MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 38/1993

Special Complex Varieties

22.–28.08.1993

Die Tagung fand unter der Leitung von W. Barth (Erlangen) und A. Van de Ven (Leiden) statt. Sie beschäftigte sich mit der Untersuchung spezieller komplex-algebraischer Mannigfaltigkeiten (Kurven, Flächen, dreidimensionale Mannigfaltigkeiten). Insbesondere Calabi-Yau-Mannigfaltigkeiten standen wegen ihrer Beziehung zu modernen Entwicklungen im Zentrum des Interesses, vor allem bei den stattfindenden Diskussionen.

Vortragsauszüge

P.M.H. Wilson:
Classification Problems for Calabi-Yau Threefolds

We review first some previous work of the speaker on Calabi-Yau threefolds, and Calabi-Yau models (i.e. complex projective normal threefolds which are dominated by a smooth Calabi-Yau). For X a Calabi-Yau model, we have defined on $Pic(X)$ a cubic form $(D \mapsto D^3)$ and a linear form $(D \mapsto f^*D \cdot c_2(\tilde{X}))$; here $f : \tilde{X} \to X$ denotes a desingularisation of X.

In $Pic_{\mathbb{R}}(X) = Pic(X) \otimes \mathbb{R}$ we have the cubic cone $W^* = \{D;\ D^3 = 0\}$ and the nef cone $\bar{\mathcal{K}}$. Away from W^*, $\bar{\mathcal{K}}$ is locally rational polyhedral, with codim r faces $(\not\subset W^*)$ corresponding to birational contractions $\Phi : X \to \bar{X}$ with $\rho(\bar{X}) = \rho(X) - r$.

We propose that one should study Calabi-Yau minimal models (i.e. with $\partial\bar{\mathcal{K}} \subset W^*$) and use the linear form c_2 for classification purposes. For X a Calabi-Yau minimal model, we have 3 possibilities:

(1) c_2 is the zero form on $Pic(X)$.

(2) c_2 non-zero but $c_2 \cdot D = 0$ for some D.

(3) $c_2 \cdot D > 0\ \forall D \in \bar{\mathcal{K}}$ — "minimal CY model of general type".

We briefly discuss (2) and (3) (except possibly in very special cases, (2) will yield a fibre space structure on X), before describing the proof of the following result (which implies that in case (1), X is the finite quotient of a torus).

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Theorem. (Shepherd-Barron, Wilson) *If X is a complex projective 3-fold with canonical singularities and with K_X numerically trivial and the Q-Chern class $\tilde{c}_2(X)$ trivial as a form on $Pic(X)$, then X is isomorphic to the quotient of an abelian 3-fold by a finite group acting freely in codim 1. Moreover, if c_2 as defined above is trivial as a form on $Pic(X)$, then the same result holds but with the group action being free in codimension 2.*

A. Teleman:
Moduli spaces of stable holomorphic bundles over non-Kähler surfaces

We introduce the concept of stability for bundles over arbitrary hermitian compact manifolds and we explain the Kobayashi-Hitchin correspondence in the general hermitian case. We show that this correspondence gives a real analytic open embedding of the moduli space $\mathcal{M}_g^{HE}(E,h)$ of irreducible HE connections (in a hermitian C^∞-bundle (E,h)) in the moduli space $\mathcal{M}^{\mathrm{simple}}(E)$ of simple holomorphic structures in E. As application we point out the openness properties of the stability concept: with respect to the bundle, the hermitian metric on the base and the complex structure on the base.

We give, finally, explicit descriptions for the Donaldson compactifications of the moduli spaces of instantons over primary Kodaira surfaces, elliptic fibre bundles and over the blown up (standard) Hopf surface.

M. Zaidenberg:
Exotic structures on $\mathbb{C}^n$, $n \geq 3$, of positive Kodaira dimension

An exotic $\mathbb{C}^n$ is an affine algebraic variety which is diffeomorphic but not isomorphic to $\mathbb{C}^n$. By a theorem of C.P. Ramanujam (1971) there exist no exotic $\mathbb{C}^2$; however, there is a lot of examples of smooth contractible affine algebraic surfaces non-isomorphic to $\mathbb{C}^2$ (Ramanujam, 1971; Gurjar & Miyanishi, Sugie, tom Dieck & Petrie, Zaidenberg, 1987-1992). First examples of exotic $\mathbb{C}^n$-s for $n \geq 3$ arized as product-structures $X \times \mathbb{C}^{n-2}$, where X is a surface as above, due to a remark of Ramanujam. They are of logarithmic Kodaira dimension $\bar{\kappa} = -\infty$, while for X of log-general type $X \times \mathbb{C}^{n-2}$ is even non-biholomorphic to $\mathbb{C}^n$ (Zaidenberg, 1990). Non-product exotic $\mathbb{C}^n$-s have been constructed recently by several different methods (Dimea, Kaliman, Russell, tom Dieck, 1990-1993). Most of them, being quasi-homogenous hypersurfaces in $\mathbb{C}^{n+1}$, have positive Kodaira dimension; in particular, there are exotic $\mathbb{C}^3$-s in $\mathbb{C}^4$ of $\bar{\kappa} = 2$ (Kaliman, Russell). tom Dieck remarked that a product of several contractible smooth surfaces of log-general type is a log-general type exotic $\mathbb{C}^n$; however, no example of exotic $\mathbb{C}^n$ being a hypersurface in $\mathbb{C}^{n+1}$ is known, that has log-general type.

Most of known hypersurfaces in $\mathbb{C}^4$ diffeomorphic to $\mathbb{C}^3$ have been proven to be exotic even having $\bar{\kappa} = -\infty$ (Kaliman, Makar-Limanov, 1993). But there are still some of them which are not distinguished from $\mathbb{C}^3$; for example, this is $X_0 = \{(x,y,z,t) \in \mathbb{C}^4 \mid x + x^2y + z^3 + t^2 = 0\}$. If X_0 is isomorphic to $\mathbb{C}^3$ (or at least some of these quasi-homogenous 3-folds), it gives counterexamples to both of the linearization problems for regular embeddings $\mathbb{C}^2 \hookrightarrow \mathbb{C}^3$ and for $\mathbb{C}^*$-actions on $\mathbb{C}^3$. Otherwise, an important reduction of the linearization problem for $\mathbb{C}^*$-actions would hold (Russell).

R. Brussee:

(-1)-curves, (-1)-spheres, and the Van de Ven conjecture

A (-1)-sphere is an oriented smooth 2-sphere in a 4-manifold M with self intersection -1.

Conjecture 0.1. *(Friedman and Morgan) Let X be an algebraic surface with Kodaira dimension $\kappa \geq 0$, then every (-1)-sphere is rationally homologous to a (-1)-curve up to sign.*

Theorem 0.2. *Conjecture 0.1 is true if $p_g > 0$ and one of the following conditions holds*

(i) $b_1(X) = 0$, $p_g K_{min}^2 \equiv 0$ (mod 2) and $|K_{min}|$ contains a reduced divisor.

(ii) there is no class in $\overline{\mathrm{NE}}(X_{min})$ with self intersection (-1).

Here $\overline{\mathrm{NE}}$ is the Mori cone, i.e. the closure of the positive cone generated by effective divisors. More generally we have

Theorem 0.3. *If X is an algebraic surface with $p_g > 0$ then up to sign every (-1)-sphere is rationally homologous to either a (-1)-curve or a class $C \in \overline{\mathrm{NE}}(X)$ with $C^2 = -1$.*

The proof is based on two observations: the divisibility of the Donaldson-Kotschik Φ_k-polynomials by (-1)-spheres, and the pureness of Φ_k in the natural Hodge structure on $S^d H^2(X)$. The theorems then follow from an application of the Donaldson and O'Grady non vanishing theorems.

The pureness of the Donaldson polynomials is also useful for understanding of the Kronheimer-Mrowka classes K_i, on algebraic surfaces. One can prove that the Kronheimer-Mrowka classes are in the Neron Severi group, and that they are closely related to the canonical class.

K. Oguiso:

Calabi-Yau Moisheson 3-folds whose cubic forms vanish identically

Theorem. *Generic Calabi-Yau threefolds of type (2,4) contain smooth, infinitesimally rigid rational curves of each degree.*

As an application, we construct a Moishezon Calabi-Yau threefold whose cubic form is identically zero and covered by effective algebraic one-cycles homologous to zero.

V. Lin:

Holomorphic mappings between manifolds of polynomials without multiple roots

For each natural $m \geq 3$, consider the domain $G_m \subset \mathbb{C}^m$ consisting of all points $z = (z_1, \ldots, z_m)$ for which the polynomial in one variable t $p_m(t,z) = t^m + z_1 t^{m-1} + \ldots + z_m$ has no multiple roots. The following two problems arise naturally in a connection with a part of the 13th Hilbert Problem concerning superpositions of algebraic functions:

A. Describe holomorphic mappings $f : G_n \to G_k$ for all pairs $n, k \in \mathbb{N}$, $n, k \geq 3$.

B. Describe holomorphic mappings $f = (f_1, \ldots, f_k) : G_n \to G_k$ which satisfy the following condition (*):

(*) For every point $z = (z_1, \ldots, z_n) \in G_n$ the polynomials $p_n(t,z) = t^n + z_1 t^{n-1} + \ldots + z_n$ and $p_k(t, f(z)) = t^k + f_1(z) t^{k-1} + \ldots + f_k(z)$ are coprime.

A mapping $f : G_n \to G_k$ is called splittable if it is homotopic to a composition $g \circ d_n : G_n \xrightarrow{d_n} \mathbb{C}^* \xrightarrow{g} G_k$, where $d_n : G_n \to \mathbb{C}^* = \mathbb{C}\backslash\{0\}$ is the standard polynomial mapping defined by the discriminant $d_n(z)$ of the polynomial $p_n(t, z)$, and g is a continuous mapping. Holomorphic splittable mappings have a simple explicit description, and the Main Conjecture is that (with a few exceptions) the condition $n \neq k$ implies that any holomorphic mapping is splittable. This Conjecture is proved for $n > \text{maximum}(k, 4)$, and also for $4 < n < k$ under condition $k \not\equiv n, n(n-1), n(n-1)+1, (n-1)^2 \bmod n(n-1)$. An explicit description of all unsplittable holomorphic self-mappings $G_n \to G_n$ $(n \neq 4)$ is obtained as well.

As to the problem B, it is proved that for $n > 3$ there is no holomorphic mapping $f: G_n \to G_k$ which satisfies (*). The proof involves some analytic properties of the Teichmüller space $T(0, n+1)$.

S. Popescu:

Some non general-type threefolds in $\mathbb{P}^5$

The purpose of the talk, which outlines joint work in progress with Wolfram Decker, is to describe the construction of some new examples of 3-folds in $\mathbb{P}^5$, which are not of general type. Motivation for our search are the existent classification of 3-folds in $\mathbb{P}^5$ (complete up to degree 11) and the following finiteness result, in the vein of the result proven by Ellingsrud and Peskine for smooth surfaces in $\mathbb{P}^4$:

Theorem. (Braun, Ottaviani, Schneider, Schreyer) *There are only finitely many families of smooth 3-folds in $\mathbb{P}^5$, which are not of general type.*

A precise bound for the degree d_0 of the non general-type 3-folds in $\mathbb{P}^5$ is not known. The evaluations used in the proof of the above theorem yield only something like $d_0 \leq 100000$. In any case, $d_0 \geq 18$ since we succeeded in constructing the following examples:

Theorem. *There exist families of smooth 3-folds $X \subset \mathbb{P}^5$ with the following invariants:*

1) $d = 13$, $\pi = 18$, $p_g = 1$, which are blown-up Calaby-Yau 3-folds,

2) $d = 17$, $\pi = 32$, $p_g = 1$, which are blown-up Calaby-Yau 3-folds, with a minimal model a complete intersection $(1,1,1,1,1)^2$ in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$,

3) $d = 18$, $\pi = 35$, $p_g = 0$, $h^2(\mathcal{O}_X) = 1$, $\chi = 2$ and with Kodaira dimension $\kappa(X) = -\infty$.

F. Catanese:

Nodal surfaces and irregular surfaces

Let X be a projective surface $/\mathbb{C}$ with singularities $p_1, \ldots, p_r$ nodes $(\approx \mathbb{C}^2/\pm 1)$, let $\pi: S \to X$ be a minimal resolution of singularities, $A_i = \pi^{-1}(p_i)$, $X^\# = X - \{p_1, \ldots, p_r\}$. A subset $\mathcal{N}$ is said to be 1) *even* if $\sum_{i \in \mathcal{N}} A_i \equiv 0\ (2)$ in $H^2(S, \mathbb{Z})$, 2) *topologically half-even* if $\exists D \in H^2(S, \mathbb{Z})$ with $\sum_{i \in \mathcal{N}} A_i + D \equiv 0\ (2)$, 3) *algebraically half-even* if such D is of type $(1,1)$. Sets of type 1), 2) are determined by a topological map $H^2(X^\#, \mathbb{Z}) \to (\mathbb{Z}/2)^2 = K$, and we obtain interesting binary codes, an example being given where

a) $X = M/i$, i an involution with isolated fixed-points on a smooth manifold, and

b) $M = C_1 \times C_2$, $i = (i_1, i_2)$, $C_j/i_j \cong \mathbb{P}^1$.

We call the resulting S a generalized Kummer surface $S_{n,m}$ (here $n - 1 = \text{genus}(C_1)$, $m - 2 = \text{genus}(C_2)$). For generalized Kummer surfaces we can actually compute $\pi_1(X^\#)$, which is generated by elements ε_{ij} of order 2 $(i = 1, \ldots, 2n,\ j = 1, \ldots, 2m)$. From this we determine all the conjugacy classes of $SO(3)$-representations dividing them into two categories:

I) $\rho: \pi_1(X^\#) = \pi$ is *hooked* if $\exists (i,j)$ s.t. $c_{ij} = \rho(\varepsilon_{ij}) = 1$, (i,k) s.t. $c_{ik} \neq 1$, (h,j) s.t. $c_{hj} \neq 1$.

II) non hooked, and these are either *vertical* $(c_{ij} = c_i \forall j)$ or *horizontal* $(c_{ij} = c_j \forall i)$.

Theorem. *If $\rho : \pi \to SO(3)$ is hooked, ρ factors through the Klein group $(\mathbb{Z}/2)^2 \subset SO(3)$ and is determined by its Stiefel Whitney class w_2. Else, ρ vertical $\to$ a representation of* $< \gamma_1, \dots, \gamma_{2n} \mid \gamma_i^2 = 1, \pi\gamma_i = 1 >$.

Now, to each such ρ, one associates an *orbifold bundle* $E^\#$ on X, such that $E^\#/X^\#$ has a canonical extension to S, according to Kronheimer. Actually, this construction works more generally for R.D.P.'s. We have the following:

Theorem. (Kronheimer et al.) Let X have A_n-singularities, then the canonical extension preserves V-D=VIRTUAL-DIMENSION, in case V-D=0, preserves $q_S(E) = q_X(E^\#)$ (number of points counted with orientation for a general choice of a Riemannian metric on S). Moreover, the V-D equals $-2p_1^\# - 3(b^+ - b^1 + 1) + \varepsilon k \delta^{(k)}$, $\delta^{(k)} = \#$ of points where $\rho(\text{local gen.}) = \begin{pmatrix} 1 & 0 \\ 0 & \varepsilon^{(k)} \end{pmatrix}$.

We hope to be able to complete the proof of the following

Claim. For $n = 3$, $m \equiv 2$, if $p_1 = -6\chi$, w_2 has $(w_2)^2 \equiv 1(4)$. $\exists a$ s.t. (V-D being =0) $q_S(P(p_1, w_2)) = 0, 1$ or a.

With this we hope to distinguish differentiable structures of non deformation equivalent Noether-Horikawa surfaces. What is finally the connection with irregular surfaces? It is as follows: for an even set of type 1) (or 3)), there is a canonical double cover $p : M \to X$ ramified precisely at the nodes in M (and on D in case 3)). Define $\mathcal{J}$ by $p_*(\mathcal{O}_M) = \mathcal{O}_X \oplus \mathcal{J}$, thus multiplication gives a symmetric bilinear map $\mathcal{J} \times \mathcal{J} \to \mathcal{O}_X$, and $\mathcal{J}$ is locally free except at $\mathcal{N}$.

Generalizing classical constructions, Barth showed that such $\mathcal{J}$ is gotten (if X is a surface in $P = \mathbb{P}^3$ or a 3-fold) as $\mathrm{Coker}(\alpha) : E \to E^\vee \otimes L$, where E is a vector bundle, L a line bundle, and $\alpha = {}^t\alpha$. It is still open the converse

Problem. Do all these even sets in $X \subset \mathbb{P}^3$ arise this way?

The case where E is split was solved by the author in '79, showing that then the converse holds, and this case is characterized by $H^1(\mathcal{J}(i)) = 0$ $\forall 0 \le i \le \frac{d-4}{2}$ ($d = \deg X$). A generalization of these ideas lead the author to find explicit equations for canonical rings of surfaces of general type with $q = 0$. Recently, using the method provided by the Beilinsons's spectral sequence, together with Schreyer, we proved the following:

1) Let $p : M \to X \subset \mathbb{P}^3$ be birational, and associated to $V \subset H^0(K_M)$

2) Assume X is not a quadric, write $H^0(K) \cong V \oplus W$, $H^0(2K) = \mathrm{Sym}^2(V) \oplus W'$.

3) Set $E \cong (W' \oplus \mathcal{O}_{\mathbb{P}_3}(-2)) \oplus (H^1(K) \otimes \Omega^1(-1)) \oplus W^\vee \otimes \Omega^2_{\mathbb{P}}$; then we have the following

Theorem. (–, Schreyer) $\mathcal{J} = p_*\mathcal{O}_M$ has a self-dual ($\alpha = {}^t\alpha$) resolution $0 \to \mathcal{O}(-5) \oplus E^\vee(-5) \xrightarrow{\alpha} \mathcal{O} \oplus E \to \mathcal{J} \to 0$, and α satisfies (R.C.), i.e. the sheaf of ideals generated by submaximal minors of α equals the one generated by maximal minors of α', obtained from α by removing the first row).

Conversely, if α satisfies (R.C.), $\alpha = {}^t\alpha$, $\det \alpha$ is irreducible, then $\mathcal{J}$ is a sheaf of rings, and if $\mathcal{R} = \bigoplus H^0(\mathcal{J}(i))$, $M = \mathrm{Proj}(\mathcal{R})$ is a canonical model of a surface of general type provided M has R.D.P.'s at most as singularities; plus a degree condition is verified. Can this method be applied to solve the above problem?

R.-P. Holzapfel:

Decomposition hierarchies of abelian threefolds corresponding to Picard modular surfaces

We look for a partial answer of the following general problem: Which elliptic curves sit in which abelian varieties? By means of the classification theory of endomorphism algebras the (fine) decomposition types of abelian varieties are introduced and, moreover, specialisations of them via families of abelian varieties and their restrictions to smaller base spaces. The central result in dimension 3 is the following:

Theorem. *There are precisely five decomposition types of abelian threefolds corresponding to (points of) Picard modular surfaces. The hierarchy structure of them is given by three specialisations of codimension 2 and three of codimension 1.*

The precise description of the types yield a criterion and a method for discovering non-trivial elliptic curve families sitting in families of our abelian threefolds. By example we show that it happens over curves inside of the moduli surfaces and not only over boundary components of compactifications.

Observe that the celebrated Taniyama conjecture is also a subproblem of the general one above; and also our subproblem has an arithmetic application. By additional use of Theorems of Wüstholz (Shiga), Shimura-Taniyama, Schneider and Feustel we find a rather short way to prove that the projective arithmetic degree (relation) of algebraic Picard integrals is ∞ (transcendental) or ≤ 6, where the latter case happens if and only if the Jacobian of the underlying Picard curve is of CM (complex multiplication)-type.

S. Müller-Stach:

Syzygies and the Abel-Jacobi map for cyclic coverings

There has been recent interest to give explicit bounds for the degree of ampleness needed in certain theorems of Hodge theory like infinitesimal Torelli, (generalized) Noether-Lefschetz theorem and others. We first look at the case of complete intersections on an arbitrary variety, where all the results are connected with Nori's theorem (Inv. 1993). We give an overview of known results and then give a proof of the following theorem about cyclic coverings:

Theorem. *Let X be a smooth projective 3-fold, A very ample and $f : Z \to X$ be a cyclic covering of degree N, branched along a smooth and general section $s \in H^0(X, A^{Nd})$. If either*

(a) $(X, A) = (\mathbb{P}^3, \mathcal{O}(1))$, $d \geq 5$ or

(b) $X \neq \mathbb{P}^3$, $d \geq 3r_0 + 13$, $r_0 = min\{r \in \mathbb{Z} \mid \omega_X^{-1} \otimes A^r \text{ nef}\}$, then the image of the Abel-Jacobi map for Z, $\Psi : CH_{\text{hom}} \to J^2(Z)$ is contained in $f^* J^2(X)$ modulo torsion.

C. Peskine:

Gonality and Clifford index of smooth complete intersection twisted curves in $\mathbb{P}^3(\mathbb{C})$

Let C be a smooth twisted curve in $\mathbb{P}_3$, complete intersection of two surfaces. Let L be a line l-secant to C and assume that C has no $(l+1)$-secant line. If H is a plane containing L, the divisor $D = H \cap C - L \cap C$ belongs obviously to a pencil of degree $d^oC - l$.

B. Basili (Paris 6) has proved the following results:

1) The gonality of C is $d^oC - l$.

2) If $\Delta \subset C$ is an effective divisor, belonging to a pencil computing the gonality, there exists a line L such that the positive divisor $\Delta + L \cap C$ is a plane section of C.

3) The Clifford index of C is $d^oC - l - 2$, except if $d^oC = 9$ in which case $l = 3$ and $\text{Cliff}(C) = 3$ is computed by $\mathcal{O}_C(1)$.

To establish this result, Basili studies the index of speciality of a group of points embedded in the projective space. $\Gamma \subset \mathbb{P}_n$, then $e(\Gamma)$ is the largest n such that Γ does not impose $d^o\Gamma$ conditions on hypersurfaces of d^on.

Theorem. *Let $\Gamma \subset \mathbb{P}_3$ be a group of points. Assume that Γ is analytically plane, i.e. the tangent space to Γ in a point is of dimension ≤ 2. Let t be an integer such that the surfaces of degree $\leq t$ containing Γ intersect in a scheme of dimension 0 or in a reduced irreducible curve of degree $\geq d^\circ\Gamma/t$. Assume $d^\circ\Gamma \leq it$ with $i \leq t$. Then*

1) $e(\Gamma) \leq i + t - 3$

2) $e(\Gamma) = i + t - 3 \iff \Gamma$ *is the complete intersection of surfaces of degrees* $1, i, t$.

3) $e(\Gamma) = i + t - 4 \iff$ *there exists a plane H such that $e(\Gamma \cap H) = e(\Gamma)$ with a list of exceptions all for $i \leq 4$.*

Using this theorem, Basili considers a moving divisor of degree $\leq d - 2$ on a smooth twisted complete intersection C of degree d. Assuming that all effective divisors strictly contained in this divisor do not move, she is able to prove that this divisor is contained in a plane. The residual divisor in the corresponding plane section has to be on a line. The computation of the Clifford index becomes then an easy application of a recent result of Coppens and Martens.

D. Huybrechts:

Moduli spaces of stable pairs (framed modules)

(joint work with M. Lehn) A framed module on a smooth projective variety consists of a coherent sheaf E together with an homomorphism $\alpha : E \to E_0$, where E_0 is any fixed coherent sheaf on the variety.

There is a stability condition for such objects which allows to construct moduli spaces for semistable pairs as a projective scheme by using Mumford's GIT.

A couple of examples for such moduli spaces is given by specializing E_0 to the trivial line bundle or to a trivial vector bundle on an effective divisor. These two cases correspond to Higgs pairs and framed vector bundles, resp.

The change of an extra parameter δ on which the stability condition depends is explained in the case of the Hecke correspondence for vector bundles on a curve.

S.A. Strømme:

A residue formula of Bott with applications to enumerative geometry of twisted cubic curves

(joint work with G. Ellingsrud) The formula is this: $\int_X G(\varepsilon_1, \dots, \varepsilon_n) = \sum_{x \in X^{\mathbb{C}^*}} \frac{G(\tau_1(x),\dots,\tau_n(x))}{w_1(x)\cdot\dots\cdot w_n(x)}$ where X is a nonsingular projective n-dimensional variety with a $\mathbb{C}^*$-action with isolated fixpoints, and E is an equivariant rank-r vector bundle with Chern roots $\varepsilon_1, \dots, \varepsilon_r$, and G is a symmetric homogenous polynomial of degree n in r variables. The $w_i(x)$ are the weights of $\mathbb{C}^*$ on $T_X(x)$, and $\tau_i(x)$ the weigths of $E(x)$, for any fixpoint $x \in X$ (Bott, J. diff. geometry 1, 1967).

Let H_n be the Hilbert scheme component parametrizing twisted cubic curves in $\mathbb{P}^n$ (and their flat degenerations). Then H_n is smooth projective of dimension $4n$, and has $130\binom{n+1}{4}$ fixpoints for the natural action of a maximal torus in $GL(n+1)$, hence same number of fixpoints for suff. general 1 parameter subgroup $\mathbb{C}^* \subseteq GL(n+1)$. Thus any zero-cycle on H_n which can be expressed as the top Chern class of some natural rank-$4n$ vector bundle on H_n, can be easily evaluated by Bott's formula, even without knowing the cohomology ring structure of H_n. We used this to compute the number 317206375 twisted cubics on a general quintic hypersurface in $\mathbb{P}^4$ and 1345851984605831119032336 twisted cubics on a general degree 9 hypersurface in

$\mathbb{P}^7$. During our stay here we also computed the number of elliptic quartic curves on a general hypersurface of degree $k+1$ in $\mathbb{P}^k$ ($k = 4, 5, 6$), the results are:

k	# elliptic quartic curves on hypers. of deg $k+1$ in $\mathbb{P}^k$
4	3718024750 smooth (+2875 × 1185 plane binodal quartics)
5	387176346729900
6	81545482364153841075

The first of these numbers confirmes a prediction coming from mirror symmetriy computations.

C. Okonek:

Complex 3-folds from a topological viewpoint

(joint work with A. Van de Ven) Our aim is to understand the classification of complex 3-folds from a topological point of view.

1. Topological classification: This has been developed in a series of papers by Wall, Jupp, Žabr; we reformulate their results and thereby transform the classification into an arithmetical moduli problem.

2. Arithmetic of cubic forms: The arithmetic classification of integral cubic forms is – up to finite indeterminency – equivalent to the homotopy classification of 1-connected 6-dim. manifolds with torsion free cohomology. There are two essentially different steps: i) algebraic classification, which can be handled by GIT; ii) arithmetical classification, for which we use reduction theory of algebraic groups to obtain finiteness results.

3. Cubic forms of complex 3-folds: We discuss almost complex structures and show that all triplets $(a, b, c) \in \mathbb{Z}^3$ satisfying the unitary cobordism relations can be realized by 1-connected almost complex manifolds. We prove that the existence of *Kähler structures* implies a certain non-degeneracy condition on the associated cubic form: its Hessian must be non-trivial. Finally we construct examples of complex 3-folds which give infinitely many deformation types in a fixed C^∞-type.

N. Shepherd-Barron:

Foliations of algebraic varieties

Given a foliation $\mathcal{F}$ on a complex algebraic variety X, a problem is to decide whether the leaves of $\mathcal{F}$ can be completed to projective subvarieties of X. The aim of the talk (describing some partial results of Ekedahl, R. Taylor and the speaker concerning vector fields in Abelian surfaces) was to describe how reduction mod p can sometimes be used to attack the problem, as in Miyaoka's theorem that (roughly speaking) $\mathcal{F}$ has algebraic leaves if $c_1(\mathcal{F})$ is positive and in Katz' proof of Grothendieck's conjecture for the Gauss-Manin connexion. One point that arises is that sometimes (as with the results of Miyaoka and Katz) reduction modulo an arbitrary infinite set of characteristics is sufficient, while at other times (e.g. for Abelian varieties over a number field K) it is necessary to consider almost all primes and to use the action of the full Galois group $Gal(\bar{\mathbb{Q}}/K)$.

R. Braun:

Conic bundles in $\mathbb{P}_4$

(Report on joint work with K. Ranestad.) A smooth surface S in $\mathbb{P}_4$ is called a conic bundle, if there exists a morphism $p : S \to C$ onto a smooth curve C, such that every fiber of p is embedded in $\mathbb{P}_4$ as a conic. Classically known are 2 examples: del Pezzo surfaces of degree 4 and Castelnuovo surfaces of degree 5. Some years ago Ellia and Sacchiero stated the Theorem: These are the only conic bundles in $\mathbb{P}_4$. The talk gave an outline of another proof, which is achieved in 2 steps: First we show, involving the method of generic initial ideals, that the degree of S does not exceed 42, and secondly by geometrical reasoning we exclude the remaining cases.

W. Ebeling:

Coxeter-Dynkin diagrams of the simple isolated complete intersection singularities

Let (X,x) be an isolated complete intersection singularity (icis) of positive dimension $n > 0$ defined by an analytic map germ $f : (\mathbb{C}^{n+k},0) \to (\mathbb{C}^k,0)$. The singularity (X,x) is called simple if in any deformation of (X,x) only finitely many isomorphism types of singularities occur. V.I. Arnol'd has classified the simple hypersurface singularities ($k = 1$). This result was extended to general k by M. Giusti (1977). Apart from hypersurface singularities only singularities of space curves ($n = 1$, $k = 2$) appear in Giusti's list. Giusti has given some diagrams for the simple space curve singularities. It is, however, not completely clear what his graphs mean: how is his basis related to a distinguished one? In former work, we have already shown for each singularity of Giusti's list except Z_9 and Z_{10} that there exists a Coxeter-Dynkin diagram corresponding to a dintinguished system of generators of vanishing cycles which contain Giusti's diagram as a subgraph.

In a joint work with S.M. Gusein-Zade we compute Coxeter-Dynkin diagrams with respect to distinguished systems of generators of vanishing cycles for the remaining singularities Z_9 and Z_{10} and show that the above result is also true for these singularities. The calculation is based on the use of real morsifications.

C. Peters:

Geography of simply connected spin-surfaces

We consider *minimal* complex projective surfaces of *general type*. The basic topological invariants are c_1^2 and χ and we know the restrictions on pairs $(x,y) = (\chi, c_1^2)$:

$$(*)\quad \begin{matrix} x > 0,\ y > 0 \\ 2x - 6 \le y \le 9x \end{matrix}$$

Results of Chen, Persson, and Xiao imply that almost all integral points in the (x,y)-plane satisfying (*) occur. Moreover quite a large part is realised by simply connected surfaces.

Since the topological type of such surfaces is completely determined by the triple (c_1^2, χ, w), where w is the modulo 2 reduction of the canonical class, a natural and easily verifiable question is whether the previously constructed surfaces have $w \equiv 0 \bmod 2$ (*spin surfaces*) or not. It turns out that most of them are not spin there remains the problem of constructing simply connected spin surfaces in the region (*).

Together with U. Persson and Xiao G. we have found some partial answers. First of all, the region $2x - 6 \le y < 8/3x$ is forbidden. Secondly, all surfaces in the region $16/5x < y <$

$8x$ – Const.$x^{2/3}$ occur (if some congruence relations dictated by topology are satisfied). Finally we have infinitely many simply connected spin surfaces with positive signature. For the latter, the calculation of the fundamental group is much simpler than for the hitherto known surfaces considered by Moishezon-Teicher (that also are spin and have positive signature).

J. Spandaw:

Very ample polarizations on complex abelian varieties

The results in this talk were obtained in collaboration with O. Debarre and K. Hulek. We work over $\mathbb{C}$.

The starting point is the following

Conjecture. (Debarre) Let (A, L) be the generic polarized abelian variety of $dim = g > 2$ and type $(1, \ldots, 1, d)$. If $d > g$ then $|L|$ is base point free and the associated morphism $\Phi : A \to \mathbb{P}^{d-1}$ is an embedding outside a closed subset of dimension $2g + 1 - d$.

In view of a result by Van de Ven/Debarre, this would be optimal. For $d \geq g + 2$ the statement is that Φ_L is birational onto its image. This can be proved. For $d \geq 2g + 2$ the conjecture asserts that L is very ample. Here we only have a partial result:

Theorem. *Let (A, L) be the generic polarized abelian variety of* dim $= g$ *and type $(1, \ldots, 1, d)$. If $d > 2^g - \frac{g(g-3)}{2}$ then L is very ample.*

The theorem is proved by degenerating (A, L) to a (polarized) $(\mathbb{P}^1)^{g-1}$-bundle over an elliptic curve modulo some gluing. As shown by Kollár, our result in dimension 3 implies the following version of a conjecture by Griffiths and Harris: for d odd, $d \geq 9$, the degree of any curve on a very general hypersurface of degree $6d$ in $\mathbb{P}^4$ is divisible by d.

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