MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 47/1993

Geometrie

17. 10. - 23. 10. 1993

Die Tagung stand unter der Leitung von
V. Bangert (Freiburg) und
U. Pinkall (Berlin).

In 30 Vorträgen wurde ein weites Spektrum von differentialgeometrischen Themen behandelt. Schwerpunkte waren

- Untermannigfaltigkeiten konstanter mittlerer Krümmung
- Mannigfaltigkeiten nicht-positiver Krümmung
- Geometrische Methoden beim Studium hamiltonscher Systeme

Großes Interesse fand eine Demonstration der Software GRAPE am Dienstagabend.

Die Pausen zwischen den Vorträgen und die Abende wurden zu intensiver Diskussion genützt. Mehrere der Vortragenden wiesen darauf hin, daß sie durch solche Gespräche auf vorangegangenen Geometrie-Tagungen zu ihren neuen Ergebnissen angeregt worden waren.

ABSTRACTS

C. Bär
Elliptic operators and representation theory of compact groups

Atiyah and Singer solved the problem to calculate the index of an elliptic operator over a compact manifold in topological terms. There have been numerous applications, e.g. integrality theorems. For instance, spin manifolds carry the Dirac operator and the $\hat{A}$-genus is its index, hence an integer. In general, the $\hat{A}$-genus is only rational, e.g. $\hat{A}(\mathbb{C}P^2) = -\frac{1}{8}$. To apply the Atiyah-Singer index theorem it is important to know how to construct interesting elliptic operators. In this talk I describe how homotopy classes of elliptic operators correspond to elements of a certain ideal in the representation ring of the structure group of the manifold. This ideal is finitely generated and its generators correspond to fundamental operators. Using this method one obtains easily new integrality results for example for almost quaternionic manifolds. Moreover, immersions lead to interesting structure groups whose elliptic operators can be determined by the above method. Calculation of their indices yields lower bounds on the codimension of such immersions.

H. Reckziegel
Hypersurfaces with harmonic curvature in spaces of constant curvature

Let M be a hypersurface of a standard space N of constant curvature χ, suppose that M has harmonic curvature, i.e., that the divergence of the curvature tensor vanishes, and let Λ^* denote the set of all "completely smooth" principal curvatures of M such that the trace of the shape operator of M is constant in the direction of the corresponding principal directions. M. Meumertzheim and the speaker could show: If Λ^* has k elements, then there exists an open neighbourhood U of M in N which "is" a warped product with k spherical factors and which gives M locally also the structure of a warped product; the induced spherical leaves of M are open parts of the (complete) spherical leaves of the warped product U and simultaneously integral manifolds of the principal distributions corresponding to the principal curvatures $\lambda \in \Lambda^*$. For $\chi \geq 0$ we can prove $k \leq 2$. Examples are given by hypersurfaces of revolution in N whose profile curve satisfies a special ODE of second order.

H.C. Im Hof (joint work with J. Böhm): Complex volume of orthoschemes

In order to study hyperbolic polytopes, it is most convenient to use the projective model of hyperbolic geometry. It then becomes natural to consider polytopes in projective space and to study their geometry with respect to the hyperbolic metric. For distances and angles, this has been done by Cayley and Klein. Our purpose is to define volumes of polytopes.

The length of a segment and the area of a triangle, defined by the defect formula, turn out to be complex numbers. Thus we have to settle for complex volumes.

An orthoscheme is a simplex with "mostly" right angles; its Coxeter diagram is linear. The volume of a reducible orthoscheme is defined inductively as the product of the volumes of its components. With an irreducible orthoscheme (of dimension n) we associate a configuration of $n+3$ hyperplanes, which in turn defines a set of $2n+7$ polytopes. One of these is hyperbolic, it therefore has positive real volume. It differs from the given orthoscheme by an algebraic sum of reducible orthoschemes. This allows us to assign a complex volume to any orthoscheme.

The Schläfli differential formula, holding for hyperbolic polytopes and being compatible with products and dissections, continues to hold in our more general context.

M. Min-Oo (joint work with E. Ruh): A geometric model of the space of normal distributions

We will introduce a new Riemannian metric on the space of normal distributions (Gaussians) on $\mathbb{R}^n$. This metric is different from the usual Fisher information metric that is used by statisticians. In fact, it is a Riemannian symmetric metric and has a larger group of isometries. The metric will also give us new formules for the maximum likelihood estimator used in parametric statistical inference.

J. Berndt (joint work with F. Prüfer, F. Tricerri, L. Vanhecke): Geometry of generalized Heisenberg groups and their Damek-Ricci harmonic extensions

The purpose of the talk is to treat the Riemannian geometry of generalized Heisenberg groups and their Damek-Ricci harmonic extensions. The latter ones provide the first counterexamples to the Lichnerowicz conjecture on harmonic manifolds. In the first part we give a survey about the known relations between the following classes of Riemannian manifolds: Riemannian symmetric spaces, naturally reductive Riemannian homogeneous spaces, Riemannian g.o. spaces, weakly symmetric spaces, ray symmetric spaces, commutative spaces, $\mathfrak{C}$-spaces, $\mathfrak{G}\mathfrak{C}$-spaces, harmonic spaces, D'Atri spaces. We also discuss the cases where generalized Heisenberg groups and their Damek-Ricci harmonic extensions provide examples for the strictness of several inclusions between these classes. In the second part we concentrate on spectral properties of the Jacobi operators on the above two classes of Lie groups. In particular, we prove that the eigenvalues of the Jacobi operator are constant along each geodesic for any generalized Heisenberg group, but for the Damek-Ricci examples this property characterizes the symmetric ones. Further, we also consider spectral properties for the shape operators of geodesic spheres and prove that the generalized Heisenberg groups provide examples of spaces where the principal curvatures of small geodesic spheres are invariant under geodesic symmetries.

P. Kohlmann:
A new characterization of cylinders

The following theorem ist proved.

Let $x : M \to \mathbb{R}^{n+1}$ be a hypersurface immersion of a complete manifold M with $K \geq 0$ and $\sum a_r E_r =$ const. $\neq 0$ for some fixed $a_1, \ldots, a_n \geq 0$. Then $x(M)$ is a right spherical cylinder (i.e. $x(M)$ congruent $\mathbb{S}^b \times \mathbb{R}^{n-b}$) or a sphere.

The proof ist based on a technique developed by Walter and makes use of the ellipticity and selfadjointness of mean curvature associated differential operators introduced by Voss. $x(M)$ is the boundary of a convex body C. The assumption that the recession cone of C ist not a linear subspace leads to a contradiction by two argumentation strings. The first is based on an eigenvalue inequality deduced from a theorem of Barta, the second on volume estimates for suitable compact slices of C. The case E_r =const. was already proved by Hartman in 1978.

B. Leeb (joint work with M. Kapovich):
Gromov's asymptotic cone and quasi-isometry invariants of 3-manifold groups

Let Γ be a finitely generated group. We can associate to Γ its Cayley graph which is a metric space well-defined up to quasi-isometry. (A quasi-isometric embedding is a map of metric spaces that distorts sufficiently large distances by a bounded factor.) We are interested in geometric properties of Γ i.e. quasi-isometry invariants of its Cayley graph Cayley(Γ). Wellknown examples of geometric properties of groups are "virtually nilpotent" (Gromov), "virtually abelian" and "virtually polycyclic" (Bridson, Gersten), "word-hyperbolic" (Gromov).

Quasi-isometries ignore small distances. Looking for quasi-isometry invariants we are thus led to understand the asymptotic large-scale geometry of a metric space X. An aspect of it, namely the asymptotic geometry of finite subsets of distant points in X, is encoded in the geometry of the asymptotic cone $\text{Cone}_\omega X$. This concept has been introduced by Gromov in his paper "Asymptotic properties of infinite discrete groups" (1991). Bi-Lipschitz invariants of $\text{Cone}_\omega X$ are quasi-isometry invariants of X.

If M is a closed non-positively curved Riemannian manifold, then

$$\mathrm{Cone}_\omega \tilde{M} \overset{\text{Bi-Lip.}}{\cong} \mathrm{Cone}_\omega \pi_1 M$$

reflects the interplay of totally-geodesic flat subspaces in $\tilde{M}$. The following theorem illustrates that the asymptotic cone is a non-trivial invariant:

Theorem: The quasi-isometry class of the fundamental group of an irreducible Haken 3-manifold M^3 detects whether in the topological splitting of M^3 into hyperbolic and Seifert components a Seifert piece occurs.

The link with non-positive curvature is established by

Theorem: The fundamental groups of all non-Seifert graph-manifolds (i.e. irreducible 3-manifolds built from Seifert components only) are quasi-isometric to each other.

This implies combined with a result in my thesis, saying that in presence of a hyperbolic piece an irreducible Haken 3-manifold carries a metric of non-positive curvature, the

Corollary: For every irreducible Haken 3-manifold M_1, there is a non-positively curved irreducible Haken 3-manifold M_2, s.t. $\pi_1 M_1$ and $\pi_2 M_2$ are quasi-isometric.

E. Leuzinger:
An exhaustion of locally symmetric spaces

Let X be a Riemannian symmetric space of noncompact type and rank ≥ 2 and let Γ be a nonuniform irreducible (arithmetic) lattice. On the locally symmetric quotient $V = \Gamma\backslash X$ we construct a function $h : V \to [0, \infty)$ such that the sets $\{h \leq s\}$ $(s \geq 0)$ exhaust V. The boundaries $\{h = s\}$(for $s > 0$) are parallel hypersurfaces consisting of pieces of horospheres which meet in corners. Moreover, $\{h \leq s\}$ is diffeomorphic (as a manifold with corners) to the Borel-Serre compactification of V. As an immediate application we also obtain a simple proof of the Gauss-Bonnet formula for V.

S. Buyalo (joint work with W. Ballmann):
Nonpositively curved metrics on 2-polyhedra

Research was motivated by the question of Gromov whether a word hyperbolic group admits a cocompact, discrete action on some space with a metric of negative curvature. We obtain some results in this direction; namely non-existence of a Γ-invariant metrics of negative curvature on certain spaces X (2-polyhedra) with a given cocompact and discrete action of a group Γ, even if Γ is hyperbolic. The Gauss-Bonnet formula involving the labelings of links of vertices turns out to be an essential tool. The notions of a minimal labeling of a graph and a tight metric on a 2-polyhedron is introduced. This leads to investigation of the space of tight metrics on a given 2-polyhedra which particular case is Theichmüller space of 2-torus. Examples of a totally rigid 2-polyhedra are found. Completions of some spaces of tight metrics are considered.

H.-B. Rademacher:
Twistor spinors with zeros

This is a report on joint work with Wolfgang Kühnel. The conformally invariant concept of a twistor spinor was introduced by R. Penrose in general relativity. We consider Riemannian spin manifolds carrying a twistor spinor φ i.e.

$$\nabla_X \varphi + \frac{1}{n} X \cdot D\varphi = 0 .$$

for every vector field X. Here ∇ is the spinor derivative, D the Dirac operator and "$\cdot$" the Clifford multiplication. We show that in dimension $n \geq 4$ a manifold carrying a twistor spinor with zeros and with non-trivial associated conformal vector field is conformally flat.

J. Heber:
Noncompact homogeneous Einstein spaces

Bounded homogeneous domains constitute one of the most prominent of many known classes of noncompact homogeneous Einstein spaces ("NHES"). Most structural results for NHES require additional geometric assumptions (e.g.: Kähler NHES = bounded domains; quaternionic Kähler NHES: classified by D.V. Alekseevskii).

We present uniqueness and further structural results for NHES under rather mild algebraic assumptions:

Up to isometry, all known NHES are of the form (solvable Lie group S, left invariant $< .,. >$), such that the Killing form B on the Lie algebra $\underline{s}$ satisfies

$$(*) \quad B(X,X) \geq 0 \qquad \text{for all } X \in \underline{s} .$$

For arbitrary solvable S which satisfies $(*)$, we show

(i) S admits ***at most one*** left invariant Einstein metric modulo Aut(S) and scaling (quite in contrast with the compact homogeneous case).

(ii) If S admits an Einstein metric $< .,. >$, then $\underline{a} := [\underline{s},\underline{s}]^{\perp}$ is ***abelian*** and all ad_A, $A \in \underline{a}$, are ***normal operators*** on $([\underline{s},\underline{s}], < .,. >)$. Consequently,

(iii) if $\underline{s} = \underline{a} + [\underline{s},\underline{s}]$ and some ad_A, $A \in \underline{a}$, is ***not semisimple***, then S does not admit left invariant Einstein metrics (general ***nonexistence results*** in the compact case were obtained by Wang-Ziller).

The proofs involve a modification msc of the scalar curvature functional sc on the space $\mathcal{M}$ of left invariant metrics on S. One has to investigate the ***geometry of*** msc on $\mathcal{M}$ as a symmetric space instead of its Morse theory, since in this setting, Einstein metrics are not the critical points of msc (or sc).

P. Ghanaat:
Nilpotent structures on framed manifolds

Nilpotent structures are (given by) systems of locally defined fibrations of a framed manifold M into almost flat nilmanifolds, satisfying compatibility conditions on the overlaps of fibered domains. The talk gives a description of these structures and applications to discrete groups of isometries of Riemannian manifolds and finiteness theorems. This unifies ideas and work of Cheeger, Fukaya, Gromov, Min-Oo, Ruh and myself.

G. Wiegmink:
Total bending of vector fields on Riemannian manifolds

The total bending of globally defined unit vector fields on a Riemannian manifold M is a functional which is devised to measure to what extent such a vector field X fails to be parallel. The total bending of X is defined as the integral over M of the squared norm of the Levi-Civita covariant differential of X. Many manifolds do not allow parallel vector fields.

Aiming at finding vector fields of minimal total bending, some general variational results are presented and the situation on 2-dimensional tori is discussed. For these tori rather complete results have been achieved, in particular the vector fields of minimal total bending have been determined for tori of revolution with arbitrary profile curve.

J. Cao:
Rigidity for non-compact surfaces of finite area

In this talk, we consider the rigidity of marked length-spectrum (and geodesic flows) for non- compact surfaces of finite area.

The speaker will discuss the following new rigidity result for non-compact surfaces: "Let M and N be two non-compact and complete Riemannian surfaces with finite area and strictly negative curvature. Suppose that M and N have the same marked length-spectrum (or have time-preserving conjugate geodesic flows). Then the two surfaces M and N must be isometric".

In addition, the rigidity of compact harmonic Kähler manifolds of negative curvature will be addressed.

L. Polterovich:
Unknottedness of Lagrangian surfaces

The talk which is based on a joint work with Yakov Eliashberg is devoted to the so called Lagrangian knots problem. The main result can be formulated as follows. Denote by Σ either torus $\mathbb{T}^2$ or sphere $\mathbb{S}^2$.

Theorem: Let $f : \Sigma \to T^*\Sigma$ be a Lagrangian embedding which is homologous to the zero section. Then f is isotopic to the zero section.

The proof is based on Gromov's theory of pseudo-holomorphic curves in symplectic manifolds.

N. Innami:
Natural Lagrangian systems without conjugate points

The variation vector fields through extremals of the variational principles of natural Lagrangian functions satisfy the equation of Jacobi type. By making use of Jacobi equation, we obtain the estimate of measure-theoretic entropy for natural Lagrangian systems without conjugate points. The estimates of the measure-theoretic entropy for geodesic flows were obtained by Pesin, Ossermann-Sarnak, Ballmann-Wojtkowski, Foulon, (Finsler case) and so on. The measure-theoretic entropy plays an important role in the classification problem of dynamical systems because it is a measure-theoretic conjugacy invariant.

Let M be a manifold with dimension n and TM the tangent bundle of M with natural projection $\pi : TM \to M$. Let $L : TM \to \mathbb{R}$ be a Lagrangian function given by

$$L(y) = \frac{1}{2}g(\pi(y))(y,y) - U(\pi(y)) ,$$

where g is a Riemannian metric of M and U is a function on M. Let S be an energy surface in TM with total energy $e > \sup\{U(p)|p \in M\}$. Then, the solutions of the Euler-Lagrange equation yield a flow $f^t : S \to S$ preserving the Liouville measure ω.

We prove the following estimate of the measure-theoretic entropy.

Theorem: Suppose M is compact. If the sectional curvature $\overline{K}$ with respect to the Jacobi metric $\overline{g} = (e-U)g$ satisfies $-b^2 \leq \overline{K} \leq -a^2 < 0$ $(a, b > 0)$, then the measure theoretic entropy $h_\omega(f^t)$ of the flow f^t satisfies the inequality

$$\frac{\sqrt{2}(n-1)b}{\mathrm{vol}_\omega(S)} \int_S e - U \circ \pi \, d\omega \geq h_\omega(f^t) \geq \frac{\sqrt{2}(n-1)a}{\mathrm{vol}_\omega(S)} \int_S e - U \circ \pi \, d\omega .$$

K. Leichtweiß:
Affine evolutions of hypersurfaces

At the last Oberwolfach meeting "Geometry" the question arose if the affine evolution of a strictly convex hypersurface, given by the nonlinear parabolic differential equation $\frac{\partial x}{\partial t} = y$ (y = affine normal vector) makes this hypersurface more and more "ellipsoidal". This is the affine analogue of well known results in euclidean evolution theory. It turns out that the answer to the question is positive. The proof of this fact uses the monotonicity of the affine isoperimetric quotient combined with a fundamental theorem of A.D. Aleksandrow on mixed volumes.

R. Kusner:
Conformally invariant energies of knots, links and embedded submanifolds

Following work of Michael Freedman and his collaborators we have defined a Möbius invariant energy for embedded n-dimensional submanifolds of $\mathbb{R}^{n+k} \cup \{\infty\} = \mathbb{S}^{n+k}$, with an infinite barrier to self-intersection. We shall discuss results on critical points for knots and links ($n = 1$) obtained jointly with Denise Kim, as well as an existence and regularity theory for higher dimensional (which represents joint work with John Sullivan) $M^n \subset \mathbb{S}^{n+k}$. Experimental results may also be presented.

F. Pedit (joint work with J. Dorfmeister and H. Wu):
Constant mean curvature surfaces with prescribed umbilic points

To unterstand the structure of cmc-surfaces of higher genus ≥ 2 it is preliminary to understand cmc-surfaces with umbilic points. Using an Iwasawa-type decomposition of the loop group of $SL(2,\mathbb{C})$ we discuss how every simply connected cmc-surface is obtained from certain holomorphic data. Moreover we show that "locally" those holomorphic data are described by a holomorphic function $f : D \to \mathbb{C}$, where D is either $\mathbb{C}$ or the unit disc. Thus provides a Weierstrass type representation of cmc-surfaces (where $H \neq 0$).

U. Pinkall:
Fast factorization in loop groups and the multiheaded Mr. Bubble

Dorfmeister, Pedit and Wu have given a method to construct all constant mean curvature surfaces in $\mathbb{R}^3$, where the basic choice to be made in order to construct a specific surface is to pick a holomorphic function f of one variable. For example, the well known surface usually called "Mr. Bubble" arises from the choice $f(z) =$ const. $\cdot z$.

Here we give a fast and efficient numerical algorithm to implement the construction mentioned above. The two main ingredients are:

(1) A fast algorithm to factor elements of the polynomial loop group $\Lambda_{\mathrm{pol}}SL(2,\mathbb{C})$ according to the Iwasawa decomposition.

(2) A modified Euler method to solve linear ODE's with values in $\Lambda SL(2,\mathbb{C})$.

Using this algorithm large pieces of cmc-surfaces with several umbilic points ("multiheaded Mr. Bubbles") can be computed even using Mathematica on a Macintosh PC.

M. Bialy:
Convex billiards and a theorem by E. Hopf

A theorem by E. Hopf states that the only Riemannian metrics without conjugate points on two-dimensional torus are flat ones. We define the notion of conjugate points for convex plane billiards and prove that the only billiards without conjugate points are circular billiards. Dynamical version of this result can be formulated as follows. The only billiards whose phase space is foliated everywhere by not null-homotopic invariant curves are circular billiards. This result solves in part the old conjecture by Birkhoff that the only integrable billiards are circles and ellipses.

M. Kotani:
The space of harmonic maps from 2-sphere into the unit sphere of degree d

Let us define the space $Harm_d(S^N)$ of all harmonic maps from 2-sphere into the unit N-sphere S^N of degree d. In the case of $N = 2$, this space has two connected components. One component is identified with the space of all meromorphic functions of degree d and the other is its conjugate. We can see the space is path-connected if N is greater than 2 by using a deformation method. More precisely, for an arbitrary element $g \in Harm_d(S^N)$, we construct a smooth family g_t of $Harm_d(S^N)$ such that $g_0 = g$ and $g_\infty \in Harm_d(S^{N-2})$. By inductive arguments, g can be connected to an element in $Harm_d(S^3)$, which space is connected.

S. Redjel:
Metric constructions with controlled curvature

Let $(M, \partial M, g)$ be a Riemannian manifold with compact boundary ∂M and a "curvature assumption".

Problem: Can one deform g near the boundary to a metric $\hat{g}$, which has the same "curvature assumption" and at least a prescribed 2-jet on the boundary ?

We present here some responses to this problem where the "curvature assumption" considered is an upper and lower bound for the different notions of curvature (i.e. we consider sectional, Ricci and scalar curvature). For negative Ricci curvature a similar problem is treated by Gao-Yau and Lokhamp.

In particular, this gives sufficient geometric conditions on the boundary, depending on the chosen curvature, for these classes of manifolds for which the curvature bounds are not disturbed by a doubling process.

P. Dombrowski: Remarks on the GAUSS equation of an isometric immersion of a m-dimensional riemannian manifold M $(m \geq 2)$ into spaces of constant curvature and the curvature operator of M

Examples and exercises concerning the GAUSS equation (originally prepared for teaching students on that topic) were presented, e.g.:

(i) If the curvature operator $\mathcal{R}_p^{(2)} : \Lambda^2 T_pM \to \Lambda^2 T_pM$ of M for some $p \in M$ has an eigenvalue, which does not occur as a sectional curvature value of M at p, then M does not admit an isometric immersion in any space of constant curvature with codimension ≥ 1 and flat normal bundle.

(ii) If there exists $p \in M$, such that $\mathcal{R}_p^{(2)}$ has an eigenvalue C of multiplicity $k \in \mathbb{N}_+$ with $\binom{m}{2} - k$ not being of the form $\binom{n}{2}$ with $n \in \mathbb{N}$, then M does not admit an isometric immersion in a space of constant curvature C.

(iii) If M is the riemannian product of two non-flat 2-dimensional riemannian manifolds, then M does not admit an isometric immersion in $\mathbb{E}^5$.

(iv) If M is the riemannian product ot two hypersurfaces N_i of $\mathbb{E}^{n_i+1}$, so that at some point $p_i \in N_i$ the shape operator of N_i has rank r_i $(i \in \{1,2,3\})$ with $2 \leq r_1, r_2$ and $r_1 \leq \sqrt{2(r_2+1)}$, then M does not admit an isometric immersion in $\mathbb{E}^{n_1+n_2+1}$.

(v) If $m = \dim M \equiv 3 \bmod 4$ and if C is any real number greater than the maximum of the sectional curvatures of M at some point $p \in M$, then M does not admit an isometric immersion in a (m+1)-dimensional space of constant curvature C.

Specific examples (e.g. homogeneous spaces) illustrated these results.

E. Heintze: Polar actions and s-representations

S-representations are the isotropy representations of symmetric spaces. They are polar in the sense that there exists a linear subspace which meets every orbit and is perpendicular to every orbit at each point of intersection. A theorem of J. Dadok states an almost converse: every polar action is orbit equivalent to an s-representation. A geometric proof is given using recent results of C. Olmos. As a by-product we obtain a simple classification of certain isotropy irreducible homogeneous spaces.

G. Knieper: Asymptotic geometry on manifolds of negative curvature

In this talk we considered compact Riemannian manifolds (M,g) of negative sectional curvature. Those manifolds can be written as $\tilde{M}/\Gamma$, where $\tilde{M}$ is diffeomorphic to $\mathbb{R}^n$ and Γ is the discrete group of covering transformation. The objective of this lecture was to study the asymptotic geometry of $\tilde{M}$. The main motivation for that is the following question: does the asymptotic geometry of $\tilde{M}$ determine the local geometry?

As an example we studied the counting function for lattice points on $\tilde{M}$ which is defined as follows. For two points $p,q \in \tilde{M}$ let

$$N_\Gamma(p,q,t) = \mathrm{card}\{\gamma \in \Gamma | d(p,\gamma(q)) \leq t\} .$$

If $h(g)$ is the exponential volume growth of g,

$$\lim_{t\to\infty} \frac{N_\Gamma(p,q,t)}{e^{h(g)t}} = c(p,q) .$$

In dimension 2 we obtained: $c(p,q)$ is constant if and only if the surface has constant negative curvature.

U. Simon (joint work with N. Bokan, P. Gilkey): Applications of spectral geometry to affine and projective geometry

Let M be a connected, oriented C^∞-manifold, $m = \dim M$, g a Riemannian metric and ∇ a torsionsfree, Ricci-symmetric affine connection. Consider the second order p.d. operator of Laplace type for closed M

$$D := -\mathrm{trace}_g \left(\mathrm{Hess}_\nabla + \frac{1}{n-1}\mathrm{Ric}(\nabla)\right) .$$

We investigate the asymptotic expansion of the Dirichlet series for $t \downarrow 0$

$$\sum_{\nu\geq 1} \exp(-t\lambda_\nu)\mu_\nu \quad \sim \quad \sum_{k=0}^{\infty} a_n(D)t^{(k-m)-\frac{1}{2}} .$$

Theorem: Let M be closed, $m \geq 2$. Then

1. $a_{2j+1}(D) = 0$.

2. $a_k(D)$ is locally computable.

3. $a_k(D)$ is independent of projective changes of ∇.

4. $a_m(D)$ is independent of conformal changes of g.

Corollary: $a_m(D)$ is conformally and projectively invariant.

Applications in affine and Euclidean hypersurface theory (Pick functional, Willmore functional etc.).

W. Kühnel:
Ovaloids with second fundamental form of constant curvature: The non-rigidity of the sphere

We call two ovaloids $M, M^* \subseteq \mathbb{E}^{n+1}$ I-isometric or II-isometric if I=I* or II=II*, respectively (up to a diffeomorphism of class C^1). It is well known that two I-isometric ovaloids are congruent to each other. A theorem of R. Schneider 1972 says that an ovaloid of class C^4 with constant inner curvature of II is congruent to the standard sphere:
The sphere S^n is II-rigid in the class C^4.
Based on results by Erard (Thesis ETH Zürich 1968), we show the following:
The sphere S^n is not II-rigid in the class C^2.
More precisely: There exists a 1-parameter family $M_t \subseteq \mathbb{E}^{n+1}$ of ovaloids of class C^2, piecewise analytic, such that the following holds:

- M_0 ist the unit sphere $S^n(1)$,
- M_t, M_0 are II-isometric for all t,
- M_t is not congruent to the unit sphere if $t \neq 0$.

This family sweeps out the whole space between the unit sphere and the circumscribed unit cylinder.

K. Polthier:
A priori estimates for hyperbolic mininal surfases

We presented a program to construct minimal surfaces in hyperbolic space with ends. The main results consider four topics.

1. By using geometric arguments we derive a priori gradient estimates for hyperbolic minimal surfaces.

2. These are used to prove an existence theorem for boundary contours lying in part in $\mathbb{H}^3$ and in part in the asymptotic sphere S^∞.

3. A new C^1 comparison theorem for planar hyperbolic curves is derived with which the conjugate surface construction for minimal surfaces can be controlled.

4. We applied the previous results to prove existence of new non-periodic, 1- and 2-periodic complete embedded minimal surfaces.

G. Huisken:
Mean curvature evolution of surfaces with prescribed volume

Let $F_0 : M^n \hookrightarrow N^{n+1}$ be a smooth embedding of a closed surface in a Riemannian manifold N^{n+1}. We evolve the surface according to the equation

$$\frac{d}{dt} F = (H - h)\nu \, ,$$

where H is the mean curvature of the surface, $h = h(t) = \int H \, d\mu / \int d\mu$ the average of the mean curvature and ν is the unit normal. This flow decreases the area of the evolving surfaces, but maintains a constant enclosed volume.

It is shown that for sufficiently convex initial data a solution exists for all time and converges to a constant mean curvature surface. General properties of this gradient flow for the isoperimetric problem and uniqueness of the limit surface are also discussed.

Berichterstatter: A. Hornecker

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