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Nonlinear Equations in Many-Particle Systems

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Organizers: Jürgen Batt (Munich)

Carlo Cercignani (Milano)

48 scientists from 13 countries participated in this conference, which covered the following three main topics in many-particle systems: the theory and applications of the Boltzmann equation, the nonlinear systems connected with the Vlasov equation (such as the Vlasov-Poisson and Vlasov-Maxwell systems), and the quantum mechanical counterparts of the above (such as the quantum Boltzmann equation, Wigner-Poisson and Schrödinger-Poisson systems). The 43 half-hour talks and the 2 20-minute-talks represented latest developments in the following particular research areas: Discrete velocity models and the hydrodynamic limit of the Boltzmann equation, existence theory (with particular emphasis laid on suitable boundary conditions), stability, numerical analysis and modelling, and the diffusion limit of kinetic equations. There were two lectures in the new area of the Vlasov-Einstein equations.

The conference was lively and reflected the vivid activity of research being carried out in important mathematical aspects of many-particle systems.

The following abstracts are given in chronological order of the talks given.

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Abstracts

H. CABANNES:

Review of exact solutions in discrete kinetic theory

For the simplest discrete model of the Boltzmann equation, the Broadwell model,

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=\frac{\partial v}{\partial t}-\frac{\partial v}{\partial x}=-2\frac{\partial w}{\partial t}=uv-w^2$$

an exact solution has been found by Cornille in 1986:

$$\begin{aligned} u(t,x) &= \alpha+\gamma\frac{\sinh(2t)+\sigma\sin(2\sigma x)}{\cosh(2t)+\cos(2\sigma x)} \quad v(t,x)=u(t,-x) \\ w(t,x) &= \gamma+\alpha\frac{\sinh(2t)}{\cosh(2t)+\cos(2\sigma x)}. \end{aligned}$$

After Cornille we have extended the results 1) to models with 14 velocities with binary collisions, 2) to some models with triple collisions and 3), last year, to the following two-dimensional semi-continuous model

$$\frac{\partial N}{\partial t}+c\cos\theta\frac{\partial N}{\partial x}=\frac{c}{2\pi}\int_0^{2\pi}[N(\varphi)N(\varphi+\pi)-N(\theta)N(\theta+\pi)]\,d\varphi.$$

M. SLEMROD:

Self similar hydrodynamic limits

We will consider the resolution of the hydrodynamic limit problem for the Broadwell system. The hydrodynamic limit is obtained for the system when the Knudsen number ε is replaced by εt and $\varepsilon\to 0$.

R. ILLNER:

The asymptotic behaviour of one-dimensional discrete velocity models in a slab

We prove results on the asymptotic behaviour of solutions to discrete velocity models of the Boltzmann equation in the slab $0<x<1$ with general stochastic boundary conditions. Assuming that there is a constant "wall" Maxwellian $M=(M_i)$ compatible with the boundary conditions, it follows under physically reasonable assumptions on the models that

$$\lim_{t\to\infty}\sum_i\int_0^1|f_i(x,t)-M_i|\,dx=0.$$

Under weaker assumptions on the model, a weaker type of convergence can be proved. We also notice that in the absence of collision terms (but with the same boundary conditions), renewal theory is applicable to prove similar convergence results.

G. GIROUX:

Idealized interacting particle systems leading to some nonlinear equations akin to the Boltzmann equation; LLN and CLT

I will present results about idealized models of many-particle systems. In such a framework, recent results about tightness of sequences of probability measures lead to functional Law of Large Numbers and Central Limit Theorems. In other words, the evolution of a particle, in a many-particle system, is well approximated by the evolution of a fictitious particle; this fictitious particle obeys a nonlinear equation akin to the Boltzmann equation. Hopefully, some links will be presented with more realistic models.

M. PULVIRENTI:

Boltzmann-Grad limit for one-dimensional particle systems

We consider N particles on the line with velocity $v=\pm1,\pm2$ and suppose that they collide (each pair) with probability ε whenever they met. In the limit $N\to\infty$, $\varepsilon\to0$, $N\varepsilon\to\lambda$ we prove that the empirical distribution

$$\frac{1}{N}\sum_{j=1}^{N}\delta(x-x_j)\delta(v-v_j)$$

converges weakly to $f=f(x,v,t)$, where f solves the one-dimensional Boltzmann equation

$$(\partial_t+v\partial_x)f(t,x,v)=3\lambda(f(t,x,v')f(t,x,v_1')-f(t,x,v)f(t,x,v_1))$$

with $|v-v_1|=3$; v', v_1' are the post-collisional velocities and x_j, v_j are distributed, initially, independently according to the distribution $f_0(x,v)=f(x,v,0)$. Ref. : S. Caprino, M. Pulvirenti: A cluster expansion approach to a one-dimensional Boltzmann equation, preprint 1993

G. RUSSO:
Kinetic theory for bubbly flow
A kinetic theory for incompressible dilute bubbly flow is presented. The Hamiltonian formulation for a collection of bubbles is outlined. A Vlasov equation is derived for the one-particle distribution function with a self-consistent field starting from the Liouville equation for the N-particle distribution function and using the point-bubble approximation. A stability condition which depends on the variance of the bubbles momenta and the void fraction is derived. If the variance is small then the linearized initial value problem is ill-posed. If it is sufficiently large, then the initial value problem is well-posed and a phenomenon similar to the Landau damping is observed. Numerical simulations of the Vlasov equation in 1-D are performed using a particle method. Some evidence of clustering is observed for initial data with small variance in momentum.
The Vlasov equation for bubbly flow is modified to account for local interactions between bubbles. Fluid equations are deduced in the limit where local interactions cause the system to become locally Maxwellian. The resulting fluid equations are well-posed for sufficiently large temperature. The computed void wave speeds are found to be in agreement with experiments. In the limit when the temperature is zero, fluid equations previously derived by other investigators are recovered. In this limit there are solutions of the equations that blow up in finite time.

M. R. FEIX:

Self similarity and rescaling

Equations invariant under similarity transformations (a subset of Lie groups) can be partially integrated. For ODE the order (and consequently the phase space dimension) is lowered by one. For PDE, the number of independent variables is decreased by one, leading in evolution problems, to an absorbtion and formal elimination of time; but this simplification has a heavy price: only special initial conditions (I. C.) can be treated with no way to precise the status of these I. C. and what happens for other I. C. . Rescaling provides an answer. Time varying scales are introduced for dependant and independant variables and a new time $\hat{t} = f(t)$ is introduced. The scales and $f(t)$ are selected on physical grounds in order to allow an easy guess on the asymptotic solution in the new rescaled space. For problems underlying particle (or fluid) motion we introduce: (a) a rescaled physical force (b) a transformation potential (c) a friction. Often this friction drives the system to a stationary state providing a natural, asymptotic elimination of $\hat{t}$. In many cases, the stationary equation is identical to the one obtained by the self similar technique pointing out their asymptotic, attracting character. Examples will be provided for Vlasov beam evolution and nonlinear heat diffusion.

R. ESPOSITO:

Stationary solutions of the Boltzmann equation in a slab in the hydrodynamic limit

The limit of low Knudsen numbers is discussed in the case of the stationary Boltzmann equation in a slab with Maxwell boundary conditions (complete accomodation). The two sides of the slab are at different temperatures and may translate, with velocities parallel to the planes. An external constant force, proportional to the Knudsen number and parallel to the slab, is also considered. The scattering cross section is assumed to be the one corresponding to hard spheres. If the force, the relative velocities of the planes and the difference of temperatures are small enough, it is proven that there is a solution to the stationary boundary value problem for the Boltzmann equation. Moreover the limit of vanishing Knudsen numbers does exist and is given by a local Maxwellian with parameters satisfying the stationary compressible Navier-Stokes boundary value problem with no slip conditions. The result is consequence of the proof of the validity of a truncated bulk & boundary layer expansion. In fact the L_∞-norm of the remainder of the expansion is extimated in terms of a suitable power of the Knudsen number. The result shows that the Navier-Stokes corrections are crucial for establishing a stationary solution. The details are in two joint works with J. L. Lebowitz and R. Marra, one in press, [*Commun. Math. Phys.* (1993)], and the other in preparation.

M. KUNZE:
Plasma corners and a nonlinear integral equation
The equations

$$\begin{aligned} \Delta\psi + J = 0 \quad &\text{in} \quad D_p\,, \\ \Delta\hat{\psi} = 0 \quad &\text{in} \quad D_v\,, \\ \psi = \hat{\psi} = 0 \quad &\text{on} \quad \partial D_p\,, \\ \frac{\partial\psi}{\partial n} = \frac{\partial\hat{\psi}}{\partial n} \quad &\text{on} \quad \partial D_p\,, \\ \hat{\psi} \to \hat{\psi}_{\text{ext}}\,, \quad &|(t,y)| \to \infty\,, \end{aligned} \tag{1}$$

are thought of to describe the free-boundary problem of a plasma-vacuum interface in plane geometry with constant current profile $J \geq 0$. Here $\psi = \psi(t,y)$ and $\hat{\psi} = \hat{\psi}(t,y)$ denote the flux functions in D_p and D_v, the domain occupied by plasma and by vacuum, respectively, and $\hat{\psi}_{\text{ext}}$ is a prescribed external field. The flux functions ψ and $\hat{\psi}$ are assumed to be symmetric to the t-axis and to have a stagnation point (a "plasma corner") at the origin. By a suitable ansatz for ψ and $\hat{\psi}$ by means of Green's function for Δ in $\mathbb{R}^2$, the free-boundary problem (1) can be reduced to solving a certain nonlinear integral equation of non-standard-type

$$x^2(t) - t^2 + \frac{J}{4\pi}\int_0^1 k(t,s,x(t),x(s))\,ds = 0$$

for the function $x = x(t), t \in [0,1]$, describing the upper boundary of the plasma cut off at $t = 1$.

L. TRIOLO:

A generalisation of the H-theorem in kinetic theory; non-uniform steady states

Consider the Boltzmann equation in a bounded domain $\Omega \subset \mathbb{R}^3$ with stochastic boundary conditions, and the associated stationary problem.
Let f be the solution to the time dependent problem and $\bar{f}$ the steady solution (with the same mass) to the stationary problem. Define the generalized relative entropy functional

$$\underline{\Psi}[f|\bar{f}] := \int\int_{\Omega\times\mathbb{R}^3} dx\,dv\,\bar{f}\,\Psi(f/\bar{f})$$

where Ψ is a convex function. We find the decomposition

$$\frac{d}{dt}\underline{\Psi}[f|\bar{f}] = \sigma_{\mathrm{Bdry}} + \sigma_{\mathrm{Bulk}}$$

where $\sigma_{\mathrm{Bdry}} \leq 0$ for all Ψ and $\sigma_{\mathrm{Bulk}} \leq 0$ in the special case of a linear Boltzmann equation (for any Ψ) and, for the nonlinear case, in the special case of $\Psi(z) = h(z) := z \ln z$ and $\bar{f}$ is a global Maxwellian. These results may be interesting for the asymptotic (in time) analysis of the solutions.

J. BATT:

Stability for the Vlasov-Poisson system

The use of the word "stability" in papers on the Vlasov-Poisson system or related systems (such as the Vlasov-Maxwell system) is critically reviewed. In fact, in most of the work this concept is not even precisely defined. According to its use in the context of dynamical systems, a stationary solution f_0 of the Vlasov-Poisson system is defined to be nonlinearly (linearly) stable (with respect to a normed function space X) if a solution of the nonlinear (linearized) system exists in X for all $t \geq 0$ and stays close to f_0 if it stayed sufficiently close to f_0 at $t = 0$. In this sense rigorous results seem to have been proven only by J. Batt and G. Rein, [*Annali di Mat. pura ed applicata* (1993)], K. O. Kruse and G. Rein [*Archive of Rat. Mech. Anal.* (1992)] and G. Rein [*Math. Meth. in the Appl. Sci.* (1993)].

H. ZIEGLER:

Applications of multiconstrained energy minimization techniques to the modelling of relaxation by mixing in Vlasov-Poisson and Vlasov-Maxwell systems

Minimum energy states are calculated under constraints of incompressible flow in $(\vec{x},\vec{v})$ phase-space for Vlasov-Poisson systems. Reducing the information contained in the constraints in a way consistent with an averaging process by filamentation and averaging of f can be modelled. Starting with a given non-equilibium $f(\vec{x},\vec{v},t)$ of energy E_0 we can calculate a corresponding minimum energy state f^* where the phase space constraints are reduced to such an extend that the energy of f^* equals E_0. Using e. g. the assumption of strong and uniform filamentation of f for the reduction of the constraints f^* roughly corresponds to the week limit $\langle f(\vec{x},\vec{v},t\to\infty)\rangle$ for some cases, which is shown by performing numerical simulations of the actual dynamics.

A. SINITSYN:

Some families of solutions of the Vlasov-Maxwell system and their stability

We consider a collisionless plasma described by the Vlasov-Maxwell system in a bounded domain. The theorems on existence and uniqueness of solutions to the boundary-value problem for the nonlinear elliptic system are proved by techniques using lower and upper solutions. The cases are considered where the electromagnetic fields and the distribution functions can be constructed in an explicit form.

P. A. RAVIART:

A free boundary problem for the Vlasov-Poisson equations

We consider the problem of extracting an ion beam from a neutral plasma. A physically relevant model is obtained by assuming that the electrons behave as an isothermal fluid and the ions are cold and monokinetic. Using an appropriate scaling, the equations may be written as follows:

$$n_e = \exp\left(-\frac{\Phi}{\eta}\right)$$
$$\nabla\cdot(n_i u_i) = 0$$
$$\nabla\cdot(n_i u_i \otimes u_i) - \frac{\alpha}{2\eta} n_i \nabla\Phi = 0$$

$$\Delta\Phi = \frac{\eta}{\varepsilon^2}(n_i - n_e)\,.$$

In the above dimensionless equations, ε and η are small parameters and $\alpha > 0$ is a parameter of order o(1). In the one-dimensional case, this problem reduces to the nonlinear elliptic boundary value problem

$$\frac{d^2\Phi}{dn^2} = \frac{\eta}{\varepsilon^2}\Big[\big(1+\alpha\frac{\Phi}{\eta}\big)^{-1/2} - \exp\big(-\frac{\Phi}{\eta}\big)\Big], \quad n \in]0,1[$$
$$\Phi(0)=0, \quad \Phi(1)=1\,. \tag{1}$$

In the physically interesting case where $\varepsilon^2 = O(\eta^{3/2})$, we give an asymptotic analysis of (1) as $\eta \to 0$. In particular, we characterize the limit solution of (1) as the solution of a free boundary problem. We present a multidimensional extension of this free boundary problem which provides a convenient model for ion extraction which may be used in realistic numerical simulations. Ref. : N. Ben Abdallah, S. Mas-Gallic, P. A. Raviart: Analysis and asymptotics of a one-dimensional ion extraction, preprint 1993, and S. Mas-Galic, P. A. Raviart: Mathematical models of ion extraction and plasma discharge, preprint 1993.

G. REIN:

The spherically symmetric Vlasov-Einstein system

The Vlasov-Einstein system describes a large, self-gravitating, collisionless ensemble of mass points (e. g. a galaxy). We study the system in a spherically symmetric setting, where in appropriate coordinates it takes the following form:

$$\partial_t f + e^{\mu-\lambda}\frac{v}{\sqrt{1+v^2}}\cdot\partial_x f - \big(\frac{x\cdot v}{r}\partial_t\lambda + \sqrt{1+v^2}e^{\mu-\lambda}\partial_r\mu\big)\frac{x}{r}\cdot\partial_v f = 0$$
$$e^{-2\lambda}(2r\partial_r\lambda - 1) + 1 = 8\pi r^2\varrho, \quad e^{-2\lambda}(2r\partial_r\mu + 1) - 1 = 8\pi r^2 p$$
$$\varrho(t,x) = \int\sqrt{1+v^2}f(t,x,v)dv, \quad p(t,x) = \int\big(\frac{x\cdot v}{r}\big)^2 f(t,x,v)\frac{dv}{\sqrt{1+v^2}}\,.$$

where $t\in\mathbb{R}$, $x,v\in\mathbb{R}^3$, $r=|x|$. For small data we prove existence and uniqueness of global, geodesically complete, asymptotically flat solutions. This is the first global existence result for the Einstein equation coupled to matter, and is in sharp contrast to dust, where even for small data singularities develop which are naked, i. e. violate cosmic censorship. Besides that we have results on the localisation of possible singularities for large data, on the Newtonian limit, and on the existence of static solutions. (Joint work with A. D. Rendall and in part with J. Schaeffer)

A. D. RENDALL:

The Newtonian limit of the Vlasov-Einstein system

The Vlasov-Einstein system provides a fully relativistic description of self-gravitating collisionless matter. On physical grounds one expects that it would reduce to the Vlasov-Poisson system in the Newtonian limit where the speed of light c tends to infinity. I have shown that for appropriate families of initial data for the Vlasov-Einstein system depending on an parameter $\lambda = c^{-2}$ the corresponding family of solutions converges, locally in time, to a solution of the Vlasov-Poisson system. This limit is a singular one, where a hyperbolic equation degenerates to an elliptic one as $\lambda \to 0$. A central element of the proof is a λ-independent energy estimate.

J. J. DORNING:

Plasma wave solutions that bifurcate from Vlasov equilibria; periodic waves, solitary waves and nonlinear superposition

Recent results on exact solutions to the nonlinear Vlasov-Poisson-Ampère equations will be summarized. First, a brief review will be given of exact undamped periodic travelling wave solutions obtained using classical bifurcation theory in a not-so-conventional way, see Holloway/Dorning, [*Phys. Rev.* (1991)] and [*Operator Theory: Advances and Applications*, Birkhäuser 1991]. Then, a summary will be given of related periodic wave solutions, solitary wave solutions, and travelling double-layer solutions arrived at using simple techniques base on nonlinear dynamical systems and classical mechanics see Buchanan/Dorning, [*Phys. Lett. A* (1993)]. Like the original periodic wave solutions, all these solutions also bifurcate from the infinite manifold of spatially-uniform equilibrium solutions (Vlasov equilibria) of the V-P-A equations. Finally, details will be presented of very recent results, see Buchanan/Dorning [*Phys. Rev. Lett.* (1993)], on the use of Hamiltonian pertubation theory and approximate time-dependent invariants introduced to avoid singularities associated with nonlinear resonances and to accomodate stochastic layers, and thereby develop a nonlinear superposition principle for superimposed BGK plasma waves, see Bernstein/Green/Kruskal, [*Phys. Rev.* (1957)], observed as final states in recent numerical simulations, see Demeio/Zweifel, [*Phys. Fluids* (1990)], of nonlinear Landau damping.

P. J. MORRISON:

<u>A new energy expression for the linear Vlasov-Poisson equation: a transformation to action-angle variables in an infinite dimensional Hamiltonian system</u>

The energy content of an electrostatic pertubation about an homogeneous Vlasov-Poisson equilibrium was discussed. The calculation leading to the well-known dielectric energy was briefly revisited and interpreted in light of Vlasov theory. It was argued that the dielectric energy is deficient because resonant particles are not treated correctly in the derivation of this quantity. A new class of linear integral transforms was presented and important identies were derived [Morrison and Shadwick (1993)]. It was shown that an element of this class solves the linear Vlasov-Poisson system. This is a generalization of the method of Van Kampen (1955). The solution obtained by the integral transform was then substituted into the Kruskal-Oberman (1958) energy expression and a new expression in terms of the electric field alone was obtained [Morrison and Pfirsch (1992)]. It was described how the integral transform amounts to a change to normal or action-angle variables in the infinite dimensional Hamiltonian description of the Vlasov-Poisson system [Morrison (1980)]. <u>Ref. :</u> P. J. Morrison and B. Shadwick, [*Acta Physica Polonica (1993)*], to appear; N. G. Van Kampen, [*Physica* **21**, 949 (1955)]; M. Kruskal and C. Oberman, [*Physics of Fluids* **1**, 275 (1958)]; P. J. Morrison and D. Pfirsch, [*Physics of Fluids* **B4**, 3038 (1992)]; P. J. Morrison, [*Physics Letters* **80A**, 383 (1980)].

H. D. VICTORY, JR.:

<u>Particle methods for periodic Vlasov-Poisson systems</u>

We have extended and refined the theory obtained in [*SIAM J. Numer. Anal.* **26** (1989), pp. 249-288] for the Vlasov-Poisson Cauchy problem to include multidimensional periodic systems. This work directly generalizes the analysis by G. H. Cottet and P. A. Raviart [*SIAM J. Numer. Anal.* **21** (1984), pp. 52-76] and earlier work by H. Neunzert and J. Wick [*Numer. Math.* **21** (1973), pp. 234-243] on one-dimensional Vlasov-Poisson systems. We have also relaxed the requirement that the problem data possess compact support with respect to velocity. This is accomplished by the use of a weighted discrete L^p-Norm which involves discrete velocity moments of the errors in the Hamiltonian trajectories. Such an idea was motivated by the clever analysis by P. L. Lions and B. Perthame [*Invent. Math.* **105** (1991), 415-430] treating global existence of solutions to the three-dimensional Vlasov-Poisson systems. These authors have shown that control of certain velocity moments of the distribution for all time leads to the existence of global classical solution. Analogously, we can show that if we can control the weighted discrete L^p-errors in the particle trajectories with respect to time, then we can get globally uniform estimates for the errors.

J. WECKLER:

The Vlasov-Poisson system on a bounded domain

The initial-boundary value problem for the Vlasov-Poisson system in 3 dimensions is investigated. To get strong solutions of the linear Vlasov equation it is first shown that reflected characteristics do generically exist. Existence of strong solutions of the mollified Vlasov-Poisson system is proved. If the mollifying parameter tends to zero, a weak solution of the Vlasov-Poisson system is obtained as a weak limit of strong solutions of the mollified problem.

Y. GUO:

Singular solutions of the Vlasov-Maxwell system on half-line

We study the Vlasov-Maxwell system in a half line with initial-boundary conditions. It is shown that if the signs of charges of the particles are different, no classical solution exists unless a neutral plasma specularly reflects at the boundary. These weak solutions are generally functions of bounded variation. Therefore they are uniquely determined by the initial-boundary conditions. On the other hand, solutions are classical, if the particles have one sign of charge.

P. DEGOND:

Kinetic equations for plasmas and semiconductors

Plasmas and semiconductors are subject to various transport phenomena which do not occur in the dynamics of neutral fluids. Some of these phenomena can be explained by asymptotic pertubations in the basic kinetic equations, the Vlasov or the Boltzmann equation, depending on the physics involved. In this talk, we shall try to show how a mathematical analysis of these pertubation problems can provide relevant physical information.

H. LANGE:

Periodic Schrödinger-Poisson equations

We consider the periodic Schrödinger-Poisson system

$$\begin{aligned} i\partial_t\Psi_m &= -\tfrac{1}{2}\Delta\Psi_m + V(\Psi)\Psi_m, \quad x\in\mathbb{R}^3, t\in\mathbb{R} \\ -\Delta V &= n(x,t)-q(x); \quad \Psi_m(x,0)=\varphi_m(x) \end{aligned} \tag{1}$$

where the density $n(x,t)$ is given by

$$n(x,t)=\sum_{k=1}^{\infty}\lambda_k|\Psi_k(x,t)|^2$$

with occupation probabilities λ_k. (1) is considered with space-periodic boundary conditions on the unit cube $Q=[0,1]^3$. We can prove global (in time) existence of unique strong solutions and error estimates for the approximating Galerkin sequence of 1-periodic functions in $L^2(Q)$. Also we can prove that there is an infinity of stationary solutions (bound states) for (1) of type

$$\Psi(x,t)=e^{i\omega t}\Phi(x), \quad \omega\in\mathbb{R},$$

with a real function Φ. These results transfer to the Wigner-Poisson system by using the Wigner-Weyl transform.

A. ARNOLD:

Absorbing boundary conditions for the Wigner (-Poisson) equation

Recently C. Ringhofer et. al. derived a hierarchy of absorbing boundary conditions (ABC) for the Wigner equation (WE), a kinetic pseudo-differential

equation in quantum mechanics. These ABC relate the time derivative of the Wigner function on the inflow boundary to the outflow data.
In this talk, we first show the strong well-posedness of the resulting IBVP on a one-dimensional interval in the positon variable, when using 1st and 2nd order ABC. The proof relies on a contractive iteration for the inflow data. This approach is then extended to the relaxation-time WE and to a (simplified) first order ABC for 2 spacial dimensions.
When using the 1st order ABC in conjunction with the quasi-linear Wigner-Poisson system, only local-in-time existence and uniqueness of the resulting IBVP have been obtained so far. In this case, the lack of a (for the WE usual) L^2 a priori estimate complicates the analysis. Ref. : A. Arnold: On absorbing boundary conditions for quantum transport equations, submitted 1993.

F. POUPAUD:

Energy transport equations for the Maxwell system in periodic structures

The behaviour of weak limits of the electromagnetic fields in periodic structures is well-known when the typical length of periodicity tends to zero. However, the weak limit of the energy density is not the energy of the weak limit of the electromagnetic field. Using Bloch's decomposition of Maxwell's system and Wigner series we prove that the limit energy is given by the sum of the densities of distribution functions which solve transport equations. The velocities of these transport equations are given by the gradients with respect to k eigenvalues of Maxwell's system with k-quasiperiodic conditions.

P. A. MARKOWICH:

Boltzmann equations and drift-diffusion models for phonon scattering

This talk is concerned with recent results on the Boltzmann equation of solid state physics and its drift-diffusion limit, obtained jointly with C. Schmeiser and F. Poupaud.
The phonon scattering Boltzmann operator has an infinite dimensional kernel. A precise analysis of this kernel in combination with the entropy inequality are used to prove the convergence of a solution to a steady state.
Also, it can be shown that the scattering operator is compact in the angular direction but non-compact in the energy direction. This fact is used explic-

itly to diagonalize the collision operator in a spherical harmonics basis and to determine the drift-diffusion limit by taking the particle mean free path to zero. The main ingredient for the proof is a precise control of the range of the collision operator and a coercivity inequality. The resulting drift-diffusion equation is parabolic in the time-position space and degenerates to a hyperbolic equation in the time-energy direction.

C. TOEPFFER:

Analytical methods for the semiconductor Boltzmann equation

The manifold of solutions of the Boltzmann equation in the relaxation time approximations for particles under a constant external force is obtained in closed form. The Green's function for the infinite domain and all moments of the probability distribution are calculated explicitely. The second moment accounts for the Joule heating due to collision processes. In submicron semiconductor devices the mean free path of the electrons becomes comparable to the dimension of the device. One observes, in particular, a failure of the current-voltage (J-V) characteristic given by the conventional drift-diffusion equation assuming a velocity distribution according to the lattice temperature. Including a position-dependent Joule heating according to the local self-consistent second moment of the velocity distribution leads to an agreement with the J-V results of a numerical solution of the Boltzmann equation.

C. BARDOS, F. GOLSE:

Diffusive limits of kinetic equations

The purpose of this talk was to show an example of a kinetic equation generating a transformation group on the phase space that preserves the Liouville measure, but converges in the long-time/ small-scale limit, to the heat equation (which plays here no other role then that of an irreversible dynamics). The example is as follows: consider two parallel plates and consider the flow of particles between the plates with velocity parametrized by the 2-torus $a : \Pi^2 \to \mathbb{R}$. At each collision with one of the plates, the velocity is changed according to the rule $a(\omega) \to a(T\omega)$ where T is the hyperbolic automorphism of the torus known as Arnold's Cat Map

$$T=\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \bmod 2\pi\,.$$

Assume a is smooth and has mean zero, i. e. $\langle a\rangle=0$. Let $f_{\pm}(t,x,z,w)$ be the particle densities. Set

$$F_{\pm}^{h}(t,x,z,w)=f_{\pm}(t/h,x,hz,w)\,.$$

As $h\to 0$, $F_{\pm}^{h}$ converges to the solution of a heat equation in the weak-$*$-L^{∞} topology. The diffusion coefficient is

$$D(a)=\frac{1}{2}|a|_{L^2}^2+\sum_{k\geq 1}\langle a\circ T^k a\rangle\,.$$

H is >0 unless a is a coboundary, i. e.

$$\exists\varphi:\Pi^2\to\mathbb{R}\Big/(\varphi\circ T-\varphi=a)\,.$$

The proof is based on the mixing properties of the map T that are analyzed with the Fourier transform on Π^2.

S. UKAI:

Global weak solutions of the compressible Euler equation with spherical symmetry

There are two kinds of solutions known so far for the compressible Euler equation: 1) local classical solutions for any space dimension $n\geq 1$, and 2) global weak solutions for $n=1$.

1) Lax and Kato proved the local existence, applying the theory of quasilinear symmetric systems with a symmetrizer which gives solutions such that the density is bounded from below. We will show that there is another symmetrizer which allows the density to tend to zero as $|x|\to\infty$ or to have a compact support.

2) There are many works on this subject, using Glimm's method (Nishida, Smoller-Nishida, Liu, ...), or the method of compensated compactness

(Di Perna, Chen, Takeno, Lions-Perthame, ...). However, no global solutions have been known in the case $n \geq 2$. Applying Glimm's method with a non-uniform mesh, we will construct spherically symmetric weak global solutions for a class of initial data which includes trivial (i. e. constant) stationary solutions.

K. ASANO:

Fluid dynamical limit of the Boltzmann equation with an external force

We consider the Boltzmann equation with a small external force. We study the behaviour of the solution of the initial value problem when the mean free path tends to zero. If we assume the analyticity of the external potential force, the analyticity of the initial data in both space and velocity variables, and Grad's angular cut-off hard scattering potential condition, then the solution of the Boltzmann equation exists in a uniform time interval independent of the small mean free path, when the initial data are sufficiently close to an equilibrium state. As the mean free path tends to zero, the solution tends to a local Maxwellian with the macroscopic fluid dynamical quantities (mass, velocity and temperature), which solves the compressible Euler equation.

B. PERTHAME:

A N-particle system approximating scalar conservation laws

In this joint work with M. Pulvirenti, we prove the convergence of the N-particles density function associated with a random process. Particles move with free transport and, according to a Poisson stopping time with intensity λ, the velocity jumps to sample uniformly on $[0, \varrho^{\Delta}(x, X_n)]$ where x is the position of the particle at the jumping time. $\varrho^{\Delta}(x, X_n)$ is the density of particles in the same cell Δ than x, where the subdivision of the domain $x \in (0,1)^d$ in cells Δ is a priori given. As $N \to \infty$, $|\Delta| \to 0$, $N|\Delta| \to \infty$ we prove that the N-particle density function tensorized to the solution of kinetic equation. As $\lambda \to \infty$ additionally (and slowly enough) the N-particle density function tensorized to an equilibrium associated with the solution of a conservation law related to the dynamics of particles. This proves also the convergence of a Monte-Carlo procedure.

H. NEUNZERT:

Numerical matching of Boltzmann and Euler codes

In many fields of applications as spaceflight, semiconductors etc. kinetic equations are only necessary in rather small regions, while in rest limit equations (like Euler equations) are sufficient and much cheaper to solve numerically. A simple one-dimensional example may illustrate the problems: consider the slab $I=[-L,L]$ and an $x_0 \in [0,L[$. Imagine that we have to solve "aerodynamic equations" in $A=[-L,x_0]$ and a Boltzmann equation in $B=[-x_0,L]$. We need therefore influx conditions at $-x_0$ for B and boundary conditions at x_0 for A. They have to be chosen such that the overall picture is near to the "truth" (i. e. the Boltzmann solution) in $[-L,L]$. We call these conditions "coupling boundary". There are at x_0 3 choices: continuity of moments, Marshak conditions and jump conditions from a layer analysis. To realize these coupling conditions one has two possibilities: Schwarz iteration or direct coupling. A. Klor showed that the Schwartz iteration converges for linearized Boltzmann with linearized Euler and Marshak coupling; for a simple linear transport equation with diffusion and Marshak he proofs moreover, that the coupled solution converges (for mean free path ε tending to zero) to the solution of the $[-L,L]$ diffusion only if $x_0>0$, i. e. for matching. For palding $x_0=0$ he gives a counterexample. This shows that an additional condition is needed if one wants to pald. This additional condition seem to be the closure relation for the kinetic density at $x_0=0$, which was proposed together with the direct coupling in a paper by R. Illner and H. Neunzert.

E. PRESUTTI:

Non local, nonlinear equations arising from spin systems with Kac potentials

The equation

$$\frac{dm}{dt}=-m+\tanh\Big(\beta[T*m+h]\Big),$$

(where $m=m(v,t):\mathbb{R}^d\times\mathbb{R}_+\to[-1,1]$, $\beta>0$, $h\geq 0$ and $T\in C^2(\mathbb{R}^d)$ spherical symmetric and compactly supported as well as $\int T\,dv=1$, $T*m$ denotes the convolution of T and m) exhibits phenomena of phase separation, development of interface dynamics when $\beta>1$ and $h\geq 0$ is small enough. These results have been proven in several papers by different authors, namely P. Buttà, A. De Masi, T. Gobzon, M. A. Katsoulakis, E. Orlandi, E. Presutti, P. E. Souganidis and L. Triolo.

P. VAUTERIN, H. DEJONGHE:

Nonlinear solutions of the collisionless Boltzmann equation for stellar systems

We present a theory for bars in stellar disks, which we consider as pertubations of an axisymmetric basis distribution. Observations suggest that these pertubations are nonlinear. We construct them in the form of a series, based on a harmonic expansion of the angular coordinate and the time, power expansion of the radius and power expansion of the velocity coordinates. In the linear regime, this series can be determined recursively, and we took care in checking that this series is consistent with the direct numerical integration of the PDE. In the nonlinear regime, it is still possible to obtain the series, with comparably small extra effort. We use our method to assess the limitations of the linear approach by combining the power of the linear mode analysis and the extra information provided by nonlinearity.

G. TOSCANI:

Lyapunov fuctionals and the central limit theorem

In a recent series of papers, we investigated the convergence towards equilibrium of the solutiton to the Boltzmann equation for Maxwell pseudomolecules. The results were based on the monotonicity properties of different Lyapunov functionals, including Boltzmann's H-functional and Fisher's measure of information. Here we apply this physical idea to give a proof of the central limit theorem of probability theory. By means of new convex Lyapunov functionals, we prove convergence to Gaussian density in various norms. The results are easily extended to investigate convergence towards equilibrium for Kac's caricature of a Maxwell gas.

B. GUO:

Global smooth solutions and its asymptotic behaviour of the Cauchy problem for Benjamin-Ono type equations

The purpose of this work is to establish the solutions for the Cauchy problem for the general equation

$$u_t + 2uu_x + \alpha H u_{xx} - \beta H u_x + \gamma(x,t) H u + b(x,t) u_x + c(x,t) u = f(x,t) \qquad (1)$$

of Benjamin-Ono type, where $\alpha > 0$ and $\beta \geq 0$ are constants. This is a nonlinear partial differential equation with singular integral operators. The solutions of the problem for the above equation are approximated by the solutions of the Cauchy problem for the nonlinear parabolic equation

$$u_t - \varepsilon u_{xx} + 2uu_x + \alpha H u_{xx} - \beta H u_x + \gamma(x,t) H u + b(x,t) u_x + c(x,t) u = f(x,t)$$

with Hilbert transform terms, which is obtained by addition of a diffusion term εu_{xx} with small coefficient $\varepsilon > 0$ in the equation (1). The solutions of the Cauchy problem for the nonlinear equation (1) are obtained by the limiting process of the vanishing of the diffusion coefficient $\varepsilon \to 0$. In addition, we are going to consider the large-time behaviour of the global solutions of the Cauchy problem for the equation of Benjamin-Ono type. A series of large-time global estimates for the solutions of the problems for the nonlinear parabolic equations with Hilbert operators and the corresponding nonlinear equations of Benjamin-Ono type are constructed. By means of these global estimates, the attractors of the Cauchy problems for the mentioned nonlinear equations are considered. Furthermore, the dimensions of the global attractor are estimated.

C. CERCIGNANI:

Some new inequalities for the solutions of the Boltzmann equation

A few years ago, R. Di Perna and J. P. Lions provided an existence theorem (without uniqueness) for the Boltzmann equation in the case of rather general inhomogeneous data. Their solutions are rather weak and it is difficult to handle them for further developments. Thus it appears desirable to have stronger solutions even if these may be proved to exist under more restrictive conditions. A case which appears to be promising is that of solutions depending on just one space variable, say x. Here we want to prove that in this case an inequality holds, which guarantees that both the gain and the loss term in the collision integral are in L^1, under a truncation which amounts to an acceptable assumption on the collision kernel $B(\theta, |v - v_*|)$. As a corollary we shall prove that the solutions of Di Perna and Lions do not require normalization under the stated assumptions. We shall also discuss other interesting inequalities for the solution of the Boltzmann equation in both 1- and 3-d.

L. ARKERYD:

Diffusion reflection for the BGK equation (in memoriam Nina Maslova)

The talk discusses the BGK equation

$$(\partial_t + \xi \cdot \nabla_x) f = \mathcal{M}f - f, \quad x \in \Omega.$$

Here $\mathcal{M}f$ is the Maxwellian with the same first moment as f. The initial value is f_0 with

$$f_0(1+|\xi|)^\beta \in L^\infty(\Omega \times \mathbb{R}^3)$$

for some $\beta > 5$, and the boundary condition is (ingoing density)

$$\gamma^+ f(x,\xi) = \int_{\text{outgoing}} \theta^2 (2\pi)^{-2} \exp\left(-\frac{\theta}{2}|\xi|^2\right) |\xi' \cdot n(x)| \gamma^- f(x,\xi') d\xi'.$$

It is proved that if the initial value is such that the equation with no gain term has strictly positive velocity integral (a condition easily checked as described), then there is exactly one solution with

$$f_t(1+|\xi|)^\beta \in L^\infty(\Omega \times \mathbb{R}^3).$$

Moreover,

$$\|f\|_{L^\infty_\beta(\Omega\times\mathbf{R}^3\times(0,T))} + \|\gamma^- f\|_{L^\infty_\beta(\partial\Omega\times\mathbf{R}^3\times(0,T))} \le C(T).$$

The proof employs some new L^1 and L^∞ estimates of the boundary terms.

R. PETTERSSON:

On the solution of the linear Boltzmann equation (with general boundary conditions, external forces and infinite range collisions)

1) First mild L^1-solutions are constructed by iterates $f_{n+1} \ge f_n$ using the exponential form of the equation in the cut-off case.
2) On collision invariants: There are no other measurable functions than the Maxwellians satisfying detailed balance relation.
3) A general H-theorem for convex functions is proved (giving an estimate for the collision integral).
4) Strong L^p-convergence to a Maxwellian is proved for both hard and soft

potentials in the cut-off case.
5) In the case of infinite range forces the equation is studied in a weak form. Existence of solutions is proved (a. e.) by a compactness lemma, and then strong L^p-convergence to a unique Maxwellian function is proved.
Ref. : R. Petterson: On weak and strong convergence to equilibrium solutions to the linear Boltzmann equation, [*J. Stat. Phys.* **72** (1993), pp. 355-380]

M. L. EKIEL-JEZEWSKA :
Global existence proof for the relativistic Boltzmann equation
Consider the Cauchy problem for the Boltzmann equation with the relativistic kinematics of particles. A set of relativistic collisional invariants related to conservation laws does not contain an analogue of $(\vec{x}-\vec{v}t)^2$. This lack does not allow us to apply directly techniques developed by Di Perna and Lions for the nonrelativistic Boltzmann equation. However, we constructed a relativistic modification of their method, based on a causality proof for the existence of solutions. Finally, we proved global existence in L^1_{loc} and L^1 and also local-in-time existence for systems close to equilibrium. Physically natural (time-independent) a priori bounds in L^1 of the solutions are given.
Ref. : M. Dudynski, M. L. Ekiel-Jezewska, [*J. Stat. Phys.* **66** (1992)]

M. DUDYNSKI :
Existence of solutions to the quantum Boltzmann equation
Global existence of solutions to the quantum Boltzmann equation for fermions is proved for a wide class of physical cross-sections. The partial regularity of solutions is shown. Extensions of these results to relativistic quantum Boltzmann equations as well as to systems of equations describing many particles are also considered.

R. GATIGNOL :
Boundary conditions in discrete kinetic theory and applications
The discretization method of the velocities allow to replace the usual Boltzmann equation by a system of partial differential equations which is more tractable. By using simple models without spurious invariants, we specify the boundary conditions in two cases: first on an impermeable wall, and second

on an interface with the condensation and evaporation phenomena. As applications, we consider the Couette flows between two parallel infinite plates, and the evapuation and condensation problems between two interfaces. With some particular models, we observe the phenomenon of temperature inversion: the temperature is strictly increasing from the hot interface to the cold interface.

G. WOLANSKY:

Weak nonlinear stability of the Vlasov-Poisson equation

The mass distributions of a large class of stationary solutions of the Vlasov-Poisson equation can be considered as functions of the energy and, under certain symmetry assumptions, of the angular momentum as well. Because of the Hamiltonian structure of the dynamics, linear stability analysis of such stationary solutions is not sufficient to guarantee genuine stability. In this research the stability problem of stationary solutions of the VP equation with Newtonian interactions is approached by introducing Lyapunov functionals over the space of mass distributions on the phase space. These functionals admit non-quadratic leading order terms, an thus cannot be imbedded in a proper functional space as C^2-functionals. The study and the application of these functionals in order to obtain non-linear stability criteria is carried out by convexity arguments and dual formulation of the variational problem. Some a priori estimates on time-dependent solutions of the VP equation are obtained as well, using the same methods.

N. MAUSER:

(Semi-) classical limit of the quantum Vlasov equation

We present the Wigner-Poisson equation modelling quantum transport both for vacuum electrons and electrons in a periodical potential. Our main goal is a mathematically strict limit for vanishing Planck's constant which yields the classical Vlasov equation. For the crystal case this gives a strict derivation of the semiclassical equation as it is used for semiconductor modelling. Without any mathematical details we present the main features of this (weak) limit. One of the techniques is the use of a Husimi function, i. e. a smoothing of the Wigner function in order to get a nonnegative classical distribution function as the limit of the oscillatory real-valued quantum mechanical function.

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