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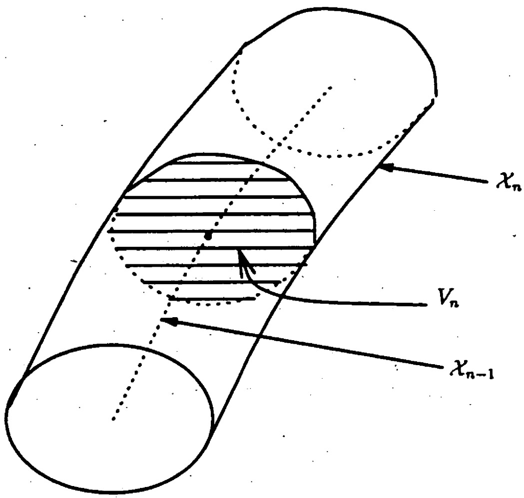


The conference was organized by J.D. Jones (Warwick), I. Madsen (Aarhus) and E. Vogt (Berlin). 51 participants from Europe and the United States attended the conference.

Among the 20 talks there was a series of 3 talks, given by M. Hopkins, on connections between algebraic geometry, formal groups and stable homotopy theory. Besides stable homotopy theory, the main topics of interest were low dimensional topology (3– and 4–manifolds, knot theory) and K-theory and its relation to (topological) cyclic– and Hochschild–homology.

In a short talk A.S. Mishchenko described the project of the Encyclopaedia of Mathematical Sciences.

Vortragsauszüge

M. Hopkins:Moduli Spaces of formal groups and Stable Homotopy Theory

This was a series of talks beginning with a general description after Quillen, Morava, Lubin–Tate, etc. of the moduli space of commutative, 1–dimensional formal group laws, and the orbit structure of the action of the formal diffeomorphism group of the line, and ending with applications to stable homotopy theory. The applications included

(1) A formula (due to Devinatz–Gross–Hopkins) for the dualizing complex in $K(n)$–local homotopy theory.

(2) A discussion of how, by systematically demanding that all objects have E_∞–structures, and that all maps between them respect the E_∞–structure, the correspondence between formal group laws and spectra could be rigidified. This leads to the result (Hopkins–Miller) that the stabilizer group S_n acts on E_n, and leads to the construction of new cohomology theories. (EO_n, etc.).

(3) The work of Ando and Ando–Strickland–Hopkins relating Dyer–Lashof operations to isogenies.

W. Lück:L^2–methods and applications to K–theory and topology

We introduce basic notions about von Neumann algebras like von Neumann trace, von Neumann dimension, Fuglerde–Kardison determinant and give applications. The algebraic K_0 and K_1 of a von Neumann algebra are computed by Roendam and Lück. As a consequence one gets:

Theorem. Let H be a finite normal subgroup of the (arbitrary) group Γ. Then the map induced on Whitehead groups

$$Wh(H)^\Gamma \to Wh(\Gamma)$$

is rationally injective.

We introduce for a finite connected CW–complex X with vanishing L^2–Betti numbers and positive Novikov–Shubin invariants its combinatorial L^2–torsion $\rho^{(2)}(X) \in \mathbf{R}^{>0}$. It behaves like a multiplicative Euler characteristic. Given an irreducible 3-manifold with infinite fundamental group M which is non–exceptional, it is the product of the invariant applied to the hyperbolic pieces in its decomposition by incompressible tori. It is conjectured that the logarithm of the invariant is $-\frac{1}{3\pi}\cdot$ Volume. This would follow from the conjecture that analytic and combinatorial L^2–torsion agree and would relate these invariants to Gromov's simplicial volume.

T. tom Dieeck:Knot theories and root systems

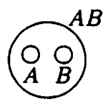
Classical knot theory is related to the series (A_n) of Dynkin diagrams. There are similar theories belonging to other series of (affine) root systems and Weyl groups and more general Coxeter groups. In the talk, the case of the series (B_n) was presented in detail. A diagrammatic description of B-knots via symmetric knots was given. There exists a Temperley-Lieb algebra of type B_n and a Kauffman bracket for B-knots. More general invariants are constructed with the help of representations of Hecke algebras.

There exists a natural class of such representations on tensor powers of modules which are contructed using R-matrices. They lead via quantum traces to two variable polynomial invariants of B-knots. The Hecke algebra representations have a natural generalization to representations of tangle categories.

B. Kirby:Dedekind sums and the signature cocycle

If $s(a,c)$ denotes the classical Dedekind sum, then we give a new, more topological, definition of a function $S(a/c)$, $S : \mathbf{Q} \cup \infty \to \mathbf{Q} \cup \infty$, where $S(a/c) = 12\,\mathrm{sign}(c)s(a,c)$. Let $a/c = -1/a_1 - 1/a_2 - 1/a_3 - \ldots - 1/a_n$ be a continued fraction. Let M be the intersection matrix for the 4–manifold defined by plumbing according to the weighted graph $\overset{a_1}{\bullet}\!-\!\overset{a_2}{\bullet}\!-\!\overset{a_3}{\bullet}\!-\cdots-\!\overset{a_n}{\bullet}$. Then $S(a/c) = 3\mathrm{signature}(M) - \sum_{i=1}^{n} a_i + \frac{a+d}{c}$, where $b/d = -1/a_1 - 1/a_2 - \ldots - 1/a_{n-1}$. Note that $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in SL(2,\mathbf{Z})$ and that $\varphi(A) = 3\sigma(M) - \sum a_i$ is the negative of the classical Rademacher φ function, $\varphi : SL(2,\mathbf{Z}) \to \mathbf{Z}$, which satisfies $\varphi(A) + \varphi(B) - \varphi(AB) = -\,\mathrm{sign}(c_A c_B c_{AB})$ where c_A is the lower left corner of $A \in SL(2,\mathbf{Z})$.

The signature 2–cocycle σ, $\sigma \in H^2(SL(2,\mathbf{Z});\mathbf{Z}) = \mathbf{Z}/12$ is defined as $\sigma(E)$, where E is the 2–torus bundle over a pair of pants with monodromies A and B.



$$\text{Let } \nu(A) = \begin{cases} \mathrm{sign}\, b & \text{if } A = \left(\begin{smallmatrix} 1 & b \\ 0 & 1 \end{smallmatrix}\right) \\ \mathrm{sign}\, c(a+d-2) & \text{otherwise} \end{cases}$$

Theorem. $\sigma(A,B) = -\,\mathrm{sign}\,(c_A c_B c_{AB}) + \nu(A) + \nu(B) - \nu(AB)$.

The proof involves calculating the μ-invariants of the torus bundles T_A, T_B and T_{AB} over the circle ($= \partial E$) which then gives $\sigma(E)$ modulo 8. But $|\sigma(E)| \leq 3$ and $|\sigma(A,B)| \leq 4$ so mod 8 implies equality. $\nu(A)$ is the signature of the obvious bordism between T_A and the lens space L_A. $\varphi(A)$ is related to the μ-invariant of L_A.

W. Meyer and R. Sczech have made calculations via algebraic methods which can be reduced to that of the Theorem.

J.P. May:Commutative algebra in stable homotopy theory

This talk was based on joint work with Tony Elmendorf and Igor Kriz and with John Greenlees.

Work in stable homotopy theory requires that one have an additive, triangulated, stable homotopy category of spectra. It must be symmetric monoidal under the smash product. This allows one tho define ring spectra R in terms of unit maps $S \to R$ and products $R \wedge R \to R$, and it allows one to define R-module spectra M in terms of maps $R \wedge M \to M$.

For more refined work, one wants ring and module spectra that are defined in terms of diagrams that commute on the pointset level rather than just up to homotopy. The appropriate notions are A_∞ and, in the commutative case, E_∞ ring spectra and their modules.

A new category of S–modules was described. It has an associative and commutative smash product, $\wedge_S$, and it gives a model for the stable homotopy category. In it, one can redefine A_∞ and E_∞ ring spectra in terms of maps $S \to R$ and $R \wedge_S R \to R$ of S–modules. One can redefine E_∞ modules in terms of maps $R \wedge_S M \to M$. Moreover, when R is an E_∞ ring spectrum, there is a commutative and associative smash product over R, $\wedge_R$, that takes values in R–modules. In fact $M \wedge_R N$ is defined via a coequalizer diagram exactly like that defining the tensor product of modules over a commutative algebra.

This theory yields quick new constructions of the spectra usually constructed from MU by the Baas–Sullivan theory of manifolds with singularities or the Landweber exact functor theorem.

Moreover, these constructions work equally well for G–spectra, where G is any compact Lie group. One obtains equivariant versions of BP, $E(n)$, $k(n)$, $K(n)$, and so on in this fashion.

Further, when G acts freely on a finite product of unit spheres of representations, for example when G is nilpotent, there is a completion theorem for the computation of $M_*(BG)$ and $M^*(BG)$ in terms of "local cohomology and homology groups" for any E_∞ module over MU_G with underlying nonequivariant spectrum M. This applies to $M = MU$, BP, $E(n)$, $k(n)$, $K(n)$, etc. Understanding of calculations will require a better understanding of equivariant cobordism groups than is presently available.

I. Madsen:Topological cyclic homology

For an equivariant S^1–spectrum T and a subgroup C of S^1 there are two kinds of fixed point spectra, both equivariant S^1/C–spectra:

$$T^C(W) = T(W)^C \quad \text{and} \quad \Phi^C T(W) = \varinjlim_V \Omega^{V^C} T(V \oplus W).$$

Here W is a C–fixed S^1–representation and V runs over all S^1–representations. There is an obvious map of spectra from T^C to $\Phi^C T$. Let $\rho_C : S^1 \to S^1/C$ be the isomorphism which sends $z \in S^1$ to $\sqrt[|C|]{z}$, and let $\rho_C^\# T^C$, $\rho_C^\# \Phi^C T$ be the associated S^1–spectra

$$\rho_C^\# T^C = \rho_C^* T(\rho_C^{-1})^*, \quad \rho_C^\# \Phi^C T = \rho_C^* \Phi^C T(\rho_C^{-1})^*$$

Definition 1 T is called cyclotomic if there is an S^1-homotopy equivalence $\varphi_C : \rho_C^\# \Phi^C T \xrightarrow{\simeq} T$ such that $\varphi_{C_{rs}} = \varphi_{C_r} \circ \varphi_{C_s}$ where C_r denotes the cyclic group of order r.

There are two maps $\Phi, D : T^{C_{rs}} \to T^{C_s}$, namely

$$\Phi : T^{C_{rs}} = (\rho_{C_r}^\# T^{C_r})^{C_s} \to (\rho_{C_r}^\# \Phi^{C_r} T)^{C_s} \xrightarrow[\varphi_{C_r}]{\simeq} T^{C_s}$$

and $D : T^{C_{rs}} \to T^{C_s}$, the inclusion of fixed sets. They commute, and one defines
Definition 2 $TC(T) = \operatorname{holim}_{\Phi, D} T^{C_r}$.

The homotopy fiber of $TC(T) \to T$ is profinitely complete and $TC(T)_p^\wedge \simeq \operatorname{holim}_{\Phi, D} T^{C_{p^n}}$.

The topological Hochschild homology functor $THH(R)$ of Bökstedt's, defined for a ring R or more generally for a "functor with smash product" is a cyclotomic spectrum.
Definition 3 The topological cyclic homology of R is the (-1)–connected cover of $TC(THH(R))$. Denote it $TC(R)$.

The cyclotonic trace gives a map (of spectra)

$$trc : K(R) \to TC(R)$$

which turns out to be a strong invariant of Quillen's K–groups, or more generally of Waldhausens A–theory (which is $K(R)$ for a certain functor with smash product). If R is finite over $\mathbb{Z}$ then $TC(R)^\wedge_p \simeq TC(R \otimes \mathbb{Z}_p)$. The corresponding statement for K–theory is not true at all, so let us restrict attention to rings R of the following type:

$$R = \varprojlim R/I^n\ , \quad R/I \text{ semi-simple } \mathsf{F}_p\text{-algebra.} \tag{4}$$

These rings include the complete valuation rings of positive residue characteristic. A theorem of Gabber and Suslin asserts that $K(R)^\wedge_\ell \simeq K(R/I)^\wedge_\ell$ when $\ell \neq p$. One conjectures that

$$trc : K(R)^\wedge_p \to TC(R)^\wedge_p$$

is a homotopy equivalence for the rings in (4). The conjecture is true when $R = \mathsf{F}$ is a finite field, and can thus be stated in relative form as a cartesian square

$$\begin{array}{ccc} K(R)^\wedge_p & \longrightarrow & TC(R)^\wedge_p \\ \downarrow & & \downarrow \\ K(R/I)^\wedge_p & \longrightarrow & TC(R/I)^\wedge_p . \end{array} \tag{5}$$

It is rumored that (5) has been proved to be cartesian when I is a nilpotent ideal by Goodwillie and McCarthy.

This would give that the continuous version of the above conjecture is true: $K^C(R)^\wedge_p \simeq TC^C(R)^\wedge_p$. On the other hand,

$$K^C(R)^\wedge_p \simeq K(R)^\wedge_p \quad \text{and} \quad TC^C(R)^\wedge_p \simeq TC(R)^\wedge_p$$

when R is a discrete valuation ring and Dedekind domain of residue characteristic p, and (5) would imply that $K(R)^\wedge_p \simeq TC(R)^\wedge_p$ in this case. Application of the localization theorem in K–theory then gives a "calculation" of K–theory of local fields of characteristic zero in terms of TC of their integers.

Modulo a sticky detail, not totally resolved at this time, $TC(\mathbb{Z}_p) = (\mathrm{im}\, J \vee \sum \mathrm{im}\, J \vee \sum bu)^\wedge_p$ for $p > 0$ — this is the expected value of $K(\mathbb{Z}_p)^\wedge_p$ according to the Quillen–Lichtenbaum conjecture for this ring.

Finally it seems interesting to attempt to make TC an endo functor on the category of E_∞–ring spectra, and to evaluate $TC^{(n)}(\mathsf{F}_p)$. In particular, what is $TC^{(2)}(\mathbb{Z}_p)$?

References

1. M. Bökstedt, W.C. Hsiang, I. Madsen: *The cyclotomic trace and algebraic K-theory of spaces*, Invent. Math. (1993).

2. M. Bökstedt, I. Madsen: *Topological cyclic homology of the integers,* preprint, Aarhus Univ. (1993).

3. L. Hesselholt, I. Madson: *Topological cyclic homology of finite fields and their dual numbers,* preprint Aarhus University (1993).

C. Lescop:Generalized Surgery Formula for the Casson–Walker Invariant

The Casson-Walker invariant of a rational homology 3–sphere presented by a framed link in S^3 is given as an explicit function of Alexander polynomials and linking numbers associated with this link. This function extends naturally to all framed links in S^3 and can be directly checked to define an invariant of closed oriented 3–manifolds. This invariant is a combination (becoming simpler and simpler when the 1^{st} Betti number of the presented manifold increases) of already known invariants.

M. Boileau:Degree one maps and bundles over S^1

This is a joint work with S. Wang (Peking Univ.). We study proper non–zero degree maps between 3–dimensional compact orientable irreducible, ∂–irreducible manifolds.

If M is a bundle over S^1 with homologically irreducible rational monodromy and M and N have the same first Betti numbers, then any degree one map $f : M \to N$ is properly homotopic to a homeomorphism.

We apply the proof of this result to construct infinitely many closed hyperbolic orientable 3–manifolds with the property that no tower of abelian coverings contains a bundle over S^1. The Betti number of the manifolds can be arbitrarily large.

J. Rognes:The map $G/O \to \Omega Wh_{\otimes}^{Diff}(*)$ is an infinite loop map

We prove that the map $G/O \to \Omega Wh^{Diff}(*)$ which was constructed by Waldhausen and proved to be a rational equivalence by Bökstedt, is in fact an infinite loop map if we use a multiplicative infinite loop space structure on the target. As an application we investigate the obstruction to improving Bökstedt's two–primary results on splitting the étale K-theory space $JK(\mathbf{Z})$ off from $K(\mathbf{Z})$, to the unlooped space or spectrum level. We prove that this can be done, after completing at the prime two, if the unit map $SG \to K(\mathbf{Z})_{\otimes}$ factors through the Adams e-invariant $SG \to J_{\otimes}$. Finally we determine the algebra of spectrum self maps of $JK(\mathbf{Z})$ completed at two, and prove that two times Bökstedt's splitting map is infinitely deloopable. Hence twice the unit map $SG \to K(\mathbf{Z})_{\otimes}$ factors through the Adams e-invariant, as a spectrum map.

S. Kwasik:Unitary nilpotent groups and stability of pseudo-isotopies

Let M^n be a closed manifold and let $C(M^n)$ be the pseudo-isotopy space of M^n. The group $\pi_0 C(M^n), n \geq 5$, was computed by Hatcher-Wagoner and Igusa in terms of higher algebraic K-theory invariants.

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In particular it follows that all pseudo-isotopies on M^n are *stable* i.e. the suspension map $\Sigma^k : \pi_0 C(M^n) \to \pi_0 C(M^n \times I^k)$ is an isomorphism ($n \geq 5$).

It turns out that there is a new phenomenon in the case of low dimensional manifolds. Namely, there exist *stable* and *unstable* pseudo-isotopies. The unstable ones are not detected by the K-theoretic Hatcher-Wagoner invariants but arise from the exotic UNil groups of S. Cappell. In particular: Let $M^3 = \mathbf{R}P^3 \# \mathbf{R}P^3$.

Theorem. The suspension map $\Sigma^k : \pi_0 C(M^3) \to \pi_0 C(M^3 \times I^k)$ has infinitely generated kernel and cokernel with

$$\mathrm{UNil}_2(\mathbf{Z}_2 * \mathbf{Z}_2) = \bigoplus_{i=1}^{\infty} \mathbf{Z}_2 \subset \ker \Sigma^k.$$

G. Matić: Embedded surfaces in 4–manifolds

In this talk, I presented joint work with D. Kotschick, cf. [1].

One of the outstanding problems in four-dimensional topology is to find the minimal genus of an oriented smoothly embedded surface representing a given homology class in a smooth four-manifold. For an arbitrary homology class in an arbitrary smooth manifold not even a conjectural lower bound is known. However, for the classes represented by smooth algebraic curves in (simply connected) algebraic surfaces, it is possible that the genus of the algebraic curve, given by the adjunction formula

$$g(C) = 1 + \frac{1}{2}(C^2 + CK), \tag{6}$$

is the minimal genus. This is usually called the (generalized) Thom conjecture.

We prove several results concerning this problem.

Firstly, using branched covers and Donaldson's theorems A, B and C on spin manifolds with $b_2^+ \leq 2$, we prove a lower bound on the genera of surfaces representing certain divisible classes in 4–manifolds. As a special case, we obtain:

Theorem 1. *Let Σ_d be a smoothly embedded surface representing d times the generator of $H_2(\mathbf{C}P^2, \mathbf{Z})$. If d is even, $d > 2$ and $\frac{1}{2}d$ is odd, then $g(\Sigma_d) \geq \frac{1}{4}d^2 + 1$.*

This improves the best previously known bound by 2, and proves the Thom conjecture for degree 6 curves in $\mathbf{C}P^2$. In forthcoming work of Lee and Wilczynski it is shown that the lower bounds given by Hsiang-Szczarba and Rokhlin are realized by topologically locally flat surfaces, for all d which are even or powers of an odd prime.

Corollary 2. *For d even, $d > 2$ and $\frac{1}{2}d$ odd, the topologically locally flat surfaces of genus $g = \frac{1}{4}d^2 - 1$ representing d times the generator of $H_2(\mathbf{C}P^2, \mathbf{Z})$ constructed by Lee and Wilczynski are not smoothable.*

Secondly, if a suitable branched cover has $b_2^+ \geq 3$, then we prove that its Donaldson polynomials give obstructions to doing surgery on an embedded surface. This idea was already used by Donaldson to prove that surgery can not be done on ample complex curves. We generalize (and reprove) his result to show that under more general assumptions surgery can not be done to produce a counterexample to the Thom conjecture.

Theorem 3. *Let X be a simply connected smooth complex algebraic surface and $B \subset X$ a smooth algebraic curve. Then surgery can not be done on B if either*

1. *the selfintersection number B^2 is positive, or*
2. *$[B]$ is divisible in $H_2(X, \mathbf{Z})$.*

The second part is probably the more interesting one, because, unlike the recent results on this problem due to Kronheimer and Mrowka, it applies to complex curves of non-positive selfintersection. Although the first part looks like an immediate generalization of Donaldson's result on ample curves, his proof does not generalize, primarily because for non-ample B the fundamental group of $X - B$ is not necessarily Abelian. By necessity, our proof is rather different. The main difference is that instead of using Donaldson's vanishing theorem for his polynomial invariants of connected sums, we use the following theorem which says that certain homology classes can not be represented by spheres.

Theorem 4. *Let X be a smooth closed oriented 4-manifold with $b_1(X) = 0$ and $b_2^+(X)$ odd and > 1. Suppose X contains a smoothly embedded 2-sphere S of zero selfintersection with $[S] \neq 0 \in H_2(X, \mathbf{Q})$. Then all Donaldson invariants of X vanish.*

This theorem was first proved for the $SU(2)$ Donaldson invariants of simply connected manifolds by Morgan, Mrowka and Ruberman (unpublished).

References

[1] D. Kotschick and G. Matić, *Embedded surfaces in four–manifolds, branched covers, and $SO(3)$–invariants* (submitted).

P. Teichner: Embeddings of 2–complexes into $\mathbf{R}^4$

In 1932 van Kampen defined an obstruction $o(K)$ for the existence of an embedding of an n–dim. simplicial complex K^n into $\mathbf{R}^{2n}$. He gave an (erronous) proof that $o(K) = 0$ is sufficient for $K^n \hookrightarrow \mathbf{R}^{2n}$ if $n \geq 3$. In 1957 Wu and Shapiro gave a correct proof of this fact using the Whitney trick. In 1991 Sakaria showed that $o(K)$ is also the precise obstruction for $n = 1$ i.e. for the planarity of graphs. We (joint with M. Freedman & S. Kruskal) give an example of a 2–complex K^2 ($\simeq \bigvee S^2$) with $o(K) = 0$ but which does not embed into $\mathbf{R}^4$. This seems to be the first known example but we already get a large family of such 2–complexes so that we expect that $o(K)$ is very far from being sufficient for $n = 2$. Roughly, $o(K)$ only sees linking numbers but not the higher μ–invariants of Milnor.

A. S. Mishchenko: On the project of Encyclopaedia of Mathematical Sciences

This is some information on the project of Encyclopaedia of Mathematical Sciences. The project is divided into series of different topics on mathematics. One of them should be devoted to topology. Each series consists of several volumes, each of them approximately 240 pages long. Each volume should be published in Russian and English. The English version will be published by Springer–Verlag. As of now, I know at least 50 volumes were published in Russia and most of them translated and published by Springer–Verlag. The main conditions for authors: describe the certain part of topology with historical observation, give a complete list of necessary notions, and the most important theorems. Everything should be illustrated with examples. The number of essential examples should be as large as possible. Complete proofs should in most cases not be included. One should describe the main ideas. The text should be understandable for nonspecialists like physicists and should be interesting and useful for specialists. For more information please contact Alexander S. Mishchenko,E–mail: as@mish.mian.su

Two volumes on Topology were published.

CONTENTS of volumes "Encyclopaedia of Mathematical Sciences" on TOPOLOGY

Volume 1.

1. S.P.Novikov: A general overview of Algebraic Topology.

2. D.B.Fuks: Homologies and homotopies in certain standard classical manifolds.

Volume 2.

1. D.B. Fuks, O. Viro: General account on homology and homotopy theories

I have some suggestions but part of them may not be valid now.

Suggestions:

1. D.B. Fuks and Ju.B. Rudiak: On the calculation methods in homotopic topology, (prepared, 5 signatures).

2. M. Farber (together with Pazytnov) about Morse theory of 1–forms, vector fields, etc. (Novikov inequalities).

3. J.–Claude Hausmann: The homotopy theory of smooth manifolds. (Poincare duality, difference between Poincare duality spaces and manifolds, h and s–cobordisms, Hirzebruch signature formula, manifolds and asphericity (Novikov conjecture), etc.)

4. ???

5. V.V. Sharko: a) The functions on manifolds (The problem of minimizations of number of critical points of the Morse functions on manifolds, The Morse mapping of manifolds into circle, Manyvalued Morse–Novikov theory, family of functions on manifolds, connections with pseudoisotopy, the homotopy type of the general Morse functions, the Morse inequality with degenerated critical points, the Bott functions, equivariant Morse theory, connections with dynamical and Hamiltonian systems) (5 signatures).

   b) Homotopy type of nonsimplyconnected cellular complexes (screwed modules, homotopical systems by Whitehead, the invariants of 2–dimensional CW–complexes, classification of 2–dimensional complexes with abelian fundamental group, the Wall obstruction for dominating of cellular complexes, homotopy type and simple homotopy type of complexes, the problems of Endruce–Kertise, Whitehead, Kerver–Laudenbach, Ziman, connection with Poincare conjecture (3,5 signatures).

6. M. Farber (together with J. Levine) about multidimensional knots and links.
7. W. Lück ?
8. A.S. Mishchenko, Ju.P. Solov'ev: On topological K–theory.
9. T. Friedrich: a) Dirac operators in Riemannian manifolds,
   b) Twistor theory and field equations.
10. J.Peter May: Homotopy and homology theories.
11. A. Ranicki, S. Ferry: The finiteness obstruction. Geometric aspects arising from compact ANR spaces and non–compact manifolds, and the algebraic aspects arising from finite group actions on spheres and codimension 1 splittings.

L. Hesselholt:Topological cyclic homology of dual numbers over finite fields

Let $\mathsf{F}[\epsilon] = \mathsf{F}[t]/(t^2)$ be the ring of dual numbers over a finite field of characteristic p, and let

$$\hat{W}_n(\mathsf{F}) = (1 + X\mathsf{F}[\![X]\!])^\times/(1 + X^{n+1}\mathsf{F}[\![X]\!])^\times$$

be the (big) ring of Witt vectors of length n. The involution $\tau\colon \hat{W}_n(\mathsf{F}) \to \hat{W}_n(\mathsf{F})$ which maps $X \mapsto -X$ has a (-1)-eigenspace which we denote $\hat{W}_n(\mathsf{F})^{\langle -1\rangle}$.

Theorem. $\mathrm{TC}(\mathsf{F}[\epsilon])$ is a generalized Eilenberg-MacLane spectrum whose non-zero homotopy groups are
i) p odd: $\mathrm{TC}_{2n-1}(\mathsf{F}[\epsilon]) \cong \hat{W}_n(\mathsf{F})^{\langle -1\rangle}$ and $\mathrm{TC}_0(\mathsf{F}[\epsilon]) \cong \mathbb{Z}_p$.
ii) $p = 2$: $\mathrm{TC}_{2n-1}(\mathsf{F}[\epsilon]) \cong \mathsf{F}^{\oplus n}$ and $\mathrm{TC}_0(\mathsf{F}[\epsilon]) \cong \mathbb{Z}_2$.

The ring $\mathsf{F}[\epsilon]$ is an example where the conjecture presented by Ib Madsen in his talk applies. In other words, one expects that

$$K_*(\mathsf{F}[\epsilon])^\wedge_p \cong \mathrm{TC}_*(\mathsf{F}[\epsilon]).$$

The calculation of $\mathrm{TC}(\mathsf{F}[\epsilon])$ uses a result for topological Hochschild homology. A *pointed monoid* is a based space Π with a multiplication and unit

$$\mu\colon \Pi \wedge \Pi \to \Pi, \qquad 1\colon S^0 \to \Pi.$$

If R is a ring and Π is a discrete pointed monoid, then $R[\Pi] = R\langle\Pi\rangle/R\langle *\rangle$ is a new ring with multiplication

$$R[\Pi] \otimes R[\Pi] \to R[\Pi \wedge \Pi] \to R[\Pi].$$

Let $\Pi_2 = \{0, 1, \epsilon\}$ with 0 as basepoint and $\epsilon^2 = 0$; then $\mathsf{F}[\Pi_2] = \mathsf{F}[\epsilon]$. The cyclic bar-construction of Π is the cyclic space (Connes) with k-simplices the $(k+1)$-fold smash product $N^{cy}_{\wedge,k}(\Pi) = \Pi^{\wedge(k+1)}$ and Hochschild-type structure maps.

Theorem. $\mathrm{THH}(R[\Pi]) \simeq_{S^1} \mathrm{THH}(R) \wedge |N^{cy}_{\wedge}(\Pi)|$, where the smash product on the right hand side is that of an (equivariant) spectrum and a space.

Another ingredience in the calculation of $\mathrm{TC}(\mathsf{F}[\epsilon])$ is the following diagram in which the rows are cofibration sequences, valid for any *cyclotomic* spectrum T, and thus in particular when $T = \mathrm{THH}(\mathsf{F})$;

$$\begin{tikzcd}
T_{hC_{p^n}} \arrow[r, "N"] \arrow[d, equal] & T^{C_{p^n}} \arrow[r, "\Phi"] \arrow[d, "\Gamma"'] & T^{C_{p^{n-1}}} \arrow[d, "\hat{\Gamma}"'] \\
T_{hC_{p^n}} \arrow[r, "N^h"] & T^{hC_{p^n}} \arrow[r, "\Psi"] & \mathsf{H}(C_{p^n}; T).
\end{tikzcd}$$

The terms in the lower sequence are all approximated by spectral sequences and in favorable cases one may solve these and compute the terms in the upper sequence by an inductive procedure starting from a proof that

$$\hat{\Gamma}: T \to \mathsf{H}(C_p; T)$$

induces an isomorphism on $\pi_i(-)$ for $i \geq 0$. This is the case when $T = \mathrm{THH}(\mathsf{F})$.

J.-P. Otal:Hyperbolisation of surface bundles over S^1

We describe the proof of Thurston's hyperbolisation theorem for surface bundles with Pseudo Anosov monodromy. A new proof of the double limit theorem is given which can be applied for proving other versions of the double limit theorem, for instance, to the case of Schottky groups. The proof uses $\mathbf{R}$-trees, and the characterisation by Skona of surface group actions on $\mathbf{R}$-trees with small edge stabilizer as those which are dual to a measured lamination. The notion of realisation of a measured lamination in an $\mathbf{R}$-tree is introduced. By using the Hausdorff–Gromov topology, one shows that when the lamination λ is realized in an $\mathbf{R}$-tree which is a limit of representations of $\pi_1(S)$ in $PSL_2(\mathbf{C})$, there is a uniform control on the growth rates of closed curves near to λ in the space of measured laminations.

M. Heusener:About trace–free representations of knot groups

Let $k \subset S^3$ be a knot and $G = \pi_1(S^3 - k)$ its group. Moreover, let $m \in G$ be a meridian of the knot.

A representation $\rho : G \to SU_2(\mathbf{C})$ is called trace–free iff $tr\rho(m) = 0$. Analogous to Casson's original construction Xiao–Song Lin [J.Diff.Geo.**35**, (92)] defined an intersection number of the representation spaces corresponding to a braid representative of the knot k. This intersection number turns out to be an integer knot invariant denoted by $h(k)$.

Roughly speaking: $h(k)$ is the number of conjugacy classes of non-abelian trace–free representations of G counted with signs. Moreover, the equation $h(k) = \frac{1}{2}\sigma(k)$ holds, where $\sigma(k)$ is the signature of the knot.

It seems to be mysterious why these two quantities with apparently different algebraic-geometric contents should ever be the same.

The aim of the talk was to use the $SO_3(\mathbf{R})$-representation curves of 2-bridge knot groups in order to explain the relation between the invariants $h(k)$ and $\sigma(k)$.

R. Jung: Elliptic homotopy theories

Elliptic homotopy theories are constructed by trivializing certain fibre bundles in bordism groups. One chooses a bordism theory Ω and a category of fibre bundles with fibre F and structure group G. Then the elliptic functor $ell_*(X)$ is defined as the quotient of $\Omega_*(X)$ by the relations that $[E, f] \in \Omega_*(X)$ is identified with $[B \times F, \overline{f} \circ pr_1]$, where $F \to E \to B$ is a fibre bundle of the chosen type and the map $f : E \to X$ is constant along the fibre inducing a map $\overline{f} : B \to X$.

In all cases where one could prove that $ell_*(X)$ is a homology theory (sometimes after inverting some elements) the chosen types of bordism and fibre bundles are related to classical invariants (genera) of manifolds.

The first elliptic theory of this kind was constructed by M. Kreck and St. Stolz in 1989 using spin bordism and $\mathbf{H}P^2$ bundles. This theory was constructed away from 2 earlier by Landweber, Ravenel and Stong. Now there exist several of these theories, e.g. using unoriented bordism and $\mathbf{R}P^2$-bundles or using oriented bordism and $\mathbf{C}P^2$-bundles. These could be related to the classical homology theories. They are also connected by natural transformations which up to now do not have a geometric explanation. One can also start with stably almost complex bordism and use $\mathbf{C}P^1$- and $\mathbf{C}P^2$-bundles. Then one gets a new homology theory that is related to K-theory. The corresponding cohomology theory should have a good construction in terms of tuples of bundles with symmetries, such that complex manifolds are oriented via the symbol of the Dolbeault complex.

There are applications of elliptic theories to the problem of the existence of a metric of positive scalar curvature on a manifold and relations to the Novikov Conjecture.

D. Kotschick: 4-manifold invariants of groups

In the talk I discussed properties and examples of calculations of the invariants $p(\Gamma)$ and $q(\Gamma)$ defined below. See [1] for details and a survey of results.

Definition. Let Γ be a finitely presentable group. Define

$$p(\Gamma) = \inf\{\chi(X) - \sigma(X)\}$$

and

$$q(\Gamma) = \inf\{\chi(X)\},$$

with the infima taken over all closed oriented 4-manifolds X with fundamental group Γ.

In all the examples we know, the invariants do not depend on the category one works in: TOP or DIFF. It would be important to know how general this phenomenon is.

These invariants are related to the splitting problem for the fundamental group in dimension 4. If the fundamental group of a 4-manifold splits as a free product, then this splitting cannot always be realized topologically at the level of manifolds (Kreck-Lück), although it can be realized after stabilizing with $k(S^2 \times S^2)$ (Hillmann).

We also discussed relations of the invariants and the splitting problem with Kähler geometry and gauge theory, cf [2].

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