MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Abelsche Gruppen

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Die Tagung fand unter der Leitung von R. Göbel (Essen), P. Hill (Auburn) und W. Liebert (München) statt. Sie brachte 48 Teinehmer aus Australien, Deutschland, England, Finnland, Irland, Israel, Italien, Japan, Kanada, Rußland, Südafrika, der Tschechischen Republik und den USA nach vier Jahren zusammen.

Aktuelle Forschung auf dem Gebiet der Abelschen Gruppen zeichnet sich stark durch Wechselwirkung mit anderen Teilgebieten der Algebra, der Logik/Modelltheorie und der Mengenlehre aus, worauf beachtliche Erfolge der letzten 20 Jahre beruhen. Deshalb ist es nicht verwunderlich, daß unter den Teilnehmern namhafte Vertreter dieser Grenzgebiete zu finden sind, die nicht nur als Zaungäste ihr Interesse bekunden, sondern hier Forschung betreiben und über wichtige neue Ergebnisse vortrugen, die die Kenntnisse über Abelsche Gruppen in Breite und Tiefe weiter vergrößern. Exemplarisch sei hier auf die Berichte von Magidor und Blass verwiesen. Ersterer gab einen Überblick zu einer gemeinsamen Arbeit mit Shelah, wo gezeigt wird, daß der Funktor $\mathrm{Bext}^2(G,T)$ für Torsionsgruppen T im allgemeinen auch unter Hinzunahme der verallgemeinerten Kontinuumshypothese GCH nicht verschwindet; er ist 0 für Gruppen bis zur Mächtigkeit $\aleph_\omega$ ohne zusätzliche Mengenlehre (Dugas, Hill, Rangaswamy) und für beliebige Mächtigkeiten unter $V=L$ (Fuchs, Magidor). Dies hat großen Einfluß auf die homologischen Eigenschaften von Butlergruppen, die man mit Bext(,) definieren kann.

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Durch Fortschritte in der Darstellungstheorie partiell geordneter Mengen und bessere Kenntnis ihrer Anwendung auf Butlergruppen wurden zahlreiche neue Resultate möglich, die im Falle von endlichem und zahmem Darstellungstyp zu Klassifikationen bis auf Quasiisomorphie und bei stark unzerlegbaren Abelschen Gruppen zu solchen bis auf Isomorphie führen. Zerlegungsverhalten konnte dadurch genauer studiert werden. Es sei besonders betont, daß erstmals russische Kollegen aus Moskau und St. Petersburg über ihre interessanten Resultate zu dieser Frage vortragen konnten.

Diese genaue Kenntnis über Anwendungen aus der Darstellungstheorie machte auch die Antwort auf eine Frage von Vamos (Exeter) aus der Ringtheorie möglich, wonach "Nagata–ähnliche" Ringe nur 1, 2, 3 oder unendlich viele unzerlegbare Moduln endlichen Ranges haben. Neben neuen Resultaten über p–Gruppen konnte der Ulmsche Satz über Warfieldgruppen hinaus verallgemeinert werden, was bei den klassischen Sätzen von Ulm und Leptin Verallgemeinerungen des Satzes von Baer–Kaplansky zur Folge hat. So war es auch möglich, ein altes Problem von Kaplansky zu lösen: Isomorphie von Endomorphismenringen ist auch in Fällen, in denen der Ring die Gruppe beschreibt, nicht immer "induziert". Auf der anderen Seite konnte ein Realisierungssatz von Butler und Zassenhaus für Ringe als Endomorphismenringe auf Dedekindbereiche erweitert werden; es wurden neue Sätze zur Darstellungstheorie vorgestellt, die hier Anwendung haben sollten. Und es wurden zahlreiche Sätze auf ihre Übertragbarkeit auf Moduln über z.B. nicht–perfekten Ringen untersucht, was zu komplexen Resultaten führte.

Zusammenfassend läßt sich sagen, daß sich die durch "Wechselwirkung" entstandene, allen bekannte stürmische Entwicklung des Verständnisses Abelscher Gruppen fortsetzt. Durch die jetzt schon fast zwei Jahrzehnte anhaltende Bewegung gelangen wir zu einem grundsätzlich neuen und mehr sytematischen Verständnis dieses Gebietes. Die präsentierten Ergebnisse spiegeln diese interessante Entwicklung wider; einen Teil von ihnen werden die Tagungsleiter in einem Proceedingsband der *American Mathematical Society* in der Reihe *Contemporary Mathematics* veröffentlichen.

Vortragsauszüge

R. Göbel :

Endomorphism rings of modules of prinjective type

Simson [to appear in J. Pure Appl. Algebra] gave a characterization of the modules in question for finite representation type by means of 110 minimal partially ordered sets serving as counterexamples. If $R \neq 0$ is a commutative ring and I is a finite poset, then $\mathfrak{C}_I$ denotes the class of R-modules of prinjective type. Its objects are tuples of R-modules and distinguished submodules with given R-homomorphisms $\mathrm{M} = (M_i, \sigma_{ij} : i,j \in I,\ i \leq j)$ where $\sigma_{ij} : M_i \to M_j$, and morphisms are tuples of R-homomorphisms $\varphi = (\varphi_i)_{i \in I}$ with $\varphi_i : M_i \to M_i'$ for all $i \in I$ compatible with the $\sigma_{ij}, \sigma_{ij}'$ for any $\mathrm{M}' \in \mathfrak{C}_I$. We will show (joint work with W. May) that any $I = (I, <)$ which is not of finite representation type has the property that any algebra A over R can be realized as $\mathrm{End}\,\mathrm{M} = A$ for a suitable M.
This has many applications in model theory. The proof is based on techniques coming from abelian groups, representation theory and combinatorial methods developed by S. Shelah.

W. May :

The Baer - Kaplansky Theorem for mixed modules

A version of the Baer - Kaplansky Theorem is given for certain mixed modules.
Theorem: Let R be a discrete valuation domain and M a mixed R-module such that $1 \leq$ torsion-free rank of $M/\mathrm{p}^{\omega}M \leq \aleph_0$. If N is another R-module such that $\mathrm{End}_R\, M \cong \mathrm{End}_R\, N$, then $M \cong N$.
The isomorphism of the endomorphism algebras need not be induced, since, in answer to a question of Kaplansky, $\mathrm{End}_R\, M$ may have outer automorphisms.

B. Goldsmith :

Endomorphisms and automorphisms of pure subgroups of the Baer-Specker group

A family $\mathfrak{G}$ of $2^{2^{\aleph_0}}$ pure subgroups G_i of the Baer-Specker group $P = \mathbf{Z}^{\aleph_0}$ is constructed with the following three properties: (i) each $G_i \in \mathfrak{G}$ is slender;

(ii) $\operatorname{End} G_i = \mathbf{Z} \oplus E_o(G_i)$ (split extension), where $E_o(G_i)$ is the ideal of finite rank endomorphisms; (iii) if $i \neq j$, then every homomorphism from G_i to G_j is of finite rank.

An automorphism φ of a pure subgroup G of P is said to be a *transvection* if $\varphi - 1$ is of finite rank. The transvections form a normal subgroup T_G in Aut G. We derive the following: If H is a countable torsion (not necessarily commutative) group which is the automorphism group of a torsion-free abelian group, then there is a pure subgroup G of P such that Aut $G = T_G \circ H$ (split extension). This is joint work with A.L.S. Corner.

A. Fomin :

Torsion-free abelian groups of finite rank up to quasi-homomorphism

A description of finitely presented modules over the ring of universal numbers is given. It yields a new category which is dual to the category of torsion-free abelian groups of finite rank with quasi-homomorphisms as morphisms.

G. Viljoen :

Completely decomposable pure subgroups of completely decomposable abelian groups

A general sufficient criterion for the complete decomposability of a pure subgroup in a completely decomposable group is due to Dugas and Rangaswamy [Arch. Math. 58 (1992), 332–337]. Our objective is to establish a necessary and sufficient condition for a pure subgroup of a completely decomposable torsion-free group to be again completely decomposable. We establish this by making use of ideas developed by Bican and Fuchs [to be published]. This is joint work with L. Fuchs.

A. Mader :

Almost completely decomposable groups with cyclic regulating quotient

In joint work with C. Vinsonhaler two questions about the groups in the title were solved in the primary case: (1) The possible direct decompositions were determined and shown to be unique up to isomorphism. (2) The possible quotients modulo regulating subgroups were fully described.

O. Mutzbauer :

On the splitting of local mixed groups

This is joint work with Elias Toubassi. Local mixed abelian groups are determined by generators and relations as extensions of torsion by torsion-free groups. For the torsion a straight basis is used. There exists a necessary and sufficient splitting criterion for the special class of local groups which are extensions of a direct sum of cyclics by a torsion-free divisible group. This criterion is generalized to its utmost limit, the class of all p-local mixed groups G with arbitrary torsion tG and a torsin-free quotient G/tG of finite p-rank.

T.G. Faticoni :

Decomposable almost completely decomposable abelian groups

Let X be an almost completely decomposable abelian group of finite rank. Let $r_\tau = \mathrm{rank}(X(\tau))$ for each critical type τ of X, let $t = \#$ critical types of X, and let $(r_1,\dots,r_t) = r(X)$ be such that $(r_i) = (r_\tau)$ after suitable enumeration of the τ's. Let m denote the least number of cyclic summands in a direct sum decomposition of A/X, where A ranges over all regulating hulls of X. We use a new matrix for X to determine those rank sequences $(r_1,\dots,r_t)$ that satisfy $r(X) = (r_1,\dots,r_t) \Rightarrow X$ is decompopsable. We give a survey of the decomposable and indecomposable X's when $m = 2$, and in the case $t = 3$. This is joint work with P. Yom.

M. Droste :

McLain-groups over arbitrary rings and orderings

Given a ring R and a partially ordered set $(S,\leq)$, we can define the McLain-group $G(R,S)$ comprising all upper triangular matrices $1+(r_{\alpha\beta})_{\alpha<\beta}$ where $r_{\alpha\beta}\in R$ for $\alpha<\beta$ in S and almost all $r_{\alpha\beta}=0$. Multiplication is induced by the usual matrix multiplication. We relate group-theoretical properties of $G(R,S)$ to properties of R and $(S,\leq)$. Under suitable assumptions on R and S, we solve the isomorphism problem for McLain-groups $G(R,S)$ and determine their automorphism groups. A construction of suitable posets $(S,\leq)$ leads to the following results: (1) For each prime p, there exist continuously many countable characteristically simple locally finite 2-groups $G(\mathbf{F}_2,S)$. (2) For each countable group H, there are continuously many countable locally finite 2-groups $G \cong G(\mathbf{F}_2,S)$ such that $\mathrm{Aut}(G)/\mathrm{Linn}(G) \cong H$. Here $\mathrm{Linn}(G)$ denotes the group of all automorphisms of G which are locally inner, i.e., act like conjugation on each finite subgroup of G. This is joint work with R. Gõbel.

L. Fuchs :

Butler groups without CH

Some of the main questions in the theory of Butler groups of infinite rank are answered by using a new approach. This is based on a new concept introduced by Bican-Fuchs.

A pure subgroup A of a torsion-free group G is said to be $\aleph_0$-*prebalanced* if for each pure B with $A<B\leq G$, rk $B/A=1$, the lattice ideal $\mathfrak{J}_{A|B}$ (in the lattice of all types) generated by $t(J)$ for rank one subgroups $J \subset B\setminus A$ is countably generated. An $\aleph_0$-prebalanced chain from A to G is a continuous well-ordered ascending chain of $\aleph_0$-prebalanced subgroups A_α such that $|A_{\alpha+1}/A_\alpha| \leq \aleph_0$ for each α.

Theorem 1. There is an $\aleph_0$-prebalanced chain from A to G if and only if in a balanced-exact sequence $0 \to H \to A \oplus C \to G \to 0$ (with completely decomposable C) H is a B_2-group.

Theorem 2. A B_1-group is a B_2-group exactly if it admits an $\aleph_0$-prebalanced chain from 0.

Theorem 3. A pure subgroup A of a B_2-group G is again a B_2-group if and only if there is an $\aleph_0$-prebalanced chain from A to G.

M. Magidor :

GCH does not imply $\mathrm{Bext}^2(G,T)=0$ for torsion-free G, torsion T

In a paper by Dugas-Hill-Rangaswamy it was proved that, assuming the continuum hypothesis (CH), $\mathrm{Bext}^2(G,T)=0$ for every torsion-free G and torsion T, provided $|G|\le\aleph_\omega$. In a paper by Fuchs and Magidor the same conclusion was derived using the generalized continuum hypothesis (GCH) and the combinatorial principle $\square$ for *arbitrary* groups G. Is $\square$ necessary for this result?

Theorem (Assuming the consisteny of supercompacts). There is a model of GCH in which for some torsion-free G of cardinality $\aleph_{\omega+1}$ and some torsion T, $\mathrm{Bext}^2(G,T)\neq 0$. This is joint work with S. Shelah.

A. Blass :

Some cardinal numbers from abelian group theory

(1) Let $\Pi=\mathbf{Z}^{\aleph_0}$ and $\Sigma=\mathbf{Z}^{(\aleph_0)}$ = free group generated for $i\in\mathrm{N}$ by the sequences $e_i=(0,\dots,0,1,0,\dots)$ with 1 in the i-th component. Then:

b: = minimum size of a family of maps $f\colon \mathrm{N}\to\mathrm{N}$ unbounded w.r.t. eventual majorization

$\geq$ minimum size of $G\le\Pi$ such that every homomorphism $G\to\mathbf{Z}$ annihilates all but finitely many e_i

$\geq$ minimum size of infinite rank $G\le\Pi$ with no homomorphism onto Σ

$\geq$ add (L): = minimum number of sets of Lebesgue measure 0 whose union is not of measure 0.

(2) Suppose G is a group such that, for every countable $K\le G$, the divisible part $\mathrm{d}(G/K)$ of G/K is countable. Then, for $K\le G$ of infinite cardinality κ, we have the following upper bounds for the cardinality of $\mathrm{d}(G/K)$ under the indicated hypotheses:

a) if $\kappa<\aleph_\omega$, then $|\mathrm{d}(G/K)|\le\kappa$;

b) if $\mathrm{cf}(\kappa)\neq\omega$ and if there is no inner model with measurable cardinal (or just: covering lemma over model of GCH), then $|\mathrm{d}(G/K)|\le\kappa$;

c) if $\mathrm{cf}(\kappa)=\omega$ and if there is no inner model with measurable cardinal, then $|\mathrm{d}(G/K)|\le\kappa^+$;

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d) if $\kappa = \aleph_\omega$ and if Chang's conjecture at $\aleph_\omega$: $(\aleph_{\omega+1}, \aleph_\omega) \to (\aleph_1, \aleph_0)$ holds, then $|d(G/K)| \leq \aleph_\omega$;

e) if $\kappa = \aleph_\omega$ and if V = L (or just: $\square(\aleph_\omega)$), then $|d(G/K)| \leq \aleph_{\omega+1}$, but not $\leq \aleph_\omega$.

W. Hodges :

The role of abelian groups in first order model theory

The first major program of model theory was to classify various types of structure up to elementary equivalence. Thus W. Sznielew (1955) gave invariants characterizing elementary equivalence classes of abelian groups. Since about 1965 the emphasis has been the other way round: to classify up to isomorphism the structures in a given elementary equivalence class. Thus Vaught asked whether the number of isomorphism types of countable structures in an elementary equivalence class is either $\leq \aleph_0$ or $= 2^{\aleph_0}$. The question is still open, but the nearest approach to a counterexample is A.I. Mal'cev's Theorem (1949) that the number of orderings of countable discretely linearly ordered abelian groups is $\aleph_1$. More interestingly, abelian groups appear in B.I. Zil'ber's analysis (1984) of totally categorical structures (whose elementary equivalence classes contain one isomorphism type of each infinite cardinality). They appear both as underlying geometries and as binding groups. Some fundamental structural questions then reduce to cohomology.

J. Oikkonen :

Ehrenfeucht–Fraissé games and non–structure results for abelian groups

One can use the Ehrenfeucht–Fraissé games to measure similarity of structures. These games lead naturally to a notion of a universal equivalence tree of a structure A. If A has a universal equivalence tree, then a version of Scott height can be defined for A. But there are (assuming CH) uncountable structures with no universal equivalence trees. It can be argued that such a structure cannot have any invariants in a certain very general sense. Therefore the existence of structures with no universal equivalence tree in a class $\mathfrak{K}$ of structures is a very strong non–structure result for $\mathfrak{K}$ in the sense of the non–structure theory of S. Shelah.

We prove this kind of a non-structure result for abelian p-groups. From the point of view of Shelah's stability theory our result is interesting since we obtain a non-structure result for a stable theory with NDOP and NOTOP.

C. Vinsonhaler :

A Dedekind version of a theorem of Butler

A theorem of M.C.R. Butler says that if E is a locally-free torsion-free finite rank $\mathbf{Z}$-algebra, then $E \cong \mathrm{End}_{\mathbf{Z}}(G)$ where G is a locally free $\mathbf{Z}$-module with rank $G = \mathrm{rank}\ E$. In this talk we show that the ring of integers $\mathbf{Z}$ may, in most cases, be replaced with a subring K of an algebraic number field. The exceptional cases occur when the K-algebra E modulo its nil radical is a subring of an algebraic number field. A Galois group condition may be used to characterize these exceptional cases.

L. Nongxa :

Balanced Butler groups

Finite rank Butler groups which can be embedded as balanced subgroups of completely decomposable groups are considered. These can be subdivided into classes, called B(n)-groups, which form a strictly decreasing chain. It is shown that some well-known results for Butler groups have analogues in these classes. We show that a *minimal indecomposable* B(n)-group is an almost completely decomposable group with rank and the critical type-set both equal to $n+2$. This group is embeddable as a balanced subgroup of a completely decomposable group of rank $\binom{n+3}{2}$. A decomposition theorem is established for B(n)-groups with $n+2$ critical types. We characterize finite lattices T such that all T-Butler groups in B(n) are (almost) completely decomposable, and we derive necessary and sufficient conditions for $G[A]$'s to be B(n)-groups. This is joint work with C. Vinsonhaler.

M. Dugas :

Near–isomorphism of Butler groups

We show that the classification of Butler groups up to near–isomorphism can be reduced to isomorphism of representations of posets over rings of the form $\mathbf{Z}/p^k\mathbf{Z}$ and near–isomorphism of uniform Butler groups which in many cases are uniform a.c.d. groups. We also study uniform, block rigid a.c.d. groups with three critical types.

D. Arnold :

Indecomposable modules of finite rank over a discrete valuation ring

Co–purely indecomposable modules (duals of pure submodules of the completion) are examined. While classification is not yet complete, results up to date are sufficient to resolve a conjcture of P. Vámos: The supremum of the ranks of finite rank torsion–free indecomposables is 1, 2, or ∞. We show that the conclusion is 1, 2, 3, or ∞.

A. Giovannitti :

Pure subgroups of separable groups

We conjecture that $\mathfrak{R}^*$–groups defined in 1991 are precisely the class $\mathfrak{S}^*$ of pure subgroups of separable groups. Towards this, we show that if an $\mathfrak{R}^*$–group admits a set of cotypes that satisfy a technical condition related to properties (a)–(c) of Albrecht–Hill (1987) (called Γ–admissable), then it is an $\mathfrak{S}^*$–group. Thus direct sums of groups of this form are in $\mathfrak{S}^*$. In the countable rank case we can show that every $\mathfrak{R}^*$–group is a direct sum of such groups. For the uncountable case, we still need to show that an indecomposable $\mathfrak{R}^*$–group either admits a Γ–admissable family or is separable. If a conterexample can be found, it will have a corank one completely decomposable summand with a divisible quotient. Other related results are given about $\mathfrak{R}^*$–groups which tend to confirm our conjecture.

K.M. Rangaswamy :

Properties of B_2-groups

A number of properties of B_2-groups which are strikingly similar to free abelian groups were outlined. Examples are: (i) If $0 = G_0 < G_1 < \cdots < G_n < \cdots$ is a countable chain where, for each n, G_n is a B_2-group and is pure in G_{n+1}, then $\cup G_n$ is again a B_2-group. (ii) If a torsion-free group G is an extension of a B_2-group by a totally projective group, then G is a B_2-group. (iii) A B_2-group is always *absolutely* $\aleph_0$*-prebalanced*. A TEP subgroup of a free abelian group is always a direct summand. Investigation of the analogous property for B_2-groups leads to the following characterization: Assume CH. Then a torsion-free abelian group G is a B_2-group if and only if $\mathrm{Bext}^1(G,T) = \mathrm{Bext}^2(G,T) = 0$ for all torsion groups T.

C. Metelli :

On $B^{(1)}$-groups

Let $B^{(1)}$ be the class of abelian groups that can be obtained as quotients of a completely decomposable abelian group of finite rank over a pure subgroup of rank n. (The class of finite rank Butler groups is then the union of the classes $B^{(n)}$ for $n = 0, 1, 2, \cdots$). In a joint work with Clorinda De Vivo, we show that the strongly indecomposable members of $B^{(1)}$ are uniquely determined up to quasi-isomorphism by their rank and type-set, thus completing the characterization initiated in Fuchs-Metelli *On a class of Butler groups* [Manuscripta Math. 71, (1991)].

W. Ullery :

Quasi-isomorphism results for finite rank Butler groups

If $A_1, \cdots, A_n$ $(n \geq 2)$ are nonzero subgroups of the additive group of rationals $\mathbb{Q}$, the cokernel of the diagonal map $\cap A_i \to \oplus A_i$ is called a *bracket group*. In a recent paper, D. Arnold and C. Vinsonhaler showed that bracket groups G and H are quasi-isomorphic if and only if rank $\Sigma_{\tau \in M} G(\tau) =$ rank $\Sigma_{\tau \in M} H(\tau)$ for all finite sets of types M. In joint work with H.P. Goeters and C. Vinsonhaler we improve this result by showing that the bracket groups G and H are quasi-isomorphic if and only if rank $G(\tau) = \text{rank } H(\tau)$ for each type τ.

A. Yakovlev :

Anomalous direct decompositions of torsion-free abelian groups of finite rank

Let $\mathfrak{M}$ be a category of torsion-free abelian groups of finite rank. For every prime p we construct a new category $\mathfrak{M}_p$ and a functor $F_p : \mathfrak{M} \to \mathfrak{M}_p$. Let $\mathfrak{M}_0$ be a category with the same objects as $\mathfrak{M}$ and with quasi-homomorphisms as morphisms, and let $F_0 : \mathfrak{M} \to \mathfrak{M}_0$ be the canonical embedding. We also construct a *universal* category $\tilde{\mathfrak{M}}$ and functors $G_p : \mathfrak{M}_p \to \tilde{\mathfrak{M}}$ and $G_0 : \mathfrak{M}_0 \to \tilde{\mathfrak{M}}$. All these categories (except $\mathfrak{M}$) are Krull-Schmidt categories: their objects have unique (up to isomorphisms) direct decompositions into sums of indecomposable objects. Using some properties of the diagram of the constructed categories and functors we establish a one-to-one correspondence between groups in $\mathfrak{M}$ and vectors in the cone of a certain finite-dimensional lattice, where direct sums of groups are taken to the sums of the corresponding vectors. So we can exhibit all the anomalities of direct decompositions of torsion-free abelian groups of finite rank.

E. Blagoveshchenskaya :

Direct decompositions of torsion-free abelian groups of finite rank

This talk deals with the complete solution of problems 67 and 68 in L. Fuchs's monography *Infinite Abelian Groups*. At first these problems were solved in the class of Γ-groups introduced by the author:

Theorem 1. For the existence of a Γ-group of rank n admitting both a decomposition into sums of indecomposable summands of ranks r_i and l_j ($n = r_1 + \cdots + r_s = l_1 + \cdots + l_t$) it is necessary and sufficient that $r_i \leq n - t + 1$ and $l_j \leq n - s + 1$ for all i, j.

Theorem 2. Suppose $1 < n_1 < \cdots < n_s < n$. Then there exists a Γ-group of rank n admitting decompositions into sums of n_i indecomposable groups for all $i \leq s$ if and only if $n_1 \geq n/(n - n_s + 1)$.

The Γ-groups have played the decisive role in the solution of Fuchs's problems 68 (with A. Yakovlev) and 67 for all torsion-free abelian groups of finite rank by analogous results which have more intricate formulations than the two theorems above.

P. Hill :

Totally projective p-groups - revisited

First, a review of Axiom 3 and its contributions to the structure of abelian p-groups is given. Then a new characterization of totally projective groups is added to the list of the known results. An application of this new characterization to modular group algebras demonstrates that $V(G)/G$ is totally projective when G is an abelian p-group of cardinality $\aleph_1$.

D. Carroll :

Transitivity properties in abelian groups

In Kaplansky's definitions of transitivity and full transitivity, a p-group was (fully) transitive if whenever a pair of elements x and y satisfied certain conditions, there existed an automorphism (endomorphism) of the group mapping x to y. Here we discuss the ideas of k-transitivity and full k-transitivity (k a positive integer). A p-group is (fully) k-transitive if whenever there exist certain subsets X and Y each of cardinality k with corresponding elements satisfying certain conditions, there is an automorphism (endomorphism) of the group which maps the elements of X to the corresponding elements of Y. There exist groups which are transitive and fully transitive but neither k-transitive nor fully k-transitive for any $k > 1$.

J. Hausen :

Abelian p-groups determined by the Jacobson radical of their endomorphism rings, I

Let G and G' be two abelian p-primary groups and let $K(G)$ and $K(G')$ denote the maximal torsion subgroups of the Jacobson radicals of their endomorphism rings. It is shown that every ring isomorphism $\psi : \mathrm{K}(G) \to \mathrm{K}(G')$ is induced by a group isomorphism $\varphi : G \to G'$ provided G is unbounded modulo its maximal divisible subgroup. Thus, two p-groups with unbounded basic subgroups are isomorphic if and only if the Jacobson radicals of their endomorphism rings are isomorphic as rings.

P. Schultz :

Abelian p-groups determined by the Jacobson radical of their endomorphism rings, II

It is shown that if G has an unbounded basic subgroup, then G is determined by the torsion ideal of the Jacobson radical of its endomorphism ring. On the other hand, if G is $B \oplus D$, where B is elementary and D divisible, then G may not be so determined. In this talk it is shown that if $G = B \oplus D$ where B is bounded but not elementary, then G is determined by the Jacobson radical of its endomorphism ring.

K. Honda :

Reduced abelian p-groups, I

The purpose of this paper is to prove completely and affirmatively my conjecture in my paper *Plain Global Bases of Abelian p-Groups* in the Proceedings of the Perth Conference on Abelian Group Theory (1987).

Let A be an elementary abelian p-group and $A = A_0 \geq A_1 \geq \cdots \geq A_\alpha \geq \cdots > A_\lambda = 0$ a smooth descending chain of subgroups of A. We can consider A as a valued vector space of length λ over the field GF(p). As usual, we define the Ulm invariant u_α for every $\alpha < \lambda$ to be the rank of the factor group $A_\alpha / A_{\alpha+1}$. As in my above paper, we define the pseudo-diagonal invariant q_γ for any limit ordinal $\gamma < \lambda$. Further, for any limit ordinal $\gamma < \lambda$ denote by r_γ^* the sum of all cardinals $u_{\gamma+n}$, where n is any finite ordinal such that $\gamma + n < \lambda$.

Existence Theorem. If $q_\gamma \geq r_\gamma^*$ holds for any limit ordinal $\gamma < \lambda$, then there exists a reduced abelian p-group G such that the socle of G is isometric to A, both G and A considered as valued vector spaces.

P. Keef :

On p^α-injective abelian groups

In the category of p-local abelian groups, the injectives for the functor $p^\alpha \operatorname{Ext}$ are discussed. For example, it is shown that if $\alpha = \beta + \gamma$ and G is p^α-injective, then $p^\beta G$ is p^γ-injective and $G/p^\beta G$ is p^α-injective. Of particular importance is a long exact sequence involving the maps $p^\alpha \operatorname{Ext}^k(A/B, G) \to p^\alpha \operatorname{Ext}(A, G)$ when B is a subgroup of $p^\alpha A$.

S. & F. Kuhlmann :

Valued and ordered abelian groups - some recent developments and applications

In our joint talk, we recalled the notion of a valued abelian group and its skeleton. We hinted at connections to the theory of valued fields, in particular concerning immediate extensions. We described recent applications to two prominent problems from model theoretic algebra:

1) The unknown model theory of the power series field $\mathbf{F}_p((t))$. Following an idea of L.v.d. Dries, the key to this problem may lie in its structure as a *valued model over a ring of additive polynomials*. However, the ring in question is very bad: although being left euclidean, it is not even right Ore. We define a valued model to be a valued abelian group carrying also a module structure. This permits to introduce axioms for the compatibility between valuation and module structure according to the special situation.

2) The structure of a nonarchimedian exponential field K. The value group G of K is peculiar: all its components are equal to the additive group of the residue field, and G admits an order isomorphism from $G^{<0}$ onto $\operatorname{rank} G$. We illustrated by two examples: In the countable case, these groups have a canonical structure, enabling us to give a simple procedure to construct non-archimedian exponential fields from archimedian ones. For power series fields, the construction problem reduces to that of producing such groups which are Hahn products.

Finally, we axiomatized these groups (*contraction groups*) and mentioned their nice model theoretic properties.

L. Salce :

Totally ordered and realizable abelian groups

Report on the joint paper with S. Bazzoni: *Class semigroup of valuation domains and quotients of totally ordered complete abelian groups*. The class semigroup of a commutative domain R is the monoid $\mathfrak{F}(R)/\mathfrak{P}(R)$, where $\mathfrak{F}(R)$ is the monoid of nonzero fractional ideals with the usual multiplication and $\mathfrak{P}(R)$ its subgroup of principal fractional ideals. Denote it by $\mathfrak{S}(R)$. If R is a valuation domain, $\mathfrak{S}(R)$ is a Clifford semigroup, whose non-trivial subgroups are associated with idempotent prime ideals L of R. Such a subgroup G_L is isomorphic to $\overline{\Gamma(R_L)}/\Gamma(R_L)$, where $\Gamma(R_L)$ is the value group of the localization of R at L, and $\overline{\Gamma(R_L)}$ is its comple- in the induced order topology. The abelian groups G_L arising this way are characterized as follows: they are divisible of size $\leq 2^{\aleph_0}$, or cotorsion, or arbitrary, depending on whether P has an immediate predecessor in spec(R), or whether it is a countable or uncountable union of prime ideals.

J. Trlifaj :

Modules over non-perfect rings and their extensions

Properties of modules over a ring R differ substantially depending on whether R is a perfect ring or not. The latter alternative often bears resemblance to the particular case $R = \mathbf{Z}$. In the present talk, the dependence is studied for three important notions coming originally from abelian groups: that of a (Whitehead) test module, of an almost free module, and of a slender module.

The scheme works perfectly for the test modules: there is a proper class of test modules provided R is perfect. There is no test module provided R is non-perfect, under the assumption of a uniformization principle UP.

For almost free modules, differences may appear: there exist non-perfect rings and strongly κ-free modules which are not κ-free, for any regular uncountable cardinal κ. Of course, this does not happen for $R = \mathbf{Z}$. For the slender modules, the situation can be quite different: there exist countable hereditary and non-perfect rings such that all nonzero semisimple modules are slender (quite unlike the case $R = \mathbf{Z}$).

S. Files :

Endomorphism rings of local Warfield groups

We prove an isomorphism theorem for the andomorphism rings of p-local Warfield groups. Precisely, if G and H are reduced Warfield groups and the map φ: End $G \rightarrow$ End H is an isomorphism, then $G \cong H$ and φ is topological, i.e., bicontinuous with respect to the finite topologies. Two immediate corollaries: (1) Every automorphism of End G is topological if G is a reduced Warfield group. (2) Simply-presented groups are determined by their endomorphism rings.

W. Wickless :

A class of mixed groups

We consider the class $\mathfrak{G}$ of reduced mixed groups G defined as follows: G is self-small and G/T is divisible. The class $\mathfrak{G}$ is regarded as a full subcategory of the category WALK. For any type $\tau = [\langle k_p \rangle]$, with $\tau >$ type $\mathbf{Z}$ and $k_p < \infty$ for all p, we construct a category equivalence between that category of locally free, torsion-free groups H such that H has no rank one summand of type τ and outer type $H \leq \tau$, with maps quasi-homomorphisms, and a suitable subcategory of $\mathfrak{G}$. This equivalence gives us examples of Krull-Schmidt mixed group categories on which there is a *Warfield duality*.

R. Lafleur :

Typesets and cotypesets of finite rank torsion-free abelian groups

We give new necessary and sufficient conditions for cotype-sets of groups of rank n. Using these, we can produce for every $n \geq 3$ an example of a set of types which is the cotypeset of a rank n group but not the cotype-set of any higher (or lower) rank group. We also give for every $n \geq 3$ an example of a set of types which is the typeset of a rank n group but not the type-set of any higher (or lower) rank group. Also new necessary conditions for cotype-sets of rank n groups were given, as well as dual necessary conditions for type-sets of rank n groups.

R. Mines :

Representations and duality

Let I be a poset and let K be a field. A *topological representation* of I over K is a pair $\mathbf{V} = (V, V(\cdot))$ where V is a vector space over K having a Hausdorff linear topology and $V(\cdot)$ is a functor from I into the lattice of all closed subspaces of V. A map $f: \mathbf{V} \to \mathbf{V}$ is a continuous linear map from U to V that induces a natural transformation from the functor $U(\cdot)$ to $V(\cdot)$. A *type* is a representation $\mathbf{T} = (T, T(\cdot))$ where $T = K$. Let $\mathfrak{M}$ be a set of types. We give cardinality invariants for indecomposable cokernels of the maps $\bigcap_{\mathbf{T} \in \mathfrak{M}} \mathbf{T} \to \prod_{\mathbf{T} \in \mathfrak{M}} \mathbf{T}$.

T. Fay :

Categorical compactness relative to idempotent closure operators

We study categorical compactness with respect to a class $\mathfrak{F}$ in the category of not necessarily associative rings. In doing so, we also obtain results for other categories: associative rings, groups, R-modules, and, of course, abelian groups. If $\mathfrak{F}$ is an isomorphism closed class of objects, containing the zero object, and if A is a subobject of B, we define the $\mathfrak{F}$-closure of A in B to be the intersection of all I which are normal in B and satisfy $B/I \in \mathfrak{F}$. This notion gives rise to a Hoehnke type radical. In the case that $\mathfrak{F}$ is closed under subobjects, this closure becomes an idempotent closure operator. An object G is called $\mathfrak{F}$*-compact* provided for every object H, the second projection map $\pi_2 : G \times H \to H$ maps $\mathfrak{F}$-closed subobjects *onto* $\mathfrak{F}$-closed subobjects of H. We investigate the notion of H($\mathfrak{F}$)-closed which is a generalization of the notion of H-closed topological space and that of absolutely closed developed by Dikranjan and Giuli. H($\mathfrak{F}$)-closed objects are always $\mathfrak{F}$-compact but the converse may fail. The theory of categorical compactness with respect to the class $\mathfrak{F}$ is developed and a number of new examples are given.

S. Pabst :

An $\aleph_1$-free R-module G with trivial dual

Let R be a commutative ring with $|R| < 2^{\aleph_0}$ and let $S \subseteq R$ be a multiplicatively closed subset containing 1 such that R is S-reduced and S-torsion-free. It is shown that there exists an $\aleph_1$-free R-module G with $\mathrm{Hom}(G,R)=0$. G is constructed as a submodule of an algebraically compact R-module $\widehat{B}$ containing an R-module B freely generated by a binary tree. Moreover, $\widehat{B}$ is an $\widehat{R}$-module, where $\widehat{R}$ is a ring containing R as a subring such that $\widehat{R}$ is an algebraically compact R-module.

P. Yom :

A characterization of a class of Richman-Butler groups

In this talk, using the concept of a quasi-representing graph of a Butler group, we give the *two-vertex exchange* for a Richman-Butler group $G(A_1,\cdots,A_n)=$ the kernel of the summation map Σ: $A_1\oplus\cdots\oplus A_n\to\mathbb{Q}$ where $A_i\subseteq\mathbb{Q}$ and $n\geq 2$. This leads to a characterization theorem for a class of CT-groups. We also give an upper bound of the number of maximal types in the type-set and a conjecture for an upper bound of the number of type representations quasi-isomorphic to a given strongly indecomposable group G. This partially solves Problem 3 of the Fuchs-Metelli paper *On a class of Butler groups* [Manuscripta Math. 71 (1991)].

A. Paras :

Abelian groups as noetherian modules over their endomorphism rings

Given an abelian group G and its endomorphism ring E, equip G with an E-module structure by defining $\alpha\cdot g=\alpha(g)$ for $\alpha\in E$ and $g\in G$. A group G is an E-noetherian group if G, viewed as an E-module, is noetherian, i.e., every submodule of G is finitely generated. Up to quasi-isomorphism, the group structure of E-noetherian groups is characterized as a direct sum of strongly irreducible groups, i.e., every nonzero fully invariant subgroup is quasi-equal to the group. There are groups G whose endomorphism rings are left noetherian but are not E-noetherian. We show that the endomorphism rings of E-noetherian groups are necessarily left noetherian.

J. Reid :

Decompositions of a.c.d. groups

We consider the class of a.c.d. groups G having rigid critical type-set and having the property that for t_ν a critical type, $k_\nu = \text{rank } G(t_\nu)$, then $G(t_\nu)$ contains a summand of G of rank $k_\nu - 1$. For such groups G, any decomposition of G into t indecomposable summands must satisfy $\max(k_\nu) \leq t \leq (\Sigma k_\nu) - m$ where $m = \text{card}(T_{cr}(G))$. Such groups G have computable endomorphism rings. This leads to consideration of matrix rings Λ over pid´s R (subrings of $\mathbb{Q}$) consisting of $k \times k$-matrices $A = [a_{ij}]$ with $a_{ij} \equiv 0 \pmod q$, $j > 1$ (q a fixed prime in R). We classify the sets of mutually orthogonal idempotents with sum 1 in Λ with respect to equivalence by conjugation by elements of the unit group of Λ, and we apply the result to classify direct decompositions of certain a.c.d. abelian groups.

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