MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

TAGUNGSBERICHT 39/1993

Random graphs and combinatorial structures

29th August - 4th September

Organizers: A. D. Barbour (Zürich) and B. Bollobás (Cambridge)

The aim of the meeting was to bring together pure probabilists, interested in probabilistic combinatorics in order to test their weapons, combinatorialists, wishing to apply powerful results in probability theory to standard problems in combinatorics, people studying probabilistic combinatorics for its own sake, and computer scientists, eager to apply probability theory, extremal combinatorics and probabilistic combinatorics to their problems. The mixture worked extremely well: there was plenty of interaction, to the benefit of all the parties. The main topics discussed in the twenty seven lectures were the refinements and applications of the Stein-Chen method, the probabilistic analysis of algorithms, Ramsey and anti-Ramsey questions about graphs, percolation in the plane and in the cube, hereditary properties of graphs and hypergraphs, and applications of point processes to large subgraphs of random graphs.

ABSTRACTS

Rudolf Ahlswede (Bielefeld)
Higher order extremal problems

The theory of write-efficient memories (WEM) has led to novel combinatorial optimization problems.

I. Instead of maximizing for instance the length of antichains, we maximize now the length of "cloud-antichains" $(\mathcal{A}_i)_{i=1}^N$, where $\mathcal{A}_i \subset 2^{[n]}$ and one of the following conditions holds for all $i \neq j$:

(1) Type $(\exists, \forall)$: $\exists A_i \in \mathcal{A}_i$ with A_i not comparable to A_j for all $A_j \in \mathcal{A}_j$.

(2) Type $(\forall, \exists)$: $\forall A_i \in \mathcal{A}_i \exists A_j \in \mathcal{A}_j$ with A_i not comparable to A_j.

(3) Type $(\exists, \exists)$: $\exists A_i \in \mathcal{A}_i \exists A_j \in \mathcal{A}_j$ with A_i not comparable to A_j.

We also consider the cases where the clouds are disjoint. There are analogous problems for the relations "comparable", "intersecting" and "disjoint" and all problems can also be formulated for k-graphs.

Most cases have been settled exactly or at least asymptotically. This can be found in Preprint 92-037 of SFB 343 (with N. Cai and Z. Zheng) where also many other problems and conjectures are formulated.

II. We proved a "Diametric Theorem in the Average" for general distances in sequence spaces (Preprint 93-003 of SFB 343, with I. Althöfer) and also a "Cross-diametric Theorem" for any number of sets (Preprint 93-079, with N. Cai).

Richard Arratia (University of Southern California)

From permutations via large deviations to the Ewens Sampling Formula

The results in this talk should hold throughout the logarithmic class of combinatorial structures, but so far they have only been proven for permutations. For permutations, a construction called the *Feller Coupling* removes one layer of conditioning, and makes proofs easy. In detail, let $\mathbb{P} = \mathbb{P}_1$ be the uniform measure on S_n, with probability $1/n!$ for each permutation, and, under $\mathbb{P}_1$, let $\xi_1, \xi_2, \dots$ be independent with $p_i = P(\xi_i = 1) = 1 - P(\xi_i = 0) = 1/2$. Let $C_i(n)$ be the number of i-cycles of our permutation, so that $\sum iC_i(n) = n$. The coupling is $C_i(n) = \#i$-spacings in $\xi_1\xi_2\dots\xi_n 1$. The total number of cycles is thus $K_n = C_1(n) + \dots + C_n(n) = \xi_1 + \dots + \xi_n$. For any $\theta > 0$ consider "tilted" or "twisted" measures $\mathbb{P}_\theta$ with $dP_\theta/dP = C_{\theta,n}\theta^{K_n}$. Under P_θ, the joint distribution of $(C_1^{(n)}, \dots, C_n^{(n)})$ is called the Ewens Sampling Formula; and ξ_i is Bernoulli, with $\mathbb{P}_\theta(\xi_i = 1) = \theta/(\theta + i - 1)$, the ξ_i still independent. For simplicity, we state our results for $k = \lceil\theta \log n\rceil$ with $\theta > 1$.

Theorem. *a) If* $\log b/\sqrt{\log n} \to 0$, *then*

$$d_{TV}(L_1C_1 \dots C_h(n)) \mid K \geq k),$$

$L_\theta(C_1, \dots, C_h) \to 0$. *[Recall that* $b/n \to 0$ *iff* $d_{TV}(\mathcal{L}_\theta(C_1, \dots, C_b), \mathcal{L}_\theta(Z_1, \dots, Z_b)) \to 0$, *with* Z_i *independent Poisson variables with mean* (θ/i)*.]*

b) $\mathbb{E}(C_i(n) \mid K_n \geq k) \to \theta/i$.

c) If $\log b/\log n \to 1$, $d_{TV}(\mathcal{L}_1(C_{b+1}, \dots, C_n) \mid K_n \geq k), \mathcal{L}_\theta(C_{b+1}, \dots, C_n(n))) \to 0$.

d) For $t \in (0,1)$, $\mathbb{E}_1(\sum_{i \leq n^t} C_i(n) \mid K_n \geq k) \sim \theta t \log n$.

The results are joint work with Barbour and Tavaré.

Graham Brightwell (London School of Economics)
Random graph orders

The random graph order $P_{n,p}$ is defined by (i) putting a random graph $G_{n,p}$ on the vertex set $[n] = \{1, \dots, n\}$, (ii) interpreting an edge ij, with $i \prec j$ in $[n]$, as a relation $i < j$, and (iii) taking the transitive closure. The talk surveyed the basic structure of $P_{n,p}$ for different ranges of $p = p(n)$; the new results we presented were obtained jointly with Béla Bollobás.

The height $H_{n,p}$ of $P_{n,p}$ has been studied by Newman, by Albert and Frieze, and by Alon, Bollobás, Brightwell and Janson. Newman proved that, if $p \to 0$ with $pn/\log n \to \infty$, then $H_{n,p}$ is asymptotic to epn. If $pn/\log n \to 0$, then the height is almost determined; if $\binom{n}{h}p^{h-1} \to \infty$, then a.s. $H_{n,p} \geq h$, if $\binom{n}{h}p^{h-1} \to 0$, then a.s. $H_{n,p} < h$ and if $\binom{n}{h}p^{h-1} \to c$, then $Pr(H_{n,p} \geq h) \to 1 - e^{-c}$. If p is a fixed constant, then Alon, Bollobás, Brightwell and Janson proved that there are positive constants $\mu = \mu(p)$, $\sigma^2 = \sigma^2(p)$ such that $(H_{n,p} - \mu n)/\sigma\sqrt{n} \to N(0,1)$ in distribution. The function $\mu(p)$ is not known.

Study of the width $W_{n,p}$ of $P_{n,p}$ shows up a change in the structure near $p = c/\log n$. If $p \log n \to \infty$, then large antichains are rare, and $W_{n,p} \sim \sqrt{2\log n/\log(1/(1-p))}$. If $p\log n \to 0$, and $pn \to \infty$, then a.s. $1.455p^{-1} < W_{n,p} < 2.428p^{-1}$.

Colin Cooper (University of North London)
On the 2-cyclic property of 2-regular digraphs

A digraph D is 2-cyclic if every pair of vertices is in a directed cycle of D. Bermond and Lovász [75] asked if there was a natural number k, such that every strongly-k-connected digraph D is 2-cyclic. Thomassen [89] proved by direct construction that no such k exists. We prove that for the space $\mathbb{D}_2(n)$ of 2-regular digraphs on n vertices with the uniform measure, the following properties occur for a.e. $D \in \mathbb{D}(n)$:

(i) D is strongly-2-connected,

(ii) D is 2-cyclic.

The proof makes use of the space $S_n \times S_n$ of pairs of random permutations with suitable transformation of measure, and is effected by exploiting the cycle structure of these permutations.

Walter Deuber (Bielefeld)

Paradoxical decompositions in metric spaces

Banach and Tarski showed that the unit ball in $\mathbb{R}^3$ is paradoxical. Here we consider two subsets of a metric space as equivalent in case there exists a bijection $\phi : X \leftrightarrow Y$ with $\sup_X d(\phi(x), x) < \infty$. This equivalence has been investigated by Laczkovich (1990/1991). Here we show that a set X is paradoxical iff it has exponential growth rate, a result which we obtained jointly with M. Simonovits and V. Sós. Exponential growth rate of X is defined as follows: there is $k \in \mathbb{R}$ such that, for every finite $Y \subset X$, $|N_k(Y) \cap X| \geq 2|Y|$, where $N_k(Y)$ is the k neighborhood of Y.

Martin Dyer (Leeds)

Randomized greedy matching

Consider the following algorithm for selecting a "large" matching in a graph G. First choose a vertex of G at random, and then a random edge incident to this vertex. Delete this edge and its vertices from G. This gives the first matching edge. Now repeat until there are no edges left in G. Let $\mu(G)$ be the size of the matching produced in expectation, and $m(G)$ the size of a maximum matching in G. In joint work with Jonathan Aronson, Alan Frieze and Stephen Suen, it is shown that if $p(G) = \mu(G)/m(G)$ then $\inf_G p(G) > \frac{1}{2} + \varepsilon$ for some $\varepsilon \geq 10^{-4}$. This is rather surprising since the alternative strategy of choosing an edge at random each time is known to give $p(G) = \frac{1}{2}$, the worst case bound. The result confirms a conjecture of the speaker. Some discussion of related results is presented.

Trevor Fenner (Birkbeck College, London)

The key distribution problem for secure networks

In order to reduce the key distribution and storage costs of providing $\binom{n}{2}$ keys to enable any pair of users i and j, in an n user communication network to communicate securely, we consider the Set Intersection Schemes (SIS) introduced by Mitchell and Piper (following a selection of Blom). Each user i has a subset S_i of a global set K of k subkeys, and users i and j communicate using $S_i \cap S_j$ as the cryptographic key. This scheme is a SIS if it is secure against any other user; this requires that for any distinct users i, j and

t, we have

$$S_i \cap S_j \not\subseteq S_t. \qquad (*)$$

Previously published SIS have needed k to be of order n. We show that k must satisfy $k \geq 2\log_2 n$, and use the probabilistic method to prove the existence of an SIS with $k \leq 13\log_2 n$. This scheme can be generated randomly and checked for security in $O(n^3 k)$ time. It can also be derandomized in the same time. We give computational results to demonstrate the feasibility of these schemes and consider some improved versions. If w users collude against users i and j, condition $(*)$ is replaced by

$$S_i \cap S_j \not\subseteq S_{t_1} \cup S_{t_2} \cup \cdots \cup S_{t_w}.$$

In the talk we present joint work with Dyer, Frieze and Thomason: we show that the lower bound on k increases by a factor of w, but the SIS generated by the probabilistic method yields an increase in k by a factor of about w^3. The closure of this gap is an open problem.

Alan Frieze (Carnegie-Mellon)
Perfect matchings in random hypergraphs

Let $G(n,r,s)$ denote the set of hypergraphs $H = (V, X_1, X_2, \ldots, X_m)$, $m = rn$ such that $|X_i| = s$ for $1 \leq i \leq m$ and $|\{j : i \in X_j\}| = r$ for all $i \in V = [sn]$. A perfect matching of H is a set $\{X_i, i \in I\}$ of n disjoint edges which together cover V. The main result of the talk, proved jointly with Colin Cooper, Mike Malloy and Bruce Reed, is that if H is chosen uniformly at random from G,

$$\lim_{n\to\infty} P_r \ (H \text{ has a perfect matching}) = \begin{cases} 0 & s > \sigma_r, \\ 1 & s < \sigma_r, \end{cases}$$

where

$$\sigma_r = \frac{\log r}{(r-1)\log\left(\frac{r}{r-1}\right)} + 1.$$

The proof uses a generalization of the configuration model of Bollobás to uniform hypergraphs and the Analysis of Variance method of Robinson and Wormald.

Jennie Hansen (Heriot-Watt University, Edinburgh)

Factorization of a random characteristic polynomial over a finite field

Presenting joint work with Eric Schmutz, we consider a non-uniform measure on the monic polynomials of degree n over a finite field, where the probability assigned to a polynomial f is just the proportion of invertible matrices over the field whose characteristic polynomial is f (call this measure Q_n). We consider the (joint) distribution of the variables $\alpha_1, \alpha_2, \ldots$ wrt Q_n, where $\alpha_i(f) = \#$ irreducible factors of f of degree i.

Theorem. *There exist constants c_1 and c_2 such that for all positive integers l, n with $c_1 \log n \leq l \leq n$ and $B \leq N^{n-l}$, we have*

$$|P_n(A_n \in B) - Q_n(A_n \in B)| < c_2/l$$

where P_n is the uniform measure on the monic polynomials of degree n and $A_n = (\alpha_{l+1}, \alpha_{l+2}, \ldots, \alpha_n)$.

The "intuition" for this result comes from the observation that there exist independent $\tilde{Z}_1, \tilde{Z}_2, \ldots$ s.t. $Q_n(\alpha_1 = m_1, \ldots, \alpha_n = m_n) = P(\tilde{Z}_1 = m_1, \ldots, \tilde{Z}_n = m_n | \sum_1^n k\tilde{Z}_k = n))$, where $\tilde{Z}_m$ is"nearly" Poisson$(1/k)$ for large m. The proof relies on identities of Stong (and Kung). This work is closely related to recent work by Arratia, Barbour and Tavaré.

Svante Janson (Uppsala)

Point processes in random graph theory

Point processes are useful for describing events that may happen more than once during the evolution of a random graph. A point process is a random subset of some suitable space, usually a subset of R^d, such that there are only finitely many points in each compact subset; it is technically convenient to regard the point process as a random measure by putting a unit mass at each point. One application is to the number of multicyclic components that appear during the evolution: a point with coordinates (x, y) is placed in the upper half plane if a new multicyclic component of order $yn^{2/3}$ is created at $p = n^{-1} + xn^{-4/3}$.

Other aspects of the evolution may be studied by more general random measures where a point mass of suitable weight is put at each point. For example, the evolution of unicyclic components may be studied by putting a mass z at (x, y) if a unicyclic component of order $yn^{2/3}$ is created at $p = n^{-1} + xn^{-4/3}$, either by adding an edge to a tree (then $z = y$), or by joining a tree of order $zn^{2/3}$ to an existing unicyclic component. This leads to an easy proof that the largest unicyclic component during the evolution is $O_p(n^{2/3})$, by a simple expectation computation. The evolution of trees can be studied similarly, by putting a mass $\min(y, z)$ at $(x, y + z)$ if two trees of orders $yn^{2/3}$ and $zn^{2/3}$ are joined at $p = n^{-1} + xn^{-4/3}$.

Mark Jerrum (University of Edinburgh)
Rate of convergence of the Metropolis algorithm on random problem instances

The Metropolis algorithm (simulated "annealing" at constant temperature) is a simple stochastic strategy for searching the space of feasible solutions to a combinatorial optimisation problem. The idea is to perform a random walk on feasible solutions that is biased towards solutions of lower cost. Despite the simplicity of the algorithm, it is difficult to prove rigorous results about its effectiveness for specific optimisation problems. This talk reviews what is known about the performance of the Metropolis algorithm applied to random problem instances; the optimisation problems considered are: maximum matching, maximum clique, and minimum bisection width.

Peter de Jong (Technical University of Delft)
Random graphs and Hoeffding decomposition

A short introduction to Hoeffding decomposition was given and a central limit theorem for (a finite sum of) homogeneous sums was stated. As an example the Hoeffding decomposition was applied to the number of cliques in a random graph $G(n,p)$. (See Barbour, Jonson, Karonski, Rucinski (1990)). By calculating and comparing variances, those homogeneous sums could be identified that dominate the variance. It turns out that in some situations ("close to" Poisson), the Hoeffding decomposition does not result in a finite sum of homogeneous sums with finite variance, in which case the method for proving central limit theorems breaks down.

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Yoshiharu Kohayakawa (Sao Paulo)

The diameter and connectivity of random subgraphs of the cube

The n-dimensional cube Q^n is the graph whose vertices are the subsets of $[n]$, where two such vertices are adjacent if and only if their symmetric difference is a singleton. Let $M = |E(Q^n)|$, and write $Q^r = (Q_t)_0^M$ for a random Q^n-process. Let $t^{(k)} = \tau(\mathbb{Q}^r; f \geq k)$ be the hitting time of minimal degree at least k. In joint work with Béla Bollobás and Tomasz Luczak, we show that the diameter $d_t = \mathrm{diam}(Q_t)$ of Q_t is a.e. Q^r behaves as follows: d_t starts infinite and is first finite at $t^{(1)}$, it equals $n+1$ for $t^{(1)} \leq t \subset t^{(2)}$, and $d_t = n$ for $t \geq t^{(2)}$. Further results on the diameter and on connectivity are given.

Hanno Lefmann (Dortmund)

Some anti-Ramsey results

We consider restricted colourings of the k-sets of an n-set X and we are interested in the largest subset where the set of all its k-sets is totally multicoloured. It is shown that if every colour class is a (u, k, h) partial Steiner system, then the largest totally multicoloured subset has size at least $C_k \cdot n^{\frac{k-h}{2k-1}} (\log n)^{\frac{1}{2k-1}}$ and this is up to constant factors best possible. From this one obtains that every n-set of any group contains a Sidon-set (1st and 2nd kind) of size at least $cn^{1/3}(\log n)^{1/3}$ (joint work with V.Rödl and B.Wyocka). Related is the problem of determining the size $fd(n)$ of the largest subset of the n^d grid with mutual distinct distances. Improving earlier results of Erdős and Guy, in joint work with T. Thiele it is shown that $f_2(u) \geq c_2 n^{2/3}$ and $fd(u) \geq c_d n^{2/3}(\log n)^{1/3}$ for $d \geq 3$.

Tomasz Łuczak (Poznań)

Sparse anti-Ramsey graphs

In this joint work with Y. Kohayakawa, we show that for every k there exists a graph G such that the girth of G is k and whenever edges of G are properly coloured one may always find in G a cycle C_k of length k such that each edge of C_k is coloured with a different colour.

More generally, for given graphs G and H, write $G \rightarrow_a H$ if any proper edge-colourings of G leads to a totally multicoloured copy of H. Furthermore, let $d_2(H) = \frac{e(H)-1}{s(H)-2}$ denote the 2-density of H and call a graph H strictly 2-balanced if $d_2(H) > \max_{F \subsetneq H} d_2(F)$. We prove that for every strictly 2-balanced graph H and a natural number l there exists a graph G such that $G \rightarrow_a H$ and G does not contain any subgraph F with less than l vertices and 2-density $d_2(F) > d_2(H)$.

Hans-Jürgen Prömel (Bonn)
On sparse triangle-free graphs

In 1976 Erdős, Kleitman and Rothschild proved that almost every triangle-free graph is bipartite. This result gives essentially, only some information about graphs with approximately $m = n^2/8$ edges. The aim of the present talk is to study the behaviour of triangle-free graphs with less than $n^2/8$ edges. Let $\mathcal{F}_{n,m}(K_3)$ denote the set of triangle-free graphs on n vertices with m edges, $\mathcal{F}_n(K_3) = \bigcup_m \mathcal{F}_{n,m}(K_3)$, and let B_n be the set of all bipartite graphs on n vertices. Then the Erdös-Kleitman-Rothschild result may be rephrased by saying that $Prob[G \in B_n | G \in \mathcal{F}_n(K_3)] = 1 - o(1)$. We prove the following result.

Theorem. *There exist constants $c_1, c_2, c_3 > 0$ such that*

(1) $Prob[G \in B_n | G \in \mathcal{F}_{n,m}(K_3)] = 1 - o(1)$ for $m \geq c_1 n^{7/4} \log n$

(2) $Prob[G \in B_n | G \in \mathcal{F}_{n,m}(K_3)] = o(1)$ for $c_2 n \leq m \leq c_3 n^{3/2} \log n$.

This is a joint result with Angelika Steger.

Gesine Reinert (University of Zürich)
A weak law of large numbers via Stein's method, and applications

Random graphs and epidemic models are closely related, see e.g. the construction by F.Ball and A.D.Barbour (1990) for the Reed-Frost epidemic. Here, a general stochastic epidemic with non-Markovian transition behaviour is considered. The health states of the individuals are weakly, in a mean field sense, dependent. To obtain the average path behaviour for the population size tending to ∞, we develop a weak law of large numbers for empirical measures of independent random elements on a locally compact space with countable basis. The proof employ's Stein's method (Stein 1972) in the generator generalization by A.D.Barbour (1990). We also get the rate of convergence in a Zolotarev semimetric. Generalizations with respect to the independence are straightforward. These allow us to finally determine the average path behaviour of the general stochastic epidemics. The method may apply to random graph problems with dependent edges as well.

Malgorzata Roos (University of Zürich)
Stein-Chen method for compound Poisson approximation

One of the aspects of the compound Poisson Stein-Chen method in its 'local' formulation was presented:

$$d_{TV}(\mathcal{R}(\mathcal{W}), CP(\lambda)) \leq c_2'(\lambda) \sum_{\alpha\in\Gamma}((EI_\alpha)^2 + \sum_{\beta\in\Gamma_\alpha^{vs}\cup\Gamma_\alpha^b} EI_\alpha EI_\beta) + \sum_{\beta\in\Gamma_\alpha^b} E(I_\alpha I_\beta)) + c_1'(\lambda)\phi,$$

where Γ_α^{vs}, Γ_α^b are appropriately defined neighbourhoods of dependence, ϕ measures how strong the long range dependence is and $c_1'(\lambda)$, $c_2'(\lambda)$ are constants coming from the Stein-Chen method.

An application from reliability theory dealing with the two-dimensional m-consecutive-k-out-of-n system was given. The order of the upper bound on the failure of Poisson approximation and an improved result, obtained when the compound Poisson 'local' approach was considered, were discussed.

Andrej Rucinski (Poznań and Emory University)
Ramsey properties of random graphs

Recently, Rödl and myself proved that $p = n^{-1/m_G^{(2)}}$, where

$$m_G^{(2)} = max_{H\subset G, v_H\geq 3}\frac{e_H - 1}{v_H - 2},$$

is the threshold for the property $K(n,p) \to (G)_r^2$ for every graph G except for star forests, and for every number of colors r. During my talk I attempted to outline the ideas of the proof. Compared to Rödl's talks in Keszthely and Poznań, more details have been revealed.

In the follow-up discussion with Erdős, Lefmann and Sós we realized that the same method gives the following result: For a set of integers A we write $A \to (k)_r$ if every r-coloring of A results in at least one monochromatic k-term arithmetic progression.

Then $\forall k >= 3$ $\forall r >= 2$ $\exists c, C > 0$ such that almost all $Cn^{1-\frac{1}{(k-1)}}$-element subsets A of $\{1, 2, \ldots, n\}$ satisfy $A \to (k)_r$ and almost all $cn^{1-\frac{1}{(k-1)}}$-element subsets A do not satisfy $A \to (k)_r$.

Vera Sós (Budapest)

Hereditary extended properties and quasirandom graphs

Many attempts have been made to clarify when an individual object would be called random and in what sense and how random-like objects can be generated in a non-random way.

Chung, Frankl, Graham, Rödl, Thomason and Wilson started a new line of investigations, where they gave some characterization of random-like graph sequences (and also of random-like sequences on other structures). Chung, Graham and Wilson listed a class of graph properties, all possessed by random graphs and at the same time equivalent to each other in some well-defined sense. $(\mathcal{G}_n)$ is called quasi-random, if it has any one (and consequently all) of these properties.

In this joint work with M. Simonovits, we concentrate on the fact that sufficiently large spanned subgraphs of random graphs must also be random-like. Hence a property which is not "hereditary" in that sense cannot be quasirandom. However in some cases if we consider the "hereditary extension" of the property, it becomes quasirandom. In particular, we prove the following result.

Theorem. *Let L be a fixed sample graph with $v = v(L)$ vertices.*

If every spanned subgraph $\mathcal{G}_n$ of $\mathcal{G}_n$ contains $\lambda m^v + o(n^v)$ (not necessarily induced) copies of L, then $(\mathcal{G}_n)$ is quasirandom, with edge probability $p = p(\lambda)$. I.e. we get the following.

Corollary. *If the above holds for some L, then it holds for any other graph A.*

In our approach we use what we proved earlier – that quasirandomness can be characterized also via the Szemerédi-partition of graphs.

Angelika Steger (Bonn)
Random l-colourable graphs

In our talk we investigate properties of the class of all l-colourable graphs on n vertices, where $l = l(n)$ may depend on n. Let G_n^l denote a uniform chosen element of this class, i.e. a random l-colourable graph. For the random graph G_n^l we study the property of being uniquely l-colourable. Surprisingly, it turns out that not only does there exist a threshold function $l = l(n)$ for this property, but this threshold is the chromatic number of a random graph.

In a second step we further restrict the class of graphs under consideration to the class of all l-colourable graphs on n vertices with $m = m(n)$ edges. Here we show that it $m = cn^2$ for some constant $c > 0$ then again there exists a threshold function $l = l(n)$ for this property, where this threshold now corresponds to the chromatic number of a random graph with edge probability $p = 2c$. If $m = o(n^2)$ we prove a weaker result, namely, we show that if $m/e \geq n \log n$ then a random graph $G_{n,m}^l$ is almost surely uniquely l-colourable.

The results are joint work with H.J.Prömel.

Charles Stein (Stanford)
Exchangeable pairs with applications to random graphs

This was primarily an expository lecture on the material in my paper in the Proceedings of the Southeast Asia Probability Conference held in Singapore in 1989, with some speculative remarks on applications, rather than a presentation of results. Using an exchangeable pair, a heuristic argument was given for a result that Frieze presented at the 1989 Poznań conference, with a more precise bound for the error. Let W_H be the number of copies of a strictly balanced graph H in $G(n,p)$ and let $\lambda_H = E(W_H)$, so that $\lambda_H \approx n^{v(H)}p^{e(H)}/|\alpha(H)|$, where $\alpha(H)$ is the automorphism group of H. Then the relative error of the Poisson distribution with parameter λ_H as a pointwise approximation to the distribution of W_H out to $2\lambda_H$ is $O(e(H)^2 n^{2v(H)-2} p^{e(H)-1})$, where the implied constant is absolute. The basic idea is that the relative error (departure from Poisson) in the computation of $P\{W = w\}/P\{W = w-1\}$ is of the order of the ratio of the expected number of copies of H that share an edge to the expected number of copies. This must be multiplied by the expected number of copies of H to get the pointwise error in the Poisson approximation for $\mathbb{P}\{W = w\}$. It seems possible that the method of exchangeable pairs is applicable to a problem discussed by Prõmel.

Simon Tavaré (University of Southern California)

Random combinatorial structures and the Ewens sampling formula

I focus on the behaviour of counts $(C_1, \dots, C_n)$ whose joint law is that of $(Z_1, \dots, Z_n)$ conditioned on $\sum_1^n iZ_i = n$, for independent random variables $Z_1, Z_2, \dots$. This class includes combinatorial objects such as assemblies (permutations, mappings), multisets and selections (factoring polynomials over finite field), in which the Z_i are Poisson, Negative Binomial and Binomial respectively. The special case in which the Z_i are $P_0(\theta/i)$ for some $\theta > 0$ is known as the Ewens Sampling Formula. We consider the 'domain of attraction' of the E.S.F., those in which $i\mathbb{E}Z_i \to \theta$ and $|\frac{i\mathbb{E}Z_i}{\theta} - 1| = O(i^{-\gamma})$ for some $\gamma > 0$. Two discrete approximations are provided. For small component counts, we get $d_{TV}(\mathcal{L}(C_1, \dots, C_b), \mathcal{L}(Z_1, \dots, Z_b)) = O\left(\frac{b}{n} + \frac{1}{b^\beta}\right)$, $\beta = \min\{1, \gamma, \theta\}$. Whereas for large component counts, we have $d_{TV}(\mathcal{L}(C_{b+1}, \dots, C_n), \mathcal{L}(C_{b+1}^*, \dots, C_n^*)) = O(\log^2 b/b^\beta)$, where $(C_1^*, \dots, C_n^*)$ is the E.S.F. These results imply a wide variety of other functional limit theorems, such as functional limit laws for components of sizes $\leq n$, $0 \leq t \leq 1$, and corresponding results for the large components, where the limit law is the Poisson-Dirichlet with parameter θ. Rates of convergence are available. There is one moral to this tale: don't divide by n too soon. This is joint work with Richard Arratia and Andrew Barbour.

Andrew Thomason (Cambridge)

Projections of bodies and set systems

Presenting joint work with Béla Bollobás, we show that, given a body $K \subset R^n$, there is a rectangular parallelepiped B with $|B| = |K|$ and $|B_A| \leq |K_A|$ for all $A \subset \{1, \dots, n\}$, K_A denoting the projection of K onto the subspace spanned by the basis vectors indexed by A. In consequence, if $\mathcal{C}$ is a multiset of subsets of $\{1, \dots, n\}$, such that every element i is in exactly k members of $\mathcal{C}$, then $|K|^k \leq \prod_{A \in \mathcal{C}} |K_A|$. Discrete analogues are that $|\mathcal{F}|^k \leq \prod_{A \in \mathcal{C}} |\mathcal{F}_A|$ if $\mathcal{F}$ is a set system on $\{1, \dots, n\}$ and $\mathcal{F}_A = \{Y \cap A : Y \leq \mathcal{F}\}$. Moreover, if a subset $S \subset \{1, \dots, n\}$ is chosen by selecting each element independently with probability p, then if $Prob(S \cap A \in \mathcal{F}_A) \leq p^{c|A|}$ then $Prob(S \in \mathcal{F}) \leq p^{cn}$. Finally, it follows that if P is a hereditary property of k-uniform set systems, and if the probability that a random set system (on an n-set) have P be $p^{\binom{c_n n}{k}}$, then $c_{n-1} \geq c_n$. In particular, $\lim_{n\to\infty} c_n$ exists.

Bernd Voigt (Frankfurt)

Logit models: some remarks on theory and applications in the airline business

A short introduction into stochastic decision models is given. In particular, Probabilistic Choice Theory is explained. Probit and Logit Models are compared. Some remarks on practical applications and problems were added.

Ingo Wegener (Dortmund)

Graph representations for random Boolean functions

Ordered binary decision diagrams (OBDDs) are the state-of-the-art data structure for Boolean functions. The size of the representation may depend heavily on the chosen ordering of the variables. Hence, the sensitivity of a Boolean function f defined as the quotient of the OBDD size for the worst ordering of the variables and the OBDD size for the best ordering of the variables is of interest. The sensitivity is exponential for addition, comparison or storage access but it is very close to 1 with very high probability for random Boolean functions. For random Boolean functions the size of read-once branching programs cannot be much smaller than the size of OBDD's with high probability.

John C. Wierman (Johns Hopkins University)

Percolation theory

The critical probability or percolation threshold is an important quantity in percolation models, but the exact critical probability values are known for only a few periodic graphs in two-dimensions. A new construction is shown which produces infinitely many infinite planar self-dual graphs, which have bond percolation critical probability 1/2. Sharp bounds for critical probabilities have been obtained for several lattices using a substitution method. These bounds are sufficiently sharp to observe relationships between features of a graph and its critical probability. A good approximation may be based on the arithmetic average vertex degree, z, by $p_c \approx \frac{2}{z}$. Non-regular graphs with regular duals are empirically observed to satisfy $p_c < \frac{2}{z}$, although this has not been proved.

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