MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Dynamische Systeme

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Die Tagung fand unter Leitung von H. Hofer, J. Moser und E. Zehnder (ETH Zürich) statt.

Vortragsauszüge

S. ANGENENT:
Examples of singular solutions of mean curvature flow

If a symmetric cardiod-like convex plane curve evolves by its curvature, then it will form a cusp like singularity in finite time. The rate with which the curvature becomes ∞ is $K(t) = (1 + o(1))\sqrt{\frac{\ln\ln\frac{1}{T-t}}{T-t}}$. The shape of the cusp thus formed is $y = (\frac{\pi}{4} + o(1))\frac{x}{\ln\ln\frac{1}{x}}$ (i.e. the cusp is very flat cusp.) I also showed some embedded rotationally symmetric solutions to the mean curvature flow whose curvature does not become ∞ at the rate suggested by scaling.

V. BANGERT:
Closed geodesics on S^2

The talk complemented J. Frank's talk in that it explained the remaining part of the proof of the existence of infinitely many closed geodesics on any Riemannian S^2. Some indications of the ideas and methods in the recent results of N. Hingston were given. The results give a new variational proof for the above-mentioned fact and moreover an estimate of growth of the number of closed geodesics as a function of their lengths.

M. BIALY:
Hopf's theorem for convex billiards

In the present talk I consider billiards in convex compact planar domains. The main result states that if the phase space of the billiard ball map is foliated by not null-homotopic continuous invariant curves then the domain is circular. The result solves in part the Birkhoff's conjecture that only elliptic domains billiards are integrable. The variational nature of this result can be stated in the following form: the only convex billiards without conjugate points are circular. In this form this is analogous to E. Hopf's theorem on Riemannian metric without conjugate points on T^2.

M. CHAPERON:
Invariant manifolds and normal forms

We sketch the (simple) proof of a generalised stable manifold theorem and explain how various normal form results can be deduced from it.

A. CHENCINER:
Lagrange reduction of the N body problem and relative equilibria

This is a common work with A. Albouy, based on concepts he introduced in his thesis. We represent n masses up to translation and rotation in some Euclidean space by a symmetric tensor $B \in \mathcal{D} \otimes_s \mathcal{D}$, where the "dispositions" space $\mathcal{D}$ is defined as the space of elements $(u_1, \ldots, u_n)$ of $\mathbb{R}^n$ whose "center of mass" is at $0(\sum m_i x_i = 0)$. Endowing $\mathcal{D}$ with the scalar product $\sum_{i=1}^n m_i x_i y_i$, we think of B as the symmetric endomorphism of the Euclidean space $\mathcal{D}$ defined by the matrix

$$\begin{pmatrix} 0 & \cdot m_j r_{ij}^2 \\ & \ddots \quad 0 \end{pmatrix},$$

where r_{ij} is the distance between the masses m_i and m_j. The potential is a function on $\mathcal{D} \otimes_s \mathcal{D}$ whose derivative at B is an element of $(\mathcal{D} \otimes_s \mathcal{D})^*$, that is a quadratic form on $\mathcal{D}$. Representing it by a symmetric endomorphism A of $\mathcal{D}$ (the "Cayley matrix") we get the reduced Newton equations in

$$\mathcal{D}^2 \otimes_s \mathcal{D}^2 \ni \Phi = \begin{pmatrix} C & D \\ {}^tD & B \end{pmatrix}, D = D_s + D_a, B, C, D_1 \in \mathcal{D} \otimes_s \mathcal{D}, D_a \in \wedge^2 \mathcal{D} :$$
$$\dot{B} = 2D_s, \dot{C} = -(AD_s + D_s A) + [A, D_a], \dot{D}_s = C - \tfrac{1}{2}(AB + BA), \dot{D}_a = -\tfrac{1}{2}[A, B],$$

with first integrals $H = \frac{1}{2}$ trace $C + \omega(B)$ (the Hamiltonian) and spectrum $(J\Phi)$ (invariants of the angular momentum) ($J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$). In the case of 3 bodies, these are exactly the 10 equations and 3 integrals of Lagrange's 1772 "Essai sur le Problème de trois corps". Note that this reduction considers together n-body problems in $\mathbb{R}^p$ for all p (it is enough to take $p = 2(n-1)$). Looking at singular points, we find a necessary and sufficient condition for B to be realised in same Euclidean space as a relative equilibrium, namely $[A, B] = 0$. This is equivalent to B being a critical point of the potential function restricted to the isospectral manifold of B (i.e to the set of B'_s defining the same "solid body", i.e. having the same ellipsoid of inertia).

K. CIELIBAK:
Elliptic methods for homoclinic orbits

(Joint work with E. Séré) Let M be a compact Riemannian C^∞ manifold and $H \in C^\infty(S^1 \times T^*M, \mathbb{R})$ a timedependent Hamiltonian having a hyperbolic critical point x_0 and satisfying certain growth conditions. Associated to H and the natural symplectic structure on T^*M we have the Hamiltonian system $\dot{x} = X_H(t,x)$.
Let $C := \{x \in C^\infty_{\text{contr.}}(\mathbb{R}, T^*M) | \dot{x} = X_H(t,x), x(t)t \to +\infty \text{ as } t \to \infty\}$. The time translation defines an action of $\mathbb{Z}$ on C.
Theorem: If $\pi_1(M)$ is finite, then $C/\mathbb{Z}$ is infinite.
The theorem is proved by studying the space X of "gradients flow lines" associated to the action functional. Arguing indirectly, we assume that $C/\mathbb{Z}$ is finite. Then X turns out to be compact up to integer translations in time, which implies that the cohomology of X must be trivial above some dimension. On the other hand, by solving a boundary value problem for the Cauchy-Riemann operator, we can show that the $\mathbb{Z}_2$-Alexader-Spanier cohomology of the loop space ΩM injects into X. A result of Sullivan on the cohomology of ΩM then provides nontrivial cohomology in X for infinitely many dimensions, yielding the desired contradiction.

J. DENZLER:
On nonpersistence of breathers for the perturbed Sine Gordon equation

Up to one exception, and as a consequence of 1st order perturbation theory only, it is impossible that a large portion of the breather family

$$u^*(x,t;m) = 4 \ \arctan \frac{m \sin \omega t}{\omega \cosh mx}, m^2 + \omega^2 = 1, 0 < m, \omega < 1$$

could persist under any nontrivial perturbation of the Sine Gordon equation of the form

$$u_{tt} - u_{xx} + \sin u \ = \ \varepsilon\Delta(u,\varepsilon) \ = \ \varepsilon\Delta(u) + O(\varepsilon^2)$$

where Δ is an analytic function in a neighborhood of $u = 0$. This improves results by Birnir, McKean, Weinstein by allowing the domain of analyticity of Δ to be arbitrarily small. - For the exceptional one-dimensional space of perturbation functions, nonpersistence can also be proved by 2nd order perturbation theory, whereas all 1st order necessary persistence conditions are satisfied in this case. The persistence of any single breather remains an open question; for each, an infinite dimensional space of perturbation functions exists for which 1st order information is not sufficient to rule out persistence.

F.N. DIACU:
Collision and near-collision solutions of the 3-body problem with Maneff's gravitational law

We consider the 3-body problem given by quasihomogeneous potentials of the form $W = U + V$, where U and V are homogenous functions of degree $-a$ and $-b$ respectively, with $1 \leq a < b$. Using McGehee-type transformations we blow up the triple collision singularity and paste instead a collision manifold C. We show that the system has 10 nonhyperbolic equilibria, all on the collision manifold, which correspond to central configurations (*c.c.*), and which depend on b. V is taken of the form

$$V = \text{ const } \cdot \sum_{1\leq i<j\leq 3} \frac{\alpha(m_i, m_j)}{(\text{ dist } (m_i, m_j))^b},$$

where $m_i, i = 1,2,3$ are the masses and α is a symmetric, positive function, i.e. $\alpha(m_i, m_j) = \alpha(m_j, m_i) > 0$. There are 5 *c.c.*, 3 collinear and 2 triangular ones. The triangular *c.c.* are equilateral if and only if $\alpha(m_i, m_j) = \text{const} \cdot m_i m_j$. For $b \neq 2$ every triple-collision solution tends to a *c.c.*. For $b = 2$ we first consider the rectilinear problem. Note that the case $a = 1, b = 2$ corresponds to Maneff's gravitational law, which explains, with a very good approximation, the perihelion advance of Mercury as well as the motion of the Moon. We show that for $b = 2$ there exist solutions which tend to a triple collision without forming asymptotically a *c.c.*. The isoceles problem, of the same case $b = 2$, is further studied, and the same type of triple-collision orbits are discovered. Due to the collision manifold technique, information on the behaviour of near-triple-collision orbits is obtained. The center manifold is studied as well as the network of homoclinic and heteroclinic orbits. For an anisotropic Maneff law, we show that there is a subcritical pitchfork bifurcation of the equilibria on the collision manifold.

J.P. ECKMANN AND C.E. WAYNE:
The stability of travelling waves

We consider parabolic equations of the form $u_t = u_{xx} + uF(|u|)$ on the line, for $F(\xi)$ such as $1 - \xi$ or $1 - \xi^2$. There is a critical travelling wave $u(x,t) = f(x - ct), c = 2\sqrt{F(0)}$. To determine its stability under complex perturbations, consider initial conditions $u_0(x) = f(x)(1 + r(x))e^{i\varphi(x)}$. Define the "energies" $E_n = \int \sigma(|\partial_x^n r|^2 + |\partial_x^n \varphi|^2)$. We show that when σ satisfies $(\sigma' - (c + 2f'/f)\sigma)' \leq 0$, (for example $\sigma(x) = f^2(x)e^{cx}(1 + \int_x^\infty \frac{1}{f^2(y)e^{cy}} dy)$), then if the initial values of r and φ are small, for some $K >> 1$ one has

$$\partial_t E_0 \leq -\frac{1}{2} E_1 ; \quad \partial_t(KE_0 + E_1) \leq -\frac{1}{2}(E_1 + E_2) .$$

This implies that φ and r tend to zero in the moving frame, as $t \to \infty$ and, as a consequence, $u(x,t) \to f(x - ct)$.

Y. ELIASHBERG:
2-knots in symplectic geometry

The talk is devoted to unknotedness phenomena in 4-dimensional symplectic geometry. Typical results are:

1. Any ω-positive 2-disc Δ embedded in the standard symplectic $(\mathbb{R}^4_+, \omega)$ with the boundary in $\mathbb{R}^3 = \partial\mathbb{R}^4_+$ is unknotted.

2. Any Lagrangian torus $T \subset T^*T^2$ which is homologous to the 0-section is isotopic to the 0-section.

The second result is joint work with L. Polterovich.
One can define, using symplectic tools, a topological invariant for S^2-knots in $\mathbb{R}^4$. Its exact computation is, probably, a difficult problem.

H. ELIASSON:
Biasymptotic solutions for perturbed integrable Hamiltonian systems

An integrable Hamiltonian system in d degrees of freedom has, for a generic perturbation, an invariant hyperbolic torus of dimension $d-1$ whose stable and unstable manifolds intersect along at least d different orbits. The unperturbed Hamiltonian is supposed to be integrable in action angle variables and satisfy the usual non degeneracy condition. The existence of such invariant tori is well-known and not at all restricted to dim $d-1$. The intersection problem is totally different. It is not a perturbation problem but is solved by identifying it with the number of critical points of a function on a $(d-1)$-dimensional torus. This method only works for tori of precisely this dimension.

J.N. FORNAESS AND N. SIBONY:
Holomorphic dynamics in higher dimensions

We study Fatou Julia theory in $\mathbb{P}^k, k \geq 2$. Let $f : \mathbb{P}^k \to \mathbb{P}^k$ be holomorphic map $f = [f_0 : \cdots : f_k]$. f_i are homogeneous polynomials of degree $d \geq 2$. We define the Fatou set of t as the domain of equicontinuity of (f^n) and the Julia set as the complement of the Fatou set in $\mathbb{P}^k$. Let $G = \lim \frac{1}{d^n} \log |F^n|$ where $F = (f_0, \ldots, f_k)$. There exists a $(1,1)$ positive closed current T in $\mathbb{P}^k$ such that $n^*T = dd^cG$. Here $n : \mathbb{C}^{k+1} \to \mathbb{P}^k$.
Theorem 1. Support T = Julia set. J is connected. The Fatou components are domains of holomorphy, here $k \geq 2$.
Corollary 2. The critical set intersects J. If (f^{n_i}) is equicontinuous then (f^n) is equicontinous.
Theorem 3. If $T^l = T \wedge \ldots \wedge T$ then support T^l is connected provided $2l \leq k$. $\mathbb{P}^k|$ supp T^l is $k-l$ pseudoconvex. The Hausdorff dimension of support T^l is $> 2(k-l)$.
Theorem 4. If $\mu = T^k$, the measure μ is mixing and of maximal entropy, moreover $f^*\mu = d^k\mu$.

G. FORNI:
Destruction of invariant curves by analytic perturbations

We give converse KAM results in the analytic topology on the stability of invariant curves of exact area-preserving monotone twist maps of the annulus $S^1 \times \mathbb{R}$, following ideas and techniques developed by M. Herman and J. Mather. We obtain the following theorems:
Theorem A: Let γ be an invariant curve of rotation number $\omega \in \mathbb{R}\backslash\mathbb{Q}$. Let $(p_n/q_n)_{n\in\mathbb{N}}$ be the sequence of approximants of the continued fraction expansion of ω. Then, if

$$\text{(I)} \qquad \limsup_{n\to+\infty} \frac{\log\log q_{n+1}}{q_n} > 0$$

the curve γ can be "destroyed" by an arbitrarily "small" analytic perturbation ("small" in the analytic topology).
Theorem B: If, furthermore γ is embedded in the completely integrable family, the condition

on ω becomes

$$\text{(J)} \qquad \limsup_{n\to+\infty} \operatorname{Log} q_{n+1}/q_n > 0 .$$

J. FRANKS:

Rotation vectors for surface diffeomorphisms

This talk considers homological rotation vectors for area preserving diffeomorphisms of compact surfaces which are homotopic to the identity. There are two main results. The first is that if 0 is in the interior of the convex hull of rotation vectors for such a diffeomorphism then f has a fixed point of positive index. The second result asserts that if f has a vanishing mean rotation vector, then f has a fixed point of positive index.

E. GIROUX:

Distinguishing contact structures on T^3

The main result is the theorem which shows that on the 3-torus T^3 there exists at least 2 non-isomorphic tight contact structures. This is the first example of two non-isomorphic tight contact structures on a 3-manifold. The main idea of the proof is to pass to the symplectization and to study the set of exact embedded Lagrangian tori there via a theorem of Gromov.

A. GIVENTAL:

Mirror symmetry for projective spaces and Floer homology

Discussing quantum cohomology of Kähler manifolds constructed by string theorists, as a counterpart of Floer homology, we describe symplectic properties of correlation differential equations, relate the so called mirror conjecture, usually formulated for Calabi-Yau manifolds, to the property of the differential equations in order to have hypergeometric solutions, and verify the conjecture in this extended form for complex projective spaces.

H. HOFER:

The Weinstein conjecture in dimension three

A contact form λ on a closed three dimensional manifold M is a 1-form λ satisfying $\lambda \wedge d\lambda$ is a volume. We call λ over twisted if there exists an embedded disc $D \subset M^3$ such that $T\partial D \subset \xi|\partial D$ where $\xi = \text{kern}\ (\lambda) \to M$ is the contact structure associated to λ. Moreover one assumes $T_x D \neq \xi_x$ for all $x \in \partial D$. If such a disc does not exist we call λ tight. The following theorem is true, where X is the so called Reeb-vectorfield associated to λ defined by $\lambda(X) = 1, d\lambda(X,\cdot) = 0$:

Theorem. The ODE $\dot{x} = X(x)$ has a contractible periodic orbit provided one of the following is true

(a) $\quad M = S^3$

(b) $\quad \pi_2(M) \neq 0$

(c) $\quad \lambda$ is over twisted.

This solves many cases of the Weinstein conjecture (in dimension 3) which claims that for every pair (M, λ), M closed and λ a contact form, the associated X admits periodic solutions.

A. KATOK:
Differentiable rigidity of smooth action of $\mathbb{Z}^k$ and $\mathbb{R}^k; k \geq 2$

Consider an action of the Abelian group $\mathbb{R}^k$ or $\mathbb{Z}^k, k \geq 2$ by diffeomorphisms of a compact manifold. Among Anosov actions of $\mathbb{R}^k$ there is a natural class of principal actions which are described by a uniform algebraic construction. Prime examples are suspensions of $\mathbb{Z}^k$ actions by automorphisms of tori and nilmanifolds and Weyl chamber flows. For a principal Anasov action the first untwisted cohomology with C^∞ (Hölder) coefficients trivializes via C^∞ (Hölder) coboundaries. This implies the existence of only trivial time changes and Hölder rigidity of the action. For Weyl chamber flows and in many other cases local C^∞ rigidity is also proven.
Any Anosov action of $\mathbb{Z}^{n-1}$ on $\mathbb{T}^n, n \geq 3$ is differentiably conjugate to an action by automorphism.
Then we consider ergodic invariant measures of $\mathbb{Z}^k$ and $\mathbb{R}^k$ actiom (not necessarily Anosov) with non-vanishing Lyapunov exponents. Two crucial phenomenon which are expected to appear are
(i) cocycle super-rigidity
(ii) in the case if the measure has positive entropy in one-dimensional directions, absolute continuity of the measure itself.
Both these properties are proven for $\mathbb{Z}^2$ actions on 3-dimensional manifolds under the assumption that any two Lyapunov exponents are not proportional.

S. KUKSIN:
Capacities for PDE's

We develop a theory of symplectic capacities for infinite-dimensional phase spaces and apply it to study Hamiltonian partial differential equations. We prove a squeezing theorem for Hamiltonian PDE's and discuss physical the meanings of the assertions.

J. MATHER:
Variational construction of connecting orbits

Let M be a compact, smooth manifold, TM its tangent bundle, $\mathbb{T} = \mathbb{R}/\mathbb{Z}$, and $L : TM \times \mathbb{T} \to \mathbb{R}$ a C^2 "Lagrangian". Suppose L satisfies a Legendre condition, has superlinear growth, and its Euler-Lagrange vector field E_L is complete. For $c \in H^1(M, \mathbb{R})$ choose a closed 1-form η_c on M whose de Rham cohomology class is c. Let $\alpha(c) = \min\{\int (L - \eta_c) d\mu | \mu$ is an E_L-invariant measure $\}$. For $m, m' \in M$ set $h_c(m, m') = \min \int_0^1 (L - \eta_c)(d\gamma(t), t) dt$, the minimum taken over curves in M such that $\gamma(0) = m, \gamma(1) = m'$ Let h_c^n denote the n-fold conjunction of h_c with itself, $h_c^\infty = \lim_{n \to \infty} \inf h_c^n$, $B_c(m) = h_c^\infty(m, m), \sum_c = \{m \in M | B_c(m) = 0\}$ and $B_c^*(m) = \min\{h_c^\infty(m, \xi') - h_c^\infty(\xi, \xi') | \xi, \xi' \in \sum_c\}$. Let $W_L = \{c \in H^1(M, L) |$ the inclusion $H_1(\sum_c^*, \mathbb{R}) \to H_1(M, \mathbb{R})$ is the zero homomorphism$\}$, where $\sum_c^* = \{m \in M | B_c^*(m) = 0\}$.

Theorem. W_L is open. If c and c' are in the same connected component of W_L, the there exists an E_L-trajectory whose α-limit set is in the set M_c of c-minimal trajectories and whose ω-limit set is in $M_{c'}$.

R. MC GEHEE:
Resonance surfaces for forced oscillators

A two-parameter family of planar diffeomorphisms can be written $x \mapsto f_\mu(x)$, where $x \in \mathbb{R}^2$ is a point in the state space and $\mu \in \mathbb{R}^2$ is a point in the parameter space. The q^{th} order "resonance surface" is the set of all points (x, μ) such that x is a periodic point for $f_\mu(x)$ with least period q. The projections of these surfaces into the parameter space are standard objects of study in bifurcation theory and include, for example, the so-called "Arnold tongues". Computer visualization methods were used to generate video pictures of some resonance surfaces for a family of diffeomorphisms related to forced oscillators. These pictures illustrate some recent results concerning the global structure of resonance surfaces.

R. PEREZ-MARCO:
Fixed points, holomorphic vector fields and circle diffeomorphisms

We present some recent constructions relating three different problems: singularities of holomorphic foliations in the Siegel domain, dynamics of neutral holomorphic fixed points and analytic circle diffeomorphisms. The first construction (joint work with J.C. Yoccoz) shows the existence of singularities for holomorphic vector fields. The second construction obtains analytic circle diffeomorphisms from indifferent fixed points. These constructions had several consequences among which they avoid to make different proofs for the linearizability results and determine the optimal linearization conditions.

L. POLTEROVICH:
Symplectic packings and algebraic geometry (joint work with Dusa McDuff)

Let M be a symplectic manifold of volume 1. What is the maximal volume which can be filled inside M by k disjoint equal standard symplectic balls? This quantity, which we denote by $v(M,k)$, can be computed precisely for certain M and k. In particular, $v(B^{2n}, m^n) = 1$ (that is *full filling* exists), and $v(B^4, 8) = 288/289$ (in this case we have *packing obstructions*). It turns out that the symplectic packing problem is closely related to several subjects of algebraic geometry. Our constructions of full fillings are based on symplectic versions of complex blowing up and holomorphic branched coverings. The computation of packing obstructions in dimension 4 in many cases can be reduced to the study of rational exceptional curves on complex surfaces.

G. POPOV:
Invariants of the period spectrum of contact manifolds

Consider a contact manifold (M, σ) where M is a smooth compact manifold of dimension

$2n+1, n \geq 1$, and σ is a contact form on it. The contact vector field X is defined by the inner products $i(X)\sigma = 1, i(X)d\sigma = 0$. The period spectrum $\mathcal{L}(M,\sigma)$ is the set of periods of all periodic trajectories of X. We are concerned with invariants of the period spectrum for continuous deformations σ_s of the contact form σ_s. The main result says, that for any deformation σ_s such that the period spectrum remains constant, the KAM invariant tori of X_0 associated to an elliptic trajectory γ_0 of X_0 persist along the deformation, and the contact one-forms σ_s are conjugated to each other on the union of the tori.

J. PÖSCHEL:
Cantor manifolds of quasi-periodic oscillations for a nonlinear Schrödinger equation

We consider the nonlinear Schrödinger equation

$$(*) \qquad iu_t = u_{xx} - uu - f(|u|^2)u\,, \quad 0 \leq x \leq \pi\,,$$

with Dirichlet boundary conditions, where $m \in \mathbb{R}$, and f is real analytic near $0 \in \mathbb{C}$ with $f'(0) \neq 0$. We show that for all $m \in \mathbb{R}$ and all $n \geq 1$, through $u \equiv 0$ there exist invariant manifolds with a Cantor set structure, consisting of real analytic invariant n-tori which carry t-quasi-periodic solution of $(*)$. These manifolds are locally perturbations of corresponding invariant linear spaces of the linear equation with $f \equiv 0$, and they are asymptotically dense as one approaches $u \equiv 0$. - These results are based on an infinite dimensional KAM theory. This is joint work with Sergej Kuksin.

M. SCHWARZ:
A multiplicative structure on Floer cohomology

We consider the Floer cohomology HF^* for a closed symplectic manifold (M^{2n},ω) satisfying $\omega|\pi_2(u) = c_1|\pi_2(u) = 0$. The chain groups with coefficients in $\mathbb{Z}_2$ are generated by the 1-periodic solutions of a Hamiltonian system for $H : S^1 \times \mathbb{R}$, all assumed to be non-degenerate. The coboundary operator is defined by counting mod2 the solutions of the nonlinear PDE $\partial_s u + J\partial_t u + \nabla H(t,u) = 0$, $u : \mathbb{R} \times S^1 \to M$, which connect 1-periodic solutions.
The content of the talk was to construct a generalization of this S^1-cobordism from $\mathbb{R} \times S^1$ to any surface with cylindrical ends. Starting from a compact surface Σ_q^p of genus g with p boundary components oriented as incoming ends and q components as outgoing ends, it is possible to set up an analogous PDE consisting of the $\bar{\partial}$-operator $+a$ zero-order Hamiltonian term on each cylindrical end. Under generic conditions the set of solutions with ends fixed in $p+q$ periodic solutions is a finite dimensional manifold, compact in dimension 0. Counting these solutions finally defines an operator $Z(\Sigma_q^p) : \otimes^q HF^* \to \otimes^p HF^*$ of degree $2n(p-1+g)$. By a decomposition result this is reduced to the basic elements $e = Z(\Sigma_0^2) \in HF^0, m = Z(\Sigma_2^1| : HF^* \otimes HF^* \to HF^*)$ and $\beta = Z(\Sigma_2^0) : (HF^* \oplus HF^*)^{2n} \to \mathbb{Z}_2$, where m is a multiplication, e the unit and β a non-degenerate bilinear form. The methods are based on the analysis for the non-linear elliptic operator $\bar{\partial}_J$.

S. TROUBETZKOY:
Coding polygonal billiards

We code the forward orbit of a point by the sequence of sides which it hits. We show that the symbol sequence uniquely determines the point x if and only if the orbit of x is not periodic. As a corollary we get an improved version of a theorem of Boldrighini, Keane and Marcletti: for any x either the orbit of x is periodic or the closure of the forward orbit contains at least one vertex. As a second corollary we get that the topological entropy of polygonal billiards is zero, simplifying Katok's proof of this fact. A final corollary is the positive expansiveness of the billiard flow (up to periodic families) in phase space. In higher dimensions (polyhedral billiards) additional phenomena occur.

M. VIANA:
Persistence attractors displaying multidimensional (nonuniform) expansion

We describe a construction of smooth transformations with multidimensional expanding behaviour: several positive Liapunov exponents (at almost every point). Our construction is based on coupling uniformly hyperbolic systems (expanding maps, solenoid maps, ...) with transformations which combine some expanding behaviour with very strong contractions (quadratic maps, Hénon maps, ...). The systems obtained in this way are not uniformly hyperbolic and yet this behaviour is rather persistent: every transformation in a full neighborhood also has (the same number of) positive Liapunov exponents.

E. WAYNE:
The renormalization group and long time asymptotics of dispersive equations

We consider the nonlinear dissipative and dispersive equations

$$\partial_t u \;=\; Mu + \partial_x^3 u + u^p \partial_x u \,, \quad x \in \mathbb{R}, t \geq 1 \,.$$

The linear operator M is given by $(Mu)\hat{}(k) = -|k|^{2\beta}\hat{u}(k)$. (Here $\hat{}$ denotes Fourier transformation). Such equations arise as models of the propagation of waves on the surface of a fluid. We use renormalization group methods to prove that solutions of these equations converge to a universal form as $t \mapsto \infty$. More precisely, for sufficiently small, smooth, initial data, and for $\frac{1}{2} < \beta \leq 1, p \geq 2$, we find a ($\beta$-dependent) function $f^*(x)$ such that $u(x,t) \mapsto \frac{A}{t^{\frac{1}{2\beta}}} f^*(\frac{x}{t^{\frac{1}{2\beta}}})$ as $t \to \infty$. This is joint work with J. Bona and K. Promislow.

L.-S. YOUNG:
Lyapunov exponents of quasi-periodic cocycles

This talk is about the Lyapunov exponents of $SL(2,\mathbb{R})$-valued cocycles over rotations of the circle. We hope that analyses of this type will lead to a better understanding of whether geometric stretching corresponds to positive Lyapunov exponents for, say, area preserving diffeomorphisms of surfaces. Our main results are roughly as follows:
We consider (T_α, A_t), where $(T_\alpha : S^1 \to S^1$ is rotation by $2\pi\alpha$, and $A_t : S^1 \to SL(2,\mathbb{R})$,

$t \in [0,1]$, is a generic 1-parameter family of C^1 maps satisfying $||A_t(x)|| \approx \lambda$ and $||\frac{dA_t^{\pm}}{dx}|| \leq \lambda$ for some large λ. We prove that

1. The set $\{(\alpha, t) : (T_\alpha, A_t)$ has exponents $\approx \pm \log \lambda\}$ has nearly full measure,

and if $(T_\alpha; A_t)$ is not uniformly hyperbolic, then:

2. The closure of $\{(\alpha, t) : \alpha = \frac{p}{q}$ and $A^q(x)$ is elliptic for a positive measure set of $x \in S^1\}$ has nearly full measure.

Both sets tend to full measure as $\lambda \to \infty$.

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