MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 49/1993

Algorithmische Methoden der Diskreten Mathematik

31.10. bis 6.11.1993

Die Tagung fand unter der Leitung der Professoren Martin Golumbic (Ramat Gan, Israel), Rolf Möhring (Technische Universität Berlin) und Raimund Seidel (University of California at Berkeley) statt.

An der Tagung nahmen 39 Wissenschaftler aus 8 Ländern teil (Deutschland, Frankreich, Israel, Japan, Canada, Niederlande, Österreich, Vereinigte Staaten), die in 37 Vorträgen über neueste Forschungsergebnisse berichteten und in einer Abendsitzung ungelöste aktuelle Probleme vorstellten. Die Tagung hatte sich bewußt zum Ziel gesetzt, algroithmisch orientierte Wissenschaftler aus verschiedenen Disziplinen, insbesondere der Graphentheorie, der Kombinatorik und der Geometrie zusammenzubringen und mögliche Querverbindungen aufzuzeigen und zu diskutieren.

Diese Zielsetzung spiegelte sich wieder in den Vortragsthemen der Teilnehmer, die das Gebiet in seiner ganzen Bandbreite abdeckten: von "reinen", innermathematischen Fragestellungen über mathematische Fragestellungen, die durch anwendungsorientierte Probleme aufgeworfen wurden, bis hin zu Anwendungen der Mathematik in Industrieprojekten; von originär diskreten Fragestellungen bis hin zur Anwendung diskreter Techniken in Logik und Geometrie; von rein strukturellen Einsichten über das Wechselspiel von Struktur und Algorithmus bis hin zu „Hochleistungslösern" für diskrete Probleme.

In einer Reihe von Vorträgen wurden überraschende und aussichtsreiche neue Anwendungsmöglichkeiten der algorithmischen diskreten Mathematik angesprochen, so etwa in der systematischen Paläontologie, der künstlichen Intelligenz und im computergestützten Design. Neue Querverbindungen ergaben sich unter anderem zwischen strukturellen Resultaten und Fragen der effizienten Lösbarkeit, zwischen linearen und diskreten Modellen sowie zwischen Computergeometrie und kombinatorischer Optimierung.

Die zahlreichen Diskussionen zwischen Wissenschaftler verschiedener Ausrichtungen zeigten, daß die Tagung für alle Teilnehmer neue Einsichten und Erfahrungen mit sich brachte und sicher ein Gewinn war. Das zeigte sich auch an den regen Diskussionen und der Anbahnung von Zusammenarbeit, gerade zwischen den algorithmischen Graphentheoretikern und den algorithmischen Geometern.

Wie immer ermöglichte es die bewährte Konzeption des Hauses, in den Pausen oder am Abend die in den Vorträgen und in den Diskussionen angesprochenen Themen zu vertiefen und sich näher kennenzulernen. Dies war angesichts der Tatsache, daß verschiedene Richtungen und sehr viele "Erstbesucher" von Oberwolfach teilgenommen haben, sehr fruchtbar.

Die Veranstalter und Teilnehmer danken dem Foschungsinstitut Oberwolfach für die hervorragende Betreuung während der Tagung.

Vortragsauszüge

Avi Berman

Completely positive matrices, graphs and ranks

An $n \times n$ matrix A is *completely positive* if it can be decomposed as $A = BB^T$, where B is an $n \times k$ nonnegative matrix. The smallest k for which such a decomposition exists is called the *cp-rank* of A.

Clearly a completely positive matrix must be *doubly nonnegative*: positive semidefinite and elementwise nonnegative. We say that *a graph G is completely positive* if every doubly nonnegative matrix realization of G is completely positive. We show that G is completely positive iff it does not contain an odd cycle of length greater than 4 (i.e. G is line-perfect) and survey some results on completely positive matrices and cp-ranks.

Marshall Bern (with Scott Mitchell, Sandia Labs, and Jim Ruppert)

Non-obtuse triangulation

Given a polygon P (with holes), we would like to triangulate P, adding extra vertices interior or on the boundary as necessary, using only triangles with all angles less than 90 degrees. This problem arises in finite element methods.

In this talk I present an algorithm for this problem that uses only $O(n)$ extra vertices, improving a previous result of $O(n^2)$. The new algorithm crucially relies on a lovely geometric fact: any four circles, tangent in a cycle, have co-circular points of tangency.

Ralf Borndörfer (with Martin Grötschel, Fridolin Klostermeier, and Christian Küttner)

An application of Set Partitioning in handicapped people's transport

Telebus is a dial–a–ride transport system for handicapped people in Berlin. Given the handicapped people's requests for a particular day, the problem of scheduling and routing the special busses used in the system "in the best possible way" arises. This problem can be modelled as a *Set Partitioning problem.* In addition to different kinds of heuristic procedures this model allows a polyhedral approach. Facet defining inequalities namely of Set Packing Polyhedra can be used in a branch & cut algorithm for the solution of the problem.

In this talk we present handicapped people's transport in Berlin and the Telebus-system. We describe a Set Partitioning Model of the associated optimization problem. We discuss both our heuristics and the branch & cut algorithm and give computational results for them. These indicate that substantial improvements to both quality of transport and reduction of costs are possible.

Endre Boros (with Peter Hammer)

How to find the optimal values of some of the variables in a SAT or MAX–SAT Problem

Weak and *strong persistency* are considered, i.e. properties that ensure a given partial 0–1 assignment can be extended to an (optimal) solution of a MAX–SAT or SAT problem. A polytope is associated to any SAT or MAX–SAT problem and is shown to have half–integral vertices. Furthermore, the non–fractional components of any of the vertices of this polytope are shown to have the weak persistency (generalizing a result about vertex packing by Nemhauser and Trotter, 1975). If applied to a MAX–2–SAT problem together with a network–flow based proof–dual computation (Hammer, Hansen, and Simeone, 1984) this technique yields a 3/4–approximation for MAX–2–SAT.

Rainer E. Burkard (with V. Deineko)

Traveling salesman problems in permuted Monge arrays

In this joint work with V. Deineko (Dnepropetrovsk) we address the following problem: An $(n \times n)$ matrix $A = (a_{ij})$ is called a *Monge matrix*, if $a_{ij} + a_{kl} \leq a_{il} + a_{kj}$ for all $1 \leq i < k \leq n$, $1 \leq j < l \leq n$. Considering A as distance matrix of a TSP an optimal tour can be found in the class of pyramidal tours of the form $<1, i_1, i_2, \ldots, i_r, n, j_1, j_2, \ldots, j_s>$ with $1 < i_1 < i_2 < \cdots < i_r < n$ and $n > j_1 > j_2 > \cdots > j_s$. The optimal pyramidal tour can be determined by dynamic programming. Pank ('91) gave a recursion which uses the search in Monge arrays and finds in $O(n)$ time the optimal pyramidal tour.

A matrix A is called a *permuted Monge matrix*, if $(a_{i\tau(i)})$ is a Monge matrix. The problem of finding a shortest Hamiltonian tour in a permuted Monge matrix is $\mathcal{NP}$-hard (Sarvanov, '80). If τ, however, has a special form, the corresponding TSP can be solved polynomially. Let $\tau_1, \tau_2, \ldots, \tau_m$ be the subcycles of τ. We define the graph G_τ in the following way: The vertices of G_τ correspond to the subcycles $\tau_1, \tau_2, \ldots, \tau_m$. There is an edge between τ_i and τ_j, if city k lies in τ_i and city $k+1$ lies in τ_j. Gaikov showed that the problem can be solved in $O(nm)$ time, if G_τ is a multipath and in $O(n^7)$ time, if G_τ is a multitree. We reduce the complexity for the multipath case to $O(n)$. Deineko ('93) showed recently that also the multitree case can be solved in $O(n^2)$ time.

Derek Corneil (with Stephan Olariu and Lorna Stewart)

On the linear structure of graphs

A graph is AT-free if it contains no Asteroidal Triple (an independent triple $\{x, y, z\}$ where between any pair of vertices there exists a path that avoids the neighbourhood of the third vertex). AT-free graphs contain many well-known families of graphs such as co-comparability, trapezoid, interval, permutation and cographs. All of these families have a certain "linear structure" and the thesis of this work is that this linearity is captured by the property of being AT-free.

To support this thesis, we demonstrate various characterizations and properties of AT-free graphs. These include a forbidden subgraph characterization, a characterization with respect to dominating pairs (a pair of vertices such that every path between them dominates the graph) and a way of constructing new AT-free graphs from two given ones.

These structural results lead to new algorithmic results. For example, we present a simple linear time algorithm for the problem of finding a dominating path in an AT-free graph. No such algorithm was known for the subfamilies of co-comparability or trapezoid.

Bruno Courcelle

Construction of graph algorithms by algebraic and logical methods

Many graph problems expressible in monadic second-order logic (including $\mathcal{NP}$-complete ones) are solvable in linear time on graphs of bounded tree-width.

Algorithms can be obtained in two ways:

1. by inductive computations on decomposition trees,
2. by construction of reduction rules that can be implemented in linear time.

The first technique extends to functions on graphs defined in monadic second-order logic.

Bernd Gärtner (with Emo Welzl)

A new approach to characterizing arrangements

An arrangement of oriented pseudohyperplanes in Euclidean d-space defines on the set X of pseudohyperplanes a set system (or range space) (X, R), $R \subseteq 2^X$ of VC-dimension d in a natural way: to every cell c in the arrangement assign the subset of pseudohyperplanes having c on their positive side, and let R be the collection of all these subsets. We investigate and characterize the range spaces corresponding to *simple* arrangements of pseudohyperplanes in this way; such range spaces are called *pseudogeometric*, and they have the property that the cardinality of R is maximum for the given VC-dimension. In general, such range spaces are called *complete*, and we show that the number of ranges $r \in R$ for which also $X - r \in R$, determines whether a complete range space is pseudogeometric. Two other characterizations go via a simple duality concept and 'small' subspaces. The correspondence to arrangements is obtained indirectly via a new characterization of uniform oriented matroids: a range space (X, R) naturally corresponds to a uniform oriented matroid of rank $|X| - d$ if and only if its VC-dimension is d, $r \in R$ implies $X - r \in R$ and $|R|$ is maximum under these conditions.

Martin C. Golumbic (with Ron Shamir)

Complexity and algorithms for reasoning about time: a graph-theoretic approach

Temporal events are regarded here as intervals on a time line. This paper deals with problems in reasoning about such intervals when the precise topological relationship between them is unknown or only partially specified. This work unifies notions of interval algebras in artificial intelligence with those of interval orders and interval graphs in combinatorics.

The satisfiability, minimal labeling, all solutions, and all realizations problems are considered for temporal (interval) data. Several versions are investigated by restricting the possible interval relationships yielding different complexity results. We show that even when the temporal data comprises of subsets of relations based on intersection and precedence only, the satisfiability question is NP-complete. On the positive side, we give efficient algorithms for several restrictions of the problem. In the process, the interval graph sandwich problem is introduced, and is shown to be NP-complete. This problem is also important in molecular biology, where it arises in physical mapping of DNA material.

Michel Habib (with Alain Cournier)

Another linear substitution decomposition algorithm

Substitution decomposition has many applications and appears in many fields under various names such as: modular decomposition, X-join decomposition ..., and can be defined for many discrete structures such as: graphs, directed graphs, hypergraphs, ... We present an algorithm for undirected graphs. Prime graphs are those which have no non-trivial decomposition.

Following some nice idea or conjecture of Schmerl, it is possible to find in any prime graph G (with more than 3 vertices) a sequence of prime graphs $G_0 \subseteq G_1 \ldots \subseteq G$ such that: G_0 is a P_4 (path of length 3), from G_i to G_{i+1} one may add one or two vertices.

Our algorithm searches for such a sequence of prime subgraphs.

To obtain a linear time algorithm we merge this idea with the cograph recognition algorithm (Corneil, Perl and Stewart 1985) and avoid the neighbour of a vertex to be visited more than three times.

Another algorithm due to R. McConnell and J. Spinrad (93) runs also in linear time, but seems to be more complicated.

Peter Hammer (with Alexander Kogan)

Horn functions: structure and minimization

A Boolean function f is called *Horn* if $f(xy) \leq f(x) \vee f(y)$ for any $x, y \in B_2^n$; a Horn function is called *definite Horn* if $f(1) = 0$. The *definite Horn component* $\delta(f)$ of a Horn function f is defined as a definite Horn minorant of f, such that

(i) there exists a monotone nondecreasing function p with $f = \delta(f) \vee p$ (⋆)

(ii) if $\delta' \leq \delta$ has the above properties, then $\delta' \equiv \delta$.

It is shown that $\delta(f)$ exists and is unique, and that in every prime irredundant decomposition (⋆) the disjunctive normal forms of all the functions p have the same number of prime implicants. On this basis it is shown that finding a DNF of a Horn function, having a minimum number of literals, is $\mathcal{NP}$-complete. A polynomial time heuristic provides a solution within a guaranteed range from the optimum. A quadratic time algorithm is given for logic minimization of the special class of acyclic Horn functions — a class of problems playing a major role in expert systems/artificial intelligence.

Pavol Hell (with J. Huang, X. Deng, J. Bang-Jensen, and B. Bhattacharya.)

Local tournaments and proper circular arc graphs

Characterizing a class of undirected graphs by the existence of a suitable orientation can be useful for the design of efficient recognition and optimization algorithms for that class of graphs. I illustrated this for the class of proper circular arc graphs, obtaining O(m+n) algorithms for their recognition (and representation by circular arcs), finding a maximum clique, etc. [When the graphs are represented already, and the endpoints of the arcs sorted, the optimization algorithms are O(n).] These algorithms rely on a structural characterization of the relevant orientations, due to J. Huang. I also described a recent orientation characterization of interval graphs.

Robert E. Jamison

Antimatroids and Disjunctive Products

Antimatroids are closure systems which satisfy an änti-exchangelaw. They model any situation in which the relevant structure is given by "shellings" — e. g., assembly or disassembly of machinery.

The disjunctive product, introduced by Strahringer and Wille, for relational structures, was the first product of closures which preserved the antimatroid property.

The disjunctive product of lines provides a notion of "betweenness" for ordinal data in several variables. This notion of betweenness leads to a new and rather strange notion of convexity on euclidean space. The embedding of graphs into this structure represents an attempt to capture the 1-skeleta of polytopes in this convexity. I will discuss several Steinitz type theorems about the kinds of graphs which can occur as polyhedral 1-skeleta. I will also indicate the relevance of disjunctive products to certain optimization problems over disassembly antimatroids.

David Kirkpatrick

Computing fixed-points of the composition of monotone functions, with goemetric applications

Many problems involving convex polygons, such as funding the separation of a pair of polygons or finding the Voronoi vertex associated with a triple of polygons, have straightforward solutions using rested binary search (on the lists of polygon vertices). We describe a unifying framework for the development and explanation of optimal algorithms for these and related problems. This framework involves the computation of a fixed point of the composition of one or more monotone continuous functions that are piecewise simple (where simple implies that the composition of two or more simple functions has a fixed point that can be computed in $O(1)$ time). Some cases can be shown to require $\Theta(\log^2 n)$ time (where n describes the number of pieces in the description of the participating functions), and others have $\Theta(\log n)$ solutions using conventional prune-and-search.

Ton Kloks (with H. Bodlaender, H. Müller and D. Kratsch)

Separators in graphs

Given nonadjacent vertices a and b. An ***a, b-separator*** is a set S of vertices such that if S is removed from the graph, a and b are left in distinct connected components. The a, b-separator is called *minimal*, if no proper subset of S is also an a, b-separator.

A subset S is called a *minimal separator* if there exist nonadjacent vertices a and b such that S is a minimal a, b-separator.

In the first part of my talk I illustrate how to use minimal separators to obtain polynomial algorithms to compute the treewidth and pathwidth of permutation graphs, and the treewidth of circle graphs.

In the second part I describe an algorithm which computes all minimal separators of a graph. The algorithm can be implemented to run in time polynomial times the number of minimal separators.

Dieter Kratsch

Rankings of graphs

A vertex (edge) coloring $c : V \to \{1, 2 \ldots, k\}$ $(c : E \to \{1, 2, \ldots, k\})$ of a graph $G = (V, E)$ is a vertex (edge) *k-ranking* if for any two vertices (edges) of the same color every path between them contains a vertex (edge) of larger color. The *vertex ranking number* $\chi_r(G)$ (*edge ranking number* $\chi'_r(G)$) is the smallest value of k such that G has a vertex (edge) k-ranking.

We show how to use a minimal separator approach for designing vertex ranking algorithms on interval graphs $[O(n^3)]$, circular arc graphs $[O(n^3)]$, permutation graphs $[O(n^6)]$, trapezoid graphs $[O(n^6)]$, circular permutation graphs $[O(n^6)]$, and cocomparability graphs of dimension at most d $[O(n^{3d})]$. We show that the problem 'Given a graph $G = (V, E)$ and an integer k, is $\chi_r(G) \leq k$?' is NP-complete, even when restricted to cobipartite or bipartite graphs. On the other hand, this problem is linear time solvable for every fixed k. We characterize those graphs where the vertex ranking number χ_r and the chromatic number χ coincide on all induced subgraphs, and show that $\chi_r = \chi(G)$ implies that $\chi(G)$ is equal to the largest size of a clique in G.

Renu Laskar

Cyclic gossiping in graphs

Let G be a finite, connected, undirected graph. Gossiping is the total exchange of information between vertices in the graph. Gossiping in G occurs when the information of each vertex has been sent to all other vertices of G. The minimum time required to complete gossiping in a graph will be called the gossiping time of the graph. The color classes of a proper edge-coloring define matchings that can be used to gossip in G. Cyclic gossiping is gossiping that occurs by using those matchings in a cyclic manner.

Some theoretical and algorithmic aspects of cyclic gossiping will be discussed for certain classes of graphs.

Ulrich Lauther

Scheduling and routing of cargo trains

Management of railroad trains includes the subtasks of net management (emphasis on capacity considerations and routing), line management (emphasis on scheduling) and node management (assignment and routing of incoming trains to platforms and tracks). All these tasks come in on-line and off-line versions and are traditionally handled manually and separately.

Modeling the topological and temporal restrictions by an interval graph leads to simple algorithms for integrated net- and line management which are optimal for *one* train and can be used to build efficient heuristics for multi-train situations. Comparison to human-generated solutions is very encourageing. A prototype system for scheduling and routing of cargo trains (no predefined time-table) has been implemented and was demonstrated during the talk.

Ross McConnell

Substitution decomposition and transitive orientation

A module of an undirected graph is a set X of nodes such that for each node x not in X, either every element of X is adjacent to x or no element of X is adjacent to x.

There is a canonical linear-space representation of the modules of a graph, called *substitution*, or *modular, decomposition.* The substitution decomposition facilitates solution of a number of combinatorial problems on certain classes of graphs.

We give a linear-time algorithm for substitution decomposition, and a new bound of $O(n + m \log n)$ on transitive orientation of comparability graphs and recognition of permutation graphs. Previous bounds were $O(n + m\alpha(m, n))$ for substitution decomposition, and $O(n^2)$ for transitive orientation and permutation–graph recognition.

Kurt Mehlhorn (with Volker Priebe)

All-pair shortest path in expected time $O(n^2 \log n)$

We show how to solve the all-pair shortest path problem in expected time $O(n^2 \log n)$. The time bound holds for a wide class of probability distributions on graphs. We also show that $\Omega(n \log n)$ edges have to be inspected on average to verify a single shortest path tree. This suggests that the upper bound is optimal.

Rolf H. Möhring

Triangulating graphs without asteroidal triples

A *triangulation* of a graph G is a chordal graph H on the same vertex set that contains G as a subgraph, i.e., $V(G) = V(H)$ and $E(G) \subseteq E(H)$. An *asteroidal triple* is a triple x, y, z of independent vertices such that, between any two of them, there exists a path that avoids the neighbourhood of the third. We show that every $\subseteq$-minimal triangulation of a graph G without asteroidal triples is already an *interval graph*. This implies that the *treewidth* of a graph G without asteroidal triples equals its *pathwidth*. Another consequence is that the minimum number of additional edges of a triangulation of G *(fill-in)* equals the minimum number of edges needed to embed G into an interval graph *(interval completion number)*. The proof is based on the interesting property that a minimal cover of all induced cycles of a graph G without asteroidal triples by chords does not introduce new asteroidal triples. These results complement recent results by Corneil et al. about the linear structure of graphs without asteroidal triples.

Rudolf Müller

On polyhedral descriptions of partially ordered sets

A *poset polyhedron* is defined as the convex hull of incidence vectors of those subsets of the arcs of a digraph that induce a partial ordering on the nodes of the digraph. Poset polyhedra have applications as basis for linear programming models of many NP-hard combinatorial optimization problems, e.g., *minimum interval graph completion* and the *pathwidth* of a graph. Our aim is to describe classes of valid inequalities of such polyhedra that have a polynomial time separation algorithm.

For the general case, where the partial ordering may be any partial ordering, we introduce *odd-cycle inequalities*. The separation problem for these inequalities can be solved by solving a shortest path problem in an auxiliary graph. We then restrict the posets to *interval orders* and to *linear orders*. Using a characterization of interval orders by forbidden **2+2** we obtain a generalization of odd-cycle inequalities, called *odd-2+2-cycle inequalities*. It turns out that several well-known classes of valid inequalities of the linear ordering polytope are equivalent to odd-**2+2**-cycle inequalities, and thus can be separated in polynomial time as well.

Takao Nishizeki (with Jun–ya Takahashi and Hitoshi Suzuki)

Finding shortest non-crossing rectilinear Paths in plane regions

Let A be a plane region which is bounded by an outer rectangle and an inner one and has r rectangular obstacles inside the region. Let k terminal pairs lie on the outer and inner rectangular boundaries. This paper presents an efficient algorithm which finds k "non-crossing" rectilinear paths in the plane region A, each connecting a terminal pair without passing through any obstacles, whose total length is minimum. Non-crossing paths may share common points or line segments but do not cross each other in the plane. The algorithm runs in time $O(n \log n)$ where $n = r + k$.

William R. Pulleyblank (with Michael Jünger and Sándor Fekete)

Bounds for geometric optimization problems and avoidance of coallitions

For some geometric optimization problems (*min cost perfect matching, minimum spanning tree* etc.) there are natural dual problems whose optimum value provides (usually good) bounds on the value of an optimum solution to the original problem. We describe a *bubble-blowing* variant of the min-cost-spanning-tree algorithm and discuss what must be done to extend this bound to the minimum Steiner tree problem in the plane.

We also show that the following problem is polynomial: Given r points in the plane, assign costs W_r to the points so that ΣW_r is maximized, but for every proper subset S of the points, the solution to the travelling salesman problem is at most $\Sigma (W_r : v \in S)$.

Edgar A. Ramos

Equipartition of mass distributions by hyperplanes

The *equipartition problem* is the following: Characterize the triples (d, k, j) such that for any j mass distributions in R^d, there are k hyperplanes so that each orthant contains a fraction $1/2^k$ of each of the masses. The case (2,2,1) is very well known. The case $k = 1$ is answered by the *ham sandwich cut theorem* with the condition $d \geq j(2^k - 1)/k$. We believe that this is also sufficient. However, the only general (any k and j) sufficient condition we know is $d \geq j2^{k-1}$ (which is somewhat trivial). We are able to prove that $d \geq j(2^k - 1)/k$ is a sufficient condition for a few values of j and k; for example, $k = 2$, $j = 2^n$ (any integer $n \geq 0$) and $k = 3$, $j = 2$.

As an intermediate result we prove a Borsuk-Ulam type theorem on a product of balls (and, hence, spheres). The following is a particular case of the theorem (in which all the balls in the product are 1-dimensional): Let $f = (f_1, \ldots, f_n) : (B^1)^n \to R^n$, where $B^1 = [-1, 1]$, be such that for each i and j, either

(1) for all $(x_1, \ldots, x_{j-1}, 1, x_{j+1}, \ldots, x_n)$, $f_i(x_1, \ldots, 1, \ldots, x_n) = f_i(x_1, \ldots, -1, \ldots, x_n)$, or

(2) for all $(x_1, \ldots, x_{j-1}, 1, x_{j+1}, \ldots, x_n)$, $f_i(x_1, \ldots, 1, \ldots, x_n) = -f_i(x_1, \ldots, -1, \ldots, x_n)$.

The antipodality matrix $A(f)$ of f is an $n \times n$ matrix where $[A(f)]_{ij}$ is 0 if case (1) applies and 1 if case (2) applies. The parity Q_f of f is defined as $Q_f = (\text{permanent}A(f)) \bmod 2$. The theorem asserts that if $Q_f = 1$ then f has a zero. This is a generalization of the *intermediate value theorem*.

The motivation for this work was to resolve the case (4,4,1) (the only case open for $k = d$ and $j = 1$). Unfortunately the approach fails to give an answer in this case.

Monika Rauch

Fully dynamic graph algorithms and their data structures

A dynamic graph algorithm allows modifications of a graph by inserting and deleting edges and isolated vertices. In my talk I will present three general techniques that are used in dynamic graph algorithms. To illustrate each technique I will give a solution for dynamic connectivity in general graphs in time $O(\sqrt{n})$ per operation, for dynamic 2-edge connectivity in plane graphs in time $O(\log^2 n)$ per operation, and for dynamic 2-vertex connectivity in general graphs in time $O(m^{2/3})$ per operation, where n is the number of vertices and m is the number of edges in the graph. These are the best known algorithms for these problems.

Bruce Reed

Hamiltonian paths in interval graphs

We give an $O(n\alpha(n))$ algorithm for finding a Hamiltonian path within an interval graph which is presented as a sorted list of endpoints. We discuss a related UNION-FIND problem which may or may not be solvable in linear time.

Alexander Schrijver

Applications of cohomology to disjoint paths and time-tabling

We describe two applications of the following problem: Let $D = (V, A)$ be a directed graph and let G be a group. Let $\varphi : A \to G$ and $H : A \to 2^G$. Find a function $\psi : A \to G$ such that ψ is cohomologian to φ and $\psi(a) \in H(a)$ $\forall a \in A$. There is a polynomial-time algorithm for this problem if G is a free partially commutative group and $H(a)$ is hereditary for each $a \in A$. As application we show that for each fixed k there is a polynomial-time algorithm for the k–disjoint paths problem for directed planar graphs. We give some other applications. Moreover, we give some applications of the cohomology principle to finding the time-table for the Dutch Railways.

Otfried Schwarzkopf (with Mark de Berg and Katrin Dobrindt)

Lazy randomized incremental construction

We introduce a new type of randomized incremental algorithms, suited for computing structures that have a 'non-local' definition. We generalize traditional results on random sampling to analyze these "lazy" randomized incremental algorithms.

We apply our scheme to obtain new and efficient algorithms for the computation of single cells in arrangements of segments in the plane or triangles in three-space, and zones in arrangements of hyperplanes.

Ron Shamir (with M. Golumbic and H. Kaplan)

Sandwich and pathwidth problems in graphs

Let $G^1 = (V, E^1)$ and let $G^2 = (V, E^2)$ be a supergraph of G^1, i. e. $E^2 \supseteq E^1$. The sandwich problem for property Π is to decide if there exists a "sandwich" graph $G = (V, E)$ such that $E^1 \subseteq E \subseteq E^2$ and G satisfies the desired property Π. We investigate the complexity of the problem for various properties Π. In particular, the problem is $\mathcal{NP}$-hard for interval, unit interval, comparability and permutation graph, but easy for Π being a cograph, a threshold graph and a split graph.

Motivated by computational biology we study the sandwich problem with the property Π being a unit-interval graph with clique size at most k. We prove it is polynomial for fixed k, but the parametric problem is $W[1]$-hard. In the process, we define a new graph-theoretic parameter which we call "proper pathwidth", and prove a somewhat surprising equivalence to bandwidth.

Micha Sharir

Recent results on arrangements of surfaces in higher dimensions

Let Σ be a collection of n algebraic surfaces (or surface patches) in $\mathbf{R}^d$, of constant maximum degree. The arrangement $A(\Sigma)$ of Σ is the decomposition of $\mathbf{R}^d$ into maximal connected cells of various dimensions, induced by the surfaces of Σ (each cell is contained in the intersection of a fixed subset of the surfaces). We study the combinatorial complexity of various substructures of such an arrangement. We show:

1) The complexity of the *lower envelope* of the surfaces of Σ (i.e., pointwise minimum of the surfaces, each viewed as the graph of some (partial) function $x_d = f(x_1, \ldots, x_{d-1})$), is $O(n^{d-1+\varepsilon})$, where the constant of proportionality depends on ε, d, and the maximum degree of the surfaces, for any $\varepsilon > 0$. This is almost tight in the worst case and solves a 1-year-old open problem.

2) The complexity of a *single cell* of $A(\Sigma)$, in $d = 3$ dimensions, is $O(n^{2+\varepsilon})$, for any $\varepsilon > 0$. Again, this also solves a major open problem, is almost tight in the worst case, and has applications to robot motion planning.

3) The complexity of the union of k convex prolyhedra in $\mathbf{R}^3$, having a total of n faces, is $O(k^3 + kn\log^2 k)$, and in the worst case can be $\Omega(k^3 + kn\alpha(k))$.

4) In the previous problem, if the given polyhedra are *Minkowski sums* of the form $A_i \oplus B, i = 1, \ldots, k$, where $A_1, \ldots, A_k$ are pairwise-disjoint convex polyhedra and B is another convex polyhedron, then the complexity of their union is $O(kn\log^2 k)$, and $\Omega(kn\alpha(k))$ in the worst case.

5) We also develop algorithms for computing some of the above structures, and present many applications of these results to motion planning, visibility, Voronoi diagrams, and more.

Jach Snoeyink (with Michael McAllister and David Kirkpatrick)

A compact piecewise-linear Voronoi diagram for convex sites in the plane

In the plane, the *post-office problem*, which asks for the closest site to a query point, and *retraction motion planning*, which asks for a one-dimensional retract of the free space of a robot, are both classically solved by computing a Voronoi diagram. When the sites are k disjoint convex sets, we give a compact representation of the Voronoi diagram, using $O(k)$ line segments, that is sufficient for logarithmic time post-office location queries and motion planning. If these sets are polygons with n total vertices given in standard representations, we compute this diagram optimally in $O(k \log n)$ deterministic time for the Euclidean metric and in $O(k \log n \log m)$ deterministic time for the convex distance function defined by a convex m-gon.

Jerry Spinrad

Optimization and representation

This talk deals with optimization problems on classes of graphs which have special representations, usually but not always intersection graphs. Most work in this area uses the representation in order to solve the optimization problem. There is an implicit assumption that if the graph is given in standard adjacency list form, a first step will be finding the representation.

We will look at examples for which the optimization problem can be solved more efficiently than the recognition or representation problems, and new issues which arise in such cases. A number of open problems dealing with the interplay of optimization and representation will be discussed. For example, can you find a problem on a natural representation of a class of graphs which is $\mathcal{NP}$-complete if the representation is not given, but becomes polynomial if the representation is given.

Éva Tardos (with Bruce Hoppe)

Flow algorithms for dynamic graphs

A *dynamic network* is defined by a graph, so with a *capacity* $u(vw)$ and an integral *transit time* $\tau(vw)$ on every edge vw. If a unit of flow is entered on edge vw at time τ it arrives to w at time $\tau + \tau(vw)$, and the edge can be used to send $u(vw)$ units of flow pipelined at every unit of time.

We consider the evacuation problem no dynamic networks, in which each source s is given a flow amount v_s that needs to reach the sink in time T. We give the first polynomial time algorithm for the *multisource* evacuation problem.

Single source dynamic flow problems were considered by Ford and Fulkerson, and can be solved by a simple min–cost flow computation. All dynamic flow problems are simple flow problems on an exponentially large "time expanded graph". Most of the difficulties in solving dynamic flow problems arise from the size of this graph. E.g. in doing flow augmentations we need to do exponential number of them all at once.

Dorothea Wagner (with H. Ripphausen-Lipa and K. Weihe)

A linear–time algorithm for the vertex-disjoint Menger problem in planar graphs

We consider the problem of finding a maximum set of internally vertex-disjoint paths between two vertices s and t in planar graphs. Usually, this problem is solved by flow methods, which ends up with a running time of $O(n \cdot \sqrt{n})$ resp. $O(n \cdot k)$ where k is the number of paths. The best previously known algorithm has time complexity $O(n \log n)$ [Suzuki, Akama, Nishizeki 91] and is based on divide–and–conquer. In this talk a new algorithm is presented that has only linear running time. It is based on "right–first–search".

Karsten Weihe (with Matthias Müller-Hannemann)

Using network flows for nicer surface approximations

Input of our problem is the surface of a body in the three-dimensional space, represented by a "tiling" with spherical triangles and quadrangles. The problem is to refine the tiling appropriately so as to make a numerical analysis possible. This is one of the crucial tasks in the computer aided design of bodyworks of cars, trains, planes, machines, etc.

Surprisingly, one may disregard all geometrical and topological properties and yields, thus, a purely combinatorial problem. We show that this problem has a rich network structure, give a heuristic approach based on network flow techniques, and discuss its (encouraging) results. We also show how to incorporate the intuitive optimization criterion stated by the engineers: The result should look as "nice" as possible.

In cooperation with the *Dr. Krause Software GmbH, Berlin.*

Emo Welzl (with B. Chazelle, J. Matousek and L. Wernisch)

Geometric graphs with small stabbing numbers

A *geometric graph* is a graph with a straight line embedding. (vertices $\mapsto$ points in the plane, edges $\mapsto$ line segments connecting their endpoints.) The stabbing number of a geometric graph is the maximal number of edges intersected by a line. Every set of n points allows a spanning path with stabbing number $O(\sqrt{n})$ (which is tight). We review the proof of this result and indicate applications to range searching and (combinatorial) discrepancy. The result follows from a theorem on set systems of finite *VC–dimension*.

Ungelöste Probleme

Marshall Bern

Given a plane graph G (i.e., an embedded planar graph) we would like to draw its planar dual G' in a compatible way. Within each face of G (except the exterior face) we must choose a location for the vertex of G' so that each edge of G' may be drawn as a straight line segment that crosses only the edge to which it is dual. (Let's say we are allowed to place dual vertices even on the boundary of the face, and "crosses" means intersects at relative interiors.)

Deciding whether such locations exist is $\mathcal{NP}$-hard for G with convex faces with at most five sides. It's linear-time solvable for G with convex faces with at most four sides. (M. Bern and J. R. Gilbert, Drawing the planar dual, Inf. Proc. Lett. 43 (1992) 7–13.)

The question is: what if G may include non-convex four-sided faces? (We know that the problem is not always solvable, but we don't know an algorithm to decide.) Also open are variations such as: what if G includes only a bounded number of faces with more than four sides?

Derek Corneil

Fast recognition of AT–free graphs.

There is a straightforward AT–free graph recognition algorithm that achieves time $\mathcal{O}(n^3)$. Can one get an algorithm in time of boolean matrix multiplication or better? (That's the current best time for recognition of an independent triple.)

Bernd Gärtner

An *Abstract Optimization Problem* (AOP) is a triple $(H, <, \Phi)$ where H is a finite set, $<$ a total order on 2^H and Φ an oracle that, for given $F \subseteq G \subseteq H$, either reports that $F = \min_<\{F' \subseteq G\}$ or returns a set $F' \subseteq G$ with $F' < F$. To solve the problem means to find the minimum set in H. There is a randomized algorithm that solves any AOP with an expected number of at most $e^{2\sqrt{n}+O(\sqrt[4]{n}\ln n)}$ oracle calls, $n = |H|$. In contrast, any deterministic algorithm needs to make $2^n - 1$ oracle calls in the worst case.

Question: can one find non-trivial lower or improved upper bounds for the problem in the randomized setting? Better upper bounds would imply a new combinatorial bound for Linear Programming and related problems.

Martin C. Golumbic

In [1] we defined a *strong matching* to be an induced subgraph of an undirected graph whose connected components are disjoint edges. Let $\beta^*(G)$ denote the maximum number of edges in a strong matching in G. We are able to calculate β^* and find a strong matching for circular arc graphs and for chordal graphs in polynomial time [1]. In general, this problem would be $\mathcal{NP}$–complete for arbitrary graphs.

1. Are other results known about strong matchings?
2. What other families of graphs admit polynomial time solutions?

[1] Martin C. Golumbic and Renu C. Laskar: *Irredundancy in circular arc graphs.* Discrete Applied Mathematics 44 (1993), 79–89.

<u>Ton Kloks</u>

A graph is *dominotype* if each vertex is contained in at most two maximal cliques. Recognition of these graphs can be done in linear time. We know that pathwidth of these graphs is $\mathcal{NP}$–complete, even when restricted to chordal graphs which are dominotype.

Question: What about treewidth?

<u>Rolf Möhring</u>

When does pathwidth equal treewidth?

Characterize (or establish classes of) graphs G for which *pathwidth* (G) = *treewidth* (G). Known such classes are graphs without asteroidal triples (i.e., without three independent vertices such that, between any two of them, there exists a path that avoids the neighborhood of the third), and grid graphs $G_{m,n}$. $\left[G_{2,3} = \right.$  $\left. \right]$

<u>William R. Pulleyblank</u>

Three tree traversal problems.

Let $T = (V, E)$ be a tree rooted at a node r. We are interested in three tree traversal problems, all of which require a tour which starts at r and visits the other nodes of the tree by travelling along tree edges. In the unweighted problem, each edge is assumed to have unit length. In the weighted version, each edge has a weight which is a nonnegative integer.

The *distance* from r to a node v is the sum of the lengths of the edges traversed until the first time node v is reached.

Problem I *A team of Bruces departs the root late at night, trying to find a node at which the proprietor is willing to sell a case of Banks. By tradition, this person is normally called "Rocky". For each node v they know* a priori *the probability p_v of being successful at that node. Their tour ends when either they have made their purchase or else all nodes have been visited. In what order should they visit the nodes so as to minimize the expected length of their tour?* □

This is a specialisation of a problem told to me by Robert Cahn of IBM Research. He was interested in solving the problem for more general graphs and distance functions.

Problem II *For this problem we have no node probabilities. What order of traversal of T minimizes the average distance to every node v? Equivalently, what order of traversal minimizes the sum of the distances to all nodes v?*

This problem was told to me by Steve Hedetniemi at Clemson University. Robert Reynolds, a student of his, had obtained a "somewhat long" solution to the unweighted case. Reynolds showed that a traversal was optimum if and only if it was a depth-first traversal of the tree. Can you find a short proof of this?

Problem III *How do you solve the weighted case of the above?*

Remarks. Considering a path with four edges, having costs 1000, 1, 1, 1000 and the root in the center, you can see that depth-first traversal is not optimum. Beth Novick has produced a solution to Problem II and conjectures Problem III to be $\mathcal{NP}$-hard (see ahead). Bruce Reed (as well as others not at the workshop) noted that for the case that T is a path, Problems I and III can be solved using dynamic programming. The key observation is that at each stage, the set of visited nodes will consist of a subpath containing r and we can assume we are at one of the two ends. The number of such subpaths is $\mathcal{O}(|V|^2)$ and each one can be evaluated by considering at most $\mathcal{O}(|V|)$ alternatives.

Bruce Reed also showed that for Problem I he could obtain in polynomial time a feasible solution whose value was no worse than eight times the optimum.

For Problem I, with weights, the case of a star seems particularly interesting, and is unsolved, to the best of my knowledge.

Ron Shamir

Define a mixed graph to be a *partial interval order (PIO)* if one can complete the orientation, i.e. sets a direction on each of the undirected edges, to form an interval order.

Given a complete mixed graph, can one delete any subset of the directed edges only (keeping all undirected ones) and obtain a PIO?

The problem is currently open. An $\mathcal{NP}$-completeness proof will resolve four open problems in the paper by M.C. Golumbic and R. Shamir, "Complexity and algorithms for reasoning about time: a graph-theoretic approach" (to appear in JACM).

Jerry Spinrad

Decomposition with forbidden subgraphs.

For any bipartite graph $F = (X, Y, E)$, we can define a decomposition which asks whether vertices of an arbitrary graph G can be partitioned into V_1, V_2 such that

1. $|V_1|, |V_2| \geq \min\{|X|, |Y|\}$,
2. the graph formed by taking only edges between V_1 and V_2 contains no induced copy of F.

Either show that for every F, one can find the decomposition in polynomial time, or show that it is $\mathcal{NP}$-complete for some F.

Emo Welzl

Visibility in permutations.

Let π be a permutation of $\{1,\ldots,n\}$, $\mathcal{B} \subseteq \{1,\ldots,n\}$. We call i, $1 \leq i \leq n$, *$\mathcal{B}$-visible* in π, if no element preceding i in π is an element of $\mathcal{B}$. For example, if $\mathcal{B} = \{2,4\}$, then 1 and 3 are the $\mathcal{B}$-visible elements in 1 3 2 4 5.

Prove or disprove: For every set $\pi_1,\ldots,\pi_n$ of permutations of $\{1,\ldots,n\}$, and for k, $1 \leq k \leq n$, there is a set $\mathcal{B} \subseteq \{1,\ldots,n\}$ with $|\mathcal{B}| = \mathcal{O}(n/k)$ such that no i, $1 \leq i \leq n$ is $\mathcal{B}$-visible in more than k of the permutations. (I can prove that $|\mathcal{B}| = \mathcal{O}\left(\frac{n \log k}{k}\right)$ suffices.)

Berichterstatter: Karsten Weihe

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Prof.Dr. Pankaj Agarwal
Department of Computer Science
Duke University
202 North Building

Durham , NC 27706
USA

Prof.Dr. Rainer E. Burkard
Institut für Mathematik B
Technische Universität
Kopernikusgasse 24

A-8010 Graz

Prof.Dr. Avi Berman
Department of Mathematics
Technion
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof.Dr. Derek G. Corneil
Dept. of Computer Science
University of Toronto

Toronto Ontario M5S IA4
CANADA

Dr. Marshall Bern
XEROX PARK
3333 Coyote Hill Road

Palo Alto , CA 94304
USA

Prof.Dr. Bruno Courcelle
Mathématiques et Informatique
Université de Bordeaux I
351, cours de la Libération

F-33405 Talence Cedex

Ralf Borndörfer
Konrad-Zuse-Zentrum für
Informationstechnik Berlin
- ZIB -
Heilbronner Str. 10

D-10711 Berlin-Wilmersdorf

Bernd Gärtner
Institut für Informatik (WE 3)
Freie Universität Berlin
Takustr. 9

D-14195 Berlin

Prof.Dr. Endre Boros
RUTCOR, Centre of Operations
Research, Mathematical Sciences
Rutgers University
P.O. Box 5062

New Brunswick , NJ 08903-5062
USA

Prof.Dr. Martin Golumbic
Scientific Center
IBM Israel Ltd.
Technion City

Haifa 32000
ISRAEL

Prof.Dr. Michel Habib
CRIM
860 Avenue de Saint-Priest

F-34100 Montpellier

Ton Kloks
Dept. of Mathematics and
Computing Science
Technical University Eindhoven
Postbus 513

NL-5600 MB Eindhoven

Prof.Dr. Peter L. Hammer
RUTCOR, Centre of Operations
Research, Mathematical Sciences
Rutgers University
P.O. Box 5062

New Brunswick , NJ 08903-5062
USA

Dr. Dieter Kratsch
IRISA
Campus de Beaulieu

F-35042 Rennes Cedex

Prof.Dr. Pavol Hell
School of Computing Science
Simon Fraser University

Burnaby , B.C. V5A 1S6
CANADA

Prof.Dr. Renu Laskar
Dept. of Mathematics
Clemson University
Martin Hall

Clemson , SC 29634-1907
USA

Prof.Dr. Robert E. Jamison
Department of Mathematics
White Hall
Cornell University

Ithaca , NY 14853
USA

Dr. Ulrich Lauther
SIEMENS AG
ZTI DES V 223
Otto-Hahn-Ring 6

D-81739 München

Prof.Dr. David G. Kirkpatrick
Department of Computer Sciences
University of British Columbia

Vancouver , B.C. V6T 1Z2
CANADA

Prof.Dr. Ross McConnell
Department of Computer Science
Campus Box 430
University of Colorado at Boulder

Boulder , CO 80309-0430
USA

Prof.Dr. Kurt Mehlhorn
Max-Planck-Institut für Informatik
Geb. 44
Im Stadtwald

D-66123 Saarbrücken

Prof.Dr. William R. Pulleyblank
IBM Corporation
Thomas J. Watson Research Center
P. O. Box 218

Yorktown Heights , NY 10598
USA

Prof.Dr. Rolf Hermann Möhring
Fachbereich Mathematik - FB 3
Technische Universität Berlin
MA 6-1
Straße des 17. Juni 136

D-10623 Berlin

Prof.Dr. Edgar Ramos
Department of Computer Science
University of Illinois at
Urbana-Champaign
1304 W. Springfield Av.

Urbana , IL 61801-2987
USA

Rudolf Müller
Fachbereich Mathematik - FB 3
Technische Universität Berlin
MA 6-1
Straße des 17. Juni 136

D-10623 Berlin

Dr. Monika Rauch
Department of Computer Science
Cornell University
5133 Upson Hall

Ithaca , NY 14853
USA

Prof. Dr. Ketan Mulmuley
Department of Mathematics and
Computer Science, University of
Chicago, Ryerson Hall
1100 East 58th St.

Chicago , IL 60637
USA

Dr. Bruce Reed
Dept. of Mathematics
Carnegie-Mellon University

Pittsburgh , PA 15213-3890
USA

Prof.Dr. Takao Nishizeki
Dept. of Information Communications
Faculty of Engineering
Tohoku University

Sendai 980
JAPAN

Prof.Dr. Alexander Schrijver
Centrum voor Wiskunde en
Informatica
Postbus 4079

NL-1009 AB Amsterdam

Dr. Otfried Schwarzkopf
Faculteit Wiskunde en Informatica
Rijksuniversiteit Utrecht
Postbus 80.089

NL-3508 TB Utrecht

Prof.Dr. Eva Tardos
Dept. of Computer Science
Eötvös University
Muzeum krt 6-8

H-1088 Budapest

Prof.Dr. Ron Shamir
Department of Computer Science
Tel Aviv University
Ramat Aviv

69978 Tel Aviv
ISRAEL

Dr. Dorothea Wagner
Fachbereich Mathematik, MA 6-1
Technische Universität Berlin
Straße des 17. Juni 136

D-10623 Berlin

Prof.Dr. Micha Sharir
School of Mathematical Sciences
Tel Aviv University

69978 Tel Aviv
ISRAEL

Karsten Weihe
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136

D-10623 Berlin

Dr. Jack Snoeyink
Department of Computer Sciences
University of British Columbia

Vancouver , B.C. V6T 1Z2
CANADA

Prof.Dr. Emo Welzl
Institut für Informatik (WE3)
Freie Universität Berlin
Arnimallee 2-6

D-14195 Berlin

Prof.Dr. Jeremy P. Spinrad
Department of Computer Science
Vanderbilt University
P.O. Box 6213 - B

Nashville , TN 37235
USA

E–Mail–Adressen

P. Agarwal	pankaj @ cs.duke.edu
A. Berman	MAR64AA @ technion.technion.ac.il
M. Bern	bern @ parc.xerox.com
R. Borndörfer	borndoerfer @ zib-berlin.de
E. Boros	boros @ rutcor.rutgers.edu
R. Burkard	burkard @ kop.tu-graz.ada.at
D. Corneil	dgc @ cs.toronto.edu
B. Courcelle	courcell @ labri.u-bordeaux.fr
B. Gärtner	gaertner @ inf.fu-berlin.de
M. Golumbic	golumbic @ vm.biu.ac.il
M. Habib	habib @ lirmm.fr
P. Hammer	hammer @ rutcor.rutgers.edu
P. Hell	pavol @ cs.sfu.ca
R. Jamison	rejam @ math.cornell.edu
D. Kirkpatrick	kirk @ cs.ubc.ca
D. Kratsch	kratsch @ irisa.fr
T. Kloks	ton @ win.tue.nl
R. Laskar	rclsk @ clemson.clemson.edu
U. Lauther	lauther @ ztivax.zfe.siemens.de
R. McConnell	ross @ kalkan.cs.colorado.edu
K. Mehlhorn	mehlhorn @ mpi-sb.mpg.de
R. Möhring	moehring @ math.tu-berlin.de
R. Müller	mueller @ math.tu-berlin.de
K. Mulmuley	mulmuley @ cs.uchicago.edu
T. Nishizeki	nishi @ ecei.tohoku.ac.jp
W. Pulleyblank	wrp @ watson.ibm.com
E. Ramos	ramos @ cs.uiuc.edu
M. Rauch	mhr @ cs.cornell.edu
B. Reed	breed @ muff.cs.mcgill.ca
A. Schrijver	lex @ cwi.nl
R. Shamir	shamir @ math.tau.ac.il
M. Sharir	sharir @ math.tau.ac.il
J. Snoeyink	snoeyink @ cs.ubc.ca
J. Spinrad	spin @ vuse.vanderbilt.edu
O. Schwarzkopf	otfried @ cs.ruu.nl
É. Tardos	eva @ cs.cornell.edu
D. Wagner	wagner @ math.tu-berlin.de
K. Weihe	weihe @ math.tu-berlin.de
E. Welzl	emo @ tcs.fu-berlin.de