MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 31/1993

Methoden der Modul- und Ringtheorie
4.07. - 10.07.1993

The conference was organized by Barbara Osofsky (New Brunswick) and Robert Wisbauer (Düsseldorf).

The heyday of the use of module theoretic methods to describe and study properties of rings was from the fifties to the seventies. By the end of this period it was felt that to some extent these methods had been exploited exhaustively. At that time ring theorists began to concentrate more on special areas of the subject such as representation theory (of finite dimensional algebras), noetherian rings and group rings. This resulted in the appearance of regular conferences on these topics.

Meanwhile questions in general module theory continued to interest people worldwide. Here the emphasis was on the stucture of modules themselves independent of the structure of the underlying ring. For example, one aspect of this was to find structure theorems for classes of modules with semisimple modules as a prototype. It has turned out moreover that these general studies often have an impact upon the more specialized fields mentioned above. Unfortunately, communication in this kind of research was not very well organized. The purpose of this conference was to promote better cooperation and appreciation of current work in general module theory. This was certainly achieved.

Forty-four mathematicians from twenty-one different countries accepted the invitation to come, many of them meeting for the first time. In thirty-five lectures they presented their ideas and during a problem session open questions were discussed. All participants agreed that the conference was a great success and they expect that it will have a strong impact on their future work.

It was expressed by all that the friendly atmosphere, the excellent facilities, and the perfect service provided by the Institute made the stay most enjoyable. There was an overall desire for a similar meeting to be held in due course.

Vortragsauszüge

G. ABRAMS

Smash products and skew rings for finite semigroups

Let R be a ring graded by the finite semigroup S; let S^* denote $S \setminus \{z\}$ where z is the zero element of S. The *smash product ring* $R\#S^*$ is the subring of $M_{|S^*|}(R)$ given by

$$R\#S^* = \{\Lambda \in M_{|S^*|}(R) \mid \Lambda_{x,y} \in \sum_{t\in S,\ ty=x} R_t \text{ for each } x, y \in S^*\}.$$

If A is any unital ring and σ (resp. γ): $S^* \to \mathrm{Ring}(\mathrm{End}(A))$ is an action (resp. reversing action) of S as endomorphisms on A, we may form the *skew semigroup ring* $S *_\sigma A$ (resp. $A *_\gamma S$). We give conditions which ensure that these are normalizing extensions of A. We then describe the structure of the resulting "star-smash" rings. Moreover we investigate the structure of rings of the form "smash-star" for naturally arising actions of S on $R\#S^*$.

P. ARA

On simple regular rings

It has been proved recently by K.R. Goodearl that there exist simple regular rings R such that $K_0(R)$ has nonzero torsion. However, all known examples of simple regular rings are strictly unperforated, which means that $nA \prec nB$ implies $A \prec B$ for all finitely generated projective modules A and B ($C \prec D$ means that C is isomorphic to a proper submodule of D). In joint work with K.R. Goodearl and E. Pardo we show that the strictly unperforated simple regular rings are exactly the simple regular rings which satisfy the weak comparability condition introduced by K. O'Meara.

H. ASASHIBA

On the Nakayama conjecture

Let A be a commutative artinian local ring with maximal ideal m and assume that A contains the residue field $k = A/m$. Put $D := \mathrm{Hom}_k(-, k)$.

(1) Assume that $|A/(x)| \leq 2$ for some $x \in m$. Then $\mathrm{Ext}^1(DA, A) = 0$ implies that A is self-injective.
(2) Assume that $|A/(x)| \leq 3$ for some $x \in m$. Then $\mathrm{Ext}^i(DA, A) = 0$ for $i = 1$ and 2 implies that A is self-injective.

G. BACCELLA

Semiartinian Von Neumann regular rings

Semiartinian right V-rings, which we call *right SV-rings*, form a special class of von Neumann regular rings. We characterize these rings by the fact that every factor ring imbeds as a subring in a direct product of right full linear rings containing the socle. If R is a semiartinian ring with all primitive factor rings artinian, then the condition of being an SV-ring is left/right symmetrical and is equivalent to being regular. On the other hand, if R is a right and left SV-ring, then all primitive factor rings of R are artinian.
For right SV-rings whose proper ideals are prime we show that the condition of being unit-regular is equivalent to being directly finite. On the other hand we show that there exists a directly finite right SV-ring which is not unit-regular. Furthermore we provide two constructions. For any given ordinal ξ, the first one gives a prime, unit-regular right SV-ring of Loewy length $\xi + 1$, which is not a left V-ring, and is hereditary if ξ is a natural number; the second one gives a directly infinite right SV-ring of Loewy length $\xi + 2$, which is not a left V-ring. These constructions are general enough to produce a wide supply of SV-rings, starting from given ones.

K.I. BEIDAR

On quasi-injective modules of finite length

The report is devoted to the following question, posed by C. Faith in 1967 (LNM 49). Let M be a right R-module of finite length satisfying the double annihilator condition $S^{RM} = S$ $(*)$ for finitely generated Λ-submodules, where $\Lambda = \mathrm{End}(M)$. What can be said about the structure of M?
Let N be a simple right module and $D = \mathrm{End}(N)$. We set $\dim(N) = \dim_D(N)$. Further, let $0 = M_0 \subset M_1 \subset \ldots \subset M_t = M$ be a composition series of M. We set $\deg(M; N) = |\{1 \leq i \leq t \mid M_i/M_{i-1} \simeq N\}|$.
Theorem.
Let L be a right R-module with a finitely generated socle $\mathrm{Soc}(L)$ and $\Lambda = \mathrm{End}(L)$. Assume that L satisfies the double annihilator relation $(*)$ for cyclic Λ-submodules, and $\deg(\mathrm{Soc}(L); N) \leq 1$ for any simple R-module N with $\dim(N) = 1$. Then the mapping $\alpha : \mathrm{End}(L) \to \mathrm{End}(\mathrm{Soc}(L))$, given by $\alpha(f) = f|_{\mathrm{Soc}(L)}$, is surjective.

G.F. BIRKENMEIER

Essential supernilpotence

J. Fisher defines an ideal L of a ring R to be *essentially nilpotent* if it contains

a nilpotent ideal N of R which has nonzero intersection with each nonzero ideal of R which is contained in L. He then shows that the prime radical of an arbitrary ring is essentially nilpotent. E. Eslami and P.N. Stewart extended Fisher's results on essential nilpotency.
The purpose of the talk is to generalize the concept of essential nilpotency to a theory of essential supernilpotency. We define an ideal $\bar{\rho}(R)$ which is a closed essential extension of $\rho(R)$ (where ρ denotes a supernilpotent radical property) and develop its basic properties. We then apply our theory to several large classes of rings including Baer rings, CS-rings, FPF-rings, and perfect rings. Our theory not only encompasses the work of Fisher, Eslami, and Stewart, but also yields various decompositions of the aforementioned rings which generalize results of C. Faith, S. Page, and Y. Utumi.

G. Brodskii

On local Morita equivalences and dualities

It is proved that different Morita-type theorems on equivalences and dualities between subcategories of an abelian category and a module category have dualizable generalizations, in particular the following generalization of the Gabriel-Popescu theorem.

Theorem.

If S is a ring, U is a left S-object of a cocomplete locally small abelian category $\mathcal{C}$ with surjective map $S \to \mathrm{End}_{\mathcal{C}}(U)$, $\mathcal{K} \subseteq \mathcal{C}$ is a linearly cocompact hereditary full subcategory and $\ker(U^n \to K) \in \mathrm{Gen}(U)$ for all $n \in \mathbb{N}$ and $K \in \mathcal{K}$, then the following conditions are equivalent:

(1) U is a $\mathcal{K}$-generator (i.e. $\mathcal{K} \subseteq \mathrm{Gen}(U)$);

(2) the functor $h^U = \mathrm{Hom}_{\mathcal{C}}(U, -) : \mathcal{C} \to \text{mod-}S$ and its left adjoint functor t^U : mod-$S \to \mathcal{C}$ induce an equivalence between the full subcategories $\mathcal{K} \subseteq \mathcal{C}$ and $h^U(\mathcal{K}) \subseteq$ mod-S.

A.W. Chatters

Isomorphisms and matrices

Recent work has shown that many rings which are not originally presented as full matrix rings are, in fact, full matrix rings. We shall consider some of these examples and show how to calculate matrix units in them. This work also raises questions concerning how to decide whether two rings are isomorphic, if the matrix rings over them are isomorphic.

J. Clark

When is a self-injective semiperfect ring quasi-Frobenius?

This is a report on joint work with D. van Huynh (to appear in J. Algebra). We show that a right self-injective semiperfect ring R is quasi-Frobenius if and only if every uniform submodule of any projective right R-module is contained in a finitely generated submodule.

A. FACCHINI

Finitely generated projective modules over a self-injective simple regular ring of type II

Linear categories are those abelian categories $\mathcal{C}$ such that when we construct the Grothendieck group $\mathrm{Grot}(\mathcal{C})$ we loose as little information as possible (which happens if and only if $\mathcal{C}$ is skeletally small, two objects are stably isomorphic if and only if they are isomorphic, and all exact sequences split), and for which the Grothendieck group $\mathrm{Grot}(\mathcal{C})$ is a totally ordered archimedian complete group. A linear category $\mathcal{C}$ is either a null category (in which case $\mathrm{Grot}(\mathcal{C}) = 0$), or it is isomorphic to the category vect-k of all finite dimensional right vector spaces over a division ring k (in which case $\mathrm{Grot}(\mathcal{C}) \simeq \mathbb{Z}$), or it is isomorphic to the category proj-R of all finitely generated projective right modules over a right self-injective simple regular ring R of type II_f (in which case $\mathrm{Grot}(\mathcal{C}) \simeq \mathbb{R}$). Over such a category proj-R all the theorems (or, better, most theorems) of linear algebra hold: every object has a (finite) basis, the dimension is a *real* number, every endomorphism has a determinant with the usual properties, every matrix has a rank, this rank is the greatest order of the nonzero minors, Cramer's rule, Rouché-Capelli's Theorem...

C. FAITH

Maximal module theorems

A ring R is a *right max ring* if every right module $M \neq 0$ has at least one maximal submodule. Although there is an extensive literature on max rings it appears to have escaped notice that it suffices to check for maximal submodules of a single module and its submodules in order to test for a max ring; namely, any cogenerating module E of mod-R. Furthermore, another test is to check the submodules of the injective hull $E(V)$ of each simple module V (*). As a corollary we conclude that R is right max whenever $E(V)$ is noetherian for each simple right R-module V.

Another important test for a right max ring R is *transfinite nilpotence* of the radical of E in the sense that $\mathrm{rad}^\alpha E = 0$ for some ordinal α; equivalently, there is an ordinal α such that $\mathrm{rad}^\alpha(E(V)) = 0$ for each simple module V. This holds iff each power $\mathrm{rad}^\beta(E)$ (or $\mathrm{rad}^\beta(E(V))$) has a maximal submodule, or is zero. It follows that R is right max iff every nonzero (subdirectly irreducible) quasi-injective right R-module has a maximal submodule.

We apply (*) and a theorem of Harada and Ishii, and another of Kurata to characterize a right max ring R via the endomorphism ring Λ of any injective cogenerator E of mod-R; namely, Λ/L has a minimal submodule for any left ideal $L = \mathrm{ann}_\Lambda M$ for a submodule (or subset) $M \neq 0$ of E. We deduce that R is right max whenever Λ is a left Loewy (= semiartinian) ring.

D. van Huynh, J.K. Park

Characterizations of rings by their modules

We present results recently obtained jointly with Y. Hirano and H.K. Kim. Let R be a ring.

(1) The following conditions are equivalent:

(i) R is semilocal,

(ii) every semiprimitive right R-module is CS,

(iii) every semiprimitive finitely generated right R-module is quasi-continuous.

(2) R is semiprimary right and left SI iff every semiprimitive countably generated right R-module is a direct sum of an injective module and a projective module.

(3) R is a ring direct sum of a semiprimary right and left SI-ring and a right CS-, right SI-ring with zero right socle iff every cyclic semiprimitive right R-module is a direct sum of an injective module and a projective module.

S.K. Jain

Weakly injective modules

The concept of weak relative-injectivity of modules was originally introduced to obtain a characterization of semiperfect rings over which each cyclic right module is embeddable as an essential submodule of a projective module (CEP-rings). In analogy to a characterization of quasi-Frobenius rings, a ring R is right CEP if and only if R is right artinian and each indecomposable projective right R-module is weakly R-injective. An R-module M is called *weakly injective* if for each finitely generated R-module N, and for each R-homomorphism $\varphi : N \to E(M)$ (where $E(M)$ is the injective hull of M), there exists a submodule X of $E(M)$ such that $\varphi(N) \subset X \simeq M$.

Among others, we consider the questions: For which rings is it the case that each weakly injective module is injective and when are the direct summands of weakly injective modules again weakly injective? Rings over which direct sums of weakly injective modules are weakly injective are precisely those rings over which each cyclic module has finite uniform dimension.

Rings over which every right module is weakly injective are called *right weakly semisimple*. Weakly semisimple rings are semiprime, right noetherian, left Goldie over which quasi-injectives are injective. We also address the questions: Which abelian groups are weakly injective as $\mathbb{Z}$-modules? More generally which modules over hereditary noetherian prime rings are weakly injective?

A.I. Kashu

Morita contexts: some aspects and applications

The questions about the applications of Morita contexts to the study of torsion theories, localizations, and lattices are elucidated. If we have an arbi-

trary Morita context $(R, {}_RU_S, {}_SV_R, S)$ then it determines a lot of remarkable classes of modules in categories ${}_R\mathcal{M}$ and ${}_S\mathcal{M}$ of left R- and left S-modules. Hom-functors determine the different relations between the torsion theories of the categories ${}_R\mathcal{M}$ and ${}_S\mathcal{M}$.
The main results concern the questions on relations between the lattices of submodules of the components ${}_RR_R$, ${}_RU_S$, ${}_SV_R$, ${}_SS_S$ of the given Morita context. The different lattice isomorphisms between the lattices of submodules (defined by the associated radicals) are indicated. Some mappings which reverse the inclusions give us Galois connections. The conditions on Morita contexts to have antiisomorphisms between the distinguished subsets are given.

G. KRAUSE

Standard prime ideals of modules over noetherian algebras of finite Gelfand-Kirillov dimension

Let k be a field, let R and S be noetherian k-algebras of finite Gelfand-Kirillov dimension, and let ${}_SM_R$ be a bimodule that is finitely generated on both sides. A *right standard prime bi-factor series of* M is a sequence of S-R-subbimodules $0 = B_0 \subset B_1 \subset \ldots \subset B_{i-1} \subset B_i \subset \ldots \subset B_n = M$ such that for each i, the right annihilator in R of B_i/B_{i-1} is the unique right associated prime ideal P_i of this sub-bi-factor, and such that $\mathrm{GK}(R/P_i) \leq \mathrm{GK}(R/P_j)$, whenever $1 \leq i \leq j \leq n$.
It is shown that all unrefinable such series of M have the same length, called the *right standard length of* M, and that for each prime P_i its multiplication in any two such unrefinable series coincide. Furthermore, the right standard length is the same as the left standard length.
Applications are presented that deal with "lying over" for prime ideals for the case when S is an extension of R, and S_R is finitely generated.

L. LEVY

Torsion modules over HNP-rings

We complete the basic theory of finitely generated modules over a hereditary noetherian prime ring (*HNP-ring*) by showing that every such module is the direct sum of right ideals and homomorphic images of right ideals.
We also show why no clear picture has ever been given of what indecomposable modules of finite length over such rings look like. We do this by proving that the category of such modules (for suitable HNP's) is *wild* in a very strong sense. (Joint work with L. Klinger.)

S.R. LÓPEZ-PERMOUTH

Weakly projective and weakly injective modules

A module M is said to be *weakly N-projective* if it has a projective cover

$\pi : P(M) \twoheadrightarrow M$ and for each homomorphism $\varphi : P(M) \to N$ there exists an epimorphism $\sigma : P(M) \twoheadrightarrow M$ such that $\varphi(\ker\sigma) = 0$, or equivalently there exists a homomorphism $\hat{\varphi} : M \to N$ such that $\hat{\varphi}\sigma = \varphi$. A module M is said to be *weakly projective* if it is weakly N-projective for all finitely generated modules N. *Weakly N-injective* and *weakly injective* are defined dually. We study rings R over which every weakly injective right R-module is weakly projective. We also study those rings over which every weakly projective right module is weakly injective. Among other results, we show that for a ring R the following conditions are equivalent:

(1) R is left perfect and every weakly projective right R-module is weakly injective.

(2) R is a direct sum of matrix rings over local QF-rings.

(3) R is a QF-ring such that for any indecomposable projective right module eR and for any right ideal I, $\mathrm{soc}(eR/eI) \simeq (eR/eI)^{(\alpha)}$ for some cardinal α.

(4) R is right artinian and every weakly injective right R-module is weakly projective.

(5) Every weakly projective right R-module is weakly injective and every weakly injective right R-module is weakly projective.

L. MÁRKI

Martindale's Theorem and Posner's Theorem for GPI-rings

Martindale's Theorem characterizes prime rings with a generalized polynomial identity in terms of the central closure of the ring. We give an internal characterization of these rings by means of Goldie-type conditions. Using a new notion of order, prime rings satisfying a GPI are proved to be the same as left orders in primitive rings satisfying a GPI. The latter result is based on the following theorem:

A ring R is a left order (in our sense) in a primitive ring with nonzero socle iff R is prime, left non-singular, and has a uniform left ideal.

The notion of left order we use here is a modification of left orders in the sense of J. Fountain and V. Gould. (Joint work with P.N. Anh.)

A. MEKEI

Subalgebras of finite codimension

The main topic of the talk is the following:

Theorem.

Let R be an infinite dimensional algebra over the field F and A a subalgebra of finite codimension, $\dim_F(R/A, +) < \infty$. Then R is a semiprime algebra if and only if A is a semiprime algebra and, either R has an essential ideal which is semiprime as a ring and has non-trivial intersection with A, or R is of type $R = K \oplus I$, K, I ideals of R, $\dim_F K < \infty$, K is semiprime as a ring and $I \leq A$.

C. MENINI

Realization theorems for categories of graded modules over semigroup-graded rings

Let S be a semigroup and let R be an S-graded ring. Firstly we show that the category R-gr of S-graded left R-modules is a Grothendieck category for which we explicitly present a system of finitely generated projective generators. Afterwards we consider, for a given subset X of S, the category (R, X)-gr consisting of those graded left R-modules whose support is contained in X and we note that this is a TTF-class in R-gr. We prove that the category (R, X)-gr (and hence, in particular, R-gr) is equivalent to the category $\mathrm{Gen}({}_A I_X)$ of left A-modules generated by the idempotent left ideal I_X of A, where A is a suitable ring. When X is finite $\mathrm{Gen}({}_A I_X)$ coincides with A-mod and $A = R\#X$. These results stem from more general categorical results. Namely we prove an analogous result for any TTF-class $\mathcal{D}$ in a Grothendieck category $\mathcal{C}$ having a system of small projective objects which generate $\mathcal{D}$.

A.V. MIKHALEV

Isomorphisms and anti-isomorphisms of endomorphism rings

Let A and B be rings and M and N a right A-module and a right B-module respectively. Let $f : \mathrm{End}(M_A) \to \mathrm{End}(N_B)$ be either an isomorphism or an anti-isomorphism. The Baer-Kaplansky problem is to characterise all such mappings f. The problem has a long history including contributions from Eidelheit, Mackey, Rickart, Wolfson, Morita, Stephenson, Mikhalev and Celuskin. A related question is to describe all isomorphisms and anti-isomorphisms $g : \mathrm{GL}_m(R) \to \mathrm{GL}_n(S)$, for given rings R, S. Recent progress is reported.

B.J. MÜLLER

Self-injective and continuous rings

We are interested in rings which are, on *both* sides, self-injective or continuous or CS. We discuss briefly ring-direct decomposition properties which follow from such assumptions. In particular we are interested in the question of whether, for a twosided self-injective ring R (where $Z({}_RR) = J(R) = Z(R_R)$ holds) the equation $Z_2({}_RR) = Z_2(R_R)$ is true. We show that the existence of a counterexample is equivalent to the existence of a bimodule with many injectivity- and balanced-ness properties (reminiscent of Morita duality). We ask whether such a bimodule can exist.

S.S. PAGE

Relative (quasi-)continuous, discrete, and τ-discrete modules

We fix a torsion theory τ and define the notion of a *τ-summand* of a module

M as a submodule A such that there exists a B so that $A \oplus B$ is τ-dense. From this follows appropriate definitions of the four types of modules mentioned in the title. These then lead to many decompositions type theorems. For a ring R we denote by $K(R)$ the sum of all τ-small submodules of R. We introduce *τ-densely-projective* modules and end with the following theorem: Every R-module has a τ-densely projective cover iff
(i) $R/K(R)$ is τ-quasi-semisimple (every τ-closed submodule is a τ-summand),
(ii) $K(R)$ is τ-small,
(iii) τ-decompositions of $R/K(R)$ lift to τ-decompositions of R.
(Does $R/K(R)$ in this case have finite τ-corank?)

E. Puczylowski

On dimensions of modules and lattices

The talk consisted of two parts.

1. In 1978 Lanski asked the following questions: Suppose R is a ring with involution $*$ and $\frac{1}{2} \in R$. Denote by S the subring of R generated by the symmetric elements.
(a) Suppose ${}_RR$ has Krull dimension. Does then ${}_SS$ have Krull dimension?
(b) Let M be an artinian R-module. Is M artinian as an S-module?
In 1992 K.I. Beidar, P.F. Smith, and myself proved that if R is noetherian with respect to all twosided $*$-ideals and M is an R-module, then the Krull (dual Krull) dimensions of ${}_RM$ and ${}_SM$ coincide. In Oberwolfach we proved that the questions (a) and (b) are answered in the affirmative.
2. In the second part some results and questions related to a question of Al-Khazzi and P.F. Smith will be discussed.

F. Raggi

On a special kind of injectivity

Some relative properties on R-tors induce nice partitions of this lattice. We have studied several of these, like for instance being relatively injective with respect to a torsion theory. The classes of equivalence give us a great deal of information about the ring and about the category R-mod.

S.T. Rizvi

On continuous rings and modules

Let N_R be a fixed module, then for any module M_R we define

$$\mathcal{A}(N, M) = \{A \subseteq M \mid f(X) \subseteq_e A \text{ for some } X \subseteq N, f \in \mathrm{Hom}(X, M)\}.$$

We call a module M to be *N-continuous*, *N-quasi-continuous*, or *N-CS* if it satisfies the well-known continuous module definitions (C_1) and (C_2), (C_1) and (C_3), or (C_1) respectively for the members of the family $\mathcal{A}(N, M)$. It is easy to see that M is (quasi-)continuous iff M is N-(quasi-)continuous iff M

is M-(quasi-)continuous. We prove the following:

Theorem 1.

Let M be N_i-continuous, $i = 1, 2, \ldots, n$. Then M is $\bigoplus_{i=1}^{n} N_i$-continuous.

Theorem 2.

$M = \bigoplus_{i=1}^{n} M_i$ is N-continuous iff each M_i is N-continuous and M_i is A_j-injective $\forall A_j \in \mathcal{A}(N, M_j)$ for all $i \neq j$.

The above theorems, as a corollary, provide an alternate proof of a result of Müller and Rizvi, namely: $M = \bigoplus_{i=1}^{n} M_i$ is continuous iff M_i is continuous and M_j-injective for all $i \neq j$.

Theorem 3.

(a) Every finitely generated R-continuous module is continuous.

(b) If R is noetherian, then every R-continuous module is continuous.

For a module M we define $\bar{T}(M) = \{N \mid M \text{ is } N\text{-continuous}\}$ and $\mathbb{F}(M) = \{X \mid \mathrm{Hom}(N, X) = 0, \forall N \in \bar{T}\}$. $\bar{T}(M)$ is closed under homomorphic images, submodules, finite direct sums and group extensions.

Theorem 4.

$(\bar{T}(M), \mathbb{F}(M))$ is a hereditary torsion theory iff $M = T \oplus F$, where $T \in \bar{T}(M)$ and $F \in \mathbb{F}(M)$. In this case T is (T-continuous, hence) continuous.

Theorem 5.

R is a right V-ring iff every right R-module is S-continuous for every simple module S. (Joint work with K. Oshiro.)

H. RÖHRL

Convexity theories and Γ-convex modules

Let Γ_C be the set of all infinite sequences $\alpha_\bullet \doteq (\alpha_1, \alpha_2, \ldots)$ with entries from $\mathbb{R}$ satisfying (i) supp$(\alpha_\bullet)$ is finite, (ii) $\alpha_i \geq 0 \forall i \in \mathbb{N}$, (iii) $\sum_i \alpha_i = 1$. A *convex module* is a set X together with operations

$$\Gamma \times X^{\mathbb{N}} \ni (\alpha_\bullet, x^\bullet) \mapsto \sum_i \alpha_i x^i \in \mathbb{N} \text{ such that}$$

(1) $\sum_j \delta^i_j x^j = x^i \ \forall i \in \mathbb{N}$,

(2) $\sum_i \alpha_i (\sum_j \beta^i_j x^j) = \sum_j (\sum_i \alpha_i \beta^i_j) x^j$ for $\alpha_\bullet, \beta^1_\bullet, \beta^2_\bullet, \ldots \in \Gamma_C$ and $x^\bullet \in X^{\mathbb{N}}$.

The Klein-Hilbert parts relation $\sim$, defined by $x \sim y := \exists u, v$ such that $x, y \in (u, v) := \{\alpha u + (1 - \alpha) v : 0 < \alpha < 1\}$, is a congruence relation and is compatible with the homomorphisms between convex modules, where a homomorphism $X \to Y$ is a map $f : X \to Y$ such that

$$f(\sum_i \alpha_i x^i) = \sum_i \alpha_i f(x^i) \ \forall \alpha_\bullet \in \Gamma_C \text{ and } x^\bullet \in X^{\mathbb{N}}.$$

The category of convex modules and their homomorphisms, denoted by $\Gamma_C C$, is an abelian category. $\sim$ gives rise to a functor

$$KH : \Gamma_C C \to \Gamma_C C \text{ by } KH(X) := X/\sim .$$

The canonical map $q_X : X \to KH(X)$ is a natural transformation $q_- : \mathrm{id} \to KH$. Define $d_H(x,y)$, the *Harnack distance*, by $d_H(x,y) := \max\{\|y\|_x, \|x\|_y\}$ where $\|y\|_x := \inf\{\alpha \mid 0 \le \alpha \le 1 \text{ and } \exists z : y = \alpha z + (1-\alpha)x\}$. d_H is a pseudometric, and $d_H(x,y) = 0 \Leftrightarrow$ there is a line l (i.e. injective image of $\mathbb{R}$) in X with $x, y \in l$.

Theorem 1. Equivalent are
(i) X is *discrete* (i.e. $\sim$ is discrete on X),
(ii) $\forall x, y$ (x, y) has precisely one point,
(iii) $\mathrm{supp}(\alpha_*) = \mathrm{supp}(\beta_*) \Rightarrow \sum_i \alpha_i x^i = \sum_i \beta_i x^i$, (iv) $x \neq y \Rightarrow d_H(x,y) = 1$.

Theorem 2.
The full subcategory $\Gamma_C C^{\mathrm{dis}}$ of all discrete convex modules is an ext-epi-reflective subcategory of $\Gamma_C C$ with reflection KH and reflector q_-.

Theorem 3. Equivalent are
(i) X is *open* (i.e. $\sim$ is indiscrete), (ii) $X/\sim$ has only one element, (iii) $X = \mathrm{int}\, X := \{y \mid \forall z \in X\, \exists 0 < \alpha \le 1 \text{ and } t \in X : y = \alpha z + (1-\alpha)t\}$, (iv) $d_H(x,y) < 1\, \forall x, y \in X$.

Theorem 4.
(i) If C is open and D is discrete then each homomorphism $C \to D$ is constant, (ii) if C is such that for all discrete D every homomorphism $C \to D$ is constant then C is open, (iii) if D is such that for all open C every homomorphism $C \to D$ is constant then C is discrete.

J. Shapiro

Generating the ideal of relations of certain integral semigroup rings

We examine certain integral semigroup rings that arise in the study of Minkowski rings of polytopes. The semigroups are abelian, torsion free and finitely generated and hence the semigroup ring is naturally the homomorphic image of the integers with a finite number of indeterminates adjoined. We are interested in finding a finite generating set for the ideal of relations, in particular, determining when the semigroup ring is a complete intersection. When the semigroup is generated by four elements that satisfy rational relations of dimension two, we have a method of finding a minimal generating set. In general, we can characterize when the semigroup ring is a complete intersection in terms of the existence of a certain defining set of integer relations on the semigroup. These results extend some work of J. Herzog, "Generators and relations of abelian semigroups and semigroup rings", Manuscripta Math. 3 (1970) and H. Bresinsky, "On prime ideals with generic zero $X_i = t^{n_i}$", Proc. AMS Vol. 47, No. 2 (1975). (Joint work with K. Fischer.)

P.F. Smith

Chain conditions for free modules

Free (abelian) groups satisfy ACC_n (ascending chain condition on free n-

generated subgroups (Baumslag–Baumslag). Conversely, if A is a torsion-free abelian group which satisfies ACC_n, for every positive integer n, then every countable subgroup of A is free (Pontrjagin).
Let R be a ring (with identity) and M a right R-module. We say that M *satisfies* ACC_n, for some given positive integer n, if every ascending chain of n-generated submodules terminates. It is known that a ring R is right perfect if and only if every right R-module satisfies ACC_1, in which case every right R-module satisfies ACC_n, for every positive integer n (Jonah).
Theorem. (M. Elisa Antunes Simoẽs-P.F. Smith)
Let R be a right Goldie ring with DCC on right annihilators and let n be a positive integer such that every finitely generated free right R-module satisfies ACC_n. Then every free right R-module satisfies ACC_n.
This theorem generalizes results of Renault and Baumslag–Baumslag. It also answers an open problem in P.M. Cohn's book "Free rings and their relations".

P. VÁMOS

The structure of linearly compact integral domains

The structure of linearly compact rings (and modules) is unknown in general. This problem goes back a long way and was posed explicitly by Zelinsky in 1953. In 1970 Müller showed that rings with a Morita duality are linearly compact; recently Anh (1992) established Müller's conjecture that in the commutative case the converse is true so these two classes of commutative rings are the same.

For some time only two classes of commutative linearly compact rings were known: maximal valuation rings and complete local noetherian rings. In 1977 Vámos showed that a certain pull-back or lexicographic extension of these two is again linearly compact and conjectured that every linearly compact integrally closed integral domain arises this way. This has been proved by his student McGuire. The talk concentrated on this result and linked the problem to power-series representations and the structure of linearly compact modules, generalizing many previously known results.

N. VANAJA

Generalizations of regular modules

Suppose R is a ring with identity, M a unitary right R-module, $M^* = \mathrm{Hom}_R(M,R)$ and $S = \mathrm{Hom}_R(M,M)$. The module M is called *regular* if $m \in mM^*(m), \forall m \in M$. This has been generalised by Ramamurthi and Mabuchi. We call M a *RWR-module* (weakly regular module defined by Ramamurthi) if

$$m \in M^*(mR) = mRM^*(mR), \forall m \in M$$

and a *MWR-module* (weakly regular module defined by Mabuchi) if

$$m \in SmM^*(m) = SmM^*(Sm), \forall m \in M.$$

We study the properties of RWR-modules and compare them with those of MWR-modules. Every submodule of an RWR-module M is semiprime in M and the converse is true if M is 1-projective. For a finitely generated projective RWR-module, its endomorphism ring is a right weakly regular ring. If M_R is an $RWR(MWR)$-module then ${}_SM$ is an $MWR(RWR)$-module. We also study the properties of the modules M for which $m \in SmRM^*(SmR), \forall m \in M$.

J. Viola-Prioli

Ducompact filters and prime kernel functors

Topologizing filters of right ideals closed under taking arbitrary intersections (usually called *jansian filters*) have been extensively studied mainly in relation with torsion theories of special types. Here, for any given ring R, we introduce a map ϕ : {right ideals of R} $\rightarrow$ fil-R by means of which topologizing filters that are dual to the jansian filters are obtained. This provides a new insight into fil-R and, when restricted to right chain rings R, ϕ establishes a one-to-one order reversing correspondence between completely prime ideals of R and prime kernel functors. The distribution of prime kernel functors is then analyzed. (Joint work with J. Golan and A.M. Viola-Prioli.)

B. Wilke

Properties of a ring as a module over the skew group ring

Let R be a ring with identity and G a group acting on R as automorphisms. Then R is a left module over the skew group ring $R * G$. When G is a finite group, there are well-known results giving necessary and sufficient conditions for ${}_{R*G}R$ to be a projective $R * G$-module respectively to be a generator. We give a characterization for R to be a self-projective $R * G$-module respectively a selfgenerator. In this case the category $\sigma[{}_{R*G}R]$ is equivalent to the category of left modules over the fixed ring R^G (where $\sigma[{}_{R*G}R]$ consists of all submodules of ${}_{R*G}R$-generated modules). Further results concern factor rings R/I of R for G-invariant twosided ideals I. There are examples for a ring R and a group G such that ${}_{R*G}R$ is self-projective and a selfgenerator but not a projective generator.
Finally we consider the following question:
Which assumptions on R and G imply that we get a ring structure on the quasi-injective hull of the $R * G$-module R?

M.F. Yousif

(Quasi-)continuous rings with restricted chain conditions

A ring R is called a *left CS-ring* if every left ideal of R is essential in a direct summand of R. R is called *left continuous* if R is a left CS-ring and if every left ideal of R which is isomorphic to a direct summand of R is itself a direct summand of R. R is called *left quasi-continuous* if R is a left CS-ring and if I and J are left summands of R with $I \cap J = 0$, then $I \oplus J$ is a summand of R. Every continuous ring is quasi-continuous and there are examples of quasi-continuous rings which are not continuous. A well-known result of Y. Utumi asserts that a twosided continuous twosided artinian ring is quasi-Frobenius. We will show that Utumi's result can be extended to quasi-continuous rings with DCC on essential left ideals. Utumi's result was also extended by Jain, López-Permouth and Rizvi to rings with ACC on essential left and right ideals, by Camillo and Yousif to rings with ACC on annihilators, and by Ara and Park to rings with $R/\mathrm{Soc}_R R$ left Goldie. In all of these results the assumption that R is twosided continuous has not been weakened. On the other hand a result of Armendariz and Park asserts that a left self-injective ring with $R/\mathrm{Soc}_R R$ (or $R/\mathrm{Soc} R_R$) is left Goldie is quasi-Frobenius. We show that all of the above mentioned results can be extended to one singular unifying result. (Joint work with K. Nicholson.)

J. ZELMANOWITZ

Orders in semiprimary rings and a question of K.I. Beidar

The concept of "dominant submodules" was introduced for application to the study of orders in artinian rings (N is called a *dominant submodule of* M if $\mathrm{ann}(L) = \mathrm{ann}(L \cap N)$ for every submodule L of M). In extending consideration to orders in semiprimary rings, one encounters the question:
If R is a left order in a semiprimary ring Q and T is an ideal of R, must QT be an ideal of Q?
For brevity call a left order with this property a *steady* left order, otherwise *unsteady* left order. (For instance left orders in left noetherian rings are always steady.) Earlier, K.I. Beidar had asked a closely related question:
If R is a PI-algebra with classical quotient ring (or Ore localization) Q, must R be steady?
We can characterize steady left orders in semiprimary rings by means of chain conditions and dominance relationships of left ideals. We also produce a class of examples of PI-algebras which are unsteady orders in semiprimary rings, thus answering Beidar's question in the negative. (Joint work with M.S. Li.)

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Problem Session

G. BACCELLA

A ring is called a *right SV-ring* if it is a semiartinian right V-ring.

1. Find modules M_R (R any ring), as general as possible such that $\mathrm{End}(M_R)$ is a right or left (or both) SV-ring!
2. If R is a right SV-ring then every right R-module is QF-3′ (i.e. it cogenerates its injective hull). The converse is true if R has all primitive factor rings artinian; is it possible to remove this restriction?
3. If R is a right SV-ring and $\tau \in$ tors-R is any hereditary torsion theory, then there exists a module ${}_RA$ such that $\underline{T}_\tau = \{M_R \mid A \otimes_R M = 0\}$. Moreover ${}_RA$ can be chosen to be semisimple. Characterize those rings whose torsion theories arise in this way!
4. Are right SV-rings hereditary? (True in the case of finite Loewy length.)
5. Which right SV-rings satisfy general comparability?
6. Which right SV-rings are unit-regular?
7. Which groups can appear as $K_0(R)$ for right SV-rings?

G. BRODSKII

Characterize rings over which every right module is a direct sum of $AB5^*$-modules!

J. CLARK

Is there an example of a ring in which every left ideal is idempotent but with a right ideal which is not idempotent?

A. FACCHINI

Let R be an associative (non-commutative) ring with 1, M a right artinian R-module, $E = \mathrm{End}(M_R)$ its endomorphism ring with Jacobson radical $J(E)$. Then we know that the ring $E/J(E)$ is semisimple artinian, but we know that E is not necessarily noetherian (neither on the left nor on the right), that it can have infinite Goldie dimension, and that the semisimple $E/J(E)$-module $J(E)/J(E)^2$ can be not finitely generated.

Suppose R is a ring, M_R an artinian module and suppose that its endomorphism ring E is commutative. Must E have just a finite number of minimal prime ideals?

Reference: R. Camps, A. Facchini, *The Prüfer rings that are endomorphism rings of artinian modules*, to appear in Comm.Alg.

J.L. GÓMEZ PARDO

Let R be a left hereditary ring. It is known that if $E({}_RR)$ is finitely presented then R is left artinian (and with Morita duality).

If $E({}_RR)$ is just assumed to be finitely generated, must then R be left artinian?

Reference: J.L. Gomez Pardo, N.V. Dung, R. Wisbauer, *Complete pure-injectivity and endomorphism rings*, to appear in Proc.AMS.

S.K. Jain

1. Let R be regular. Suppose each cyclic R-module has a cyclic injective hull. Is R self-injective? It suffices to show $\mathrm{cl}(I \oplus J) = \mathrm{cl}(I) \oplus \mathrm{cl}(J)$ for right ideals I, J.
2. Suppose each proper cyclic module is quasi-injective. It is known that such a ring is either prime or semiperfect. Characterize such rings when R is prime! Complete characterization for the semiperfect case is known. Also, if R is a domain, then it is known that R is right Ore.
3. Suppose M is weakly injective and continuous. Is M injective? (True if M is finitely generated.)
4. Suppose each R-module is weakly injective. Is R hereditary? R is known to be semiprime right noetherian and left Goldie.
5. Characterize rings over which products of weakly projective modules are weakly projective!

S.R. López-Permouth

A ring is called a *right QI-ring* if every quasi-injective right module is injective.

1. Boyle's conjecture: A right QI-ring is right hereditary.
2. Is every right QI-ring a left QI-ring?
3. What are the rings over which every right module is weakly injective?

B.J. Mueller

Let R be a right and left uniserial prime ring. Does this imply that R is a domain?

S.S. Page

Find the structure of the "rest" of the FPF-rings! It is known that a noetherian FPF-ring is a product (finite) of Dedekind rings and a quasi-Frobenius ring. The commutative FPF-rings are fairly well understood as are the regular FPF-rings. All these rings have a "nice" arithmetic.

S.T. Rizvi

1. Is every right CS semiprime ring right non-singular? It is known that if R is right quasi-continuous and non-singular then R is semiprime.
2. If R is twosided CS and non-singular, is R semiprime? It it known that if R is right CS and non-singular or semiprime then R is a Baer ring.

P. VÁMOS

Major problems

1. Determine the limits of "knowable theory"!

• Undecidable problems, i.e. B. Osofsky's result on the projective dimension of rational functions.

• Kaplansky's test problems for general rings, this leads to the area of for which modules will the Krull-Schmidt theorem hold? For which ones (projective, injective, etc.) do we have a set of complete invariants?

2. Determine the extent (in terms of rings, modules) of existing effective methods, i.e. localization → completion etc.

Esoteric minor problems

1. $\sim$ will denote a Morita equivalence. If $R \sim R^{op}$ does there exist a ring S such that $S \simeq S^{op}$ and $S \sim R$?

2. Let R be a valuation ring (chain ring), commutative, with P minimal prime such that the canonical map $R \to R_P$ is injective. Is R a factor of a valuation domain?

3. Characterize commutative rings R satisfying:

(i) R has a unique (only one) prime ideal;

(ii) there is a function L : {submodules of finitely generated R-modules} $\to \mathbb{R}^+$ such that L is additive over short exact sequences and $L(R) = 1$.

4. Is there a ring extension A of $\mathbb{R}$ such that ${}_R\mathbb{R} = {}_A\mathbb{R}$ is the only simple left A-module and ${}_A\mathbb{R}$ is injective?

M.F. YOUSIF

1. Let R be a left self-injective ring such that $R/\mathrm{soc}_R R$ has a.c.c. on left annihilators. Is R a QF-ring?

2. Let R be left continuous with $R/\mathrm{soc}_R R$ left Goldie. Is R left artinian?

J. ZELMANOWITZ

A module is called a *minimal qi-module* if it contains no proper quasi-injective submodule.

1. If M is a minimal qi-module is $\mathrm{End}(M)$ a division ring?

2. If M is a minimal qi-module must M contain a non-zero compressible submodule?