MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Geometric Methods in Theoretical and Computational Mechanics

July 25 – July 31, 1993

This conference was organized by J.E. Marsden (Berkeley) and K. Kirchgässner (Stuttgart). The main topics treated were

- Mechanical/geometric aspects of numerical integrators (symplectic integrators, energy-momentum integrators, other numerical algorithms using geometric ideas).
- Ginzburg–Landau equations, modulation equations, problems with infinite space domains (including nonlinear waves, fronts, shocks, and pattern formation, both analytical and computational).
- Limiting problems (thin domains, small dissipation, surface tension, fronts).
- Dynamical systems and geometric approaches in mechanics and control theory (including bifurcation theory, constraint theory and geometric phases).
- Bridges with pure mathematical areas of geometric mechanics (including symplectic geometry and topological aspects of mechanics).

The organizers and all the participants of this conference want to thank the Oberwolfach Institute for providing a stimulating atmosphere for discussion and the exchange of ideas.

Abstracts

Solitons and geometric phases
Mark S. Alber

This paper develops a new complex Hamiltonian structure for a new family of umbilic soliton–like solutions and homoclinic orbits for a class of nonlinear integrable equations such as the nonlinear Schrödinger, Sine–Gordon and Korteweg–de Vries hierarchies of equations. In particular, this yields Hamiltonians and angle–representations on the non-compact invariant varieties for the n–dimensional separatrix solutions. These representations are used for investigating transition through a homoclinic orbit of the sine–Gordon equation and for describing breather–kink interaction. It is also shown that the Hamiltonian flow associated with homoclinic orbits introduced by Devaney for the C. Neumann problem of the motion of a particle on the n–dimensional sphere in the field of a quadratic potential coincides with the soliton x–flow of the KdV equation. This result, together with the methods of complex geometry and asymptotic reduction, leads us naturally to the introduction of homoclinic and separatrix geometric phases.

Almost Poisson Integration of Rigid Body Systems
Mark Austin

This work was completed in collaboration with P.S. Krishnaprasad and L.S. Wang. In this talk, we describe the numerical intregration of Lie–Poisson systems using the mid-point rule. Since such systems result from the reduction of Hamiltonian systems with symmetry by Lie Group actions, we also present examples of reconstruction rules for the full dynamics. A primary motivation is to preserve in the integration process various conserved quantities of the original dynamics. A main result of this paper is an $O(h^3)$ error estimate for the Lie–Poisson structure, where h is the integration step-size. Results from numerical experiments were presented, pointing out good (i.e. energy, momentum conservation), points and weaknesses of our algorithm (i.e. extension of attitude period). The talk concluded with a few remarks on our latest work - embedding extrapolation techniques inside traditional Newmark Integrators.

Nonholonomic Reduction
Larry Bates

The geometric structure of Hamiltonian systems with nonholonomic constraints, reproduced under reduction by symmetry, is described.

Several examples of such mechanical systems will also be demonstrated, and we discuss some of the subtleties of their reduction, reconstruction and dynamical behaviour.

Gradient and Hamiltonian flows in infinite dimensions
Anthony M. Bloch

In this talk, which builds on earlier work with R. Brockett, we discuss the relationships between certain classes of PDE of Hamiltonian and gradient type. In particular, we consider flows on adjoint orbits of the group of area preserving differmorphisms of the annulus. This group may be seen in some sense as an infinite–dimensional model of SU(n). We discuss flows of single bracket type which are Hamiltonian, of double bracket type, which are gradient with respect to a normal metric, and describe their relationship to the dispersionless Toda lattice equations, a completely integrable system. In addition, we describe the connection of this circle of ideas to an infinite–dimensional version of the Schur–Horn–Costant convexity theorem for SU(n).

Integrable Problems in the Dynamics of Systems of Coupled Rigid Bodies
O.I. Bogoyavlenskij

Several classical problems of dynamics are shown to be integrable for the special system of coupled rigid bodies, introduced in this work and called C^k–central configurations. It is proven that dynamics of an arbitrary quadratic potential is integrable in the Liouville sense in terms of theta–functions on Riemann surfaces. Hidden symmetry of the inertial dynamics is disclosed. Separation of rotations of a space station type orbiting system, being a C^k–central configuration of rigid bodies, is proven.

On symmetry and symmetry breaking in rigid body dynamics
A.A. Bourov
R.S. Soulikashvili

It is well known that inertial properties of a rigid body such as the location of the center of mass and the values of the inertia moments play a specific role in its dynamics. It is also well known that the equality between two or three principal moments of inertia implies specific dynamical properties of the body. Such original dynamical properties of a rigid body which admits the specified discrete group of symmetry is brightly exposed treating motion of a such body in the Newtonian force field. For this force field the expressions of the principal moments of inertia are explicitly involved both in the expressions of the moment of inertial forces and the torques of the active forces. We discuss the dynamical properties of such systems and wide opportunities in their dynamics which are interesting both from theoretical and applied viewpoint. We also discuss certain classes of discrete symmetry breaking.

Renormalization groups and front propagation
J. Bricmont

We consider the problem of the stability of fronts and profiles in the Ginzburg–Landau equation. This problem has a long history, starting with the work of Kolmogorov and Fisher on the real-amplitude, version of that equation, which is also a prototypical reaction–diffusion equation.

We prove stability of the slowest front under complex perturbations. We also study the "universal" pattern which is created by having different stationary solutions at infinity which are both stable.

Truncations of Infinite Dimensional Lie Algebras Associated with the Vlasov Equation
Paul Channell

Various symplectic integration techniques have been developed for Hamiltonian dynamics and, more recently, Lie–Poisson integration techniques have been discovered for Hamiltonian systems with noncanonical brackets. However, many Poisson Hamiltonian systems, such as the Vlasov–Poisson equations, ideal magnetohydrodynamics, and elastodynamics are infinite dimensional and the first step in numerical simulation must be truncation to finite dimensional systems that are also Poisson. Though a general solution to the truncation problem does not yet exist, we exhibit a systematic sequence of finite dimensional Poisson truncations for the Vlasov–Poisson equations. Upon implementation the truncation and Lie–Poisson integration has been benchmarked against an n-body code with great success even with low order truncations. For long time simulations the results show excellent energy stability.

Quasi–periodic solutions of the Ginzburg–Landau equation
Arjen Doelman

The Ginzburg–Landau equation is a modulation equation which appears in a very general class of nonlinear stability problems at near–critical conditions. The equation has real coefficients if it is based on an underlying problem with a reflection symmetry. The real Ginzburg–Landau equation has a family of stationary quasi–periodic solutions. We show that this is not a typical feature of the real equation: there is a family of slowly traveling quasi–periodic solutions to any weakly complex Ginzburg–Landau equation. This family merges with the stationary family in the reflection symmetric limit. Furthermore, it is shown that all quasi–periodic solutions are unstable solutions of the full PDE. The relation between quasi–periodic solutions and solitary, front–like, waves is discussed briefly.

Connections on Frame Bundles of Higher Order Contact and its applications to the Theory of Uniform Material Structures

Marek Elzanowski

We deal with linear connections on the bundles of holonomic frames of a differentiable manifold. Given a k–order connection on the bundle of holonomic k–frames we discuss the conditions for its simplicity and local flatness, specially if a given connection is locally generated by a section. We prove that not only any curvature free k–order connection on a holonomic frame bundle which is a prolongation has a vanishing torsion but also that any simple k–order connection which is either curvature free or has a vanishing torsion is locally flat. We also show how these connections induce lower order connections and what are the relations between them in the context of simplicity, prolongation and local flatness. We show how linear connections of k–order appear in the theory of continuous material structures.

Constructive Motion Planning and Time–Optimal Control of a Satellite–Rotor System with Drift

Mike Enos

We consider those motions of an asymmetric rigid body, with an internally controlled rotor spinning about its long principal inertial axis, for which the system has a prescribed, constant, and nonzero total angular momentum.

The problem we consider is one of constructive controllability, i.e., we seek to explicitly construct a simple motion of the body with given endpoints in $SO(3)$, while additionally demanding that the motion "looks" a certain way: In particular, we look for a path that is in a given homotopy class of an appropriate restriction of $SO(3)$.

We show that with appropriate feedback from the rotor we can "straighten out" the dynamics and explicitly obtain such a motion, which is expressible in terms of elementary functions of time.

We also give a full analysis of the time–optimal problem with this system on a class of paths of the above type.

Convection Patterns and the Geometry of Singularities for the Phase Diffusion Equation

Nick Ercolani

We consider the phase diffusion equation for pattern forming PDE's in two spatial dimensions. Such equations arise as modulation equations for periodic structures such as rolls in Rayleigh–Benard Convection. In this work the microscopic equations are modelled by the real Swift–Hohenberg pde: $w_t + (\Delta + 1)^2 w - Rw + w^3 = 0$;
the associated stationary phase diffusion is

$$\nabla \cdot \left(\vec{k} B(h)\right) = 0, \qquad \nabla \times \vec{k} = 0.$$

when $\vec{k} = \nabla\Theta$. The level curves $\Theta(x, y) = const$ describe the rolls. Pattern defects occur at places where $\nabla\Theta = 0$ or is undefined. We find that generically such defects are Legendrian singularities of type A_1, A_2, $D_4^{\pm}$ in Arnold's classification. These singularities are also shown to be quasi-conformally equivalent to generic singularities of quadratic differentials in the regime where the phase diffusion equation is elliptic. The classification is based on the method presented in "Multi-valued Solutions and Branch Point Singularities for Nonlinear Hyperbolic on Elliptic Systems", R. Caflisch, N. Ercolani, Hou, Landis; CPAM, Vol XLVI, 453–499 (1993).

Hamiltonian Structure of generalized affine–scaling vector fields

L. Faybusovich

We consider dynamical systems that solve matrix semidefinite linear programming problems. We discuss geometric properties of these vector fields. Hamiltonian structure, invariance properties etc. We also consider algorithms based on these geometric properties and discuss their complexity estimates.

Local Stability of Critical fronts in Non–linear Parabolic Equations

T. Gallay

For the real Ginzburg–Landau equation and similar non–linear parabolic equations on the real line, we study in some details the (non–linear) stability of the slowest monotonic front solution. In particular, we show that sufficiently small initial perturbations of the front decay to zero like $t^{-3/2}$(as $t \to \infty$) in appropriate weighted norms, and we compute explicitly the leading term in this asymptotic expansion. The proof is based on the Renormalization Group method for parabolic equations (see the talks by J. Bricmont and A. Kupiainen), and requires a careful analysis of the linear operator $\partial_x^2 + \gamma(x)\partial_x$ in the equation for the perturbation.

Singularities of solutions of partial differential systems and Lagrangian varieties

A. Givental

We classify singularities of multiple–valued solutions of higher order systems of partial differential equations in the case when projections of the corresponding integral manifolds of the Cartan distribution in the jet space to the space of independent variables have only Whitney singularities.

Beyond the Eckhaus instability in Ginzburg–Landau's equation

A. van Harten

Ginzburg–Landau's equation is found in non–linear stability theory for a large class of problems from hydrodynamics, reaction diffusion theory, etc. In the system a control parameter is taken just above a critical level. The basic state is weakly unstable and it is perturbed with a pattern proportional to the critical mode. The amplitude modulation on a long time – and space scale is described by Ginzburg–Landau's equation, which contains only two parameters a, $b \in \mathbb{R}$. Special solutions to Ginzburg–Landau's equation are known: space periodic solutions if $a = b = 0$, space–time periodic solutions if $a^2 + b^2 > 0$. These solutions are stable, if their amplitude R satisfies a condition defining the Eckhaus stability band. Crossing the stability limit for R gives rise to interesting new phenomena, which will be discussed in this lecture. The instability mechanism has an intrinsic relation with the dynamics of solutions beyond this stability limit. In case of side–band instability the governing equation becomes either a gradient–like phase equation, if $a = b$, or a perturbed Korteweg–de Vries equation, if $a \neq b$. In case of non side–band instability a modified non–local Ginzburg–Landau type of equation is found.

A new completely integrable shallow water equation

D.D. Holm

By using an asymptotic expansion directly in the Hamiltonian for Euler's equations in the shallow water regime, we derive a new completely integrable equation,

$$u_t + Ku_x - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx},$$

where u is the fluid velocity in the $x-$ direction, K is a constant related to the critical shallow water wave speed, and subscripts denote partial derivatives. This equation is bi–Hamiltonian and thus possesses an infinite number of conservation laws in involution. It follows from isospectrality and compatibility of a linear system of equations for a complex function $\Psi(x, t)$,

$$\Psi_{xx} = \left(\frac{1}{4} - \frac{u - u_{xx} + K}{2\lambda}\right)\Psi, \quad \Psi_t = -(u + \lambda)\Psi_x + \frac{1}{2}u_x\Psi.$$

The soliton solution for this equation has a limiting form as $K \to 0$ that has a discontinuity in the first derivative at its peak.

Conditional stability of solitary waves propagating in elastic beams

A. Il'ichev

The nonlinear transverse oscillations of an unextensible elastic beam are considered. The Hamiltonian form of the governing equations reads

$$(1)\quad \left(\begin{array}{l} \tau^i_t = \frac{\partial}{\partial\xi}\frac{\delta E}{\delta v_i} \\ v^i_t = \frac{\partial}{\partial\xi}\frac{\delta E}{\delta \tau_i} \; for \; \tau^i\tau_i = 1, \\ E = \frac{1}{2}\int_{-\infty}^{\infty}(v^i v_i + \tau^i_\xi \tau_{i\xi})d\xi, \end{array}\right)$$

$(\tau^i, v^i \in X$ (Hilbert space)). Here τ^i, $i = 1, 2, 3$ are the components of the vector tangent to the beam, v^i–of the velocity vector, ξ is the Lagrange length of the beam and summation is assumed under repeating indices. The system (1) has solitary wave solutions

$$\Phi_v = \{\tau^{\circ i}, v^{\circ i}\}, \quad \tau^{\circ 1} = 1 - 2\cosh^{-2} m\xi, \qquad \tau_2^\circ = 2\cosh^{-2} m\xi \sinh m\xi$$
$$\tau_3^\circ = 0, \qquad p^\circ = p_\infty - Gm^2 \cosh^{-2} m\xi, \quad m^2 = p_\infty^\circ - v^2 > 0$$

(p–pressure in the beam, $v > 0$–velocity of solitary wave)
$p^\circ \longrightarrow p_\infty^\circ$ at infinity
By stability of a solitary wave we mean the following: a solitary wave ϕ_v is called stable if for all $\varepsilon > 0$ there exists $\delta > 0$, having the following property: If $||U(0, \xi) - \phi_v||_x < \delta$ and $U(t, \xi)$ is a solution of (1) in some interval $[0, t_0)$ then $U(t)$ can be continued to a solution on $0 \le t < \infty$ and

$$\sup_{t>0} \inf_{s \in R} \inf_{\varphi \in S'} ||U(t), T(s)G(\varphi)\phi_v||_X < \epsilon$$

Where $P(s)$ is a translation and $g(\varphi)$ is a rotation around the x' axis of cartesian Euler coordinate system.

Theorem: If the solution of Cauchy problem for (1) exists (in a certain sense), then the solitary wave in question is stable.

Local and Global Bifurcations in Multibody System Dynamics

E. Kreuzer

Multibody system dynamics is world wide a very active area of research for both improving and extending modeling and simulation procedures as well as analysing the stability with respect to variable design parameters. Different types of nonlinearities in engineering mechanical systems cause problems in the stability analysis. Many mechanical systems consist of linear components with local nonlinearities introduced for example by nonlinear springs, nonlinear dampers, dry friction and backlash. Such nonlinearities may be desirable to avoid excessively high responses or stresses but also undesirable because of extensive wear or noise problems. Global nonlinearities originate from geometrically or kinematically nonlinear behavior often observed in mechanisms, robot, and vehicle dynamics. The performance of a technical system depends on its dynamical behavior which can be predicted using mathematical models for the multibody system and the excitation. Neither the system parameters nor the excitation are precisely known; both may even vary during operation. Moreover, if the structure is to require a control of system parameters there may be smooth or drastic changes in the qualitative behavior: so–called local and global bifurcations.

The stability analysis of periodic motions is based upon Poincaré maps which have to be approximated analytically if local bifurcations are studied in a general form. The global analysis requires normally large scale numerical simulations. By means of a combined computer aided numerical–symbolical approach a systematic analysis of bifurcation phenomena is visible. A single degree–of–freedom oscillator with backlash and a complex model of a crane ship serve as examples for the different tools discussed in the lecture.

Constraints, Reduction and Control
P.S. Krishnaprasad

In the realm of classical mechanics with exogenous variables, the Lagrange–d'Alembert principle occupies a central place. Recent investigations of this principle in the setting of nonholonomic constraints have lead to a deep understanding of the geometric underpinnings of the subject. Specifically, it has become possible to treat in an intrinsic manner, questions of reduction in the presence of nonholonomic constraints with symmetries. The reduced systems so obtained, are subject to both external forces and fictitious forces (e.g. forces associated to the curvature of a connection that encodes the constraint). In this talk, we present the general theory under two key hypotheses and discuss a variety of examples. The theory of principal bundles with connections is used in an essential way in this theory. While prior work in this area due to Koiller, and due to Marsden and Scheurle has been a source of inspiration, the work reported here is joint with Fui Yang and W.P. Dayawansa, and a basic reference is the 1992 Ph.D. thesis of R. Yang.

Nonintegrability of classical Zeeman Hamiltonian and related Hamiltonians
M. Kummer

We prove that the Hamiltonian H of the three dimensional hydrogen atom in a uniform static magnetic field B does not have an integral which (i) is real analytic on the phase space M of the system; (ii) is in involution with the component M_3 of the angular momentum along B; (iii) is functionally independent of H and M_3 and (iv) has a meromorphic (single–valued) extension to the complexification of M in $\mathbb{C}^6$. This follows from the fact that the Hamiltonian K_M for two degrees of freedom obtained by fixing M_3 at certain nonzero values M and reducing H w.r. to the rotational symmetry about the magnetic field, has a complexification which is nonintegrable in the Ziglin sense. We prove this nonintegrability by demonstrating that for each such M the monodromy group of the normal variational equation along a certain complexified phase curve of K_M is not Ziglin, using Churchill and Rod's adaptation of Kovacic's algorithm to the Ziglin analysis. Analogous arguments prove that the Hamiltonian of the Stormer problem is nonintegrable in the same sense.

Renormalization groups and nonlinear PDE's
A. Kupiainen

We use renormalization group (RG) ideas, borrowed from Statistical Mechanics, to study long time existence and asymptotics of solutions of parabolic nonlinear PDE's. The long time asymptotics is governed by fixed points of the RG transformation defined in a space of initial data and equations. Data and equations lying on the stable manifold of a particular fixed point have the same asymptotics. A detailed analysis of the diffusive decay and blow–up of the nonlinear heat equation is presented.

Conserving algorithms on Lie groups
D. Lewis

We consider two classes of conserving schemes for Hamiltonian systems on Lie groups: symplectic integrators and energy–monentum conserving algorithms. We derive general conditions for conservation of the energy and/or the symplectic form for a family of momentum conserving algorithms and use these conditions to design conserving algorithms for several specific Hamiltonian systems, including systems with highly nonlinear potentials. We note that conserving algorithms on vector spaces need not generalize to conserving algorithms on nonlinear manifolds. For example, one possible generalization of the mid–point rule leads to a symplectic algorithm, while another results in an energy-conserving algorithm. For the free rigid body, we construct a simple algorithm which conserves the energy, momentum, and the symplectic form. For non-integrable systems, such algorithms are generally believed not to exist (Ge and Marsden 1990). In fact, we see that for a class of momentum conserving algorithms, conservation of the symplectic form implies conservation of kinetic energy – such algorithms are clearly inappropriate for systems with a nontrivial potential.

The reduced Euler-Lagrange equations and nonholonomic systems
Jerrold Marsden

The Euler–Poincaré equations for a Lagrangian l on a Lie algebra g are shown to be the reduction of the Euler–Lagrange equations for the corresponding left invariant Lagrangian $L : TG \to I\!R$. The work of Marsden and Scheurle on Lagrangian reduction as a generalization is reviewed.

The second part of the lecture concerned the link between the reduced Euler–Lagrange equations and the equations of a nonholonomic system. We consider the Lagrange–d'Alembert equations for such a system with symmetry. Assuming only that the constraint distribution and the orbit of the group action intersect nontrivially, a synthesis of the constraint connection and the mechanical connection and a covariant–constant conservation law associated to the symmetry are constructed.

Towards a Universal Description of Patterns
A.C. Newell

Patterns of an almost periodic nature appear all over the place. One sees them in cloud streets, in sand ripples on flat beaches and desert dunes, in the morphology of plants and animals, in chemically reacting media, in boundary layers, on weather maps, in geological formations, in interacting laser beams, in wide gainband lasers, on the surface of thin buckling shells, and in the grid scale instabilities of numerical algorithms. This lecture, which will use examples from fluids and optics, deals with the class of problems into which these examples fall, namely with pattern formation in spatially extended, continuous, dissipative systems which are driven far from equilibrium by an external stress. Under the influence of this stress, the system can undergo a series of symmetry breaking bifurcations or phase transitions and the resulting patterns become more and more complicated, both temporally and spatially, as the stress is increased. The goal of theory is to provide a means of understanding and explaining these patterns from a macroscopic viewpoint that both simplifies and unifies classes of problems which are seemingly unrelated at the microscopic level.

Asymptotic Stability of Solitary Waves
R.L. Pego

We show that the family of solitary waves (1–solitons) of the KdV equation is exponentially asymptotically stable, when measured in a local sense relative to a frame moving with the solitary wave. Our methods yield the same result for a class of generalized KdV equations with flux $f(u) = u^{p+1}/(p+1)$, for $p = 2$ and generic p with $3 \leq p < 4$. (The solitary waves are unstable for $p > 4$). The solution is decomposed into a modulated solitary wave, with time–varying speed $c(t)$ and phase $\gamma(t)$, and a radiating perturbation. As $p \to 4^-$, the local decay or radiation rate must decrease due to the presence of a *resonance pole* associated with the linearized evolution equation for solitary wave perturbations.

Symmetry and Instability
T. Ratiu

In this talk the role of symmetry and dissipation in instability phenomena for relative equilibria of Hamiltonian systems will be explored.

A Gauge Theory of the Cosserat Continuum and the Direct Approach to Shell Theory
Carlo Sansour

The deformation of classical continua is completely described by the determination of the displacement field. Contrasting this, non–classical continua exhibit a microstructure the description of which necessitates the inclusion of further fields independent of displacements. Cosserat continua are characterized by a microstructure which is described by means of an independent field of rotations. The talk is intended to discuss the structure of the strain measures of such continua.

First, the rule of rotations within classical continua is discussed. Hereby a basic geometric structure of the stress and strain tensors inherent in classical theory is made apparent. A standard approach to Cosserat continua is then contrasted by that of gauge theory. The strain measures of the Cosserat continuum are achieved as gauge transformation of a connection defined on the tangent space of a set representing the body. The fact that the Lagrangian is no more independent of the gauge transformations within continuum theories which necessary differ from that well established in theoretical physics.

Further, it is shown that shell theory, as a method of dimensional reduction, can be achieved by considering a two dimensional Cosserat continuum. The strain measures of the shell are then taken to be the Cosserat deformation tensors.

Symplectic integration from a numerical analyst's point of view
J.M. Sanz–Serna

I presented numercal experiments that compare the performance of symplectic integrators with that of standard codes. While currently available symplectic methods do not appear to be competitive in quantitive problems, they outperform standard software in qualitative simulations.

I advocated the backward error interpretation as a rationale for using symplectic integrators. Finally I reported work by L. Jey (Geneve) on high–order symplectic integration of constrained mechanical problems.

Quantum computation in ultrasonic chaos physics
– From nonlinear macroscopic phenomena to quantum chaos –
W. Schempp

Among macroscopic quantum phenomena, the quantum Hall effect is best known. It is less known that magnetic resonance imaging in computer assisted biomedical diagnostics and acoustic cavitation noise in ultrasonic acoustic physics are macroscopic quantum phenomena, too.

In the Esche experiment of irradiating a liquid with sound of high intensity at a single frequency, a period doubling route to chaos can be observed. The response of the nonlinear dynamical system to the ultrasonic pumping oscillations admits a well structured broadband Fourier spectrum.

It is the purpose of the paper to mathematically explain quantum computation in ultrasonic acouustic chaos physics and to describe the transition from nonlinear macroscopic phenomena to quantum chaos in terms of tori actions on the compact Heisenberg nilmanifold which is a principal circle bundle over the flat two–dimensional torus.

Numerical Integration of Mechanical Systems with Constraints
W. Schiehlen

Mechanical systems like vehicles, engines, robots or spacecrafts may be modeled as multibody systems consisting of rigid material bodies, joints, bearings or wheels and springs, dampers and controlled actuators. Multibody systems with closed kinematical loops may be described by diffeent sets of coordinates and constraint equations. For the dynamical analysis the loops are cut and differential–algebraical equations of index 3 are obtained. The transformation to differential–algebraical equations of index 1 results in numerical instabilities.

Another possibility is to apply a representation of the multibody system in the Riemannian space with the metric of the system's inertia matrix M. Coordinate partitioning leads then to a tangent subspace of motion and a constraint subspace of reaction. In particular, the inertia matrix is transformed to a reduced inertia matrix of motion and a reaction matrix of the constraint forces. The open loops may be closed by motion coupling or reaction coupling, respectively.

The minimal number of coordinates of the tangent space results in singularities which means loss of controllability. For singularity avoidance a projective criterion due to Blajer (1991) and Schirm (1993) is presented. The projective criterion uses scalar products on the inverse metric. According to the projective criterion the minimal coordinates chosen for the numerical integration code by Shampine–Gordon (1975) is modified properly to overcome a restart when changing coordinates. As an example a spatial torus chain is considered. The projective criterion indicates clearly the singularities.

The Ginzburg–Landau approximation

Guido Schneider

We consider parabolic systems close to the threshold of instability defined on cylindrical domains. The unstable Fourier modes are concentrated at non–zero wave numbers and the real part of the associated eigenvalues is positive of order $O(\varepsilon^2)$.

It is known that the set which can be described by the so called Ginzburg–Landau formalism is attractive. Solutions of the original system can be approximated via solutions of a formally derived Ginzburg–Landau equation on a time scale of order $O(1/\varepsilon^2)$.

Here we construct a Generalized Ginzburg–Landau equation for which the approximation property holds on time scales of length $O(1/\varepsilon^3)$.

Hodge Theory on manifolds with boundary and applications

G. Schwarz

The Hodge decomposition theorem for differential forms $\omega \in \Omega^k(M)$ provides a useful tool for solving boundary value problems. Here the problem $\delta\omega = \Psi$ with the (non–standard) boundary condition $\omega|_{\partial M} = \Phi|_{\partial M}$ is studied, and a solution theorem is given.

The problem is motivated by a special question in elastostatic; its reformulation in terms of vector fields is important in the study of the Navier Stokes equations.

Conserving Algorithms of Hamiltonian and Dissipative Systems

J.L. Simo

This talk addresses the design of numerical schemes that inherit the conservation laws present in Hamiltonian systems and the dissipative property, characterized by the presence of absorbing sets and an attractor, for dissipative dynamical systems.

For Hamiltonian systems, two general classes of methods are described: (1) Symplectic integrators, which are schemes designed to preserve the symplectic character of the Hamiltonian flow, and (2) Energy–momentum algorithms, which are schemes designed to preserve the Hamiltonian and the momentum map (for systems exhibiting symmetry induced by the symplectic action of a Lie group in phase space). It is shown that symplectic algorithms which we A–stable (and B–stable) for any Δt can exihibit severe Blow–up leading therefore to useless results. Energy–momentum methods, on the other hand, produce stable solutions for any Δt. This behavior is verified by a comprehensive set of numerical experiments in infinite dimensional systems varying from nonlinear elasticity and nonlinear rods to nonlinear shells. The same behavior is observed in finite dimensional systems.

For dissipative systems, the incompressible Navier–Stokes equations in particular, schemes are described which are *linear* within the time step and inherit *the same* attractor as the exact dynamical system. The performance of these schemes is tested by numerical experiments.

References:

1. Simo & Tarnow (1992), "Exact energy and momentum algorithms for nonlinear Elasto–dynamics." ZAMP

2. Simo & Arnmo (1993), "Stability and long term behavior of transient algorithms for the incompressible Navier Stokes equations." CMAME

Triangulated Vortex Methods for 2-D Euler

J. Strain

We presented a new type of vortex method for computing 2-D incompressible inviscid fluid flows. Our "triangulated vortex method" moves N points with the (approximate) fluid velocity, so the vorticity is constant at each point, and reconstructs the velocity with a triangulation–based approximation to the Biot–Savart law. Our method has three major components; adaptive initial triangulation, fast Delaunay triangulation, and velocity evaluation with a modified fast multipole method. Numerical experiments show that the TVM achieves smaller errors in less CPU time than either standard vortex–blob methods or Larangian finite element methods. Our test cases include both smooth and discontinuous initial vorticity fields and both stationary and nonstationary solutions.

Spectral stability of Boussinesq solitary waves
M.I. Weinstein

Consider the problem of long, small–amplitude surface waves in shallow water for a 2-D incompressible, irrotational and inviscid fluid. A system of equations which approximates the dynamics is the Boussinesq system, which governs $u(x, t)$, the vertically averaged horizontal fluid velocity, and $\zeta(x, t)$, the free surface elevation.

We prove the linear spectral stability of small amplitude solitary waves of this system. The eigenvalue problem is studied using the Evans function, $D(\lambda)$, an analytic function whose zeros in the right half plane correspond to eigenvalues, and whose zeros in the left half plane correspond to "resonance poles". Small amplitude solitary waves correspond to the "KdV (Korteweg–de Vries) limit". In the KdV limit, we prove that the Evans function for Boussinesq converges to that for KdV.

The latter we have shown, in previous work, to have no nonzero eigenvalues of resonance poles.

Local and global structures of caustics in constrained variational problems
Zhong Ge

In a variational problem, the caustics are the bifurcation set of the extremals. The caustics in constrained variational problems enjoy many special properties. The relation with the curvature of the variational problem is studied.

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