MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 55/1993

General Principles of Discretization Methods, Theory and Applications

Dec 12 to Dec 18, 1993

The meeting was organized by Philip M. Anselone (Corvallis, Oregon) and Rainer Ansorge (Hamburg).

From the beginning of the 70's on, a general theory of discretization methods has been developed. The aim of this theory is to prove theorems for a very wide range of problems under minimal conditions, to generalize them and to present results from different areas of applications of dicretization methods in a general setting.

The intention of this meeting was to bring together specialists from this field as well as scientists who are working on applications of this theory and to exchange new ideas. A variety of subjects containing, e.g., integral and differential equations and eigenvalue problems, were discussed, mostly on an abstract and general level, but applications were also treated. Many interesting talks led to a lot of interaction between the participants of the meeting.

At the same time another conference, „Methoden und Verfahren der Mathematischen Physik“ organized by R.E. Kleinman (Newark), R. Kreß (Göttingen) and E. Martensen (Karlsruhe), took place at Oberwolfach. Since the contents of both meetings were rather close to each other, most of the people participated in parallel lectures as well.

The harmonic atmosphere in the Oberwolfach-Institute (which has often been described and is well known among mathematicians) was one of the reasons for the great success of the conference, especially for the numerous discussions and for private and scientific contacts.

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Abstracts

P.M. ANSELONE AND M.L. TREUDEN:

Spectral Analysis of Asymptotically Compact Strongly Convergent Operator Sequenced

Asymptotically compact operator sequences arise from the approximate solution of various integral equations. A sequence of bounded linear operators K_n on a Banach space X is asymptotically compact if, for any bounded sequence $\{X_n\}$ in X, every subsequence of $\{K_n X_n\}$ has a convergent subsequence. We write $K_n \xrightarrow{ac} K$ if $\{K_n\}$ is asymptotically compact and $K_n \longrightarrow K$ strongly. Then K is compact. We compare the spectra of K and K_n as $n \longrightarrow \infty$. The results closely parallel the more completely studied case with $\{K_n\}$ collectively compact. This is not surprising since, in all common spaces, $\{K_n\}$ asymp. compact implies that $K_n = L_n + T_n$ for some L_n and T_n with $\{L_n\}$ coll. compact and $\|T_n\| \longrightarrow 0$.

R. ANSORGE:

Iterated Discretization

During the past 17 years, the idea of an improvement of projection methods — used for the numerical treatment of integral equations — by implementation of an additional iteration step was developed and led to very satisfactory results. Particularly, superconvergence of these „iterated projection methods“ could be shown under certain conditions. In this talk, the idea is extended to more general discretization procedures (not only projection methods) and to more general classes of problems. So, the idea of a general „iterated discretization“ is presented, and also in this more general setting, sometimes superconvergence occurs.

CHR. T.H. BAKER:

Discretization of certain evolutionary problems

We discuss two aspects of the question „How robust are discretizations based on linear multistep and Runge–Kutta formulae?“ In the first we consider the application of formulae to nonlinear differential equations, point out that convergence cannot be uniform with restart to $y(0)$, that spurious steady states and chaos can arise and (significantly) that true steady states may be associates with the 'wrong' (non-convergent) branch of a multivalued flow until the step is sufficiently small. In the second we discuss stability of extensions of LMFs to delay and Volterra integro differential equations and the role of

$$c_1(\Lambda) \; = \; \inf_{|\mu|=1} |s(\Lambda;\mu)| \, , \qquad c_2(\Lambda) \; = \; \inf_{\lambda \in \partial S^{*}} |\Lambda - \lambda|$$

where $s(\Lambda;\mu) = \rho(\mu) - \Lambda\,\sigma(\mu)$ and $\lambda \in S^{\#} \;\Rightarrow\; \{s(\Lambda;\mu) = 0 \;\Rightarrow\; |\mu| < 1\}$

HERMANN BRUNNER:

Open problems in the discretization of Volterra integral equations

In this talk I shall discuss a number of open problems in the discretization of Volterra-type integral equations of the first and second kind. The discretization methods underlying these problems include certain classes of collocation methods as well as continuous (implicit and explicit) Volterra-Runge-Kutta methods. In addition, I shall touch upon Volterra equations with various types of delay arguments.

IVAN G. GRAHAM:

Parametrization methods for first-kind integral equations on polygons

We discuss the convergence of the collocation method for first kind integral equations with logarithmic kernels using splines of any order k on polygons in $\mathbb{R}^2$. Before discretization the equation is transformed to an equivalent equation on $[-\pi, \pi]$ using a nonlinear parametrization of the polygon which varies more slowly than arc-length near each corner. This produces a transformed equation with a smooth solution which is shown to be well-posed on appropriate Sobolev spaces. We are then able to show that the collocation method using splines of order k (degree $k-1$) converges with optimal order h^k. The collocation points are the mid points of subintervals when k is odd and the break-points when k is even and stability is shown under the assumption that the method may be modified slightly.

R.D. GRIEGORIEFF AND I. SLOAN:

Spline Petrov Galerkin Methods with Quadrature

A spline method for linear m-th order multipoint integro-differential equations is analyzed. The method is a Petrov-Galerkin method using smoothest splines of order $m+2$ resp. $m+3$ as trial space and piecewise linear resp. quadratic splines as test space combined with quadrature rules for approximating the integrals. Optimal order convergence of the approximate solutions and its derivatives up to order m and also supraconvergence at the breakpoints is proved. The grids can be arbitrarily nonuniform. Similar results are obtained for the corresponding eigenvalue problem.

STEFAN HEINRICH:

Complexity of approximate solution of Fredholm integral equations

The numerical solution of Fredholm integral equations is analyzed from the point of view of information-based complexity theory. A survey of recent results in this direction is given. These results allow to compare complexities of:

- using function values in the discretization vs. using Fourier coefficients
- seeking the full solution of the equation vs. seeking the value of the solution in one point
- using deterministic vs. using stochastic (Monte Carlo) methods.

LEI JINGAN:

Finite dimensional approximate theory for bifurcation of obstacle problems

In this work we are concerned with approximate theory of the bifurcation for obstacle problems. The existence theorems of bifurcation points and approximate bifurcation points are given. Moreover the convergence of approximate method is discussed, and numerical results are presented.

The idea of this paper depends on the Lagrange multiplier rule and compactness principles, as well as the set convergence.

OTTO KARMA:

On Regular Approximation in Eigenvalue Problems

Let the eigenvalue problem $A(\lambda_0)\,u^0 = 0$, $u^0 \neq 0$ be given, for the linear Fredholm operators $A(\lambda)$ depending analytically on the complex parameter λ $(A(\lambda) = A - \lambda I$ is a particular care).

We consider the convergence of the eigenvalues λ_n of the approximating problems $B_n(\lambda_n)\,x_n^0 = 0$, $x_n^0 \neq 0$ to the eigenvalues λ_0 of the original problem. (In applications, $B_n(\lambda)$ are operators on finite-dimensional spaces.)

Let ε_n be the approximation error on the generalized eigenspace of A at λ_0, and let κ be the order of the pole λ_0 of A^{-1}. Then the following asymptotic estimations hold:

1) for individual eigenvalues λ_n converging to λ_0:

$$|\lambda_n - \lambda_0| \;\leq\; c\varepsilon_n^{-\frac{1}{\kappa}}\,,$$

2) for the arithmetic mean $\overline{\lambda}_n$ of all the eigenvalues λ_n converging to λ_0:

$$\left|\overline{\lambda}_n - \lambda_0\right| \leq c\varepsilon_n .$$

3) for eigenvectors $x_n^0 \in N(B_n(\lambda_n))$, $\|x^0\| = 1$:

$$\text{dist}\left(x_n^0,\, N(A(\lambda_0))\right) \leq c\,(|\lambda_n - \lambda_0| + \varepsilon_n)\,.$$

MARIAN KWAPISZ:

General priciples of convergence of discretization methods for differential-delay, differential-functional equations

In the talk it will be shown that rather simple facts such as comparison theorems for discrete inequalities are very usefull in applied mathematics especially in the theory of discrete dynamical systems, theory of convergence of iterative methods of solving fixed points equations and the convergence of finite-difference methods for solving problems for ODE's, PDE's, IDE's, FDE's and others.

An abstract result involving partially ordered spaces will be formulated and it will be shown the error estimations for approximate solutions of corresponding discrete equations. The convergence results can be obtained as consequences of these estimations.

WILLIAM MCLEAN:

Fully-discrete spline methods for boundary integral equations of the first kind

The talk deals with a class of numerical methods that can be viewed as perturbations of Petrov-Galerkin schemes using splines as trial and test functions. The methods apply to periodic pseudodifferential equations such as arise from reformulating two-dimensional elliptic boundary value problems as boundary integral equations of the first kind. By emploiting translation-invariance of the singular term in the kernel, one can characterize the stability and convergence properties of the method in terms of the behaviour of a certain function D, whose definition depends on the particular method. The anaylsis yields a means of designing special, very simple numerical intergration techniques that take care of the non-smoothness in the kernel.

HANS JOACHIM OBERLE:

Numerical Treatment of Optimal Control Problems with Application to the Optimal Control of CO_2-Emissions

In this lecture a simplified model is considered which describes the interaction of climate changes (represented by the influences due to the enhanced emission of CO_2 and the resulting increase of the averaged surface air temperature) on one side, and the economy (i.e. the abatement costs for reduction of emissions and the damage costs due to the increased temperatur) on the other side.

The model is formulated as a linear-quadratic optimal control problem with a compact control region. Applying the standard necessary conditions a multipoint-boundary-value problem is derived and it's numerical solutions obtained by multiple-shooting techniques are presented. Special attention is paid to the computation of the reachable region of the system and to the dependence of the control structure on the final state prescribed.

This talk is based on joint work with H. and J. von Storch and O. Tahvonen.

ECKEHARD PFEIFER:

Verallgemeinerte Taylorformeln in der Numerischen Mathematik

Polynome spielen in der Numerischen Mathematik wegen ihrer Abgeschlossenheit hinsichtlich verschiedenster mathematischer Operationen und dem Vorhandensein der klassischen Taylorentwicklung eine herausgehobene Rolle.

Ersetzt man den Begriff der Ableitung durch einen linearen Operator S, so läßt sich die Taylorentwicklung zu

$$x = s\,x + T\,s\,S\,x + \ldots + T^n\,s\,S^n\,x + T^{n+1}\,S^{n+1}\,x$$

verallgemeinern. Dabei ist T eine Rechtsinverse zu S und s bezeichnet die Differenz zwischen Einheitsoperator und $T\,S$. Die Rolle der Polynome wird also durch Elemente des Nullraums von S^{n+1} übernommen.

Im Vortrag wird die Anwendung dieser Idee auf

- die Lösung von gewöhnlichen Anfangs- und Randwertaufgaben,
- die Interpolation mit verallgemeinerten quadratischen und kubischen Splinefunktionen,
- den Einsatz verallgemeinerter Newtonverfahren zur Nullstellenbestimmung

skizziert.

FLORIAN A. POTRA:

A Geometric Theory of Discretization Algorithms for Differential-Algebraic Equation

Following some ideas developed in joint work with Werner C. Rheinboldt we view differential-algebraic equations as differential equations on manifolds and we use local charts to reduce the differential-algebraic system to a system of ordinary differential equations in the local, unconstrained, coordinates. The numerical solution of the latter system is then mapped via the local parametrization into a new point on the manifold which provides a numerical solution of the initial differential-algebraic system. As shown in a joint paper with Linda R. Petzold this numerical solution can be interpreted as a generalized solution of the nonlinear system obtained by discretizing an overdetermined differential algebraic system.

SIEGFRIED PRÖSSDORF:

Wavelets methods for pseudodifferential equations

The talk is concerned with generalized Petrov–Galerkin schemes for elliptic periodic pseudodifferential equations in $\mathbb{R}^n$. This setting covers classical Galerkin methods, collocation, and quasi-interpolation. The numerical methods are based on a general framework of multiresolution analysis. In the first part we give necessary and sufficient stability conditions in terms of the „symbol“ of the methods under consideration and establish error estimates in the scale of Sobolev spaces. In the second part we analyse compression techniques for the resulting stiffness matrices relative to wavelet-type bases. It will be shown that, although these stiffness matrices are generally not sparse, the order of the overall comptuational work which is needed to realize a certain accuracy is of the form $O(N(\log N)^b)$, where N is the number of unknowns and $b \geq 0$ is some real number. The theoretical results are confirmed by numerical experiments for the double layer potential integral equation over the surface of the cube. The talk is based on a joint work with W. Dahmen and R. Schneider.

J. SARANEN AND G. VAINIKKO:

Fast solution of boundary integral equations

We propose two-grid iteration methods for a large class of the boundary integral equations on closed smooth curves. The equation is given in the form $Au + Bu = f$ such that the main part A is a convolutional integral operator and B has better smoothing property than A. Our basic method is the trigonometric collocation such that the perturbation B is discretized by applying the product integration. If B has a smooth kernel our methods include variants where already the first iteration is of the optimal order and requires $O(N \log N)$ arithmetic operations. In the general case our schemes need work of the order $O(N^2)$.

Bernd Silbermann:

Algebraic Techniques in Stability Analysis for Spline Approximation Methods

The main aim of my talk is to present how one can apply some ideas from algebra in order to study the following invertibility problem:

Let (for simplicity) $\mathcal{H}$ be a Hilbert space and $\{\mathcal{H}_n\}$ be a sequence of closed subspaces of $\mathcal{H}$. Denote by $\mathcal{D}^\infty$ the collection of all operator sequences

$$\{A_n\},\ A_n \in \mathcal{L}(\mathcal{H}_n),\ \sup \|A_n\| < \infty .$$

$\mathcal{D}^\infty$ actually forms a Banach algebra (with componentwise algebraic operations). In $\mathcal{D}^\infty$ there is a closed two–sided ideal $\mathcal{G}$,

$$\mathcal{G} \quad := \quad \{\{A_n\} \in \mathcal{D}^\infty : \|A_n\| \to 0 \quad \text{as} \quad n \to \infty\} .$$

Given a sequence $\{A_n\} \in \mathcal{D}^\infty$ we ask the following question: Is there a sequence $\{B_n\} \in \mathcal{D}^\infty$ such that

$$A_n B_n \;=\; I_n + C_n, \qquad B_n A_n \;=\; I_n + D_n \tag{1}$$

and $\{C_n\}$, $\{D_n\} \in \mathcal{G}$?

This invertibility problem is of great interest since such operator sequences occur in discretizing processes and the invertibility problem (1) is completely equivalent to the well–known stability problem, one of the corner stones in Numerical analysis.

Ian H. Sloan:

A marriage of boundary integrals and finite elements

In this talk (which describes joint work with D.W. Kelly and S. Wang) boundary integral and finite element techniques are combined, to study 2-dimensional Neumann problems for the Laplace equation. The aim is to compute reliable first derivatives, together with error bounds that are both useful and rigorous. The first step is a finite element solution. The derivatives are then recovered by a boundary integral technique, and error bounds then determined by the solution of an auxiliary problem and a complementary analysis. Numerical results confirm that very satisfactory results can be obtained, for both derivatives and their error bounds, even when the derivatives are sought close to the boundary, and even when the boundary is curved, provided it is locally smooth.

ERNST P. STEPHAN:

The *hp*-version of the boundary elment method

We investigate the *hp*-version of the boundary element method for strongly elliptic boundary integral equations on polygonal and polyhedral domains where both the mesh size h and the polynomial degree p are changed to improve the accuracy of the Galerkin solution. Using precise results for the singular behaviour of the solution of the integral equations near corners and edges of the domain we find that the rate of convergence for the p-version (h fixed) is twice that of the h-version (p fixed) for most problems. In the *hp*-version one combines both approaches. With a geometric mesh refinement near corners/edges we obtain exponentially fast convergence for the Galerkin solution. Our numerical results for first kind integral equations with weakly singular and hypersingular kernels underline our theoretical results. We present also adative algorithms for the *hp*-version where the refinement process is steered by local error indicators.

G. VAINIKKO:

Collective spectrum of disretely converging operators

Let E and $E_n\,(n \in \mathbb{N})$ be complex Banach spaces, $p_n \in \mathcal{L}(E, E_n)$ and $p_n' \in \mathcal{L}(E^*, E_n^*)$ satisfy $\|p_n u\| \to \|u\|$, $\|p_n' u^*\| \to \|u^*\|$, $n \to \infty$, $\forall u \in E$, $u^* \in E^*$. Let operators $T \in \mathcal{L}(E, E)$ and $T_n \in \mathcal{L}(E_n, E_n)$, $n \in \mathbb{N}$, be given such that $T_n \xrightarrow{\mathcal{P}} T$, $T_n^* \xrightarrow{\mathcal{P}'} T^*$, i.e. $u_n \xrightarrow{\mathcal{P}} u \Rightarrow T_n u_n \xrightarrow{\mathcal{P}} T u$, $u_n^* \xrightarrow{\mathcal{P}'} u^* \Rightarrow T_n^* u_n^* \xrightarrow{\mathcal{P}'} T^* u^*$ where $E_n \ni u_n \xrightarrow{\mathcal{P}} u \in E$ means that $\|u_n - p_n u\|_{E_n} \to 0$, $n \to \infty$. Elementary examples show that the convergence properties of the spectra $\sigma(T_n)$ may be rather poor even if the stability condition $\|(\lambda - T_n)^{-1}\| \le c_\lambda$ $(n \ge n_\lambda)$ is fulfilled for any $\lambda \in \rho(T) = \mathbb{C}\backslash\sigma(T)$. On the other hand, we show a natural way how to define the collective spectrum $\Sigma((T_n)_{n\in\mathbb{N}})$ of the sequence $(T_n)_{n\in\mathbb{N}}$ so that, under conditions introduced above, it occurs that $\Sigma((T_n)) = \sigma(T)$. Applications to Krylov subspaces methods to find $\sigma(T)$ are discussed.

The talk is based on joint work with O. Nevanlinna.

Reported by: S. Fellehner

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