiber of the map $W^{t+1} \to W^t$) is the geometric realization of a simplicial abelian group, namely

$$W^t/W^{t+1} = |d \mapsto K_0^\oplus(R\mathbb{A}^d, \mathbb{G}_m^{\wedge t})|,$$

and thus fits in well with the program of Quillen and Beilinson to find chain complexes which would provide a motivic cohomology theory, as exposed, for example, in the speaker's paper "Weight filtrations in algebraic K–theory", to appear in "Motives, Proceedings of the Seattle Conference", Proceedings of Symposia in Pure Mathematics, AMS.

We mentioned the possibility of using the resolution theorem of Quillen in this context, arising from the fact that a module with commuting automorphisms has a projective resolution with commuting automorphisms.

We hope that the Adams operations will act on the filtration, and act purely on W^t/W^{t+1} at least when R is local, but all we know is a bit about W^1/W^2, namely $\pi_0 = 0, \pi_1 = K_1R_1$ and $\pi_2 =$ the quotient of K_2 by the central commutators. Also, the map $\pi_2(W^2/W^3) \to K_2R$ is an isomorphism if R is an algebraically closed field, and is a split surjection if R is a field which is perfect or not of characteristic 2.

Panin's result on K-theory of homogeneous varieties

A. Merkurjev (Louvan-La-Neuve)

Let F be a field, X be a projective variety defined over F. X is called *homogeneous* if there exists an algebraic linear group G acting transitively on X ($G(F_{sep})$ acts transitively on the set $X(F_{sep})$).

Theorem. Let X be a homogeneous variety over a field F. There exist a separable algebra A over F and a canonical isomorphism $K_*X \cong K_*A$.

Algorithm to derive $Z(A)$ from X and a group G acting on X: We assume G is semisimple and adjoint. Let $P \subset G_S$ be a parabolic subgroup (not defined over F) such that $X_S \cong G_S/P$. The *center* of A is a commutative étale algebra E over F corresponding to the $\Gamma = \mathrm{Gal}\,(F_{sep}/F)$–set W_G/W_P.

Examples.

1) Severi–Brauer variety (Quillen's computation),
2) Generalized Severi–Brauer variety (Panin, Levine–Srinivas–Weyman),
3) Projective quadrics (Swan),
4) Involution varieties (Tao).

Topological filtration for some Severi–Brauer varieties

N. Karpenko (Münster)

Let X be a Severi–Brauer variety corresponding to a simple central algebra $A, K(X) = K_0'(X)$ the Grothendieck group of X with the topological filtration

$$\cdots \subset K^{(i+1)}(X) \subset K^{(i)}(X) \subset \cdots,$$

$$G^i K(X) := K^{(i/i+1)}(X).$$

Theorem 1: If $\mathrm{ind}\,(A) = \exp(A) = r$, then

$$G^i K(X) \cong \frac{r}{(i,r)}\mathbb{Z} \quad \text{for each } i\,,\ 0 \leq i \leq \dim X\,.$$

Theorem 2: If $A = B_1 \otimes B_2$ is a division algebra, $\mathrm{ind}\, B_j = \exp B_j$, and $\mathrm{ind}\, B_1$ is squarefree, then $G^* K(X)$ is torsion free.

Corollary: For such A, $\quad CH^2(X) \cong \dfrac{r}{(2;r)}\mathbb{Z}$.

An interesting example of a division algebra A which does not satisfy the above conditions is a tensor product

$$A = Q_1 \otimes Q_2 \otimes Q_3\,,$$

where Q_i are quaternions. Let Y_i be the Severi–Brauer variety of Q_i, Y that of $Q_1 \otimes Q_2$.

Consider the cycle $t := (Y_1 \times Y_2 \times Y_3) - 3 \cdot (Y \times (2pt))$ on X, where $2pt$ denotes a closed point on Y_3 of degree 2.

Theorem 3: $[t] \in G^\bullet K(X)$ generates $(G^\bullet K(X))_{\mathrm{tors}}$, and $2 \cdot [t] = 0$.

Question: Is $[t]$ equal to 0 or not?
If yes, then there is no torsion, and $G^\bullet K(X)$ can be described precisely.
If no, there is only one non-trivial torsion element, and $G^\bullet K(X)$ can be described as well.

Polylogarithmic construction of mixed sheaves on mixed Shimura varieties

J. Wildeshaus (Münster)

It is hoped (Beilinson and others) that the K-groups of varieties over $\mathbb{Q}$ have a natural interpretation as Ext-groups of "mixed motivic sheaves", a category that is not yet found but should be equipped with "realization functors" e.g. into the category of mixed algebraic Hodge modules à la Saito or of mixed l-adic perverse sheaves (categories which do exist (in the first case) or can be reasonably approximated (in the latter)).

In a few cases (Spec ($\mathbb{Q}$), CM-elliptic curves, modular curves) it was (at least) proven that there is a subgroup of K-theory whose image in Deligne cohomology ($\hat{=}$Ext-groups of Hodge modules) has the properties predicted in Beilison's conjectures.

A common feature of these subgroups (or rather, their images under "realization") is that they arise from an interpolation process. E.g., the numbers $-Li_k(\varphi) \in \mathbb{C}/\mathbb{Q}(k), k \in \mathbb{N}, \varphi$ a root of unity, are "values" of a one-extension of Hodge modules (which happen to be variations of Tate-Hodge structure) on $\mathbb{P}^1 \setminus \{0; 1; \infty\}$. This interpolation process was formalized and generalized to arbitrary mixed Shimura varieties.

e-Bass

R. Keith Dennis (Ithaca)

A project to distribute mathematical books and lecture notes electronically was described. A few of the many advantages were given (low cost, wide availability, use by the blind etc). A proposal to distribute an enlarged and enhanced version of H. Bass's book "Algebraic K-Theory", was made.

Aid in the form of corrections, proofreading, updates, and finances are needed. For information, write to dennis@math.cornell.edu

Divisibilty in the Chow group of zero–cycles on a singular surface
C. Pedrini (Genova)

This is a report on a joint work with C. Weibel. We study the divisibility of the Chow group of 0–cycles on a surface X over a field k, thus extending several results known for X smooth.

Then we apply these results to the particular cases: $k = \bar{k}, k =$ algebraic number field, $k = \mathbb{R}$.

The Chow group of 0–cycles on a singular surface $CH^2(X)$ is defined as in Levine–Weibel [Journ. Reine angew. Math. 359 (1985), 106–120]. Then one has the following isomorphisms:

$$CH^2(X) \cong SK_0(X) \cong H^2_{\mathrm{Zar}}(X, \mathcal{K}_2).$$

We first extend to singular surfaces a result proved by Bloch–Ogus [Am. Sc. Ec. Norm. Sup. (1974), 181–202] in the smooth case.

Theorem A: Let X be a quasi–projective surface over a field containing $1/n$. The Chern class $c_{22} : K_2(U) \to H^2_{et}(U, \mu_n^{\otimes 2})$ induces an isomorphism

$$CH^2(X) \otimes \mathbb{Z}/n \cong H^2_{\mathrm{Zar}}(X, Sh^2).$$

Here Sh^2 is the Zariski sheaf associated to the presheaf $U \to H^2_{et}(U; \mu_n^{\otimes 2})$, where μ_n denotes the étale sheaf of n^{th} roots of unity.

Next step is to relate $CH^2(X) \otimes \mathbb{Z}/n = CH^2(X)/n$ with $CH^2(\overline{X}) \otimes \mathbb{Z}/n$, where $\overline{X}$ is the normalization of X.

Theorem B: Assume either that n is prime or that $\mu_n \subset R$ and $1/n \in R$. Then there is an exact sequence

$$H^1(\overline{X}, Sh^2) \oplus H^1(Y, Sh^2) \to H^1(\overline{Y}, Sh^2) \to H^2(X, Sh^2) \to H^2(\overline{X}, Sh^2) \to 0$$

where $\pi : \overline{X} \to X$ is the normalization, $Y = \operatorname{Sing} X, \overline{Y} = \pi^{-1}(Y)$.

Using theorem A and the isomorphism $SK_1(Z)/n \cong H^1(Z, Sh^2)$ valid for any Z of dim ≤ 1 we also get the following exact sequence:

$$H^1(\overline{X}, Sh^2) \oplus SK_1(Y)/n \to SK_1(\overline{Y})/n \to CH^2(X)/n \to CH^2(\overline{X})/n \to 0.$$

Applications:
I) If $k = \bar{k}$ the exact sequence in theorem B yields an isomorphism

$$CH^2(X)/n \cong CH^2(\overline{X})/n$$

Hence $CH^2(X)/n \cong (\mathbb{Z}/n)^c$ where c is the number of proper irreducibole components of X. In particular $CH^2(X)$ is divisible if X is affine.
II) If k is an algebraic number field then the above exact sequences show that for a singular surface X both $CH^2(X)/n$ and ${}_nCH^2(X)$ may be infinite. This

contrasts with the smooth case where ${}_nCH^2(X)$ is a finite group (at least if $H^2(X, O_X) = 0$), due to a result of Colliot-Thélène and Raskind [Inv. Math. 105 (1991) 221–245].

III) $k = \mathbb{R}$. In this case it is possible to relate $CH^2(X)/2$ with topological invariants of $X(\mathbb{R})$ (equipped with strong topology). In order to do that we first extend to any m-dimensional variety such that $Y = \operatorname{Sing} X$ has codimension ≤ 2 a theorem of Colliot-Thélène–Ischebeck [C.R. Acad. Sc. 292 (1981), 723–725] for smooth varieties. Then, in the case of a real surface, we use Theorem B to compute $CH^2(X)/2$. The result is the following:

Theorem C: Let X be a real surface with normalization $\overline{X}$. Let E be the number of proper components of X having no smooth real point. Then

$$CH^2(X)/2 \cong (\mathbb{Z}/2)^{t+E+\varepsilon}$$

where $t = H^2(X(\mathbb{R}), \mathbb{Z}/2)$ and ε is the rank of the cokernel of the map $H^1(\overline{X}, Sh^2) \to H^1(\overline{X}(\mathbb{R}), \mathbb{Z}/2)$. ε is 0 if $\overline{X}$ is smooth.

Using the above result it is possible to compute $CH^2(X)$ of the so called "real umbrellas" i.e. real surfaces whose singular locus is a line sticking out of the smooth locus.

Finding elements of the tame kernel

F. Keune (Nijmegen)

Two methods have been described to produce elements of the tame kernel of a number field F.

Let p be an odd prime.

1st method: special case: $\mu_p \subset F$.
Let a be an ideal of o_F. The automorphism $\zeta \oplus 1 \oplus \zeta^{-1}$ of $a \oplus b \oplus o_F$, where $\zeta \in \mu_p$ and b an inverse ideal of a, determines an element A of $\mathrm{SL}_3(o_F)$ satisfying $A^p = I$. Since $SK_1(o_F) = 0$ by Bass–Milnor–Serre, it can be lifted to $x \in St(3, o_F)$. Then $x^p \equiv K_2(o_F)$. This construction determines a map $\mu_p \otimes Cl(o_F) \to K_2(o_F)/p$. A generalization is given by Thom Mulders in his thesis (Nijmegen 1992).
general case: use the transfer after adjoining μ_p:

$$\mu_p \otimes Cl(o_{F(\zeta_p)}) \to K_2(o_{F(\zeta_p)})/p \xrightarrow{\mathrm{tr}} K_2(o_F)/p\,.$$

Note that if you can find this way generators for $K_2(o_F)/p$ you also have generators for the p-primary part of the tame kernel.

2nd method: special case: from the short exact sequence

$$0 \to K_2(o_F) \to K_2(F) \to \bigoplus_k k(p)^* \to 0$$

extract the exact sequence

$$0 \to \mu_p \otimes Cl(o_F[\tfrac{1}{p}]) \to K_2(o_F)/p \to \bigoplus_{f \mid p} \mu_p \to \mu_p \to 0\,.$$

(This has been done by Tate, 1976.)
general case: as with the first method, use the transfer. [By using a power p^r instead of p this leads to

$$0 \to (\mu_{p^r} \otimes Cl(o_{F(\zeta_{p^r})}[\tfrac{1}{p}]))_\Gamma \to K_2(o_F)/p^r \to (\ldots) \to 0\,.$$

This makes class field theory an appropriate tool for studying the tame and wild kernel. For $p^r = 4$ this has been done by Boldy (thesis Nijmegen 1991) for the quadratic case.]
Also in this case we end up with a map $\mu_p \otimes Cl(o_{F(\zeta_p)}) \to K_2(o_F)/p$.

Theorem (Keune, Mulders) These maps coincide.
Call this map J.
Theorem (Mulders, based on an idea by Geijsberts (thesis Nijmegen 1991)) The image of J consists (under mild conditions on F) of products of Dennis–Stein–symbols.
Theorem (Mulders) For F not imaginary quadratic the p–primary part of the wild kernel consists of Dennis–Stein–symbols. (For $p = 2$ there remains a little problem in some cases.)

Stable K–theory is topological Hochschild homology
R. McCarthy (Brown Univ.)
(joint work with B. Dundas)

Theorem: (conjectured by T. Goodwillie)
For $R.$ a simplicial ring and $M.$ a simplicial $R.$–bimodule, the natural map

$$K^s(R., M.) \to THH(R., M.)$$

is a homotopy equivalence.

Two immediate applications of this are:

1) (after work of L. Hesselholt)
If $R. \xrightarrow{f} S.$ is a map of simplicial rings st. $\pi_0 f$ is a surjection with nilpotent kernel, then for all primes p,

$$K(f)^\wedge_p \xrightarrow{\sim} TC(f;p)^\wedge_p\,.$$

2) $H_*(\mathrm{GL}_\infty(R.), M_\infty(M.)) \cong \bigoplus_{p+q=*} H_p(\mathrm{GL}_\infty(R.), THH_q(R., M.))$ for all simplicial rings $R.$ and bimodules M. The action on the left is by conjugation, and on the right one has the trivial action.

The theorem is proved by writing THH as the Goodwillie derivative of a functor which approximates $K(R. \oplus M.)$. This model is motivated by writing THH

as $HH(\mathbb{Z}[M_\infty(R.)], M_\infty(R.))$ which was done by Pirashvili and Waldhausen.

The Chern character in periodic cyclic homology

C. Weibel (Strasbourg)

Theorem 1: The Karoubi–Connes "Chern character"

$$ch : K_n(A) \to HP_n(A)$$

is compatible with the λ-operations defined on $K_*(A)$ and the λ-operations defined on periodic cyclic homology $HP_*(A)$, in the sense that it sends

$$K_n^{(i)}(A)_{\mathbb{Q}} \text{ to } HP_n^{(i)}(A) := \{x \in HP_n(A) \mid \psi^k(x) = k^{i+1} \cdot x\}.$$

This holds for any commutative $\mathbb{Q}$-algebra A, and also for quasi-compact quasi-separated schemes X. Induction reduces to the case $n = 0$.

Theorem 2: There is a theory of Chern classes

$$c_i : K_0(A) \to HP_0^{(i)}(A),$$

and a rank map

$$K_0(A) \to HP_0^{(0)}(A),$$

such that the associated Chern character

$$ch : K_0(A) \to \prod_{i=0}^{\infty} HP_0^{(i)}(A) = HP_0(A)$$

is the Karoubi–Connes map.

A simple reduction allows us to assume A is smooth over $\mathbb{Q}$. In this case $HP_0^{(i)}(A) = H_{dR}^{2i}(A)$, and Hartshorne has constructed a theory of Chern classes

$$c_i : K_0(A) \to H_{dR}^{2i}(A).$$

We observe that Hartshorne's c_i are the Chern classes associated to the Karoubi–Connes map.

Question: Is $HP_0^{(i)}(A)$ the same as Hartshorne's $H_{dR}^{2i}(\operatorname{Spec} A)$?

On Loday homology of Lie algebras

T. Pirashvili (Tbilisi)
(joint work with J.-L. Loday)

For any Leibniz algebra g (defined as a κ-vector space g equipped with a bilinear map

$$[\,,\,] : g \otimes g \to g$$

satisfying the Leibniz rule) Loday constructed a new homology theory $HL_*(g)$ as the homology of the chain complex

$$\ldots \to g^{\otimes n} \xrightarrow{d} g^{\otimes(n-1)} \to \ldots$$

where $d(x_1 \otimes \ldots \otimes x_n) := \sum_{i<j}(-1)^{j+1}(x_1 \otimes \ldots \otimes [x_i, x_j] \otimes \ldots \otimes \widehat{x_j} \otimes \ldots \otimes x_n)$.

The speaker constructed a universal enveloping algebra $UL(g)$ st. one has an isomorphism

$$HL_i(g) = \mathrm{Tor}_i^{UL(g)}(U(g_{\mathrm{Lie}}), \kappa).$$

Here, $g_{\mathrm{Lie}} := g/\langle [x,x] \rangle$, and $U(\cdot)$ means the usual universal envelope of Lie algebras. The speaker proved that if $\mathrm{char}(\kappa) > 0$ and g_{Lie} is semisimple then

$$HL_1(g) = T^1((\ker(g \to g_{\mathrm{Lie}}))_g).$$

It is interesting that the universal central extension of $sl(A)$ in the category of Leibniz algebras has kernel $HH_1(A)$.

Higher K–theory of modules over finite EI–categories

A.O. Kuku (Idaban)

Let C be a finite EI–category, i.e.
i) every endomorphism in C is an automorphism,
ii) the isomorphism classes of C–objects and $C(x,y), x,y \in C$, are finite.

An RC–module is a contravariant functor

$$C \to R-\mathrm{mod}$$

where R is a commutative Noetherian ring with 1. For all $n \geq 0$, let $G_n(RC)$ (resp. $K_n(RC)$) be the Quillen–K_n of the category of finitely generated (resp. finitely generated projective) RC–modules.

Theorem A: Let C be a finite EI–category, R the ring of integers in a number field F. Then for all $n \geq 0, K_n(RC), G_n(RC)$ are finitely generated Abelian groups.

Theorem B: Let C be a finite EI–category, k a field of characteristic p. Then the Cartan homomorphisms

$$K_n(kC) \to G_n(kC)$$

induce isomorphisms after applying $\otimes \mathbb{Z}[\frac{1}{p}]$.

Remarks: The study of K–theory of modules over EI–categories is necessitated by known topological applications for $n = 0, 1$. For example, if G is a Lie group, then $\pi/(G,X)$, the discrete fundamental category induced by the fundamental category $\pi(G,X)$ for a finitely dominated G–space X is an EI–category and finiteness obstructions exist in $K_0(\mathbb{Z}\pi/(G,X))$. Also, if $C' = \operatorname{Or} G$ is the orbit category of G, and X is a $G-CW$–complex with round structure, then the Reidemeister torsion exists in $Wh(\mathbb{Q}\operatorname{Or} G)$, a quotient of $K_1(\mathbb{Q}\operatorname{Or} G)$.

For details see: W. Lück, "Transformation groups and Algebraic K–theory", Springer 1989.

Products of conjugacy classes of two by two matrices

L.N. Vaserstein (University Park)
(joint work with E. Wheland)

The covering number $cn(G)$ and extended covering number $ecn(G)$ were introduced in [1] for a simple non–commutative group G. We define $ecn(G)$ for an arbitrary group G as the least k st. for any non–trivial conjugacy classes $C_1, \ldots, C_k$ in G st. the normal subgroup generated by each C_j contains $[G,G]$ and any $g_0 \in G$ st. $g_0 \equiv C_1 \cdot \ldots \cdot C_k \operatorname{mod} [G,G]$ we have

$$g_0 \in C_1 \cdot \ldots \cdot C_k \,.$$

For $cn(G)$, we take $C_j = C_1, j = 1, \ldots, k$.

We compute $cn(G)$ and $ecn(G)$ for

$$G \in \{\mathrm{SL}_2F, \mathrm{GL}_2F, PSL_2F, PGL_2F \mid F \text{ a field}\}\,.$$

For finite F, this was done in [1].

Products of conjugacy classes in $\mathrm{SL}_n\mathbb{C}$ were studied in [2] in connection with Higgs bundles, hypergeometric functions, and Hodge structures.

A rigidity theorem was proved and used in our computation.

References: [1] – Z. Arad and M. Herzog, LNM 1112 (1985).
[2] – C.T. Simpson, Canad. M.S. Conf. Proc. 12 (1992).

On K_1 of an Exact Category

C. Sherman (Southwest Missouri St. Univ.)

Let P be an exact category. In the mod–60's Bass defined a group which has come to be denoted $K_1^{\det}(P)$, as follows: Let $\operatorname{Aut} P$ denote the category whose objects are pairs (A,α), where α is an automorphism of A; then $K_1^{\det}(P) = \frac{K_0(\operatorname{Aut} P)}{\langle [(A,\alpha_1\alpha_2)]-[(A,\alpha_1)]-[(A,\alpha_2)]\rangle}$. When R is a ring, and $P(R)$ the category of finitely generated projective R–modules, then $K_1^{\det}(P(R)) \cong K_1(R)$, but in general the definition was flawed. The correct definition of $K_1(P)$ was, of course, given by Quillen in '72 when he defined $K_i(P) = \pi_{i+1}|NQP| \; \forall i \geq 0$. For $i = 0$, he proved that $\pi_1|NQP| = K_0(P)$ (defined algebraically) by using standard techniques concerning the fundamental group. Unfortunately, there are no analogues of these techniques for π_2, so it was not clear how to describe $K_1(M)$ in algebraic terms.

It was realized early on that there is a natural transformation $\psi : K_1^{\det}(P) \to K_1(P)$, which is an isomorphism when P has the property that all short exact sequences split (see [Sh1] for proof). However, Murthy showed by an example (based on computations in Lam's thesis) that ψ need not be surjective (see [Ge]).

The first step in understanding $K_1(P)$ was made by Grayson in [Gr1]. By drawing an appropriate commutative diagram in QP, he constructed an element $T(E,F) \in K_1(M)$ whenever one is given two short exact sequences of the form

$$\begin{array}{llllllllll} E & : & 0 & \to & L & \xrightarrow{\alpha} & M & \to & N & \to & 0 \\ F & : & 0 & \to & N & \xrightarrow{\beta} & M & \to & L & \to & 0 . \end{array}$$

As an example, let p be a prime, c_p the cyclic group of order p, $R = \mathbb{Z}[C_p] \cong \mathbb{Z}[X]/(x^p - 1)$, $G_i(R) = K_i(M(R))$, where $M(R)$ denotes the category of all finitely generated R-modules. Webb's computation of $G_1(R)[w]$ shows that $G_1(R)$ is the direct sum of $G_1^{\det}(R)$ and an infinite cyclic group; then it is not hard to prove that the cyclic group is generated by $T(E_p, F_p)$, where E_p and F_p denote the following short exact sequences:

$$\begin{array}{llllllllll} E_p & : & 0 & \to & R/(x-1) & \xrightarrow{\cdot\varphi_p(X)} & R & \to & R/(\varphi_p(x)) & \to & 0 \\ F_p & : & 0 & \to & R/\varphi_p(x) & \xrightarrow{\cdot(X-1)} & R & \to & R/(X-1) & \to & 0 . \end{array}$$

The next step was taken by Gillet and Grayson in [GG], where they constructed a simplicial set GP such that $|GP| \cong \Omega|NGP|$, so that $K_1(P) \cong \pi_1|GP|$. Using this, they were able to show that any element of $K_1(P)$ can be described by specifying two objects M, N, and filtrations

$$\begin{array}{ll} 0 = M_0 \hookrightarrow M_1 \hookrightarrow \ldots \hookrightarrow M_n = M & 0 = M'_0 \hookrightarrow M'_1 \hookrightarrow \ldots \hookrightarrow M'_n = M \\ 0 = N_0 \hookrightarrow N_1 \hookrightarrow \ldots \hookrightarrow N_n = N & 0 = N'_0 \hookrightarrow N'_1 \hookrightarrow \ldots \hookrightarrow N'_n = N \end{array}$$

equipped with isomorphisms $M_{i+1}/M_i \cong N_{i+1}/N_i, M'_{i+1}/M'_i \cong N'_{i+1}/N'_i \ \forall i$.

The speaker's result is a considerable simplification of this description. First, suppose that we are given two short exact sequences

$$0 \to A \xrightarrow{\alpha} X \to C \to 0, 0 \to B \xrightarrow{\beta} Y \to D \to 0$$

along with an isomorphism $\Theta : A \oplus C \oplus Y \to B \oplus D \oplus X$. Then we can form the following loop in $|GP|$:

$$\begin{pmatrix} 0 \hookrightarrow A \\ 0 \hookrightarrow A \end{pmatrix} \begin{pmatrix} A \overset{i_1}{\hookrightarrow} A \oplus C \oplus Y \\ A \overset{i_1 d}{\hookrightarrow} X \oplus Y \end{pmatrix}$$

$$\begin{pmatrix} A \oplus C \oplus Y \xrightarrow{\Theta} B \oplus D \oplus X \\ X \oplus Y \xrightarrow[\text{switch}]{} Y \oplus X \end{pmatrix} \begin{pmatrix} B \oplus D \oplus X \overset{i_1}{\hookleftarrow} B \\ Y \oplus X \overset{i_1\beta}{\hookleftarrow} B \end{pmatrix} \begin{pmatrix} B \hookleftarrow 0 \\ B \hookleftarrow 0 \end{pmatrix} .$$

It is not hard to show that the homotopy class of this loop is independent of the choice of cokernels for α and β, so we will use the notation $G(\alpha, \beta, \Theta)$ for the element of $\pi_1|GP| \cong K_1(P)$ defined by this loop (where Θ is a family of compatible isomorphisms, one for each choice of cokernels). Then we have:

Theorem: Every element of $K_1(M)$ is of the form $G(\alpha, \beta, \Theta)$ for some α, β, Θ.

The proof is based on a long exact sequence developed by Grayson in [Gr2], and appears in [S2]. In the case that P is an abelian category the speaker gave another proof in [S1], which is based on some old work of Auslander and uses Quillen's localization sequence for abelian categories rather than Grayson's sequence.

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Structure of exceptional groups over rings

N. Vavilov (Bielefeld)

Let Φ be a reduced irreducible root system of rank ≥ 2, R a commutative ring with 1,

$E(\Phi, R)$ the elementary Chevalley group,

$G(\Phi, R)$ the Chevalley group.

The qualitative behaviour of the functors

$$\begin{aligned} K_1(\Phi, R) &:= G(\Phi, R)/E(\Phi, R), \\ K_2(\Phi, R) &:= \ker(St(\Phi, R) \to E(\Phi, R)) \end{aligned}$$

introduced by M. Stein for the cases when Φ is exceptional was discussed. (G is assumed to be simply connected.)

Questions: 1) Is $E(\Phi, R)$ normal in $G(\Phi, R)$?

2) Is $K_2(\Phi, R)$ central in $St(\Phi, R)$?

1) was answered by G. Taddei using a variation of the "localization and patching" method. The speaker sketched another proof based on "decomposition of unipotents" imitating Stepanov's simplified proof of Suslin's normality theorem. This proof was obtained jointly with E. Plotkin.

It was then shown how the techniques used can be modified to obtain an analogue of the van der Kallen–Tulenbaer–theorem on the centrality of $K_2(\Phi, R)$

in $St(\Phi, R)$ for the types E_l.

Stability questions for the K_1–functor of Chevalley groups

E. Plotkin

Let $E(\Phi, R), G(\Phi, R), K_1(\Phi, R)$ be as in the last talk. The problem is connected with M. Stein's approach to stability of $K_1(\Phi, R)$, described in the paper "Stability questions of K_1, K_2–functors, modeled on Chevalley groups and related topics", 1978. The stability question for K_1 looks as follows:

Let $\triangle \subset \Phi$ be a root embedding. It induces a homomorphism

$$\nu : K_1(\triangle, R) \to K_1(\Phi, R).$$

Find conditions on R (depending on $\triangle \subset \Phi$) st. ν is surjective (resp. injective)!

In the talk, the answer for surjectivity was given for all natural maximal embeddings: Surjective stability takes place under stable range conditions, absolute stable range conditions and additional Vaserstein type conditions.
Proofs are based on Stein's diagram technique and the Chevalley–Matsumoto decomposition.

The same problem was then discussed for twisted Chevalley groups and the functor ${}^2K_1$, the twisted variant of K_1.

Several problems remain in this field. The first of them is to accomplish the stability of K_1 for all embeddings under some "good" conditions.
Another one is: Is it true that $ASR(R_0) = ASR(R)$, where ASR denotes the absolute stable range condition, and R_0 is the subring of fixed points under involution.

K–theory with Positioning Map

M. Morimoto (Okayama)
(joint work with A. Bak)

Let G be a finite group and X a closed, 1–connected smooth G–manifold of dimension $n = 2k \geq 6$. We discuss the surgery obstruction under the weak gap hypothesis: $\dim X \geq 2 \dim X^g$ $(\forall g \in G, g \neq 1)$. Our surgery obstruction group $W_n(G)$ is determined by the data:

$$\begin{aligned}
Q &= \{g \in G \,|\, g^2 = 1 \text{ and } \dim X^g = k-1\}, \\
S &= \{g \in G \,|\, g^2 = 1 \text{ and } \dim X^g = k\}, \\
\Theta &= \{X^H_\alpha \,|\, H \leq G, \dim X^H_\alpha = k\}/X^H_\alpha \sim X^K_\beta \text{ if } X^H_\alpha = X^K_\beta \text{ (as sets)}.
\end{aligned}$$

Rough Theorem: Let $f : X \to Y$ be a degree 1 G–framed map. Then f determines an element $\sigma(f) \in W_n(G)$ having the property:

$\sigma(f) = 0 \iff f$ is G–framed cobordant rel. to the singular set $\bigcup_{g\neq 1} X^g$ to $f' : X \to Y$ being a homotopy equivalence.

For the study of the surgery obstruction, in particular for applications of the theory to topology, the action of the Burnside ring $\Omega(G)$ on $W_n(G)$ is very important. For this reason, we introduce the Grothendick–Witt ring $GW_0(\mathbb{Z}, G, \Theta)$ and the special Grothendieck–Witt ring $\nabla GW_0(\mathbb{Z}, G, \Theta)$. Then $H \mapsto W_n(H)$ is a Green module over the Green functor $H \mapsto \nabla GW_0(\mathbb{Z}, G, \Theta)$ under a certain hypothesis.

Applications: (1) If $n \geq 6$ then S^n admits a smooth one fixed point action from A_5.
(2) (Laitinen–Morimoto) If $G \notin g_p^q(\forall p, q)$, then G acts smoothly on a sphere with exactly one fixed point.

Simplicial determinant map and the second term of the weight filtration

A. Nenashev

We have the maps

$$\det : K_0 X \to \operatorname{Pic} X \text{ and } \det : K_1 X \to \Gamma(X, O_x^*]),$$

where X is an irreducible scheme. Let $W^0 = GP_X$ denote the G–construction and W^1 be the union of components of rank zero in it. The speaker constructed a simplicial set T such that

$$\pi_0 T \cong \operatorname{Pic} X, \pi_1 T \cong \Gamma(X, O_X^*) \text{ and } \pi_m T = 0 \text{ for } m \geq 2$$

and a simplicial map

$$\det : W^1 \to T$$

which induces the above two determinant maps on the homotopy groups. The homotopy fibre of this map was described to be the simplicial set W^2. A vertex in W^2 is a pair (P, P', ψ) where P and P' are vector bundles on X such that $\operatorname{rank} P = \operatorname{rank} P'$ and $\psi : \det P \xrightarrow{\sim} \det P'$ is an isomorphism ($\det P$ denotes $\Lambda^{\operatorname{rank} P} P$); a vertex connection $(P_0, P_0'; \psi_0)$ to (P_1, P_1', ψ_1) is a pair of short exact sequences

$$(P_0 \to P_1 \to P_{1/0}; P_0' \to P_1' \to P_{1/0})$$

with equal cokernels such that the diagram

$$\begin{array}{ccc} \det P_1 & \xrightarrow{\sim} & \det P_0 \otimes \det P_{1/0} \\ \psi_1 \downarrow \wr & & \downarrow \psi_0 \otimes 1 \\ \det P_1' & \xrightarrow{\sim} & \det P_0' \otimes \det P_{1/0} \end{array}$$

commutes, where the horizontal isomorphisms are naturally induced by these short exact sequences. Thus W^2 yields the SK–theory of X, i.e.
$\pi_0 W^2 \cong \ker((\operatorname{rank}, \det) : K_0 X \to \mathbb{Z} \otimes \operatorname{Pic} X)$,

$$\pi_1 W^2 \cong \ker(\det : K_1 X \to \Gamma(X, O_X^*)),$$
$$\pi_m W^2 = K_2 X \text{ for } m \geq 2.$$

Mixed motives and filtrations on Chow groups

S. Saito (Orsay)

Let X be a smooth projective variety over $\mathbb{C}$ and let $CH^r(X)$ be the Chow group of cycles of codim. r on X modulo rational equivalence. We are interested in the structure of $CH^r(X)$. The case $r = 1$ is well understood since Abel and Jacobi: let $CH^r(X)_{\text{hom}} \subset CH^r(X)$ be the subgroup of cycles homologically equivalent to zero. Then $CH^r(X)/CH^r(X)_{\text{hom}}$ is finitely generated and $CH^1(X)_{\text{hom}}$ has a natural structure of abelian variety.

The case $r \geq 2$ in general turns out to be very complicated by the following theorem of Mumford.

Theorem (Mumford): Assume $\dim X = 2$ and $p_g = \dim H^0(X, \Omega^2) \neq 0$. Then $A^2(X) := CH^2(X)_{\text{hom}}$ cannot have a geometric structure in any reasonable sense.

In contrast with this Bloch proposed the following

Conjecture: Let X be as above and assume $p_g = 0$. Then the Albanese map induces

$$A^2(X) \cong \text{Alb}\,(X)\,.$$

(In particular $A^2(X)$ has a geometric structure.)

By these two statements one may naturally wonder if there exists a way for $CH^r(X)$ to be influenced by cohomological invariants of X. This guess is precisely formulated by Beilinson in terms of mixed motives. His conjectural formulation has may interesting consequences on $CH^r(X)$, among which the most notable is the existence of a filtration

$$CH^r(X) = F_M^0 CH^r(X) \supset F_M^1 CH^r(X) \supset \ldots$$

which satisfies

1) $F_M^1 CH^r(X) = CH^r(X)_{\text{hom}}$,
2) $F_M^2 CH^r(X) = \text{Ker}\,(CH^r(X)_{\text{hom}} \xrightarrow{P} J^r(X))$, where $J^r(X)$ is the intermediate Jacobian and P is Griffith's period map.
3) $\text{Gr}^{\nu}_{F_M} CH^r(X)_{\mathbb{Q}}$ is "controlled" by $H^{2r-\nu}(X, \mathbb{Q})$,
4) $\exists_{N>0}\ F_M^N CH^r(X)_{\mathbb{Q}} = 0$.

In the talk a certain filtration

$$CH^r(X) = F^0 CH^r(X) \supset F^1 CH^r(X) \supset \ldots$$

was defined in an *explicit manner* and the following were proved:
(I) The above filtration satisfies all the expected properties except possibly

one admits the Hodge conjecture then this is the only possible filtration which can arise from Beilinson's story of mixed motives. In particular one conjectures

$$D^r(X) := \bigcap_{\nu \geq 0} F^\nu CH^r(X)_{\mathbb{Q}} = 0 .$$

(II) In the context of $CH^r(X)_{\mathbb{Q}}/D^r(X)$ several conjectures on algebraic cycles which seem intractable at present can be proved. In particular, Bloch's conjecture is easy.
(III) Mumford's non-representability result is generalized in this context.

For example we prove

Theorem: Let $X = X_1 \times X_2 \times \ldots \times X_n$, where X_i is one of the following:
1) $\dim X_i \leq 2$,
2) abelian variety,
3) smooth complete intersection $\subset \mathbb{P}^N$,
4) flag variety.

Then the following conditions are equivalent for $\nu \geq 1$:
1) $H^0(X, \Omega_X^\nu) = 0$,
2) $\mathrm{Gr}_F^\nu CH_0(X) \geq 0$,
2') $\dim_{\mathbb{Q}} \mathrm{Gr}_F^\nu CH_0(X)_{\mathbb{Q}} < \infty$,
3) There exists $f : Y \to X$ s.t. $\dim Y \leq \nu - 1$ and that

$$CH_0(X)_{\mathbb{Q}} \subset \mathrm{Im}(f_* : CH_0(Y) \to CH_0(X))_{\mathbb{Q}} + F^{\nu+1} CH_0(X)_{\mathbb{Q}} .$$

Corollary: For X as above the following are equivalent:
1) $A_0(X)/D_0(X)$ is representable, i.e. $\exists$ Abelian variety A s.t. $A_0(X)/D_0(X) \cong A$.
2) $H^0(X, \Omega_X^\nu) = 0 \ \forall \nu \geq 2$.

A simple counterexample to the Hambleton–Taylor–Williams conjecture

D. Yao (Ithaca)
(joint work with D. Webb)

Let G be a finite group, $\mathbb{Q}G = \prod_i \mathrm{End}_{D_i}(V_i)$ the Wedderburn decomposition where V_i runs over all distinct irreducible $\mathbb{Q}G$–modules and $D_i := \mathrm{End}_{\mathbb{Q}G}(V_i)$.

Let Δ_i be a maximal $\mathbb{Z}$–order in D_i. The HTW–conjecture says that there is an isomorphism

$$G_n(\mathbb{Z}G) \xrightarrow{\sim} \bigoplus_i G_n\left(\Delta_i\left[\tfrac{1}{w_i}\right]\right) , \; n \geq 0 ,$$

where $w_i := h_i/\chi_i$, h_i is the order of the image of G in $\mathrm{End}_{D_i}(V_i)$ and χ_i is the dimension of any component in the complexification of V_i. The conjecture was made based on similar formulas obtained for finite abelian groups, nilpotent groups, dihedral and quaternion groups and groups of square-free order given by Lenstra, Webb and Hambleton, Taylor, Williams themselves. However, a simple counterexample is provided by S_5: Using Kenting's results, the speaker computed the rank of $G_1(\mathbb{Z}S_5)$. It is 7, but the conjectural value of the rank is 13. So the conjecture does noch hold in general.

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