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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Nichtkommutative Algebra und Darstellungstheorie

15.08. bis 21.08.1993

Die Tagung wurde organisiert von G.Michler (Essen) und L.Small (San Diego). Teilgenommen haben 50 Mathematikerinnen und Mathematiker aus 10 Ländern (Belgien, Deutschland, Frankreich, Großbritannien, Israel, Mexiko, Norwegen, Rußland, USA und Weißrußland).

Die behandelten Themen waren ungewöhnlich breit gefächert und regten intensive Diskussionen und vielfältige Zusammenarbeit an.

Die (insgesamt 32) Vorträge (die weiter unten genauer dokumentiert sind) und die informellen Ankündigungen beschäftigten sich mit Ringtheorie, endlich-dimensionalen Algebren und ihrer Darstellungstheorie, endlichen und unendlichen Gruppen, Lie-Algebren und dem neuen und sehr aktuellen Gebiet der Quantengruppen, vor allem aber auch mit Zusammenhängen zwischen diesen Gebieten und Anwendungen von Ergebnissen aus einem diegruppen wurden unter sehr verschiedenen Fragestellungen und Sichtweisen betrachtet und der bisherige Stand dieser Theorie wurde kritisch diskutiert. Die eingesetzten Methoden entstammten neben den genannten Gebieten vor allem der Kombinatorik, der algebraischen Geometrie und der Theorie der quadratischen Formen.

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Vortragsauszüge

J. Alev: Rigidity of finite group actions on U(g), g semisimple

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Let A be a C-algebra and G a finite group of C- automorphisms of A. We say that A does not admit Galois embeddings into itself if A^G is not isomorphic to A for any G. The classical theorem of Chevalley-Shephard-Todd asserts that $\mathbb{C}[X_1, \ldots, X_n]$ (polynomial algebra) admits Galois embeddings into itself (when G is generated by pseudo-reflections). A result by Dicks and Formanek shows that the tensor algebra of a finite dimensional space V does not admit Galois embeddings into itself. We show the same result for the Weyl algebra $A_n(\mathbb{C})$ (algebra of differential operators on affine space \mathbb{A}^n) and for $U(\mathfrak{g})$, the enveloping algebra of a semi-simple Lie algebra \mathfrak{g} .

S. A. Amitsur: Applications of polynomial identities to group representations (joint work with L. Small)

Let k be a field and $G = \langle g_1, \dots, g_r \rangle$ be a finitely generated group (not necessarily finite). The study of the finite dimensional representations of G is reduced to the study of maximal ideals of PI-images of the group ring kG, and finiteness problems of affine PI-rings.

Consequences of this approach yield general results like Weil's theorem on the finite number of representations of groups with completely reducible representation (Farkas) and the generalization of Vinberg's theorem on the field of representations of a group.

C. Bessenrodt: Spin representations of symmetric groups at characteristic 2

Let $\widehat{S_n}$ be a double cover of the symmetric group S_n , i.e. $\widehat{S_n}$ has a central involution z such that $\widehat{S_n}/\langle z \rangle \simeq S_n$. An irreducible character of $\widehat{S_n}$ is called ordinary or spin according to whether it has z in its kernel or not. The associate classes of spin characters of $\widehat{S_n}$ are labelled canonically by the partitions of n into distinct parts. For an odd prime p, Morris conjectured a combina-

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torial algorithm on the labels giving the distribution of these characters into p-blocks which was proved by Humphreys and Cabanes; then the number of spin and modular spin characters was computed by Olsson, and for p = 3 and p = 5 results on the shape of the decomposition matrix were obtained in work of Bessenrodt, Morris and Olsson resp. Andrews, Bessenrodt and Olsson.

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In recent joint work with Olsson, such results were now also obtained for the case p = 2, where the previous methods could not be applied. An explicit formula for the number of spin characters in a given 2-block was presented. Using this, we proved a conjecture of Knörr and Olsson, describing the 2-block distribution of spin characters combinatorially. Based on this, we obtained an analogue of James' result for the decomposition matrix of S_{nix} generalizing also a theorem of Benson; in particular, the position and value of the least (w.r.t. lexicographical ordering of the column labels) non-zero entry in each row was determined.

A. Boldt: Characteristic polynomials of Coxeter matrices

If a path algebra with relations is built up from two subalgebras in a certain natural fashion, there is an easy relation between the involved Coxeter polynomials. This leads (especially in the hereditary case) to several formulas for those polynomials and also to efficient ways to compute them.

To be more precise, the main result is as follows: If k is a field, if Γ_1 and Γ_2 are quivers having exactly one common point r, $\Gamma := \Gamma_1 \cup \Gamma_2$, and $I \subset k\Gamma$ is an ideal generated by relations which do not involve any paths properly passing through r, then we have (using the notations $\Lambda := k\Gamma/I$, $\Lambda_i := \frac{k\Gamma_i}{I\cap k\Gamma_i}$, $\bar{\Lambda}_i := \frac{k(\Gamma_i \setminus r)}{I\cap k(\Gamma_i \setminus r)}$, and χ_A for the characteristic polynomial of the Coxeter matrix of an algebra A)

$$\chi_{\Lambda} = \chi_{\Lambda_1} \chi_{\Lambda_2} + \chi_{\Lambda_1} \chi_{\Lambda_2} - (T+1) \chi_{\Lambda_1} \chi_{\Lambda_2}$$

provided the Cartan matrices of the algebras Λ_1 and Λ_2 are nonsingular.

A. Braun: Localization, completion and the AR property

The relations between the properties mentioned above are investigated primarily in the context of Noetherian P.I. rings.

R. Cannings: Differential operators on curves (joint work with M. P. Holland)

Suppose R is the coordinate ring of an affine curve singularity over C. Let \tilde{R}, R^+ be the integral and unramified closures of R respectively and let R^- be the conductor of R^+ into R. We construct invariants of the curves as follows:

(1) the subspace system:

$$\mathfrak{R} = \bigwedge_{\substack{\underline{R} \cap R^-(m_i) \\ \overline{R^-}}} \dots \underbrace{\underline{R} \cap R^-(m_n)}_{\overline{R^-}}$$

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where the $R^{-}(m_i)$ are the primary components of R^{-} in R^{+} .

(2) the finite dimensional factor of the ring of differential operators on R, $D(R)/minideal \cong End(\mathfrak{R})$.

Main Theorem: Fix \tilde{R} , $m \ge 1$ and A a finite dimensional algebra then there exists R such that

$$End(\mathfrak{R})\cong\left(\begin{array}{c|c}M_m(\mathbb{C})&0\\ \hline *&A\end{array}\right).$$

T. Dana-Picard: Deformations of group algebras of low dimension

According to Maschke's theorem, if G is a finite group and K a field of characteristic p not dividing |G|, the group algebra KG is semi-simple. In 1974, Donald and Flanigan conjectured that if p||G|, then KG deforms to a semi-simple algebra, and proved it for abelian G.

In our talk, we recall the results obtained since then (Schaps 1989, Michler 1990, Gerstenhaber-Schaps 1992, Erdmann-Schaps 1992, Meir-Schaps 1993) in various cases. Then we give explicit examples of semi-simple deformations

for $G = A_4$ and $G = (C_3 \times C_3) \rtimes C_2$, the latest with 2 different actions of C_2 .

Finally we expose a possible strategy for searching answers to the Donald & Flanigan problem, using the knowledge on local subgroups of G.

R. Dipper: Representations of Hecke algebras of type B_n and D_n (joint work with G. James and G. Murphy)

The simple modules for Hecke algebras $\mathcal{H}_{Q,q}$ of type B_n were constructed, under certain restrictions on Q and q, in a previous paper by G. James and myself. Using new ideas, introduced by G. Murphy for the type A case, we now remove these restrictions. We present a collection of modules, of which the nonzero ones form a complete set of pairwise nonisomorphic simple modules. A conjecture is formulated as to which of these modules are nonzero, and is proved in many special cases.

Further results: 1) If $\mathcal{H}_{Q,q}$ is semisimple, a complete set of primitive orthogonal idempotents is calculated and Young's seminormal form (Hoefsmith's thesis) is derived.

2) The decomposition matrix of $\mathcal{H}_{Q,q}$ is unitriangular.

3) (Conjecture) Block structure of $\mathcal{H}_{\mathcal{O},g}$.

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Similar methods apply to Hecke algebras of type D_n . Here one may also use an embedding of $\mathcal{H}_q(D_n)$ into $\mathcal{H}_{1,q}(B_n)$.

P. Dräxler: On tameness of not locally support finite k-categories

Let A be a finite dimensional algebra over an algebraically closed field k and P an indecomposable projective A-module satisfying $dim_k End_A(P) = 1$. We refer to two previous results of ours. Firstly tame representation type of A can be characterized by the tameness of the subspace category $U(K, Hom_A(P, -))$ where K denotes the full subcategory of A - mod given by all V such that $Ext_A^1(V, facP) = 0$. Secondly under suitable assumptions about P the vectorspace category $(K, Hom_A(P, -))$ decomposes into two parts. These parts are equivalent to $(A_s - mod, R^- \otimes_{A_s} -)$ resp. $(A^s - mod, Hom_{A^*}(R^+, -))$ where A_s, A^s are proper subalgebras of A dependent.

ding on P.

DFG Deutsche Forschungsgemeinsch We explain and illustrate on examples how these results can be applied to prove the tameness of not locally support finite k-categories. Categories of this kind occur frequently as coverings of finite dimensional k-algebras.

D. Farkas: Ring theory in symplectic geometry

A project undertaken with G. Letzter is described which looks at algebraic constructions in classical and noncommutative symplectic geometry. A 'bifurcation' theorem for Poisson algebras is proved and results about various types of derivations for rings of differential operators are presented. As an illustration, a 'symplectic argument' is used to show that if B is a regular affine domain then $1 \in \{grB, grB\}$.

K. Goodearl: Is it time to define algebraic quantum groups?

I discuss the current lack of any axiomatic definition of algebraic quantum groups (i.e., quantum coordinate rings of algebraic groups) and related problems, illustrated by the case of compact quantum groups, for which an axiomatic definition is known.

J. Gräter: Bezout orders in semisimple Artinian rings

An order R in a semisimple Artinian ring Q is called valuation order or order of higher rank if R is Bezout and R/J(R) (semisimple) Artinian. These orders generalize many other types of valuation orders defined before, e.g. R is a total valuation ring if Q and R/J(R) are division rings and R is a Dubrovin valuation ring if Q and R/J(R) are simple Artinian. The results obtained so far deal with localizations of orders of higher rank, the connection between the overrings and semiprime ideals, and the decomposition of these orders. If Q is finitely generated over its centre then a precise description of valuation orders in terms of their centres can be given.



E. L. Green: Quantum algebras

We present a construction of a Hopf algebra structure on certain path algebras. We show that if Λ is a finite dimensional Hopf algebra over a field K such that $\Lambda/rad(\Lambda) \cong K \times ... \times K$ then there is a path algebra and ideal I such that $\Lambda \simeq K\Gamma/I$ and there is a Hopf algebra structure on $K\Gamma$ given by the construction such that I is a Hopf ideal and the Hopf structure on $K\Gamma/I$ induced by $\Lambda \simeq K\Gamma/I$ is the 'same' as the construction up to first order error terms.

D. Happel: Piecewise hereditary algebras

Let Λ be an artin algebra over a commutative ring R. Let $mod\Lambda$ be the category of finitely generated left Λ -modules. For an abelian category \mathcal{A} we denote by $D^b(\mathcal{A})$ the bounded derived category of \mathcal{A} . We call Λ piecewise hereditary if there exists a hereditary abelian category \mathcal{H} such that $D^b(mod\Lambda)$ and $D^b(\mathcal{H})$ are equivalent as triangulated categories.

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We will present certain restrictions both on Λ and \mathcal{H} in this situation. For example \mathcal{H} has almost split sequences and its Grothendieck group is free of finite rank. And Λ is of finite global dimension and representation-directed if it is representation-finite.

Let \mathcal{H} now be in one of the following classes of examples. Either \mathcal{H} is the module category of a hereditary artin algebra H or is the category of coherent sheaves associated to a weighted projective curve in the sense of Geigle/Lenzing. The main theorem asserts that a piecewise hereditary algebra Λ is tilting-cotilting equivalent to a quasi-tilted algebra. Recall that an artin algebra Λ is called a quasi-tilted algebra if it is the endomorphism algebra of a tilting object in \mathcal{H} . This generalizes previous results by Happel/Rickard/Schofield and Assem/Skowronski.

T. J. Hodges: Multi-parameter quantum groups

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Let G be a simply connected, connected, semi-simple complex algebraic group. For each cocycle p on the integral weight lattice P and for each $q \in \mathbb{C}^{\times}$

we define a multi-parameter quantum group $\mathbb{C}_{q,p}[G]$. Assume q is not a root of unity. The group of one-dimensional representations of $\mathbb{C}_{q,p}[G]$ is isomorphic to the maximal torus H. The H-orbits in Prim $\mathbb{C}_{q,p}[G]$ are indexed by the double Weyl group $W \times W$.

H.-J. von Höhne: Bipartite posets of finite prinjective type (joint work with D. Simson)

Let k be a field and (I, \leq) a finite partially ordered set equipped with a bipartition $I = I_1 \cup I_2$ such that the incidence algebra kI has the form $kI = \binom{AM}{0B}$ where $A = kI_1$ and $B = kI_2$. Then prin kI denotes the category of finite dimensional prinjective right kI-modules X that is where $X\binom{10}{00}$ is projective over A and $X\binom{00}{01}$ is injective over B. Our main result is the following:

Theorem: The following conditions are equivalent:

i) The category prin kI has only finitely many indecomposable objects (up to isomorphism).

ii) The quadratic form $q^I : \mathbb{Z}^I \to \mathbb{Z}$

$$q^{I}(x) := \sum_{i \in I} x_{i}^{2} + \sum_{\substack{i < j \\ i, i \in I : \text{ or } i, j \in I_{2}}} x_{i}x_{j} - \sum_{I_{1} \ni i < j \in I_{2}} x_{i}x_{j}$$

is weakly positive, that is $q^{I}(x) > 0$ for all $0 \neq x \in \mathbb{N}^{I}$.

iii) The poset *I* does not contain as a full bipartite subposet a critical bipartite poset (which are described completely).

A. Joseph: Encoding the Cartan matrix

Let C be an $n \times n$ symmetric integer-valued matrix conveniently realized in the form $\{(\alpha,\beta)\}_{\alpha,\beta\in\pi}: \pi \subset \mathfrak{h}^{\bullet}$. If $(\alpha,\alpha) \in 2\mathbb{N}^{+}$ for all $\alpha \in \pi$ then the Kac-Moody Lie algebra \mathfrak{g}_{C} is defined. I.M.Gelfand has asked if one may construct a quantum deformation $U_{\mathfrak{q}}(\mathfrak{g}_{C})$ of $U(\mathfrak{g}_{C})$ as for the integrable case $(2(\alpha,\beta)/(\alpha,\alpha) \in \mathbb{N}^{-}, \forall \alpha \neq \beta)$ considered by Drinfeld and Jimbo.

Consider the associative algebra on generators $x_{\alpha}, y_{-\beta} : \alpha, \beta \in \pi$ with relati-



$$x_{\alpha}y_{-\beta}-q^{-(\alpha,\beta)}y_{-\beta}x_{\alpha}=\delta_{\alpha,\beta}.$$

The subalgebra \tilde{U}^- (resp. \tilde{U}^+) generated by the $y_{-\alpha}$ (resp. x_{α}): $\alpha \in \pi$, is a free algebra graded by $-\mathbb{N}\pi$ (resp. $\mathbb{N}\pi$). Define a skew derivation $\delta_{-\alpha}$ on \tilde{U}^+ by $x_{\nu} \mapsto x_{\nu}y_{-\beta} - q^{-(\nu,\beta)}y_{-\beta}x_{\nu}$. Then $y_{-\alpha} \mapsto \delta_{-\alpha}$ extends to an algebra homomorphism of \tilde{U}^- into $End(\tilde{U}^+)$ and U^- is defined to be its image. Similarly U^+ is defined. This leads to a non-degenerate bilinear form φ on $U^- \times U^+$. From this one shows that U^{\pm} admit spezializations to enveloping algebras $U(\mathfrak{n}^{\pm})$ and that $\mathfrak{g} := \mathfrak{n}^- \oplus \mathfrak{h} \oplus \mathfrak{n}^+ \twoheadrightarrow \mathfrak{g}_C$. Combining φ with the Drinfeld-Rosso construction gives a Hopf algebra $U_q(\mathfrak{g})$ specializing to $U(\mathfrak{g})$. On joint work with G. Letzter the Shapovalev determinants of $U_q(\mathfrak{g})$ were shown to factor and their factors almost completely determined. Their finer analysis should settle if $\mathfrak{g} \xrightarrow{\sim} \mathfrak{g}_C$. Conversely this hypothesis is shown to remove the ambiguities in the above factors.

S. König: Exact Borel subalgebras of quasi-hereditary algebras

Exact Borel subalgebras of quasi-hereditary algebras are designed to play a role analogous to that of Borel subalgebras and Borel subgroups in Lie theory. The defining properties are in analogy to solvability and Poincaré-Birkhoff-Witt theorem. Strong exact Borel subalgebras moreover encode character theory.

Strong exact Borel subalgebras are shown to exist for the algebras to blocks of category O of a semisimple complex Lie algebra and for generalized Schur algebras to semisimple algebraic groups.

The proof goes in three steps: Firstly, necessary and sufficient conditions are proved for a subalgebra to be an exact Borel subalgebra of a given quasihereditary algebra. Secondly, explicit constructions of exact Borel subalgebras are given for the algebras mentioned above (for generalized Schur algebras this construction is due to Leonard Scott). The third step is a general construction having an exact Borel subalgebra as input and a strong exact Borel subalgebra as output.

Kazhdan-Lusztig conjecture and Lusztig conjecture are reformulated as statements about the structure of these subalgebras. This is an application of

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abstract Kazhdan-Lusztig theory of Cline, Parshall and Scott.

H. Lenzing: Wild canonical algebras and automorphic forms

Let k be a field, \mathfrak{A} a full subcategory of modules over a k-algebra Λ . For each k-linear endofunctor $F: \mathfrak{A} \to \mathfrak{A}$ and object $X \in \mathfrak{A}$

$$\mathbb{A}(F;X) = \bigoplus_{n=0}^{\infty} Hom(X,F^nX)$$

is a \mathbb{Z}_+ -graded algebra with multiplication $u_n \cdot v_m := (F^m u_n) \circ v_m$.

Theorem 1. Let $k = \mathbb{C}$ and Δ be a Dynkin diagram, G a corresponding binary polyhedral group, Λ a path algebra of an extended Dynkin quiver $\tilde{\Delta}$, τ_{Λ}^{-} the inverse Auslander-Reiten translation for Λ -modules, X a projective Λ module of defect -1. Then the completion $A(\tau_{\Lambda}^{-}, X)$ is the surface singularity of type Δ , describing the singularity of \mathbb{C}^2/G in the origin. Also $A(\tau_{\Lambda}^{-}, X) \cong \mathbb{C}[X, Y]^G$.

Theorem 2. Let $k = \mathbb{C}$ and Λ be a canonical algebra (sense of C. M. Ringel) attached to a weight sequence $(p_1, \ldots, p_t) \in \mathbb{N}^t$ and a sequence $\lambda_1, \ldots, \lambda_t$ of pairwise distinct members from $\mathbb{P}_1(k)$. Assume Λ is wild.

Let G be a Fuchsian group of the first type with data $p_1, \ldots, p_t, \lambda_1, \ldots, \lambda_t$, i.e. G is a discrete subgroup of the automorphism group of $\mathbb{H}_+ = \{z \in \mathbb{C} \mid Im(z) > 0\}, \mathbb{H}_+/G = \mathbb{P}_1(k)$, there are exactly t orbits $\lambda_1, \ldots, \lambda_t \in \mathbb{P}_1(k)$ with non-trivial stabilizer group that are all cyclic of order p_1, \ldots, p_t respectively. Let $F = \tau_{\Lambda}$ be the Auslander-Reiten translation for Λ and X a rank one module over Λ (s.t. $\tau^n X \neq 0$ for $n \geq 0$). Then

 $\mathbf{A}(\tau, X)$ = algebra of entire G-automorphic forms on \mathbb{H}_+ . In particular, $\mathbf{A}(\tau, X)$ is commutative, finitely generated over k and Gorenstein of Krull dimension 2.

E. S. Letzter: Prime ideals in quantum matrices at p^{th} roots of unity

When q is a primitive t^{th} root of unity over a field k there is a copy of

 $\mathcal{O}(M_n(k))$, the classical coordinate ring of $n \times n$ matrices, embedded within the center of $\mathcal{O}_q(M_n(k))$, the quantum coordinate ring of $n \times n$ matrices. Letting t = p be an odd prime number, we study the resulting surjection from $spec \mathcal{O}_q(M_n(k))$ onto $spec \mathcal{O}(M_n(k))$, proving that the fibers are exactly the orbits of the $(\mathbb{Z}_p)^n \times (\mathbb{Z}_p)^n$ action on $spec \mathcal{O}_q(M_n(k))$ arising from row and column multiplication of the variables by powers of q. The same conclusion holds for $\mathcal{O}_q(GL_n(k))$.

G. Letzter: What bilinear forms tell you about quantum groups (joint with A. Joseph)

Let $U_q(\mathfrak{g})$ be the quantized enveloping algebra associated to the Kac-Moody algebra \mathfrak{g} and indeterminate q. We discuss two bilinear forms on $U_q(\mathfrak{g})$: Rosso's form and Shapovalev's form. Using these forms, we factor the Shapovalev determinant and then prove the following conjecture of Drinfeld: simple $U_q(\mathfrak{g})$ modules of highest weight q^{λ} specialize to simple $U(\mathfrak{g})$ modules of highest weight λ for all $\lambda \in \mathfrak{h}_0^{\bullet}$.

M. Lorenz: Grothendieck groups of invariant rings (joint work with K. A. Brown)

Let $S = k[x_1, ..., x_n]$ be the polynomial algebra over the field k and let G be a finite subgroup of $GL_n(k)$. Then G acts on S via linear transformations of the space of variables $V = \sum_{i=1}^{n} kx_i$, and we let $R = S^G$ denote the algebra of G-invariants in S. Our main result is a description of the Grothendieck group $G_0(R)$ under the hypothesis that chark $\bigwedge |G|$:

Theorem. $G_0(R) \cong G_0(kG) / \sum_{1 \neq H \leq G} Ind_{N_G(H)}^G(\alpha_H \cdot G^{\bullet}(H)).$

Here, $G^{\bullet}(H) = \langle [M] \in G_0(kN_G(H)) \mid M^H = (0) \rangle_{\mathbf{Z}} \subseteq G_0(kN_G(H))$ and $\alpha_H = \sum (-1)^i [\Lambda^i V(H)] \in G_0(kN_G(H))$ where $V(H) = \langle v - v^h \mid v \in V, h \in H \rangle_k \subseteq V$. We discuss some applications and special cases of this result.

J. Moody: Braid representations

Let L_i be a B_i -equivariant local system on $X_i = \mathbb{C} \setminus \{1, 2, ..., n\}$, with all compatibly nested $L_1 \subset L_2 \subset L_3 \subset ...$

Let e = number of values of *i* such that L_i defines an effective intersection theory for curves in X_i , $f_0 =$ number of values of *i* such that B_i acts faithfully on a fibre of L_i , f = number of values of *i* such that B_i acts faithfully on $H_c^1(X_i, L_i)$. Then $e \leq f \leq e + 2f_0$. So, when the fibre actions aren't already faithful, *e* determines *f* to within a finite range.

The case of abelian monodromy has $f_0 = 1$. Details of the generalization to nonabelian monodromy can be found in an upcoming paper of D.Jong.

S. Montgomery: Indecomposable coalgebras and pointed Hopf algebras

For any coalgebra C, we construct a graph Γ_C as follows: a) vertices are the simple subcoalgebras of Cb) edges $S_1 \to S_2$ if $\Delta^{-1}(C \otimes S_1 + S_2 \otimes C) \neq S_1 + S_2$ C is link-indecomposable if Γ_C is connected.

Theorem: $C = \bigoplus C_{\alpha}$, where the C_{α} are link-indecomposable.

Corollary: C is indecomposable if and only if C is link-indecomposable. Thus any C is a direct sum of indecomposable coalgebras.

Theorem: Let H be a pointed Hopf algebra with G = G(H), the grouplike elements. Let $N = \{x \in G \mid x \in H_{(1)}, \text{ the indecomposable com$ ponent of <math>H containing 1}. Then $N \triangleleft G$, $H_{(1)}$ is a subHopfalgebra, and $H \cong H_{(1)} \#_{\sigma} k(G/N)$.

This theorem can be applied to the pointed Hopf algebra $U_a(g)$.

The results generalize classical work of Cartier–Gabriel and Kostant when H (and C) are cocommutative.

M. L. Nazarov: Young's symmetrizers for projective representations of the symmetric group

As early as in 1911 Issai Schur discovered a non-trivial central \mathbb{Z}_2 -extension T_n of the symmetric group S_n . That is, the group S_n possesses projective representations which cannot be reduced to linear ones. However, no explicit construction of the irreducibles had been known until the recent time. The analogue of Young's orthogonal form was produced in [M. L. Nazarov, J. London Math. Soc. 42 (1990) 437-451]. The analogue of Young's symmetrizers has been still unknown.

As recently as in 1986 Ivan Cherednik found an alternative description of Young's symmetrizers based on the representation theory of the affine Hecke algebra of the series A. This approach provides new multiplicative expressions for the symmetrizers. The same approach allows to find their projective counterparts. But instead of the group T_n one should consider a central extension of the hyperoctahedral group $S_{nP} < \mathbb{Z}_2^n$, and construct an appropriate version of the affine Hecke algebra.

F. van Oystaeyen: Schematic algebras: Grothendieck topologies and quantum sections (joint work with L. Willaert)

Hoping to provide a first answer to a question of M. Artin we define a class of noncommutative graded algebras having a geometric theory of Proj. Most algebras recently studied in quantum-ring theory are in this class, e.g. Weyl algebras, rings of differential operators on varieties, enveloping algebras of Lie algebras, colour Lie super algebras, certain gauge algebras and Witten gauge algebras, innocent quantum spaces and gauge algebras iterated from it, twisted homogeneous coordinate rings The covering property in terms of Ore sets defining schematic algebras allows to obtain a structure sheaf on a Grothendieck topology defined on the set of 'words' in Ore sets where $ST = \{st, s \in S, t \in T\}$ for Ore sets S and T. This topology is 'noncommutative' because the exact functor on R - mod, $Q_T Q_S(:=Q_{ST})$ is in general different from $Q_S Q_T$ and $Q_{S \vee T}$ where $S \vee T$ is the Ore set generated by S and T. For a word w we have Q_w , for w' < w we have a ring (or module) morphism $Q_{w'}(R \text{ or } M) \to Q_w(R \text{ or } M)$. The section theorem then states $Q_{k_+}(R \text{ or } M) = \lim_{x \to \infty} Q_w(R \text{ or } M)$ and $Q_w(R \text{ or } M) = \lim_{x \to \infty} Q_{ww'}(R \text{ or } M)$

expressing the fact that we do have a sheaf on the Grothendieck topology and a correspondence 'coherent sheaves on this site' \leftrightarrow Proj (R).

The topology is 'quantum-commutative' because for quantum-sections: $(Q_SQ_T(R \text{ or } M))^{\wedge} = Q_{S\vee T}^q(R \text{ or } M) > (Q_TQ_S(R \text{ or } M))^{\wedge}.$

The sheaf of quantum sections on the site is easily defined and it is a (negatively) filtered sheaf with a coherent ideal in degree -1 such that modulo this coherent ideal we get the projective structure sheaf of the associated graded ring. Since properties of being schematic, or of being an Auslander regular ring, lift from the graded ring G(R) to the Rees rings \tilde{R} , it is clear that the class of schematic algebras and the subclass of regular schematic algebras are probably the desired classes of non-commutative 'geometric' rings.

C. M. Ringel: The Hall algebra approach to quantum groups

Let Δ be the Cartan matrix of a Lie algebra of type $A_n, \mathbb{D}_n, \mathbb{E}_6, \mathbb{E}_7, \mathbb{E}_8$. The isomorphism

$$U_{\mathfrak{g}}(\mathfrak{n}_{+}(\Delta)) \cong H_{\bullet}(\overline{\Delta})$$

(here, $\overline{\Delta}$ is obtained from Δ by choosing an orientation, and $H_{\bullet}(\overline{\Delta})$ denotes the corresponding twisted generic Hall algebra) is used in order to derive properties of $U_q(\mathbf{n}_+(\Delta))$. By definition, $H_{\bullet}(\overline{\Delta})$ has a free $\mathbb{Z}[v, v^{-1}]$ -basis consisting of the isomorphism classes of finite length $k\overline{\Delta}$ -modules (where k is some fixed field), thus we obtain in this way a basis of $U_q(\mathbf{n}_+(\Delta))$ which turns out to be a PBW-basis. These basis elements are iterated v-commutators, starting from the simple modules. In the case of A_3 , we deal with the problem of describing Lusztig's canonical basis explicitly. In particular, the tight monomials can be related to certain tilting sets in the stable module category of the preprojective algebra of type A_3 .

J. C. Robson: Hidden matrix rings (joint work with L. S. Levy and J. T. Stafford)

We investigate subrings of an $n \times n$ -matrix ring which, despite appearing otherwise, are themselves full rings of $n \times n$ -matrices; that is, are hidden



matrices. In general, this problem is subtle, but we give fairly complete results in a number of situations. For example, we prove:

Theorem A. Let K be an ideal of a ring R, let $T = (R_{ij})$ be a tiled subring of $M_n(R)$ containing $M_n(K)$, let $R_{ii} = R_{jj}$ for all i and j and $R_{ii}/K \simeq M_n(D)$ for some ring D. Then $T \simeq M_n(S)$ for some specific ring S.

The subtleties are illustrated by:

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Theorem B. Let \mathbb{H} be the ring of integer quaternions, p be an odd prime and $R = \mathbb{H} + M_2(p\mathbb{H}) \subseteq M_2(\mathbb{H})$. Then $R \simeq M_2(S)$ for some ring S if and only if $p \equiv 1 \pmod{4}$.

K. W. Roggenkamp: On the global structure of regular orders of dimension two (joint work with Y. A. Drozd)

Let \mathcal{O} be a regular domain of dimension two with field of fractions K, and Λ an \mathcal{O} -order in a separable K-algebra Λ . Λ is said to be endo-regular (semi-endo-regular) if gldim $End_{\Lambda}(M) = 2$ for every finitely generated (indecomposable) Cohen Macaulay module M. These conditions are inherited by localizations and completions at $max(\mathcal{O})$. For endo-regular orders the converse also holds, and we give a complete description of them. In case \mathcal{O} is local we also describe the semi-endo regular orders. Globally we give examples, based on algebraic geometry, which show that the converse implications, are not true.

M. Schaps: Hecke algebras and liftable deformations of group algebras (joint work with M. Gerstenhaber)

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Let k be a unitary commutative ring (e.g. Z) and $k_n = k[q, q^{-1}][[2]_{q^2}^{-1}, ..., [n]_{q^2}^{-1}]$. We give a simplified approach to the q-Schur decomposition of $V^{\otimes n}$, $V = \langle x_1, \ldots, x_d \rangle$, by constructing an orthogonal basis (with invertible square norms) for $V^{\otimes n}$ which is compatible with the action of the q-Hecke algebra \mathcal{H} and of $U_q(\mathfrak{sl}_d(k_n))$, thus proving that \mathcal{H} is semisimple over all of $Spec(k_n)$, without the need to construct idempotents and matrix units explicitly. Either approach demonstrates that the Hecke algebra is a global solution of the Donald Flanigan problem for S_n , i.e. a deformation of kS_n which is semisimple at the generic point of the fiber over each prime. By contrast, for D_n , there is a global DF-deformation, but it is a nontrivial deformation of the Hecke algebra.

A. Schofield: Moduli spaces of representations of quivers

Let Q be a quiver; α an indivisible Schur root, $\langle \alpha, \alpha \rangle < 0$. Then there are smooth projective moduli spaces of representations of dimension α . Work in progress suggests that these have tilting bundles. Thus we realize the diagonal $\Delta : M \to M \times M$ as the degeneracy terms of a map between tensor-decomposable bundles on $M \times M$; from this we construct a resolution of Δ by tensor decomposable bundles. It remains to calculate suitable Ext groups. This reduces to a calculation of local cohomology in the unstable locus which has to be completed.

S. O. Smalø: Quasitilted algebras

A finite dimensional algebra Λ is called almost hereditary if it satisfies the following properties, (i) $gldim \Lambda \leq 2$, (ii) for each finitely generated indecomposable Λ -module X, either $pd_{\Lambda}x \leq 1$ or $id_{\Lambda}x \leq 1$. This talk gives different characterizations of these algebras both in homological and also in nonhomological terms tying them to tilting theory in locally finite dimensional hereditary abelian k-categories for a field k, and thereby identifying them with the class of quasitilted algebras. Results showing that this class of algebras is stable under skew group constructions and by forming endomorphism rings of projective modules, are also given.

S. P. Smith: The center of the 3-dimensional and 4-dimensional Sklyanin algebra (joint work with J. Tate)

Let $d \in \{3,4\}$, let *E* be an elliptic curve over a fixed algebraically closed field *k*, fix an identity $0 \in (E, +)$, and let $\tau \in E$ be such that $d\tau \neq 0$. Let $A = A_d(E, \tau)$ be the d-dimensional Sklyanin algebra associated to this data. Let Z(A) denote the center of *A*. Then *A* is a finite Z(A)-module if and only if τ is of finite order. Suppose that τ is of finite order, *n* say.



Define $S = \operatorname{Proj} Z(A)$, let A be the quasi-coherent sheaf of O_s -algebras determined by A, let Z denote the center of A and let $\operatorname{Spec} Z$ denote the projective scheme determined by glueing the local sections of Z. We describe Z(A), $\operatorname{Spec} Z$ and the Azumaya locus of A on $\operatorname{spec} Z$. For example, if d = 3, $\operatorname{Spec} Z \simeq \mathbb{P}^2$ and the non-Azumaya locus is isomorphic to the isogenous curve $E/\langle \tau \rangle$. When d = 4, $\operatorname{Spec} Z$ is a singular, normal, rational 3-fold. We describe $Sing(\operatorname{Spec} Z)$ which is also the non-Azumaya locus of A.

J. T. Stafford: Graded rings of Gelfand-Kirillov dimension 2 (joint work with M. Artin)

Let $R = \bigoplus_{i \ge 0} R_i$ be a graded domain with $\dim_k R_i < \infty$ where k is an algebraically closed field write gr Q(R) for the graded quotient ring of R; thus $gr Q(R) \cong D[zz^{-1}\sigma]$ for a division ring D.

Theorem 1. If R is as above and finitely generated as a k-algebra, with $2 \leq GKdimR \leq 2 + \epsilon$ ($\epsilon = \frac{1}{24}$ will do) then D is the function field of a (commutative projective nonsingular) curve X.

Moreover $GKdimR \leq 2$; indeed for some constant $c \ dimR_i \leq ci \forall i$. In a manner analogous to Serre's Theorem in the commutative case one can describe R geometrically. For example:

Theorem 2. Let R be as in Theorem 1. If $R_0 = k$ and R is generated by R_1 , then R is a 'twisted homogeneous coordinate ring':

 $R \cong \bigoplus_{n\geq 0} H^0(Y, \mathfrak{L} \otimes \cdots \otimes \mathfrak{L}^{\sigma^{n-1}})$ up to a finite dimensional vector space. Here Y is a curve birational to X and \mathfrak{L} is an invertible sheaf over X.

L. Unger: The simplicial complex of tilting modules

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Let A be a finite dimensional algebra over a field k and modA the category of finitely generated A-left-modules.

 $T \in modA$ is called a tilting module if the projective dimension of T is finite, if $Ext_A^i(T,T) = 0$ for all i > 0 and if there is an exact sequence $0 \to {}_AA \to T^1 \to \dots \to T^t \to 0$ with T^i in the additive closure of T. The set of tilting modules forms a simplicial complex Σ , and we report on properties of Σ such as shellability, connectedness, structure of links and

constructability of \sum . We also mention the relation between tilting modules and functorially finite subcategories of modA and answer two questions of Auslander and Reiten.

A. Zalesskii: Group rings of locally finite groups and representation theory

Let $G_1 \,\subset\, G_2 \,\subset\, \cdots$ be an infinite sequence of finite groups and F a field. Let FG denote the group algebra of G over F. If M is an FG-module then we write Irr(M) for the collection of all regular constituents of M (not counting the multiplicities) and we write Irr(FG) for M = FG, the regular FG-module. Let $S_i \subset Irr(FG_i)$ be a subset. We say that $\{S_i\}_{i=1,2,\ldots}$ is an inductive system if $S_i = \{Irr(\rho \mid G_i) : \rho \in S_{i+1} \text{ for all } i = 1, 2... (\rho \mid G_i \$ denotes the restriction of ρ to G_i). Several results are presented which assert (under various assumptions on G_i) that S_i is either 1_{G_i} or $Irr(FG_i)$ for all i (we call such systems trivial). Inductive systems are of some interest from the point of view of the representation theory. Another motivation to study them is the connection with ideals of group algebras of locally finite groups. Put $G = Lim G_i$ (the direct limit).

Theorem. There exists a natural 1-1 correspondence between two-sided ideals I of FG with FG/I semisimple and the inductive systems.

B. Zimmermann-Huisgen: Affine varieties of uniserial modules over finite dimensional algebras

Given an infinite field K, the finite dimensional path algebras modulo relations, $\Lambda = K\Gamma/I$, allowing only finitely many isomorphism types of uniserial modules are characterized. The problem arose in connection with approximations of modules by modules of a simpler structure. The question is not new, however, but has been raised by Auslander on many occasions, the first time (so he told me) at the AMS winter meeting in 1975.

The crucial preparatory result if of a general nature: For any algebra $\Lambda = K\Gamma/I$, it assigns to each path $p \in K\Gamma\backslash I$ an affine algebraic variety V(p), together with a natural surjection

 $\Phi: V(P) \longrightarrow$ (iso types of uniserials U in $\Lambda - mod$ with $pU \neq 0$ and length (R)



=length (p) + 1},

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which, in many cases is a bijection. Conversely, each variety V occurs in this fashion, and its geometry strongly impinges on the structure of the corresponding family of uniserial modules.

The description of those algebras Λ for which a) each of these varieties V(p) is either empty or consists of a single point, or b) there are only finitely many isomorphism types of uniserials, includes explicit structural descriptions of the uniserial modules arising in these cases.

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én s sar s

Berichterstatter: Steffen König

Tagungsteilnehmer

Prof.Dr. Jacques Alev Institut de Mathématiques Pures et Appliquées, UER 47 Université de Paris VI 4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Shimshon A. Amitsur Institute of Mathematics and Computer Science The Hebrew University Givat-Ram

91904 Jerusalem ISRAEL

Prof.Dr. Michel Van den Bergh Limburgs Universitaire Centrum (LUC) Universitaire Campus

B-3590 Diepenbeek

Prof.Dr. Christine Bessenrodt Fakultät für Mathematik Otto-von-Guericke-Universität Magdeburg Postfach 4120

D-39016 Magdeburg

Axel Boldt Dept. of Mathematics University of California

Santa Barbara , CA 93106 USA Prof.Dr. Amizam Braun Dept. of Mathematics and Computer Sciences University of Haifa Mount Carmel

Haifa 31905 ISRAEL

Prof.Dr. Ken A. Brown Dept. of Mathematics University of Glasgow University Gardens

GB-Glasgow , G12 80W

Prof.Dr. Robert Cannings Dept. of Mathematics Imperial College of Science and Technology Queen's Gate, Huxley Building

GB-London , SW7 2BZ

Dr. Theirry Dana-Picard Dept. of Mathematics Bar-Ilan University

52 100 Ramat-Gan ISRAEL

Prof.Dr. Carolyne Dean Department of Mathematics University of Michigan

Ann Arbor , MI 48109-1003 USA



4.

Prof.Dr. Richard Dipper Mathematisches Institut B Universität Stuttgart

D-70550 Stuttgart

Dr. Peter Dräxler SFB 343 - Diskrete Strukturen in der Mathematik Fakultät für Mathematik Universität Bielefeld Postfach 100131

D-33501 Bielefeld

Prof.Dr. Daniel R. Farkas Dept. of Mathematics Virginia Polytechnic Institute and State University 460 McBrvde Hall

Blacksburg , VA 24061-0123

Prof.Dr. Kenneth R. Goodearl Department of Mathematics UCSB

Santa Barbara CA 93106 USA

Dr. Joachim Gräter Institut für Analysis TU Braunschweig Pockelsstr. 14

D-38106 Braunschweig

Prof.Dr. Edward L. Green Dept. of Mathematics Virginia Polytechnic Institute and State University 460 McBryde Hall

Blacksburg , VA 24061-0123 USA

Prof.Dr. Dieter Happel Fachbereich Mathematik Technische Universität Chemnitz-Zwickau Reichenhainer Str. 41

D-09126 Chemnitz

Prof.Dr. Timothy Hodges Dept. of Mathematical Sciences University of Cincinnati

Cincinnati , OH 45221-0025 USA

Dr. Hans-Joachim von Höhne Institut für Mathematik II Freie Universität Berlin Arnimallee 3

D-14195 Berlin

Prof.Dr. Martin P. Holland Dept. of Pure Mathematics University of Sheffield

GB-Sheffield S3 7RH



Prof.Dr. Anthony Joseph Dept. of Mathematics The Weizmann Institute of Science P. O. Box 26

Rehovot 76 100 ISRAEL

Prof.Dr. Ellen E. Kirkman Dept. of Mathematics Wake Forest University Box 7311 Reynolda Station

Winston-Salem , NC 27109 USA

Dr. Steffen König Fakultät für Mathematik Universität Bielefeld Postfach 100131

D-33501 Bielefeld

Dr. Henning Krause Fakultät für Mathematik Universität Bielefeld Postfach 100131

D-33501 Bielefeld

Prof.Dr. Thomas H. Lenagan Dept. of Mathematics & Statistics University of Edinburgh James Clerk Maxwell Bldg. Mayfield Road, King's Building

GB-Edinburgh , EH9 3JZ

Prof.Dr. Helmut Lenzing FB 17: Mathematik/Informatik Universität Paderborn Warburger Str. 100

D-33098 Paderborn

Prof.Dr. Edward Letzter Dept. of Mathematics Texas A & M University

College Station , TX 77843-3368

Prof.Dr. Gail R. Letzter Department of Mathematics University of Virginia

Blacksburg , VA 24061 USA



Prof.Dr. Lieven Le Bruyn Dept. of Mathematics Universitaire Instelling Antwerpen Universiteitsplein 1

B-2610 Wilrijk

Prof.Dr. Martin Lorenz Department of Mathematics Temple University TU 038-16

Philadelphia , PA 19122 USA



Prof.Dr. John C. McConnell School of Mathematics University of Leeds

GB-Leeds , LS2 9JT

Prof.Dr. Maxim L. Nazarov Mechanical and Mathematical Faculty Moscow State University Department of Mathematics

Moscow 119899 RUSSIA

Prof.Dr. Gerhard Michler Institut für Experimentelle Mathematik Universität-Gesamthochschule Essen Ellernstr. 29

D-45326 Essen

Prof.Dr. Susan Montgomery Dept. of Mathematics, DRB 155 University of Southern California 1042 W 36 Place

Los Angeles , CA 90089-1113 USA Prof.Dr. Freddy van Oystaeyen Dept. of Mathematics Universitaire Instelling Antwerpen Universiteitsplein 1

B-2610 Wilrijk

Prof.Dr. Claus Michael Ringel Fakultät für Mathematik Universität Bielefeld Postfach 100131

1.4

D-33501 Bielefeld

Prof.Dr. John Moody Mathematics Institute University of Warwick

GB-Coventry CV4 7AL

Prof.Dr. J.Chris Robson School of Mathematics University of Leeds

GB-Leeds , LS2 9JT

Prof.Dr. Wolfgang Müller Fakultät für Mathematik und Physik Universität Bayreuth

D-95440 Bayreuth

Prof.Dr. Klaus W. Roggenkamp Mathematisches Institut B Universität Stuttgart

D-70550 Stuttgart

DFG Deutsche Forschungsgemeinschaft

Prof.Dr. Malka Schaps Dept. of Mathematics Bar-Ilan University

52 100 Ramat-Gan ISRAEL Prof.Dr. J. Toby Stafford Department of Pure Mathematics Leeds University : 4

.

GB-Leeds LS2 2JT

Prof.Dr. Aidan H. Schofield Department of Mathematics University of Bristol University Walk

GB-Bristol . 858 1TW

Dr. Martha Takane c/o Prof. Dr. Claus Michael Rin Fakultät für Mathematik Universität Bielefeld Postfach 100131

D-33501 Bielefeld

Prof.Dr. Lance W. Small Dept. of Mathematics University of California, San Diego

La Jolla , CA 92093-0112 USA Dr. Luise Unger FB 17: Mathematik/Informatik Universität Paderborn Warburger Str. 100

D-33098 Paderborn

Prof.Dr. S.O. Smalo Institutt for Matematikk og Statistikk Universitetet i Trondheim, AVH

N-7055 Dragvoll

Prof.Dr. Alexander E. Zalesskii Institute of Mathematics Academy of Sciences of Belarus ul. Surganova 11

Minsk 220604 BELORUSSIA

Prof.Dr. S. Paul Smith Dept. of Mathematics University of Washington C138 Padelford Hall, GN-50

Seattle , WA 98195 USA Prof.Dr. Birge Zimmermann-Huisgen Dept. of Mathematics University of California

Santa Barbara , CA 93106 USA

DFG Deutsche Forschungsgemeinschaft

E-mail Adressen:

J.Alev S.A.Amitsur M.van den Bergh C.Bessenrodt A.Boldt A.Braun K.Brown R.Cannings Th.Dana-Picard C.Dean **R**.Dipper P.Dräxler D. Farkas K.R.Goodearl E.Green D.Happel T.J.Hodges H.J.von Höhne M.P.Holland A.Joseph E.Kirkman S.König H.Krause L.LeBruyn T.Lenagan H.Lenzing E.S.Letzter G.Letzter Maria Lorenz Martin Lorenz J.McConnell G.Michler S.Montgomery J.Moody

ile@ccr.jussieu.fr amitsur@sunrise.huji.ac.il vdbergh@math.u-strasbg.fr bessen@dmdtull.urz.tu-magdeburg.de axel@uni-paderborn.de rsma785@haifauvm.bitnet k.a.brown@cms.glasgow.ac.uk r.cannings@ma.ic.ac.uk dana@bimacs.cs.biu.ac.il cdean@math.lsa.umich.edu rdipper@tunix.mathematik.uni-stuttgart.de draexler@mathematik.uni-bielefeld.de farkas@math.vt.edu goodearl@math.ucsb.edu green@math.vt.edu happel@mathematik.tu-chemnitz.de hodges@ucbeh.san.uc.edu hoehne@math.fu-berlin.de m.holland@sheffield.ac.uk mtjoseph@weizmann.ac.il kirkman@mthcsc.wfu.edu koenigs@mathematik.uni-bielefeld.de henning@mathematik.uni-bielefeld.de lebruvn@wins.uia.ac.be t.lenagan@edinburgh.ac.uk helmut@uni-paderborn.de letzter@math.tamu.edu letzter@math.vt.edu angelone@mern.math.pitt.edu lorenz@euclid.math.temple.edu pmt6jcm@leeds.ac.uk mat4bb@vm.hrz.uni-essen.de smontgom@mtha.usc.edu moody@maths.warwick.ac.uk

DFG Deutsche Forschungsgemeinschaft M.Nazarov F.van Oystaeyen C.M.Ringel C.Robson K.W.Roggenkamp M.Schaps A.Schofield L.W.Small S.O.Smalø P.Smith J.T.Stafford M.Takane L.Unger A.E.Zalesskii B.Zimmermann-Huisgen nz@ipmce.su francin@wins.uia.ac.be ringel@mathematik.uni-bielefeld.de pmt6jcr@leeds.ac.uk kwr@machnix.mathematik.uni-stuttgart.de mschaps@bimacs.cs.biu.ac.il na_as@cms.ac.uk lwsmall@ucsd.edu sverre.smalo@avh.unit.no smith@math.washington.edu jts@math.lsa.umich.edu takane@math.eatik.uni-bielefeld.de luise@uni-paderborn.de imanb%imanb.belpak.minsk.by@demos.su birge@math.ucsb.edu

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