MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 45/1993

Invariant ordering in geometry and algebra

10.–16.10.1993

Die Tagung fand statt unter der Leitung von KARL HEINRICH HOFMANN, Darmstadt, JIMMIE D. LAWSON, Baton Rouge, and ERNEST B. VINBERG, Moskau. Die Tagung behandelte vornehmlich Themen aus dem Bereich der Geometrie und Analysis, die mit Halbgruppen in der Theorie der LIE-schen Gruppen und ihrer Darstellungstheorie in Hilberträumen verbunden sind.

Jeden Tag wurden drei einstündige Vorträge gehalten, die in einen größeren Themenkreis einführten und bis zu den neuesten Entwicklungen überleitete, davon zwei am Morgen, einer am Nachmittag. Dazwischen wurden kürzere Fachvorträge von etwa vierzigminütiger Dauer angesetzt, welche Spezialthemen in ihrer Tiefe behandelten. Es wurde genügend Zeit für Diskussionen veranschlagt, die in kleinen Gruppen und informell auch außerhalb des geplanten Programms weiterliefen. Die unten wiedergegebenen Vortragsauszüge sind entsprechend dem Programm wiedergegeben.

The conference was organized by KARL HEINRICH HOFMANN, Darmstadt Institute of Technology, JIMMIE D. LAWSON, Lousiana State University at Baton Rouge, and ERNEST B. VINBERG, the Chair of Algebra, Moscow University. It primarily concerned the occurence of semigroups in the theory of LIE groups and their representation theory in Hilbert space, and is also covered their applications in geometry and analysis.

Three one hour lectures were scheduled for every day, two in the morning and one in the afternoon. They introduced the audience to one particular subject matter area and led up to latest developments. These were interspersed by lectures of about 40 minutes duration which covered technical matters in depth. There was enough time for discussions which were continued in small groups outside the formal program. The abstracts presented below are arranged according to the program.

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Als besonders erfreulich wurde es empfunden, daß rund ein Drittel der Teilnehmer aus Rußland kamen und über den hohen Stand der mathematischen Forschung in ihrem Land im Bereich des Themas der Tagung berichteten. Es ist geplant, in einem Sammelband die wichtigsten Themen in einer unabhängig lesbaren Darstellung zu veröffentlichen. Es ist dabei nicht an einen Tagungsband im üblichen Sinne gedacht. Für die Veröffentlichung technischer Beiträge in Zeitschriften wurde anderweitig gesorgt, insoweit Herausgeber solcher Zeitschriften an der Tagung teilnahmen.

It was considered fortuitous that about a third of the entire group of participants came from Russia and reported on recent research in their country in the domain of the conference. This research has a traditionally high standard well known to people in the area. There are plans to publish a collection of surveys, each independently readable. This publication will not be a proceedings volume in the usual sense but hopefully will be of the character of a monograph in the area covered by the conference. Technical contributions will appear elsewhere. Such publications may be expedited by the presence of editors of relevant journals among the participants of the meeting.

Alle Teilnehmer waren einhellig begeistert von der Infrastruktur des Instituts und der ausgezeichneten Organisationsarbeit der Verwaltung und der Belegschaft.

The participants expressed unanimously their enthusiasm about the infrastructure of the Mathematische Forschungsinstitut and the efficiency and dedication of the technical administrators and their support staff.

Invariant order in geometry and algebra: The topics of the conference

The conference focussed on recent developments relating ordered structures and Lie theory. At the Lie *algebra* level, a concept of order is expressed by a convex cone of positive elements, at the Lie *group* level by a semigroup of positive elements, and at the level of *homogeneous spaces* by a partial order invariant under the action of a Lie group. These orders are frequently called *homogeneous causal orders.* The development of a substantial mathematical theory of such interrelated structures as *invariant cones*, *Lie semigroups*, and *ordered homogeneous spaces* seems to be "an idea whose time has come" since prominent researchers from such diverse areas as mathematical physics (e.g. relativity theory), representation of groups, geometric control theory, probability, harmonic analysis, and topological semigroups reported on developments and applications of these ideas in their respective disciplines. This confluence of themes from varying viewpoints in diverse contexts provided a broad perspective that conference participants found most stimulating , and which will undoubtedly have a positive impact on future research in the area.

Noncommutative complex analysis

Classically, domains for Hardy spaces and representation theory have been constructed from a closed convex cone C in $\mathbb{R}^n$ by forming the wedge $\mathbb{R} + \oplus i \cdot C \subseteq \mathbb{C}^n$. As an appropriate noncommutative analog, GEL'FAND and GINDIKIN proposed in

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the late seventies the consideration of certain complex domains D in the complexification $G_{\mathbb{C}}$ of a Lie group G such that G is contained in the boundary of D. It was the important insight of G. I. OL'SHANSKIĬ (who participated in this conference) in the early eighties that if G is a simple hermitian Lie group, then its Lie algebra $\mathfrak{g}$ contains an invariant cone C, and the sought-after domains are the interiors of semigroups S of the form $G \exp i{\cdot}C$ in $G_{\mathbb{C}}$. The presence of such semigroups S, now generally called *Ol'shanskiĭ semigroups*, has stimulated a substantial research effort that has resulted in major developments in the fields of noncommutative harmonic analysis, invariant cones and Lie semigroups, and the development of appropriate tools for handling such theories, e.g. linear and nonlinear convexity theorems, generalizing earlier results by KOSTANT and ATIYAH.

Harmonic Analysis on causal symmetric spaces

In recent years a thorough structure theory of symmetric spaces endowed with a causal order has been developed and some of the basic results were presented at the conference. This has laid the foundations for new undertakings for the harmonic analysis on such spaces such as generalisations of spherical functions, Laplace transforms, Volterra kernels, and Volterra algebras.

Geometric control theory

Other significant impulses and appropriate mathematical tools for the development of the theory have arisen from geometric control theory. The concern with the *future reachable set* gives a certain asymmetry to the theory, which translates in the context of Lie theory to notions of conal (sometimes invariant) sets of controls and semigroups (namely, the reachable set from the identity). Of particular interest were reports at the conference on the reformulation of problems in classical mechanics and optimisation as control problems on Lie groups and the development of a sophisticated theory of *control sets* in flag manifolds of semisimple Lie groups for the study of the control semigroup.

Causality and relativity

The future set on a Lorentzian manifold (or, more generally, a manifold endowed with a cone field) and the resulting causal order structure provide the motivation for another synthesis of order theory and manifold geometry. Three lectures at the conference discussed relativity theory from this viewpoint. Again transitive actions of Lie groups on manifolds endowed with invariant cone fields played a fundamental role in the theory.

Semigroups enveloping groups

Surprisingly, one has made the discovery in recent years that certain semigroup completions and compactifications of groups in a variety of different contexts furnish important tools for understanding the structure of the groups and their corresponding representations. Lectures demonstrated how this principle applies to the creation of the enveloping semigroup of a reductive algebraic group, of "big" groups (e.g. the infinite dimensional classical groups), and how certain notions from category theory and multilinear algebra contribute to concrete constructions of enveloping semigroups.

Lie semigroup theory

New developments in the Lie theory of semigroups permit sharp existence theorems for Ol'shanskiĭ semigroups and heighten our knowledge of their structure and representation theory. Progress was reported on the globality of Lie wedges and and an outline was given of a final characterisation of divisible semigroups, a project that has stretched over the past ten years of Lie semigroup theory. As an application it was pointed out it was pointed out how notions of Lie semigroups and Lie wedges occur naturally in certain aspects of probability theory.

Abstracts

J. D. LAWSON:

Polar decompositions of Ol'shanskiĭ semigroups

In a semigroups with involution, polar decompositions are defined. Examples illustrate how this relates to the classical situation of matrices. A basic example arises from considering an invariant cone C in the Lie algebra $\mathfrak{g}$ of a a hermitean Lie algebra. The involution is given by $g^* = (\overline{g})^{-1}$. In the complexification $G_{\mathbb{C}}$ of a suitable Lie group for $\mathfrak{g}$ the set $S = \big(\exp(i \cdot C)\big)G$ is a closd subsemigroup called the *complex Ol'shanskii semigroup*. It admits polar decompositions. This concept applies to contraction semigroups with respect to nondegenerate sequilinear forms on $\mathbb{C}^n$, where the Ol'shanskiĭ semigroup is constructed from the minimal invariant cone, and to compression semigroups in which the maximal invariant cone plays this role.

K.-H. NEEB:

Holomorphic extension of unitary representations

The problem of extending a continuous unitary representation $\pi: G \to U(\mathcal{H})$ of a connected Lie group to a holomorphic representation of a suitable "complex" semigroup. The domains of such holomorphic extensions a covering semigroups $\Gamma(\mathfrak{g}, W)$ of Ol'shanskiĭ semigroups constructed from an invariant not necessarily pointed cone in $\mathfrak{g}$. (1) The unitary representation which extend holomorphically to $\Gamma(\mathfrak{g}, W)$ are characterized. (2) Holomorphic representations are highest weight representations in a general sense. (3) The corresponding highest weights are described. (4)The holomorphic representations separate the points iff $W \cap -W$ is a compact Lie algebra and $\mathfrak{g} \oplus \mathbb{R}$ contains a pointed generating invariant cone.

V. JURDJEVIC:

Variational problems on Lie groups

Optimal geometric control theory, through Pontryagin's Maximum Principle, liberates geometry and mechanics from its Riemannian and holonomic context. Kirchhoff's Theorem on the equilibrium configurations of an elastic rod illustrates this point well. Here each configuration (appropriately defined) yields a curve $t \mapsto g(t) = (x(t), R(t))$ in the group of motions of Euclidean 3-space such that $\frac{dx}{dt} = Re_1$ for an orthonormal basis e_1, e_2, e_3. One considers a first order system for the curve R in SO(3) and the total energy functional of the *strain* curve $u(\cdot)$ which enters the differential equation for R.The basic principle which governs the equilibrium configurations is that they minimize the total elastic energy over all possible configurations with the prescribed boundary conditions. The Lie theoretic treatment of this optimisation problem is described in detail.

Y. A. NERETIN:

The oscillator semigroup

Let $F(\mathbb{C}^n)$ denote FOCK space and let $\begin{pmatrix} K & L \\ L^t & M \end{pmatrix}$ be a symmetric block matrix S. The we get an operator $B(S)\colon F(\mathbb{C}^n) \to F(\mathbb{C}^n)$ defined by

$$(B(S)f)(z) = \int \exp\left\{\frac{1}{2}(z,\overline{u})\begin{pmatrix} K & L \\ L^t & M \end{pmatrix}\begin{pmatrix} z^t \\ \overline{u}^t \end{pmatrix}\right\} f(u)e^{-|u|^2}\,du\,d\overline{u}.$$

According to OL'SHANSKIĬ, the operator $B(S)$ is bounded iff $\|S\| \le 1$ and $\|K\|$, $\|M\| \le 1$. Also, the product of two composable operators $B(S_2)B(S_1)$ is of the form $\lambda{\cdot}B(T)$ with a nonzero $\lambda \in \mathbb{C}$. One now defines a category $\mathcal{K}$ whose objects are the Fock spaces $F(\mathbb{C}^n)$ and the morphisms are the operators $B(S)$ up to scalar nonzero factors. Next one defines a category **Sp** of finite dimensional vector spaces $V_{2n} = \mathbb{C}^n \oplus \mathbb{C}^n$ equipped with two nondegenerate forms, one skew symmetric, one skew hermitean. Morphisms $V_{2n} \to V_{2m}$ are suitably selected binary relations $P \subseteq V_{2n} \oplus V_{2m}$. Composition of morphisms is the composition of binary relations. A basic result is that the categories $\mathcal{K}$ and **Sp** are equivalent. $\mathrm{Aut}(V_{2n}) \cong \mathrm{Sp}(2n, \mathbb{R})$ and $\Gamma_n = \mathrm{End}(V_2 n)$ contains an Ol'shanskiĭ semigroup but is not itself one such. (See Compt. Rend. (Paris) **309** (1989), and Funct. Anal. Appl. **24:2** (1990).

A. K. GUTS:

Topoi, ordering and the theory of relativity

A topos-theoretic approach is described for a unified axiomatic description of special and general theories of relativity, i.e., a categorical causal theory of Lorentzian manifolds is constructed.– An affine object in an elementary topos containing a STOUT real number object $\mathbb{R}$ is defined in which a conal order is introduced. Now a *Lorentz object* is an affine object with a conal order satisfying certain additional conditions. Various topoi are relevant: In the topos of sets a Lorentzian object is indeed an affine space with a pseudo-euclidean structure; in the topos of sheaves over a space I a Lorentz-object is a fiber bundle with base space I and with fibers endowed with an affine structure and a Lorentzian form.

A. K. STROPPEL:

Ordered projective planes

Ordered projective planes whose order on a coordinate line is archimedean and complete, then each line is homeomorphic to a circle and the plane is homeomorphic to $P_2\mathbb{R}$ (SALZMANN). Some of the earliest known examples of non-Desarguesian planes are of this type. Using only general order properties a proof is given that the examples by HILBERT (1899) and MOHRMANN (1922) yield projective planes. The procedure is described. It is observed that the construction yields planes with large endomorphism semigroups and small automorphism groups.

A. LEVICHEV:

Invariant elliptic cones in chronogeometry and I. Segal's chonometry

There are only four 4-dimensional Lie groups with an invariant Lorentzian cone: the abelian one, $\mathbb{R} \oplus \mathrm{sl}(2,\mathbb{R})$, $\mathrm{u}(2)$, and osc (GUTS and LEVICHEV, 1984; LEVICHEV, 1985). The according simply connected Lie groups yield the "worlds" of relativity: M_0 is the world of special relativity, $U(2)$ is the compact cosmos of chronometry, the oscillator Lie group W is one of the interesting solutions of the Einstein equations in general relativity. The fourth one does not satsify the energy condition. The universal covering $M \cong \mathbb{R} \times \mathrm{SU}(2)$ of $U(2)$ is the cosmos of SEGAL's chronometry (SEGAL 1976). The action of the conformal group on M is discussed as well as the corresponding representations U, in particular those for which the invariant cone C_U of all $X \in \mathfrak{g}$ for which $i \cdot dU(X)$ is a nonnegative self-adjoint operator, is nonzero and pointed.

F. COLONIUS:

The spectrum of linear differential equations

The Lyapunov spectrum of families of linear ordinary differential equations indexed by time varying coefficient functions and the associated generalized versions of eigenspaces can be approximated from the interior by the Floquet spectrum and the exterior by the Morse spectrum. The Floquet spectrum is obtained from the periodic solutions in the control sets of the corresponding semigroup in the Lie group acting on projective space. The Morse spectrum is constructed over the chain recurrent components of an associated skew product flow. These components can be described since the base flow is chain recurrent. Under an inner pair condition, both generalized versions of eigenspace—control sets and projected chain recurrent components—coincide up to closure and the corresponding spectra coincide. Hence in this case, the Lyapunov spectrum which lies in between coincides with the Floquet spectrum.

E. B. VINBERG:

The enveloping and asymptotic semigroups of a semisimple algebraic group

Any connected semisimple algebraic group G over an algebraically closed field k is canonically embedded into a reductive algebraic semigroup $E(G)$, called the *enveloping semigroup of* G with the following properties: (1) The group of invertible elements of $E(G)$ is $(G \times T)/Z$ where $T \cong (k^*)^n$ is a maximal torus and Z the diagonally embedded center of G. (2) $E(G)$ has a zero. (3) Any linear algebraic representation of G can be canonically extended to a linear representation of $E(G)$. (4) There is an algebraic surjective homomorphism from $E(G)$ onto k^n mapping T onto $(k^*)^n$ such that the inverse image of the identity is exactly G, and that all point inverses of this homomorphism have the same dimension (namely, $\dim G$).— The inverse image $\mathrm{As}(G)$ of zero in $E(G)$ is called the *asypmtotic semigroup* of G. The enveloping semigroup together with this homomorphism may be regarded as a contraction of G to the asypmtotic semigroup. Further details of the asymptotic semigroup and the construction of $E(G)$ are given.

G. I. Ol'shanskiĭ:

Enveloping semigroups of big groups

The term "big" groups is a nontechnical common name for many infinite dimensional groups which are not locally compact. Among these, the *infinite dimensional matrix groups* are defined as direct limits $\varinjlim G(n)$, where $\{G(n)\}$ stands for a natural sequence of classical groups or finite symmetric groups or else some wreath products. Examples: $\mathrm{O}(\infty) = \varinjlim \mathrm{O}(n)$, $\mathrm{U}(\infty) = \varinjlim \mathrm{U}(n)$, $\mathrm{Gl}(\infty) = \varinjlim \mathrm{Gl}(n)$, $\mathrm{S}(\infty) = \varinjlim \mathrm{S}(n)$, where $\mathrm{S}(n)$ denotes the symmetric group on n elements. In such a group one can choose (in several ways) a certain subgroup K and a chain of subgroups $K \supseteq K_1 \supseteq K_2 \supseteq \cdots$ such that $\bigcap_n K_n = \{e\}$ and that for each n the group $G(n)$ commutes with K_n. Any such chain $\{K_n\}$ defines a totally disconnected group topology on the group G. For a wide class of examples the following theorem holds: Let G, K, K_n be as above and let $\Gamma(n)$ stand for the space of double cosets $K_n \backslash G / K_n$. Set $\Gamma = \varprojlim \Gamma(n)$. Then there is a natural semigroup structure on $\Gamma(n)$ and Γ. There exist idempotents $\varepsilon_n \in \Gamma$ and bijections between $\Gamma(n)$ and $\varepsilon_n \gamma \varepsilon_n$ such that the natural projection $\Gamma \to \Gamma_n$ corresponds to $\gamma \mapsto \varepsilon_n \gamma \varepsilon$. With respect to the projective limit topology, Γ is a semitopological semigroup and G is dense in Γ. For certain groups G of classical type, the semigroups $\Gamma(n)$ are Lie semigroups, locally. Further details are given.

D. Mittenhuber:

Globality of Lie wedges

A Lie wedge W in the Lie algebra $\mathfrak{g}$ of a simply connected Lie group G is called *global* if $W = L(\langle \exp W \rangle)$. A classification of all global Lie wedges of all solvable Lie algebras of dimension up to 3 is given. The theorems on which this classification is based are explained.

B. Ørsted:

Reproducing kernels and composition series

Let G be a simple Lie group and K a maximal compact subgroup such that the symmetric space G/K is a bounded Hermitian domain of the tube type, e.g. $G = \mathrm{SU}(2,2)$ or $G = \mathrm{O}^*(8)$. Let S be the Shilov boundary of G/K. Now S is a symmetric space K/L which is also a homogeneous space G/P for a maximal parabolic subgroup $P = MAN$. In the first example $S = \mathrm{U}(2)$, the conformal compactification of Minkowski space, and in the second $S = \mathrm{U}(4)/\mathrm{Sp}(2)$. For a complex linear functional ν on $\mathfrak{a}$ the generalized spherical principal series representation $I(\nu) = \mathrm{Ind}_P^G(1 \otimes e^\nu \otimes 1)$ is considered. The general problem is addressed of giving the explicit expansions of reproducing kernels for unitary representations in terms of the K-types in the representation. We determine the composition series of the $(\mathfrak{g}, K)$- module $I(\nu)_K$, the vector space of K-finite elements of $I(\nu)$. The details are discussed and explicit formulae involving the spherical polynomials on S are presented.

Y. A. NERETIN:

Universal completions of complex classical groups

For finite dimensional vector spaces V and W a vector subspace of $P \subseteq V \oplus W$ is called a *linear relation* $P: V \Rightarrow W$ For and such one defined kernel, image, domain and indefinity (the latter is $P \cap W$). A *simple hinge* is a sequence $\mathcal{P} = (P_1, \ldots, P_n)$ of linear relations $P: C^n \Rightarrow C^n$ such that $\mathrm{dom} P_{j+1} \subseteq \ker P_j$ and $\mathrm{im}\, P_j \subseteq \mathrm{indef} P_{j+1}$. If equality holds always, and if further $\mathrm{indef} P_1 = \{0\}$ and $\ker P_m = 0$, then the hinge is called *exact*. The language of hinges gives and elementary description of some classical objects such as complete quadrics, complete symmetric varieties, Satake-Fürstenberg compactifications of symmetric spaces. One succeeds to define a semigroup multiplication on the set of hinges in $\mathbb{C}^n$ and obtains a semigroup $\overline{\mathrm{PGl}_n}$. It acts naturally in the set of all irreducible representations of Sl_n. For each irreducible representation ρ of Sl_n the closure of the set $\mathbb{C} \cdot \rho(\mathrm{Sl}_n)$ in the set of all endomorphisms of the representation space is the image of $\overline{\mathrm{PGl}_n}$. Hinge completions exist for all classical groups. (See Funct. Anal. Appl. **26:4** (1992)).

A. K. GUTS:

Left invariant conal ordering in affine 3-space

Let G be a simply connected solvable 3-dimensional Lie group which acts on simply transitively on $\mathbb{R}^3$ by an affine action. The action is said to be normal is a maximal abelian subgroup acts by translation.—THEOREM 1. (Guts) Suppose that A_1, A_2 are two affine structures on $\mathbb{R}^3$ induced by two actions of G. Suppose there exists an elliptic conal ordering with respect to A_1 and A_2 which is left invariant under both actions. If one of the actions is by parallel translations, then the two affine structures are affinely equivalent.—THEOREM 2. (Shalamova) If an elliptic conal ordering is invariant under a normal action, then every order continuous automorphism is an affine transformation.—Another theorem of Shalamova's is presented which says that for a normal action on an elliptic conal ordering not invariant under all translations, the group cannot act homogeneously on the interior, exterior, or boundary of a future set. These results are used for axiomatic descriptions of Pseudo-Euclidean geometry on $\mathbb{R}^3$.

Z. JUREK:

Probability and semigroups

The Central Limit Problem in probability theory on vector spaces is the characterization of *all* limit distributions in the following scheme:

$$A_n(\xi_1 + \ldots + \xi_n) + a_n, \; n \geq 1, \tag{1}$$

where $\xi_1, \xi_2, \ldots$ are independent random variables into a Banach space X, where $a_n \in X$, and $A_n \in \mathrm{Aut}(X)$, and where the following conditions are satisfied: The triangular array $A_n \xi_j$, $1 \leq j \leq n$, $n \geq 1$ is uniformly infinitesimal, and the semigroup generated by all $A_n A_k^{-1}$ is compact in the semigroup $\mathrm{End}(X)$ of all bounded linear operators of X equipped with the norm topology. (This last condition is automatically satisfied if X is finite dimensional). We note that it is crucial to consider only *full* measures, i.e., those whose support is not contained in any hyperplane. The *decomposability semigroup* $D(\mu)$ of a probability measure μ

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is defined by

$$D(\mu) = \{A \in \mathrm{End}(X) : (exists \nu_A) \quad \mu = A\mu * \nu_A\},$$

where $*$ is ordinary convolution of measures and $A\mu$ is the image of μ under the mapping A. If the probability measure ν_A may be chosen to be a point measure, then A is said to belong to the *symmetry semigroup* $A(\mu)$. The latter turns out to be the group of units in $D(\mu)$. The two are compact if and only if μ is full, in the case $X = \mathbb{R}^n$. A theorem of URBANIK asserts that a measure μ is a limit of equation (1) above if and only if there exists $Q \in \mathrm{Aut}(X)$ such that (i) $\exp(-tQ) \in D(\mu)$ for all $t \geq 0$, and (ii) $\lim_{t\to+\infty} \exp(-tQ) = 0$. Such measures are said to be *operator-selfdecomposable*. Let Q be called a U-exponent of μ if $\exp(-tQ) \in D(\mu)$ for all $t \geq 0$. Then from definition the set $E(\mu)$ of U-exponents of μ it follows that $E(\mu)$ is (the negative of) the tangent wedge of the closed semigroup $D(\mu)$, and thus is a Lie wedge. One can average any member of this wedge over the adjoint action of the compact group $A(\mu)$ and obtain members of the wedge commuting with the symmetry group $A(\mu)$. These concepts are illustrated for a full Gaussian measure on $\mathbb{R}^n$.

L. SAN MARTIN:

Control sets and semigroups in semisimple Lie groups

Let G be a semisimple Lie group with finite center, S a subsemigroup with interior, and M a flag manifold of G on which G and hence S operate. There is a natural relation defined on M by $x \leq y$ if $y \in xS$, and the level sets of this relation are called *control sets*. They can be characterized by means of Weyl chambers in G meeting $\mathrm{int}(S)$, and hence the Weyl group can be exploited in the study of these control sets. With this machinery, it can be shown that there is a unique invariant control set, and that in any flag manifold this control set is proper if and only if S is proper in G. Thus maximal semigroups must be compression semigroups of subsets with dense interior in the minimal boundaries.

V. GICHEV:

Invariant algebras on Lie groups

Certain invariant subalgebras A of the algebra of continuous complex-valued functions vanishing at ∞ on a Lie group G are considered. The maximal ideal space $\mathcal{M}_A$ of such a subalgebra admits in a natural way the structure of a locally compact semigroup. It is possible to show that the Lie algebra $\mathfrak{g}$ of G contains an invariant cone when the algebra A contains an approximate identity and to characterize this cone in terms of A. Various results about the idempotents in the maximal ideal space are given, and their structure is linked to that of the group G. It is also to be observed that the semigroup contains a dense Ol'shanskiĭ semigroup.

G. 'OLAFSSON:

Causal homogeneous spaces

Let $M = G/H$ be an irreducible semisimple symmetric space corresponding to an involution τ commuting with a Cartan involution θ. The Lie algebra $\mathfrak{g}$ decomposes into eigenspaces $\mathfrak{h}+\mathfrak{q}$, where $\mathfrak{q}$ may be identified with the tangent space of M at $o = eH$. The manifold M is *causal* if there exists an invariant (pointed) cone field $m \mapsto C_m$ on M. The causal structures may be naturally identified with H-invariant cones in $\mathfrak{q}$. This is the case if and only if $H \cap K$ fixes a non-zero vector in $\mathfrak{q}$. A different analysis results according to whether M is *compactly causal* (the interior of the invariant cone meets $\mathfrak{q} \cap \mathfrak{k}$) or is *non-compactly causal* (the interior of the cone meets $\mathfrak{q} \cap \mathfrak{p}$), where $\mathfrak{k}$ and $\mathfrak{p}$ arise from the Cartan involution. There is a duality between the two notions arising from the dual Lie algebra, $\mathfrak{g}^c = \mathfrak{h} + i{\cdot}\mathfrak{q}$ that makes possible the classification of both cases, the compactly causal spaces being classified from the known classifications of anti-holomorphic involutions on G/K. The invariant cones in $\mathfrak{q}$ are uniquely determined by intersections with maximal abelian subalgebras. In the case of a non-compactly causal cone C, there is a corresponding Ol'shanskiĭ semigroup $H(\exp C)$, and a corresponding ordering of M that is globally hyperbolic.

J. FARAUT:

Analysis on causal symmetric spaces

An extension of two topics from classical analysis, Hardy spaces and the Laplace transform, to noncommutative analysis on complex Ol'shanskiĭ semigroups is considered. The appropriately square integrable holomorphic functions on the interior of $\Gamma(C) = G\exp(i{\cdot}C)$ are taken as the Hardy space $H^2(C)$, this embeds via the boundary value operator into $L^2(G)$, and the image can be decomposed using irreducible representations of G belonging to the holomorphic discrete series. Next a Volterra algebra $V(M)$ consisting of causal kernels is considered on a noncompactly causal symmetric space $M = G/H$. The subalgebra of invariant causal kernels is commutative, and the spherical Laplace transform performs in the appropriate way with respect to composition on these invariant causal kernels.

V. MOLCHANOV:

Holomorphic discrete series for hyperboloids of Hermitian type

The hyperboloid G/H where $G = SO_o(p,2)$, $H = SO_o(p,1)$ is a causal semisimple symmetric space of Hermitian type. The quasiregular representation of G on $L^2(G/H)$ contains holomorphic discrete series, which can be realized on Hardy spaces on appropriate complex manifolds. Explicit expressions for the corresponding Cauchy-Szegö kernels have been obtained, for the projections of $L^2(G/H)$ onto the subspaces of the discrete series representations. The difference of the projections for the holomorphic and antiholomorphic representations is an analogue of the Hilbert transform and various of its properties can be described.

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J. HILGERT:

Convexity theorems and Lie semigroups

In the classification of invariant convex cones, an important question is to determine the projection of a group orbit to certain subspaces. Kostant's linear convexity theorem gives the answer for compact groups, and Paneitz proved a corresponding non-compact linear theorem appropriate for classifying invariant cones. A general theorem which generalizes the result of Paneitz to proper moment mappings for Hamiltonian torus actions on non-compact symmetric spaces is presented, much as the Atiyah-Guillemin-Sternberg theorem in the compact case generalizes the result of Konstant. Other applications and corresponding non-linear theorems are discussed.

K. H. HOFMANN:

The divisibility problem

The following problem is discussed: Let G be a connected real Lie group. Classify all closed subsemigroups $S \subseteq G$ which are divisible, i.e., have roots of all positive integral orders. It has been known for 10 years that each element in such a semigroup must Lie on a one parameter subsemigroup of S. The outstanding conjecture to establish has been that the tangent wedge $L(S)$ is a semialgebra, i.e., that S is locally divisible near the identity. A proof of this conjecture would make the known classification of Lie semialgebras available to complete a solution of the classification problem. The lecture contains various instructive examples, motivation and history of the problem, and some hints toward the final solution, which are more fully discussed in the presentation of W. Ruppert.

M. ZELIKIN:

Synthesis of optimal trajectories on representation spaces of Lie groups

Let X be a representation space of a Lie group G, ω a differential form of the first degree on X, and $K(x)$ a field of closed convex cones on X. The main problem considered is the minimization of the integral of the differential form ω along curves which satisfy certain boundary conditions and are solutions of $\dot{x}(t) \in K(x(t))$. This problem is assumed to be appropriately equivariant with respect to the action of G. For this problem the concept of a totally extremal space, an analogue of a totally geodesic submanifold, is introduced, and some theorems about the existence of totally extremal manifolds in the Lie group setting are given. These theorems are used to construct a synthesis of optimal trajectories for certain multidimensional equivariant problems. The totally extremal manifolds exhibit properties akin to Cartan subalgebras.

W. RUPPERT:

The solution of the divisibility problem

Let $S = \overline{\exp L(S)}$ be a closed subsemigroup of a connected Lie group G, such that $W = L(S)$ has interior points in $\mathfrak{g} = L(G)$ and such that $W \cap -W$ is ideal free. Then $S = \exp W$ and the following assertions hold: (i) $\mathfrak{g}$ is the ideal direct sum $\mathfrak{s} \oplus \mathfrak{d} \oplus \mathfrak{k}$, where $\mathfrak{s}$ is a direct sum of copies of $\mathrm{sl}(2, \mathbb{R})$, $\mathfrak{d}$ is diagonalizable metabelian, and $\mathfrak{k}$ is a compact Lie algebra. Moreover, $\mathfrak{d}$ is center free, and the center of $\mathfrak{g}$ is contained in $\mathfrak{k}$. (ii) W can be written as a direct sum $(W \cap \to) \oplus (W \cap (\mathfrak{d} \oplus \mathfrak{k})$, where the first summand again decomposes over the $\mathrm{sl}(2, \mathbb{R})$ factors in an explicit fashion, while W_0 is the intersection of half-spaces and an invariant cone. (iii) $\mathfrak{k} \cap H(W) = 0$, further $H(W) \cap \mathrm{comp}\mathfrak{g} = 0$, and finally $H(W)$ is metabelian.

V. GICHEV:

Cauchy kernels for Hardy spaces on oscillator groups

The problem of describing the bounded rational inner functions on a Ol'shanskiĭ semigroup $S = G\exp(i \cdot C)$ is considered, where the inner functions are those which are smooth up to the Shilov boundary and satisfy $-i \cdot df(X) > 0$ for all $X \in C$, and $-i \cdot d\log f|G > 0$. The particular case considered is that where G is a solvable Lie group admitting an invariant cone in its Lie algebra, i.e., the Lie algebra is a generalized oscillator algebra. An appropriate Hardy space $H^2(S)$, which can then be realized $L^2(G)$, is defined and the problem is analyzed by passing to the Lie algebra endowed with the transfer of Haar measure. An appropriate division operator then defines an isometry between the corresponding Hardy spaces on $\mathfrak{g}$ and G. The problem is also raised of relating Cauchy kernels on $\mathfrak{g}$ equipped with the usual Lesbegue measure and the Cauchy kernels of $\mathfrak{g}$ with respect to the transfer of Haar measure.

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