

## MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 40/1993

## NOVIKOV CONJECTURES, INDEX THEOREMS AND RIGIDITY

5-11 SEPTEMBER 1993

The meeting was organized by S. Ferry (Binghamton, NY, USA), A. Ranicki (Edinburgh, Scotland, UK) and J. Rosenberg (College Park, MD, USA). The object of the meeting was to bring together topologists, analysts, and geometers with related interests. There were 38 participants from Australia, Canada, France, Germany, Great Britain, Hong Kong, Poland, Russia, Switzerland, and the United States.

There were 24 official lectures, 3 informal evening lectures, and a (marathon) problem session. The organizers were pleased with the lively discussions which accompanied many of the talks. The meeting proved to be timely, since many participants were using analogous methods in different contexts. In particular, the relation between the index theoretic and bounded topological approaches to the Novikov Conjectures became much clearer during this meeting. This interaction has resulted in a program for uniting the analytic, topological, and algebraic approaches to the subject. In contrast with some "interdisciplinary" meetings, there was a great deal of informal interaction between experts in very different fields.

A volume of conference proceedings is planned which will include both research and expository papers, together with an annotated problem list and bibliography.

## VORTRAGSAUSZÜGE - ABSTRACTS OF TALKS

O. ATTIE :

### The Borel conjecture for manifolds with bounded geometry

One can pose the question: given a map between manifolds of bounded geometry, i.e. sectional curvature bounded in absolute value, injectivity radius below, when is this map deformable to a smooth quasi-isometry? This question is in fact related to a form of the Borel conjecture. One can define a homology theory  $H_*^{u,f}(X; G)$  where  $G$  is either a normed group or metrized spectrum, and a version of surgery theory  $L_*^{bg}(X)$ , both of which depend on the Riemannian metric of  $X$ . If  $X$  is uniformly contractible and the assembly map

$$A : H_*^{u,f}(X; \mathbb{L}) \longrightarrow L_*^{bg}(X)$$

is an isomorphism then the answer to the question above is "always" if the map between manifolds is surjective and effectively proper Lipschitz. This "Borel conjecture" is true for  $X = \mathbb{R}^n$ .

J. BLOCK :

### Variations of Roe's coarse homology theory

Putting  $\mathcal{L}$  type uniformity conditions on the cycles in the coarse homology theory there is a relationship between the vanishing of the homology groups and the presence of isoperimetric inequalities. For example, Weinberger and I prove that  $H_n^{u,f}(X) = 0$  if and only if  $X$  is not amenable ( $H^{u,f}$  corresponds to the  $\mathcal{L}^\infty$  condition), in the sense that for all  $U \subseteq X$

$$\frac{\text{area } \partial U}{\text{vol } U} \geq \delta > 0$$

with  $\delta$  independent of  $U$ . The vanishing of the higher-dimensional homology groups is related to isoperimetric inequalities for higher codimension subspaces.

U. BUNKE :

### On the Glueing Problem for the Index and the $\eta$ -Invariant

We contribute to the index theory of Dirac operators equivariant with respect to the action of some  $C^*$ -algebra. We extend the theory of Mishchenko-Fomenko to real operators on complete manifolds and real graded algebras. We prove a relative index theorem. Let  $M_i = V_i \cup_N W_i$ ,  $i = 1, 2$  be partitioned manifolds with Dirac operators as above which are invertible at infinity, and assume that neighbourhoods of the partitioning manifolds  $N$  (together with all relevant structures) are isomorphic. Then one can form  $M_3 = V_1 \cup_N W_2$  and  $M_4 = V_2 \cup_N W_1$ . We obtain

$$\text{index}(D_1) + \text{index}(D_2) = \text{index}(D_3) + \text{index}(D_4).$$

An application is to show that the index of the spin-Dirac operator twisted with a flat  $C^*$ -bundle on a compact manifold does not change under isometric cutting and pasting at hypersurfaces carrying a metric of positive scalar curvature. Another application is to show that the map  $R_*(\pi) \rightarrow K_*(C_r^*(\pi))$  suggested by S. Stolz is well defined. Here  $\pi$  is a finitely presented group and  $R_*(\pi)$  is the group of bordisms of positive scalar curvature metrics in dimension  $*$ . Similar analytic techniques can be applied in order to prove a glueing formula for  $\eta$ -invariants. In fact, consider in the geometric situation above all manifolds to be compact and all Dirac operators to be the usual selfadjoint ones. Then the reduced  $\eta$ -invariants are defined as

$$\bar{\eta}(D) = \eta(D) - \dim(\ker D).$$

We prove

$$\bar{\eta}(D_1) + \bar{\eta}(D_2) + \bar{\eta}(D_3) + \bar{\eta}(D_4) = I \in 2\mathbb{Z}.$$

Under certain conditions we can show the vanishing of  $I$  in the adiabatic limit.

F. CONNOLLY :

Ends of  $G$ -manifolds and Stratified Sets

This is joint work with B. Vajiac.

When is a  $G$ -manifold the interior of a compact  $G$ -manifold?

This is equivalent to asking when the stratified space  $M/G$  (in the sense of F. Quinn) is the interior of a compact stratified space. Since stratified spaces have good induction properties, the right question to ask is:

Let  $X$  be a stratified space with a single tame,  $\pi_1$ -stable, end  $\epsilon$ . Assume  $\sigma X$  (the union of all strata except the top stratum) is already the interior of a compact stratified space  $Y$ . Is  $X = \text{Int } \bar{X}$ , for some compact stratified space  $\bar{X}$ , for which  $\sigma \bar{X} = Y$ ?

The answer will involve  $\tilde{K}_0(\mathbb{Z}\pi_1(\epsilon))$ . Here  $\pi_1(\epsilon)$ , the fundamental group of  $\epsilon$ , is defined as  $\text{Lim } \pi_1(N \setminus \sigma N)$ , as  $N$  ranges over neighborhoods of  $\epsilon$ . Also,  $\sigma\epsilon$ , the "singular part" of  $\epsilon$  has controlled  $K$ -groups,  $\tilde{K}_i(\sigma\epsilon, p)$ ,  $i \leq 1$ . Here  $p$  is the projection  $\text{Holink}(X, \sigma X) \rightarrow \sigma X$ . The inclusion  $i : \text{Holink}(X, \sigma X) \rightarrow X \setminus \sigma X$  induces a map  $a : \tilde{K}_i(\sigma\epsilon, p) \rightarrow \tilde{K}_i(\mathbb{Z}\pi_1(\epsilon))$ .

It would seem natural to follow Siebenmann and Quinn, by taking stratified space neighborhoods  $N$  of  $\epsilon$  such that  $\sigma N$  is an open collar for  $\sigma\epsilon$ . (A stratified space neighborhood of  $\epsilon$  is a neighborhood  $N$  of  $\epsilon$ , which is also a stratified space with compact boundary; the boundary is the frontier of  $N$  in  $X$ .) It would then seem natural to try to make handle exchanges across  $\partial N \setminus \sigma \partial N$  in the hope of making  $(N, \sigma N)$  acyclic. But this process fails at the first step as is seen in the first part of:

**MAIN THEOREM** *Let  $X$  be a stratified space with a single tame,  $\pi_1$ -stable, end. Assume  $\sigma X = \text{Int } Y$  for some compact stratified space  $Y$ . Assume  $\dim(X) \geq 6$ .*

*A. There is an obstruction  $\sigma_{-1}(\epsilon)$  in  $K_{-1}(\sigma\epsilon, p)$  which vanishes iff  $\epsilon$  admits arbitrarily small stratified space neighborhoods  $N$ . In this case,  $N$  can be taken so that  $N \cap \sigma X$  is a collar for  $\sigma\epsilon$ .*

*B. If  $\sigma_{-1}(\epsilon) = 0$ , there is a second obstruction  $\sigma_0(\epsilon)$  in  $\tilde{K}_0(\mathbb{Z}\pi_1(\epsilon))/a(\tilde{K}_0(\sigma\epsilon, p))$ . This obstruction vanishes iff  $X = \text{Int } \bar{X}$  for some compact stratified space  $\bar{X}$  for which  $\sigma \bar{X} = Y$ .*

One could state the theorem for more than one end, and also without assuming a completion of the singular set has been chosen. This would make the statement more difficult to read.

J. CUNTZ :

The excision problem in periodic cyclic cohomology

This is joint work with D. Quillen.

Given a (non-unital) algebra  $\mathcal{J}$  over a field of characteristic 0 set

$$C^n(\mathcal{J}) = \{(n+1) - \text{linear functionals on } \mathcal{J} \times \mathcal{J} \times \dots \times \mathcal{J}\},$$

$$C^n(\mathcal{J}^\infty) = \varinjlim_k C^n(\mathcal{J}^k)$$

where  $\mathcal{J} \supset \mathcal{J}^2 \supset \mathcal{J}^3 \supset \dots$  are the powers of  $\mathcal{J}$ . Extending the concept of  $H$ -unitality introduced by Wodzicki we say that  $\mathcal{J}$  is approximately  $H$ -unital if the complex  $(C^n(\mathcal{J}^\infty), b')$  with the usual  $b'$ -operator is acyclic, and prove the following:

**THEOREM** *For every exact sequence  $0 \rightarrow \mathcal{J} \rightarrow A \rightarrow A/\mathcal{J} \rightarrow 0$  with  $\mathcal{J}$  approximately  $H$ -unital there is a six-term exact sequence*

$$\begin{array}{ccccc} HP^0(\mathcal{J}) & \longleftarrow & HP^0(A) & \longleftarrow & HP^0(A/\mathcal{J}) \\ & & \downarrow & & \uparrow \\ HP^1(A/\mathcal{J}) & \longrightarrow & HP^1(A) & \longrightarrow & HP^1(\mathcal{J}) \end{array}$$

where  $HP^*$  denotes periodic cyclic cohomology.

We are able to show that virtually all algebras of interest do satisfy the condition of approximate  $H$ -unitality. Specifically, the following algebras are approximately  $H$ -unital (but not  $H$ -unital):

- principal ideals in polynomial algebras,

- algebraic or  $C^k$ -suspensions of approximately  $H$ -unital algebras
- the Schatten class ideals  $\mathcal{L}^p(H)$
- the ideals  $IA$  and  $QA$  in the universal extension  $0 \rightarrow IA \rightarrow RA \rightarrow A \rightarrow 0$  and the universal split extension  $0 \rightarrow QA \rightarrow QA \rightarrow A \rightarrow 0$ .

These facts imply important, previously unknown, properties of  $HP^*$  like Bott periodicity and existence and isomorphism properties of Chern characters.

J. EICHHORN :

Rigidity of the index on open manifolds

Let  $(M^n, g)$  be open complete,  $(S, (\cdot, \cdot), \nabla) \rightarrow M$  a Clifford bundle,  $D$  the generalized Dirac operator.  $D$  depends on  $\nabla, g, D = D^{\nabla, g}$ . Let  $\mathcal{I}_D$  be the corresponding index form,  $\mathfrak{m}$  a functional on  $\mathcal{I}_D$ ,  $\mathcal{A}$  an operator algebra,  $\zeta$  a cyclic cohomology class,  $\text{Ind } D \in K_1 \mathcal{A}$ ,  $\text{ind}_i D = (\mathcal{I}_D, \mathfrak{m})$ ,  $\text{ind}_a D = (\text{Ind } D, \zeta)$  the topological or analytical index respectively. Rigidity amounts to the description of the admissible variations of  $\nabla, g$  such that  $[\mathcal{I}_D]$ ,  $\text{Ind } D$ ,  $\text{ind}_i D$ ,  $\text{ind}_a D$  remain unchanged. We always assume  $g, \nabla$  of bounded geometry up to order  $k$ ,  $\nabla \in \mathcal{C}(k)$ . The key approach is to introduce suitable Banach or Sobolev topologies by means of uniform structures on  $\mathcal{C}(k)$ , thus obtaining spaces  $\mathcal{C}^{2,r}(k)$ . The choice of the uniform structure has to be adapted to the choice of  $i, \mathcal{A}, \mathfrak{m}, \zeta$ . Then we obtain many rigidity theorems, e.g. the following:

**THEOREM** Assume  $(M^n, g)$  open, complete, of bounded geometry up to order  $k$ ,  $k \geq r > \frac{n}{2} + 1$ ,  $\nabla, \nabla_1 \in \mathcal{C}(k)$ ,  $\nabla_1 \in \text{component of } \nabla \subset \mathcal{C}^{2,r}(k)$ ,  $D = D^\nabla$ ,  $D_1 = D^{\nabla_1}$ ,  $S$  and  $D$  graded,  $D : \Omega^{0,2,1}(S, \nabla) \rightarrow \Omega^{0,2,0}(S) = L_2(S)$  Fredholm. Then  $D_1$  is Fredholm too and

$$\text{ind}_a D^+ = \text{ind}_a D_1^+.$$

**REMARK:** For  $\nabla, \nabla_1 \in \mathcal{C}(k)$  arbitrary or other topologies this is in general definitely wrong. On compact manifolds this is always trivially true.

S. FERRY (chair) :

Informal session on the topological invariance of rational Pontrjagin classes

The theorem under discussion is:

**THEOREM** (Novikov) If  $f : M \rightarrow N$  is a homeomorphism between closed smooth manifolds, then  $f^*(p(\tau_N)) = p(\tau_M)$ , where  $p(\tau_M)$  and  $p(\tau_N)$  are the total Pontrjagin classes of the tangent bundles in  $H^*(M; \mathbb{Q})$  and  $H^*(N; \mathbb{Q})$ , respectively.

By material in Milnor-Stasheff, the theorem follows immediately from the following claim:

**CLAIM A** If  $f : W' \rightarrow W = V \times \mathbb{R}^k$  is a homeomorphism between smooth manifolds with  $V$  closed, then the signature of the transverse inverse image of  $V$  is that of  $V$ .

By "the transverse inverse image of  $V$ ", we mean the inverse image  $\hat{f}^{-1}(V)$ , where  $\hat{f}$  is a smooth approximation to  $f$  which is transverse to  $V$ .

Claim A follows from the stronger claim below:

**CLAIM B** If  $f : W' \rightarrow W = V \times \mathbb{R}^k$  is a bounded homotopy equivalence (over  $\mathbb{R}^k$ ) between smooth manifolds with  $V$  closed, then the signature of the transverse inverse image of  $V$  is that of  $V$ .

Claim B follows from a bounded version of Browder's  $M \times \mathbb{R}$  Theorem. This theorem is more-or-less equivalent to the simply-connected version of Quinn's End Theorem.

**BOUNDED SPLITTING THEOREM** If  $f : W' \rightarrow W = V \times \mathbb{R}^k$  is a bounded homotopy equivalence between smooth manifolds with  $V$  closed and simply connected,  $\dim V \geq 5$ , then  $f$  is boundedly close to  $f' : W' \rightarrow V \times \mathbb{R}^k$  so that  $f'$  is transverse to  $V \times \mathbb{R}^{k-1} \times \{0\}$  and

$$f'| : f'^{-1}(V \times \mathbb{R}^{k-1} \times \{0\}) \rightarrow V \times \mathbb{R}^{k-1} \times \{0\}$$

is a bounded homotopy equivalence over  $\mathbb{R}^{k-1}$ .

For general  $V$ , the splitting obstruction lies in  $K_{-1}(\mathbb{Z}\pi_1(V))$  which is zero for  $V$  simply connected by Bass-Heller-Swan. Claim B follows by splitting  $k$  times and using the homotopy invariance of signatures of closed manifolds. If the dimensions get too low, we cross with  $\mathbb{C}P^2$  and proceed.

REMARK. Choosing to prove Claim B instead of proving Claim A directly confers the technical advantage that all constructions are carried out in the smooth category.

A detailed discussion of S. Weinberger's analytic proof of the same theorem followed. Key points in the discussion included the observation that the topologists'  $L_{*,M}^{bdd}(\mathbb{Z})$  and the analysts'  $K_*(C^*(M))$  (= the  $K$ -theory of the bounded propagation algebra) are closely related, as well as the following theorem:

**THEOREM** *The Novikov Conjecture is true for  $\pi$  if and only if there is a nontrivial finite group  $G$  so that for every free homologically trivial action of  $G$  on a connected smooth manifold  $M$  with  $\pi_1(M) = \pi$ , the higher signature of  $M$  vanishes.*

M. GROMOV :

#### Geometric reflections on the Novikov conjecture

The simplest manifestation of rough geometry in the Novikov conjecture appears when one looks at a fibrewise rough (coarse) equivalence between two vector bundles over a given base, where the fibres are equipped with metrics of negative curvature. Such an equivalence induces a fibrewise homeomorphism of the associated sphere bundles and hence, by Novikov's theorem, an isomorphism of the  $\mathbb{Q}$ -Pontrjagin classes of the bundles.

What is relevant of the negative curvature, as we see it nowadays, is a certain largeness of such spaces. More generally, if  $X^n$  is a contractible manifold admitting a cocompact group of isometries (or more general uniformly contractible space) one expects it to be rather large in many cases, e.g. admitting a proper Lipschitz map into  $\mathbb{R}^n$  of non-zero degree. An example of an  $X$  without such maps to  $\mathbb{R}^n$  may eventually lead to a counterexample to the Novikov conjecture. On the other hand, there is a growing list of spaces where such a map is available.

A purely analytic-geometric counterpart of the Novikov conjecture for  $X$  is the claim that the non-reduced  $L_2$ -cohomology  $L_2H^*(X)$  does not vanish. A similar conjecture can be stated for the Dirac operator (instead of the deRham complex): the square of the Dirac operator on  $X$  contains zero in its spectrum. Both properties express the idea of the "spectral largeness" of  $X$ , and the latter is closely related to the non-existence of a metric with positive scalar curvature quasi-isometric to  $X$ . The non-existence of such a metric on  $X$  is yet another version of the Novikov conjecture which is often somewhat easier than the original Novikov conjecture, as one can combine here operator-theoretic techniques with the minimal surface approach of Schoen and Yau.

S. HURDER :

#### Exotic index theory and the Novikov conjecture

The "method of descent" using operator algebras from the "coarse Novikov Conjecture" to the Novikov Conjecture for groups is described. This proves:

**THEOREM.** *Suppose  $B\Gamma$  is a finite complex and the universal covering  $E\Gamma$  admits a compactification  $X$  so that*

- (a)  $\Gamma$  acts continuously on  $X$
- (b)  $X$  is contractible
- (c) compact subsets of  $E\Gamma$  translate to small subsets near  $\partial X$
- (d)  $X$  is metrizable.

(These four conditions are the same as in Erik Pedersen's talk. For this talk, we add the extra hypothesis)

- (e)  $\Gamma$  is a generalized duality group; that is,  $\partial X$  is a (Cantor continuum possibly) wedge of spheres.

Then the operator algebra assembly map of Mishchenko and Kasparov  $\mu: K_*(B\Gamma) \rightarrow K_*(C_r^*(\Gamma))$  is injective.

REMARKS.

- A. Conditions (b), (c), & (d) imply that the coarse Novikov Conjecture of John Roe is true for  $E\Gamma$ .
- B. It is likely that the homology condition (e) above is not necessary - a suitable use of Poincaré duality should eliminate this.
- C. Hypotheses (a), (b) & (d) can be weakened (see the Remark below), but no examples are known where the more general hypothesis is needed.

D. Injectivity of  $\mu$  is stronger than the rational injectivity of the algebraic assembly map; for example, it has applications to the non-existence of metrics of positive scalar curvature on closed manifolds  $M$  with  $\pi_1(M) = \Gamma$ .

Idea of proof - 1) Introduce families of Roe algebras associated to the balanced product  $\pi: V_\Gamma = E\Gamma \times_\Gamma E\Gamma \rightarrow B\Gamma$  to obtain a  $C^*$ -algebra  $C^*(\Gamma, \pi)$ . Then

THEOREM. There is a natural map of algebras  $\chi: C_r^*(\Gamma) \rightarrow C^*(\Gamma, \pi)$ .

2) Form the separable corona  $\partial_\pi \Gamma = \partial X \times_\Gamma E\Gamma \rightarrow B\Gamma$  which is a fiberwise compactification of  $V_\Gamma$ . Then

THEOREM. There is a natural (transgressed) index pairing

$$K^{*+1}(\partial_\pi \Gamma) \times K_*(C_r^*(\Gamma, \pi)) \longrightarrow KK_{*+*}(V_\Gamma, B\Gamma)$$

which can be evaluated via the Connes-Skandalis foliation index theorem for the fibration  $\pi: V_\Gamma \rightarrow B\Gamma$ .

3) For each  $a \in K^{*+1}(\partial\Gamma)$  we obtain the exotic index map

$$\rho(a) = \chi \times \delta a: K_*(C_r^*(\Gamma)) \rightarrow K_*(C^*(\Gamma, \pi)) \rightarrow KK_{*+*}(V_\Gamma, B\Gamma)$$

Hypotheses (b) & (e) imply there are sufficient classes  $a \in K^*(\partial_\pi \Gamma)$  to detect all of the classes in the image of the  $\mu$ -map, proving injectivity.

REMARK. The above method applies to any separable corona compactification of  $V_\Gamma$ , from which more general results possibly follow.

P. JULG :

#### KK-groups and the Baum-Connes conjecture for $SU(n, 1)$

This is joint work with G. Kasparov.

Kasparov has introduced the ring  $R(G) = KK_G(\mathbb{C}, \mathbb{C})$  for locally compact group  $G$ . We show that if  $G = SU(n, 1)$  and  $K = U(n)$  a maximal compact subgroup of  $G$ , the restriction map  $R(G) \rightarrow R(K)$  is an isomorphism. This proves in particular the Baum-Connes conjecture for discrete subgroups of  $SU(n, 1)$ .

The proof uses the geometry of the natural boundary (contact structure, Rumin complex of differential forms) and a Szegő map sending some forms in the Rumin complex to the  $L^2$ -cohomology of the symmetric space.

R. JUNG :

#### Relations between elliptic homology and the Novikov conjectures

Let  $M$  be a closed oriented manifold with fundamental group  $\pi$ , and  $f: M \rightarrow B\pi$  be the classifying map of the universal covering. The rational form of the Novikov conjecture asks if  $f_*(L(M) \cap [M])$  is a homotopy invariant (where  $L(M)$  is the total Hirzebruch L-class of  $M$ ).

This expression is obviously a bordism invariant which can be written as a natural transformation  $\Omega_*^{\text{SO}}(B\pi) \rightarrow H_*(B\pi; \mathbb{Q}[t])$ ,  $[M, f] \mapsto L_t(M, f)$ . The index  $t$  indicates that one keeps track of the dimension of the manifold using an indeterminate  $t$  of degree 4. The total L-class has excellent multiplicative properties as e.g. given by the 1957 theorem of Chern, Hirzebruch, and Serre. From this one gets rigidity for the total L-class in fibre bundles: If  $p: E \rightarrow B$  is a fibre bundle with fibre  $F$  s.t.  $\pi_1(B)$  acts trivially on  $H^*(F; \mathbb{Q})$ , then  $p_*(L_t(E)) = \text{sign}(F) \cdot L_t(B)$ , i.e. the invariant does not see the non-triviality of the fibre bundle.

Definition: Let  $T_*(X)$  be the submodule of  $\Omega_*^{\text{SO}}(X)$  consisting of singular manifolds  $f: M \rightarrow X$  in  $X$  s.t.  $M$  is a fibre bundle with fibre  $\mathbb{C}P^2$ , base  $B$ , and structure group the full group of isometries of  $\mathbb{C}P^2$ . Furthermore the map  $f$  has to be constant along the fibres, inducing a map  $\bar{f}: B \rightarrow X$ , and the singular manifold  $\bar{f}: B \rightarrow X$  has to be zero-bordant in  $X$ . Denote by  $\text{ell}_*^{\text{SO}}(X)$  the quotient  $\text{ell}_*^{\text{SO}}(X) := \Omega_*^{\text{SO}}(X)/T_*(X)$ .

Then  $L_t$  is trivial on  $T_*$ , so it gives an invariant of the classes in  $\text{ell}_*^{\text{SO}}(X)$ .

THEOREM  $\text{El}_t^{\text{SO}}(X) := \text{cl}_t^{\text{SO}}(X)[(\mathbb{C}P^2)^{-1}]$  is a multiplicative generalized homology theory with coefficients  $\text{El}_t^{\text{SO}} = \mathbb{Z}[t, t^{-1}]$ . Away from the prime 2 there is an isomorphism  $\text{El}_t^{\text{SO}}(X)[\frac{1}{2}] \cong KO_*(X)[\frac{1}{2}]$  given by the Sullivan orientation class. At the prime 2 after modifying the invariant  $L_t$  by 2-torsion it gives an isomorphism  $\text{El}_t^{\text{SO}}(X)_{(2)} \cong H_*(X; \mathbb{Z}_2[t, t^{-1}])$ .

This means that the class of  $[M, f]$  in  $\text{Ell}_*^{\text{SO}}(X)$  is rationally equivalent to the invariant  $L_1$ . But since  $\text{Ell}_*^{\text{SO}}$  is integrally defined there is additional torsion information in this class. So one can ask the following

Question: Suppose the Novikov conjecture is true for the group  $\pi$ . Is the class  $[M, f]$  in  $\text{Ell}_*^{\text{SO}}(X)$  a homotopy invariant?

Examples:

- 1)  $\pi = \mathbb{Z}^n$  : There is no torsion in  $\text{Ell}_*^{\text{SO}}(B\pi)$ , so the answer is yes.
- 2)  $\pi = \mathbb{Z}/2\mathbb{Z}$  :  $\text{Ell}_*^{\text{SO}}(B\pi)$  has only 2-torsion that can be detected in  $H_*(B\mathbb{Z}/2\mathbb{Z}; \mathbb{Z}/2\mathbb{Z})$  and is given by Stiefel-Whitney classes, so again the answer is yes.

This idea of defining elliptic homology theories was first used by M. Kreck and S. Stolz in defining an elliptic homology theory starting from spin bordism and using fibre bundles with fibre  $\mathbb{H}\mathbb{P}^2$ .

J. KAMINKER :

Duality for  $C^*$ -algebras of hyperbolic dynamical systems and the Novikov conjecture

This is joint work with I. Putnam.

A hyperbolic dynamical system, such as a subshift of finite type, has two  $C^*$ -algebras,  $R^u$  and  $R^s$  associated to it. They satisfy a form of Spanier-Whitehead duality. This says that there is an element  $\Delta \in KK(R^u \otimes R^s, \mathbb{C})$  which induces an isomorphism

$$\otimes \Delta : K_i(R^u) \rightarrow K^{i+1}(R^s).$$

This duality is an isomorphism for sub-shifts of finite type, generalized solenoids and Anosov diffeomorphisms of the torus. The algebras  $R^u$  and  $R^s$  are generalizations of Cuntz-Krieger algebras.

The relation of this to the Novikov conjecture is the following. The Novikov conjecture, in a strong form, can be viewed as the statement that the  $C^*$ -algebra  $C_0(B\Gamma)$  is Spanier-Whitehead dual to  $C_r^*(\Gamma)$ . If  $\Gamma$  is a finitely presented Fuchsian group, with limit set  $S^1$  and containing parabolic elements, then the action of the group  $\Gamma$  on (a slightly modified)  $S^1$  is orbit equivalent to a dynamical system of the type considered above. There is a diagram relating the Novikov conjecture duality between  $C_0(B\Gamma)$  and  $C_r^*(\Gamma)$  to the duality between the algebras  $R^u$  and  $R^s$  obtained from the dynamics on the boundary.

G. KASPAROV :

Groups acting on bolic spaces and the Novikov conjecture

This is joint work with G. Skandalis.

Let  $\delta$  be a positive real number. Consider a  $\delta$ -geodesic metric space  $X$ . We will call  $X$   $\delta$ -bolic if the following two conditions are satisfied:

$B_\delta 1$ . Let  $x, y, z \in X$  and  $a$  be a  $\delta$ -middle point of  $\{x, y\}$ . Then  $\rho(a, z) \leq \max(\rho(x, z), \rho(y, z)) + 2\delta$ . Moreover, for any  $r > 0$  there exists  $R(r) > 0$  such that for any  $R \geq R(r)$ , if  $\rho(x, z) \leq R$ ,  $\rho(y, z) \leq R$ , and  $\rho(x, y) \geq R/2$ , then  $\rho(a, z) \leq R - r$ .

$B_\delta 2$ . For any  $r > 0$  there exists  $R > 0$  such that for any quadruple of points  $a, b, c, d \in X$  satisfying the conditions:  $\rho(a, b) + \rho(c, d) \leq r$ ,  $\rho(a, c) + \rho(b, d) \geq R$ , and  $\rho(a, c) \leq \rho(a, d)$ , one has:  $\rho(b, c) \leq \rho(b, d) + 2\delta$ .

**MAIN THEOREM** *Let  $X$  be a discrete bolic metric space such that for any  $r > 0$  the number of points in each ball of radius  $r$  is bounded by a constant which depends only on  $r$ . Then for any discrete group acting isometrically and properly on  $X$  the Novikov conjecture is true.*

T. KOŹNIEWSKI :

On the Nil and Unil groups

This is joint work with F. Connolly.

We describe the Nil and UNil groups  $(Nil(R, \alpha)$  of Bass and Farrell,  $Nil(R; B_1, B_2)$  of Waldhausen,  $UNil_n^*(R; B_1, B_2)$  of Cappell) in terms of a polynomial extension category  $\mathbf{A}_\alpha[\beta]$ . The description implies:

1° Both Nil groups above are special cases of the same construction.

2° In the geometric cases there is an isomorphism

$$UNil_{2n}^h(R; B_1, B_2) = L_\epsilon(A_\alpha[t])$$

where  $L_\epsilon(\ )$  is the Wall-Ranicki  $L$ -group.

In the "universal case"  $UNil_{2n}^h(R) := UNil_{2n}^h(R; R, R)$  we obtain the following computations:

3° Let  $R$  be a division ring with involution.

- (i) If the characteristic of  $R$  is  $\neq 2$ , then  $UNil_{2n}^h(R) = 0$ .
- (ii) Let  $R$  be a perfect field of characteristic 2. Then:
  - If the involution on  $R$  is nontrivial then  $UNil_{2n}^h(R) = 0$ .
  - If the involution on  $R$  is trivial then  $UNil_{2n}^h(R) = \sum_{k=0}^{\infty} R$ .

4° Let  $R$  be a Dedekind domain of characteristic  $\neq 2$  with involution. Assume  $R/2R$  is a perfect ring. Then:

- (i) If the involution is nontrivial or  $n$  is even then  $UNil_{2n}^h(R) = 0$ .
- (ii) If the involution is trivial and  $R/2R$  is a perfect field then the natural map  $UNil_{2n}^h(R) \rightarrow UNil_{2n}^h(R/2R)$  is an isomorphism.

W. LÜCK :

$L^2$ -Betti numbers and Novikov-Shubin invariants

We investigate the  $L^2$ -Betti numbers and the Novikov-Shubin invariants of a finite CW-complex. The  $L^2$ -Betti numbers are defined to be the von Neumann dimension of the  $L^2$ -cohomology. If  $X$  is a compact Riemannian manifold, they agree with the limit for large times of the von Neumann trace of the heat kernel of the Laplace operator on the universal covering and the Novikov-Shubin invariants measure how fast it approaches its limit and in particular whether there is a gap in the spectrum at zero.

**THEOREM 1** (joint with J. Lott) *Let  $M$  be a compact orientable connected 3-manifold with prime decomposition*

$$M = M_1 \# M_2 \# \dots \# M_r.$$

*Assume that  $\pi_1(M)$  is infinite and each  $M_i$  is finitely covered by a manifold which is Haken, Seifert or hyperbolic. Then*

(1) *The  $L^2$ -Betti numbers are such that*

$$\begin{aligned} b_3^{(2)}(M) &= 0 \\ b_2^{(2)}(M) &= r - 1 + \sum_{i=1}^r \frac{1}{\pi_1(M_i)} + \text{card}\{C \in \pi_0(\partial M) \mid C = S^2\} \\ b_1^{(2)}(M) &= b_2^{(2)}(M) - \chi(M) \\ b_0^{(2)}(M) &= 0. \end{aligned}$$

(2) *The Novikov-Shubin invariants are all positive and can be computed explicitly for  $p = 0, 3$  for all  $M$  and for  $p = 1, 2$  for Seifert and hyperbolic manifolds.*

The following three results answer questions by M. Gromov.

**THEOREM 2** *If the compact manifold  $M$  fibers over  $S^1$ , all its  $L^2$ -Betti numbers vanish.*

**THEOREM 3** *Let  $1 \rightarrow \Delta \rightarrow \Gamma \rightarrow \pi \rightarrow 1$  be an extension of infinite finitely presented groups. Suppose that  $\pi$  contains an infinite cyclic subgroup. Then:*

$$b_1^{(2)}(\Gamma) = 0.$$

**THEOREM 4** *Let  $X$  be a finite CW-complex with residually finite fundamental group  $\pi = \pi_1(X)$ , i.e. there is a sequence of nested normal subgroups  $\dots \subset \Gamma_2 \subset \Gamma_1 \subset \pi$  in  $\pi$  whose intersection is the trivial subgroup. Let  $X_m$  be the finite covering associated to  $\Gamma_m$  and  $b_p(X_m)$  be the ordinary Betti number. Then Kazhdan's inequality is actually an equality, i.e.*

$$b_p^{(2)}(X) = \lim_{m \rightarrow \infty} \frac{b_p(X_m)}{[\pi : \Gamma_m]}$$

We mention the following corollary of Theorem 3

**COROLLARY 5** Under the assumptions of Theorem 3 we have:

(1) The deficiency of  $\Gamma$  (which is the maximum of the difference of the number of generators and the number of relations of any presentation) is bounded by 1.

(2) If  $M$  is a closed orientable 4-manifold with fundamental group  $\pi_1(M) = \Gamma$ , then:

$$\text{sign}(M) \leq \chi(M).$$

V. MATHAI :

Homotopy invariance of  $\eta$ -invariants and their generalizations

Let  $X$  be a closed, oriented Riemannian manifold of odd dimension with fundamental group  $\pi$ . Let  $R\pi$  denote the space of all unitary  $p$ -dimensional representations of  $\pi$ . To each smooth map  $g : N \rightarrow R\pi$  I define a generalized  $\eta$ -invariant  $\tilde{\eta}_g$  of  $X$ . Here  $N$  is a closed, oriented Riemannian manifold of even dimension. In particular, for the map  $g : S^0 \rightarrow R\pi$ ,  $\tilde{\eta}_g = \eta_{g(N)}(0) - \eta_{g(S)}(0)$ , where  $S^0 = \{S, N\}$  and  $\eta_\phi(0)$  denotes the Atiyah-Patodi-Singer  $\eta$ -invariant for the  $\phi$ -twisted tangential signature operator  $B \otimes \phi$  of  $X$ . In this paper, I study the homotopy invariance of  $\tilde{\eta}_g$ . Recall that a smooth map  $g : N \rightarrow R\pi$  is said to be null cobordant if  $N = \partial M$  and  $g$  extends to a smooth map  $\bar{g} : M \rightarrow R\pi$ . One result which I prove is that for any null-cobordant map  $g : N \rightarrow R\pi$ ,  $\tilde{\eta}_g$  (modulo 1) is a homotopy invariant of  $(X, g)$ .

J. MILLER :

Signature operators and surgery groups

We study the connections between signature operators, surgery obstructions and  $K$ -theory in a general context. Let  $A$  be a  $C^*$ -algebra with unit. A  $\tau$ -complex is a Fredholm complex of Hilbert  $A$ -modules, equipped with an involution with the properties of the Hodge  $*$  operator. Such a complex has a Fredholm signature operator  $S$  with  $\text{Ind } S \in K_i(A)$ ,  $i \equiv n \pmod{2}$ . Let  $B^n(A)$  be the bordism group of  $n$ -dimensional  $\tau$ -complexes, and  $L^n(A)$  the symmetric surgery group. The main result is that there are isomorphisms

$$\text{Ind } S : B^n(A) \rightarrow K_i(A), \quad \sigma' : B^n(A) \rightarrow L^n(A), \quad m : L^n(A) \rightarrow K_i(A)$$

such that  $\text{Ind } S = m\sigma'$ . In the situation where  $A = C^*\pi$ ,  $\pi = \pi_1(M)$  for a closed manifold  $M^n$ , and the complex is the deRham complex of  $M$  with coefficients in a flat  $C^*\pi$ -bundle, this shows that  $\text{Ind } S$  is a homotopy invariant. Products in the various groups are also discussed.

A. MISHCHENKO :

On analytical torsion over  $C^*$ -algebras

This is joint work with A. Carey and V. Mathai.

Let  $N$  be a non simply connected compact smooth simplicial manifold, with  $\pi = \pi_1(M)$ ,  $\dim M = n$ .

Let  $\rho : \pi \rightarrow C^*[\pi]$  be the regular representation. One can associate to the representation  $\rho$  the de Rham complex, the simplicial chain complex and the de Rham homomorphism

$$I : \Omega^p(M; E) \rightarrow C^p(M; E)$$

with the cylinder  $\text{Cyl}(M, \rho)$  an acyclic complex. The problem discussed in the lecture is to extend the notion of analytic torsion due to Ray and Singer to arbitrary  $C^*$ -algebras with a trace function, especially to  $C^*[\pi]$ . We offer to consider the torsion of the acyclic  $\text{Cyl}(M, \rho)$  as an analog of the torsion of Ray and Singer. Generalizations of the technique and the independence of the choice of Riemannian metric are discussed.

E. PEDERSEN :

Controlled algebra and the Novikov conjecture

This is joint work with G. Carlsson.

If  $\Gamma$  is a group with finite  $B\Gamma$  such that  $E\Gamma$  admits a compactification  $X$  satisfying:

- (i) the  $\Gamma$ -action on  $E\Gamma$  extends to  $X$ ,
- (ii)  $X$  is contractible (at least in the Čech sense),
- (iii) compact subsets in  $E\Gamma$  translate to small sets near boundary,

then the  $K$ - and  $L$ -theory Novikov conjectures hold for  $\Gamma$ . The proof for metrizable  $X$  uses the fact that controlled algebra produces Steenrod homology theories. The metrizability condition is removed by mapping to Čech homology.

S. PRASSIDIS :

$K$ -theory rigidity of virtually nilpotent groups

Let  $\Gamma$  be a virtually nilpotent group, that is a group which fits into an exact sequence:

$$1 \rightarrow N \rightarrow \Gamma \rightarrow G \rightarrow 1$$

where  $N$  is a finitely generated, torsion free, nilpotent group and  $G$  is finite. Using Malcev's completion, we can construct a cocompact action of  $\Gamma$  on  $\mathbb{R}^n$  where  $n = \text{vcd}(\Gamma) = h(N)$ . Notice that  $N$  acts freely and the quotient  $M_\Gamma = \mathbb{R}^n/N$  is a compact manifold. The main result of the project is the following:

**THEOREM** *With the above notation*

- (a)  $K_{i,G}^{T,op}(M_\Gamma) = 0$ , for  $i \leq -1$ .
- (b) If  $|G|$  is odd then the forget control map  $K_{i,G}^{P,L}(M_\Gamma)_c \rightarrow K_{i,G}^{P,L}(M_\Gamma)$  is a split monomorphism for  $i \leq 1$ .
- (c) If  $|G|$  is odd then  $\hat{H}^*(\mathbb{Z}/2\mathbb{Z}; K_{i,G}^{T,op,p}(M_\Gamma)) = 0$  for  $i \leq 1$ .

Here the  $K$ -groups are the equivariant  $K$ -groups of Steinberger-West and  $K_1 = Wh$ ,  $K_0$  denotes the reduced  $K_0$ -group. The  $\mathbb{Z}/2\mathbb{Z}$ -action in (c) is given by inverting the  $h$ -cobordisms.

For the proof, we use the fibering apparatus construction introduced by Farrell-Hsiang to translate the problem to one with control over a crystallographic manifold. We complete the proof using the expansive maps and the classification of the crystallographic groups. Notice that part (a) is immediate consequence of the vanishing results for lower  $K$ -groups given by Farrell-Jones.

M. PUSCHNIGG :

Cyclic cohomology and the Novikov conjecture

In this expository talk the role of cyclic cohomology in several approaches to the Novikov conjecture was discussed. It can be summarized by the following commutative diagram:

$$\begin{array}{ccccc}
 L_*(C_r^*(\Gamma)) & \xrightarrow{\text{sign}} & K_*(C_r^*(\Gamma) \otimes \mathcal{X}) & \xrightarrow{\text{ch}} & HC_*(C_r^*(\Gamma) \otimes \mathcal{X}) \\
 \uparrow & & \uparrow & & \uparrow A_2 \\
 L_*(\mathbb{C}\Gamma) & & K_*(\mathbb{C}\Gamma \otimes \mathcal{X}^\infty) & \xrightarrow{\text{ch}} & HC_*(\mathbb{C}\Gamma \otimes \mathcal{X}^\infty) \\
 \uparrow & & \uparrow \text{Indez} & & \uparrow A_1 \\
 h_*(B\Gamma; \mathbb{L}) & \longrightarrow & h_*(B\Gamma; \mathbb{K}\mathbb{U}) & \xrightarrow{\text{ch}} & h_*(B\Gamma, HC_*(\mathbb{C}))
 \end{array}$$

The assembly map  $A_1$  is always injective and should be used to imply the injectivity of the corresponding assembly maps in  $K$ - and  $L$ -theory. This is obstructed however by the vanishing of cyclic homology for (stable)  $C^*$ -algebras, which forces the map  $A_2$  to be zero.

The first approach (due to Connes-Moscovici) overcomes this difficulty for hyperbolic groups by replacing the group  $C^*$ -algebra  $C_r^*(\Gamma)$  by smooth subalgebras  $C^*(\Gamma)$  ( $r \in \mathbb{N}$ ) which possess "enough" cyclic cohomology

to detect the image of the assembly map. (This requires delicate estimates using the hyperbolicity of the group.)

The second approach (due to Connes, Gromov and Moscovici) is based on the observation that in the diagram above the inclusion  $C\Gamma \rightarrow C^*(\Gamma)$  may be replaced by any asymptotic morphism  $C\Gamma \rightarrow A$  (for a suitable  $C^*$ -algebra  $A$ ) that arises as the monodromy representation of an "almost flat" bundle (i.e. a sequence of bundles  $(E_n, \nabla_n)$  of f.g. projective  $A$ -modules in a fixed  $K$ -theory class, with curvature tending to 0). A modified version of cyclic homology has to be used to make the diagram well defined in the asymptotic setting. The "asymptotic cyclic cohomology" turns out to be a sensitive invariant of  $C^*$ -algebras (in contrast to the other cyclic theories). For example, there exists a bivariant Chern character  $ch : KK^*(-, -) \rightarrow HC_*^*(-, -)$  that becomes an isomorphism under complexification on a large class of  $C^*$ -algebras. In particular, there are sufficiently many asymptotic cocycles and "almost flat" bundles over  $B\Gamma$  to verify the injectivity of the  $K$ -theory assembly map for a large class of groups  $\Gamma$  containing all the cases where the  $K$ -theory Novikov conjecture was known so far.

A. RANICKI :

#### Algebraic Novikov for analysts

The talk addressed analysts to the extent that it concerned the reduction to algebra of the original Novikov conjecture on the homotopy invariance of the higher signatures. The reduction also strips Novikov's proof of the topological invariance of rational Pontrjagin classes to its algebraic essence. The assembly map in the Wall compact surgery obstruction groups of a space  $X$

$$A : H_*(X; \mathbb{L}(\mathbb{Z})) \longrightarrow L_*(\mathbb{Z}[\pi_1(X)])$$

is defined in [2] by sending a sheaf  $\Gamma$  of  $\mathbb{Z}$ -valued quadratic forms over  $X$  with compact support to the  $\mathbb{Z}[\pi_1(X)]$ -valued quadratic form  $q, p^*\Gamma$ , with  $p : \tilde{X} \rightarrow X$  the universal covering projection and  $q : \tilde{X} \rightarrow \{\text{pt.}\}$  the unique map. The Novikov conjecture holds for a group  $\pi$  if and only if  $A$  is a rational injection for the classifying space  $X = B\pi$ . The locally finite assembly map in the  $X$ -bounded quadratic  $L$ -groups of a metric space  $X$

$$A^{lf} : H_*^{lf}(X; \mathbb{L}(\mathbb{Z})) \longrightarrow L_*(C_X(\mathbb{Z}))$$

is defined in [2] by analogy with  $A$ , with  $C_X(\mathbb{Z})$  the Pedersen-Weibel category of  $X$ -graded free  $\mathbb{Z}$ -modules and bounded morphisms, and  $L_*(C_X(\mathbb{Z}))$  the Ferry-Pedersen bounded surgery obstruction groups. If  $\pi$  is a group with finite  $B\pi$  and  $A^{lf}$  is a rational isomorphism for the universal cover  $X = E\pi$  of  $B\pi$  then the Novikov conjecture holds for  $\pi$ . In particular,  $A$  is an isomorphism for  $X = B\mathbb{Z}^n = T^n$  and  $A^{lf}$  is an isomorphism for  $X = E\mathbb{Z}^n = \mathbb{R}^n$ , giving the algebraic proof of the Novikov conjecture for the free abelian group  $\pi = \mathbb{Z}^n$  and also of the topological invariance of the rational Pontrjagin classes. See [1], [2] and [3] (perhaps in reverse order) for more extended accounts of the algebraic theory and its applications.

- [1] A.R., *Lower K- and L-theory*, London Math. Soc. Lecture Notes 178, Cambridge University Press (1992).
- [2] A.R., *Algebraic L-theory and topological manifolds*, Cambridge Tracts in Mathematics 102, Cambridge University Press (1992).
- [3] A.R., *On the Novikov conjecture* (preprint circulated at the conference, and submitted to the conference proceedings).

J. ROE :

#### Coarse geometry and index theory

*Coarse geometry* is the study of metric spaces  $X$  up to some concept of 'large scale equivalence'. For example,  $\mathbb{Z}$  and  $\mathbb{R}$  should be equivalent in coarse geometry. Given any generalized homology or cohomology theory on the category of locally compact spaces and proper maps, one can define an associated coarse theory on the category of metric spaces and coarse maps; one way to do this (though not always applicable) is to define the 'coarse  $S$ -theory'  $SX_*(X)$  of a space  $X$  to be the ordinary  $S$ -theory of any uniformly contractible space coarsely equivalent to  $X$ .

There is a natural map from ordinary  $S$ -homology to coarse  $S$ -homology. The thesis of the talk was that this map in  $K$ -homology should be thought of as a generalization of the Atiyah-Singer index. As evidence,

note that if  $M$  is a compact manifold then  $KX_*(M) = \mathbb{Z}$  and the 'coarsening' map does indeed associate to any elliptic operator (thought of as defining a class in  $K_*(M)$ ) its usual Fredholm index.

One can then make a 'meta-conjecture' to the effect that the coarsening map on  $K$ -homology should enjoy all the good properties of the usual index. For instance, the class in  $KX_*$  of an invertible elliptic operator should vanish; the class of the signature operator should be invariant under suitable homotopy equivalences. These imply more familiar versions of the Novikov and positive scalar curvature conjectures.

One can attempt to prove them by introducing an analytic model for coarse homology. This is the  $K$ -theory of the translation algebra  $C^*(X)$  of locally compact finite propagation operators on a Hilbert space  $\mathcal{H}$  over  $X$  (that is, the continuous functions on  $X$  act on  $\mathcal{H}$  by bounded operators). There is an assembly map  $\mu: KX_*(X) \rightarrow K_*(C^*(X))$  and one can conjecture that  $\mu$  is an isomorphism (or, more cautiously, that it is injective or rationally injective). This should imply the conjectures above.

One hopes that because the coarse category is much more flexible than the category of groups, more rapid progress will be possible on these conjectures. In joint work with N. Higson and G. Yu, I have verified the strong conjecture (isomorphism) for cones on finite polyhedra and for Hadamard manifolds, and the weak conjecture (rational injectivity) for hyperbolic metric spaces in the sense of Gromov.

J. ROSENBERG :

#### Analytic Novikov for topologists

We explain for topologists the "dictionary" for understanding the analytic proofs of the Novikov conjecture, and how they relate to the surgery-theoretic proofs. In particular, we try to explain the following points:

Why do the analytic proofs of the Novikov conjecture require the introduction of  $C^*$ -algebras?

Why do the analytic proofs of the Novikov conjecture all use  $K$ -theory instead of  $L$ -theory? Aren't they computing the wrong thing?

How can one show that the index map  $\mu$  or  $\beta$  studied by analysts matches up with the assembly map in surgery theory?

Where does "bounded surgery theory" appear in the analytic proofs? Can one find a correspondence between the sorts of arguments used by analysts and the controlled surgery arguments used by topologists?

E. TROITSKY :

#### On the homotopy invariance of some higher signatures

**THEOREM** Let  $\xi$  be a complex vector bundle over  $B\pi$ , such that  $\xi$  is trivial off the 1-skeleton of  $B\pi$ . Then the higher signature

$$\langle L(M)f_M^*(Ch(\xi)), [M] \rangle$$

is an oriented homotopy invariant, where  $\pi_1(M) = \pi$ ,  $f_M : M \rightarrow B\pi$  is the classifying map.

Two approaches to the proof of this theorem are discussed. One of them probably can be generalized and the author is close to proving this theorem for the 2-skeleton instead of the 1-skeleton. This is more interesting because of the homotopy structure of  $B\pi$ .

S. WEINBERGER :

#### Coarse geometry and the Novikov conjecture

I discussed joint work with Steve Ferry on the "Principle of Descent" which enables one to deduce Novikov type results (for various functors) from bounded versions on the associated metric spaces. Nonetheless, we also give an example of a uniformly contractible manifold (i.e.,  $(M, d)$  is a metric space and there is a function  $f$  such that the inclusion  $B_r(\tau) \subset B_r(f(\tau))$  is null-homotopic for all concentric balls) which is not boundedly topologically rigid. - For this example, the bounded propagation speed algebra also has interesting  $K$ -homology.

B. WILLIAMS :

Transfers:  $K$ -theory versus Homotopy Theory

This is joint work with W. Dwyer and M. Weiss.

Let  $p : E \rightarrow B$  be a fibration where the fibers are finitely dominated, and where  $B$  is a connected CW complex. Let  $S$  be an associative ring. Let  $\phi : \pi_1(E) \rightarrow GL_n(S)$  be an  $S$ -representation of  $\phi$ . The representation  $\phi$  determines an element  $\hat{\phi} \in K'(S)^0(E)$ , where  $K'(S)$  denotes the  $K$ -theory spectrum for the category of f.g. modules over  $S$  (not necessarily projective). If  $F$  is a fiber of  $p$ , then  $\tilde{B} \times_{\pi_1(B)} H^i(F, \phi) \rightarrow B$  represents an element  $h^i(p, \phi)$  in  $K'(S)^0(B)$  (where  $\tilde{B}$  denotes the universal cover of  $B$ ).

THEOREM If  $p$  is a differential fiber bundle with compact fibers, then

$$p_!(\hat{\phi}) = \sum_i (-1)^i h^i(p, \phi).$$

Here  $p_!$  denotes the Becker-Gottlieb transfer for  $p$ .

The key step follows from the Orthogonal Calculus of Weiss.

This theorem implies results of Bismut-Lott and Becker-Schultz which were originally proved using index theory. (Notice that if  $S$  is the real or complex numbers, then  $\phi$  determines a flat bundle on  $E$ .) Furthermore, this result is not true for general fibrations even when  $S$  is the integers.

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Becker, J. C. and Schultz, R. E. : The real semicharacteristic of a fibered manifold, *Quart. J. Math. Oxford* (2) 33 (1982) 385-403.

M. YAN :

The role of signature in periodicity

In the classical surgery theory, the normal invariant lives in the  $L$ -homology theory (induced from the surgery obstruction spectra), which turns out to be the real  $K$ -homology theory after inverting 2. The Bott periodicity in the  $K$ -theory then suggests that we should have a similar periodicity in the surgery theory. However, the usual Bott periodicity in  $K$ -theory is induced from the Dirac operator. This is not compatible with the periodicity in the surgery obstructions, which is accounted for by the signature. Hence in order to obtain a periodicity for the structure set, we need to use the signature operator instead of the Dirac operator. Although it does not produce periodicity in the  $K$ -theory, it does so after inverting 2. As a consequence we obtain the periodicity of the structure set after inverting 2.

Equivariantly, the equivariant Dirac operator still provides the periodicity in the equivariant  $K$ -theory. As explained before, we should use the signature operator instead. If the group is of odd order, then the signature operator indeed produces periodicity in  $K$ -theory. Moreover, for actions of finite groups, we still have the identification of the  $L$ -homology theory with  $K$ -theory after inverting 2. As a consequence, we have the periodicity of the structure set for odd order group action, after inverting 2.

Such kind of periodicity of the structure set is certainly limited: 2-torsion is ignored, the group has to be odd order. So what is the role of signature in my periodicity, which is true integrally for actions not only of odd order groups, but also even order groups, and even some compact Lie groups?

I obtain the periodicity in a rather geometrical way. A key step is the following situation: Let  $E \rightarrow M$  be a bundle of closed manifolds, and let  $M$  be the boundary of another manifold  $N$ , such that  $\pi_1 M = \pi_1 N$ . Then crossing with  $E$  kills any surgery obstruction. This is closely related to Atiyah's analysis of the nonmultiplicativity of the signature. In general, if the signature of  $M$  is zero, it does not necessarily follow that the signature of  $E$  is zero. However, Atiyah expresses the signature of  $E$  in terms of universal characteristic classes over the classifying space of the fundamental group. Therefore, the condition  $\pi_1 M = \pi_1 N$  implies that the characteristic class extends over  $N$ , and then the signature of  $E$  is zero. In fact, the fundamental  $\pi - \pi$  theorem in the surgery theory says that such is the case even when we view the signature as an element in the surgery obstruction group, instead of a simple number.

The simple situation as above can be further generalized to certain stratified union of many (isovariantly)

$\pi - \pi$  pairs. Atiyah's computation generalizes correspondingly.

I would like to emphasise that the vanishing of the signature should be for purely algebraic reason. Lacking stratified algebraic symmetric groups, we can only perform our construction in an ad hoc way. I truly wish that somebody would work out an algebraic definition of  $L_G^*$ .

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