# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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## Methoden und Verfahren der mathematischen Physik

12.12. bis 18.12. 1993

Die 13. Tagung über **Methoden und Verfahren der mathematischen Physik** wurde von R.E. KLEINMAN (Newark), R. KRESS (Göttingen) und E. MARTENSEN (Karlsruhe) geleitet. Von den 25 Teilnehmern aus 9 Ländern (Deutschland 13, Finnland 1, Frankreich 1, Griechenland 1, Großbritannien 1, Niederlande 1, Schweden 1, Tschechien 1, USA 5) wurden 21 Vorträge gehalten.

Es wurden vor allem Themen aus der klassischen Streutheorie akustischer und elektromagnetischer Wellen sowie aus der Strömungslehre, der Magnetohydrodynamik, der Potentialtheorie und der Elastizitätstheorie diskutiert. Im Mittelpunkt des Interesses standen dabei die sogenannten inversen Probleme, bei denen aus den bekannten einfallenden und reflektierten Feldern die Geometrie bzw. die Materialeigenschaften des Streukörpers bestimmt werden. Die Tagung spiegelte die Dynamik dieses Forschungsgebietes wider, deren wesentliche Ursache die große praktische Relevanz der inversen Probleme ist. So wurden u.a. Aufgabenstellungen und Lösungsverfahren aus der zerstörungsfreien Materialprüfung und der medizinischen Diagnostik vorgestellt.

Das Hauptziel der Tagung war die Förderung des interdisziplinären wissenschaftlichen Austausches, insbesondere zwischen Arbeitsgruppen der angewandten Analysis und der Physik. Wie die regen Diskussionen über die vorgestellten Probleme und Lösungsmethoden zeigten, konnte dieses Ziel voll erreicht werden.

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## Vortragsauszüge

M. FEISTAUER, G. HSIAO, R.E. KLEINMAN

### Analysis and numerical realization of coupled BEM and FEM for nonlinear exterior problems

The paper presents the analysis of the coupled BEM and FEM to a nonlinear generally nonmonotone exterior boundary value problem. The problem consists of a nonlinear differential equation considered in an annular bounded domain and the Laplace equation outside. These equations are equipped with boundary and transmission conditions. The problem is reformulated in a weak sense and combined with an integral equation on the outer part of the boundary. The discretization is carried out by the coupled finite element – boundary element method. We prove the existence of the solution and the convergence of the method and discuss the solution of the nonlinear discrete problem.

A. KIRSCH

### Scattering problems for periodic structures

The first part of the talk summarizes results for direct and inverse scattering problems by periodic boundaries under Dirichlet boundary conditions. It is shown that the full knowledge of the scattered waves above the structure for all incident waves is sufficient to determine the shape of the scatterer. If only the components of the scattered plane waves are known then – for the case of small perturbations of the plane – only the slow variations can be recovered. In the second part of the talk results of the same kind are derived for the case when the scattering structure is described by an inhomogeneous (with respect to the horizontal coordinate only) layer on a perfectly reflecting plane.

P. HÄHNER

### Reconstruction of an inhomogeneous density in inverse elastic scattering

I consider time harmonic elastic waves in an isotropic elastic medium in $\mathbb{R}^3$ which has an inhomogeneous density. I state existence and uniqueness for the direct scattering problem.
Then I reconstruct the density from the knowledge of the scattered waves on a large sphere for all incoming waves at a fixed frequency. To this end I need special solutions of the elasticity equation which were obtained for other equations by Sylvester and Uhlmann.

G. DASSIOS

Index weaving and generation of semiseparable solutions

Let $E^2$ denote the second order elliptic operator that determines the vorticity field in spheroidal coordinates for the Stokes flow problem in axisymmetric geometry. This flow problem has the unique property of separating variables for the equation $E^2\psi = 0$ and not separating variables for the equation $E^4\psi = 0$. Generalized eigenspace theory is used to obtain the complete set of solutions for the nonseparable equation $E^4\psi = 0$. It is shown that the solutions of $E^2\psi = 0$ are products of Gegenbauer functions and that an appropriate weaving of their indices taken among 3-D subspaces of either even or odd order provide the generalized eigenmodes that belong to the ker $E^4$. A spectral investigation of the weaving pattern reveals that the ordinary eigenmodes of $E^2\psi = 0$ provide the complete set of zero vorticity flow fields while the generalized eigenmodes generated by the index weaving are rotational fields with irrotational vorticities. This unique phenomenon of semiseparable behavior generated by the weaving of the index gives rise to theoretical investigations concerning the characterization of partial differential operators that possess semiseparable solutions.

W. BORCHERS, W. VARNHORN

On the boundedness of the Stokes semigroup in two dimensional exterior domains

We prove that the semigroup associated with the nonstationary Stokes equations in the exterior domains of $\mathbb{R}^2$ is uniformly bounded. That is, we show that the solution $u$ of the nonstationary Stokes initial boundary value problem is bounded on the semiaxis $[0, \infty)$ as an $L^r(\Omega)$–valued function, where $1 < r < \infty$ and $\Omega \subset \mathbb{R}^2$ is an exterior domain with a smooth boundary. Up to now, this property was known for bounded domains, in the Hilbert space case ($r = 2$), and also for exterior domains $\Omega \subset \mathbb{R}^n$ with $n \geq 3$, the two dimensional case was excluded, however.
We prove our result using potential theory for the Stokes resolvent equation and the method of boundary integral equations.

B.R. VAINBERG

On asymptotic stability of solutions of a nonlinear wave equation

We consider the nonlinear wave equation

$$u_{tt} = \Delta u + f(x,u) \quad , x \in \mathbb{R}^3 \tag{1}$$

in the whole space with a nonlinear term which is localized in $x$. Let $v = S(x)$

be a stationary solution of the equation $0 = \Delta S(x) + f(x, S)$. We find conditions when any solution $u$ of (1) with initial data close enough to the initial data of the solution $v = S(x)$ tends (in some sense) to the solution $v$ as $t \to \infty$. This property (asymptotic stability) is very unusual for problems of wave propagation because we do not have here any dissipation of energy which is a standard requirement for this property. In problems under consideration the scattering of waves plays the role of a dissipation of energy.
An asymptotic expansion for $u(x,t) - v$ as $t \to \infty$ is found. This expansion is expressed through exponential functions containing "scattering frequencies", which are independent of initial data and depend only on medium and obstacles. These frequencies were studied earlier only for linear problems.

D. Colton

Spectral theory of the far field operator

It is shown that there exist an infinite number of eigenvalues of the far field operator corresponding to the scattering of a time harmonic plane wave by either an imperfectly conducting obstacle or an absorbing inhomogeneous medium. Regions in the complex plane are found where these eigenvalues must lie and, in the case of obstacle scattering, numerical examples are given showing how precise these regions are. An application is discussed showing how leukemia can potentially be detected through the use of electromagnetic imaging.

L. Päivärinta

Inverse problems for systems

Some recent methods for solving inverse problems in two body Schrödinger scattering were discussed. It was emphasized that these methods, including non-physical exponentially growing solutions, can be useful also in solving inverse problems for systems, as for Maxwell's systems. However there are major differences between scalar and vector problems due to the first order terms. These difficulties can be handled by careful analysis as is done in recent joint works with D. Colton, P. Ola and E. Somersalo.

P.M. VAN DEN BERG, R.E. KLEINMAN

Reconstruction of the location, the shape and the composition of a scattering object

The location, the shape and the composition of a penetrable scattering object is characterized by a complex contrast. A method for the reconstruction of this contrast from measured scattered field data is discussed. The method consists of casting the inverse problem as an optimization problem in which the cost functional is the sum of two terms, one is the defect in matching measured field data with the field scattered by a body with a particular contrast and the second is the error in satisfying the equations of state, integral equations for the field produced in the body by each excitation. The contrast and the fields are each updated by an iterative method in which the updating directions are weighted by parameters which are determined by minimizing the cost functional. Some numerical examples are given indicating the limits to the contrasts which can be reconstructed.
The method is then modified to reconstruct the location and the shape of an impenetrable object. It is based on the fact that for high imaginary contrast the penetration depth of the scatterer is small, in which case the only meaningful information produced by the algorithm is the boundary of the scatterer. There are no other implicit assumptions about the location, connectivity, convexity or boundary conditions. Results of a number of numerical examples are presented which demonstrate the effectiveness of the location and shape reconstruction algorithm.

W. RUNDELL, B. LOWE

The determination of coefficients in elliptic and parabolic equations from multiple input sources

The simplest model problem is to determine the potential function $q(x)$ in

$$-u_j'' + q(x)u_j = f_j \quad , u_j(0) = u_j(1) = 0$$

from a measurement of the boundary values $\{u_j'(0)\}, j = 1, \ldots$. Here $\{f_j\}$ is assumed to be a complete family of source terms. Higher dimensioned analogues and time-dependent problems will be discussed. Uniqueness and continuous dependence results will be obtained.

F. BROUAYE, B. DUCHÊNE, M. LEFEBURE, D. LESSELIER, R. DE OLIVIERA-BOHBOT, CH. ROZIER

Eddy current imaging of defects in a conductive half-space as an inverse wave scattering problem

With adequate corrections and extensions, solution methods of inverse scattering problems can be used for low-frequency nondestructive testing of damaged metallic structures where eddy currents are the sources of the anomalous fields observed outside the structure under testing. When successful, inversion of such anomalous fields provides images of the defects.
But there is one severe challenge: the problem is strongly ill-posed due to exponential decay of the probing field in metal (skin effect) and corresponding evanescence of fields outside. In mathematical terms, an integral operator with damped kernel has to be dealt with. Also, at least in the first step, we are restricted to linearized solutions based on the Born approximation (which means testing of small defects at low frequency), the inverse problem being nonlinear with respect to the defects' conductivity.
Key aspects of the research carried out on the subject, mostly in cooperation with departments of *Electricité de France, Direction des Etudes et Recherches*, will be introduced during the talk, with attempting to point out open questions in both theory and numerical practice.

P. WERNER

Resonance phenomena in periodic structures

We study the propagation of linear waves, generated by a compactly supported time-harmonic force distribution, in an infinite string under the assumption that the material properties are $p_1$-periodic for $x > a$ and $p_2$-periodic for $x < -a$. As pointed out in a previous paper devoted to the special case of a purely periodic string (Math. Meth. Appl. Sci. 14, 227-263 (1991)), the combination of a time-periodic force and a periodic spatial structure may lead to resonance phenomena. We show that also the present configuration permits resonances of orders $t$ and $t^{\frac{1}{2}}$ for a discrete set of frequencies. The occurrence of resonances is closely related to the presence of non-trivial solutions of the corresponding time-independent homogeneous problem with certain asymptotic properties ("standing waves").

M. PETRY

Acoustic scattering in a layered medium

The direct transmission problem for the scalar Helmholtz equation in a three dimensional horizontally layered medium containing a bounded obstacle is analyzed. Uniqueness is proved with the help of Green's theorem. A system of Green's functions is constructed, which allows the transformation of the given transmission problem into an integral equation of the second kind. By application of Riesz's theory the existence of a solution follows.

P.A. MARTIN

Asymptotic approximations of functions defined by series

Mellin transforms are used to find asymptotic approximations for functions defined by series. The simplest cases are those of the form

$$\sum_{n=1}^{\infty} c_n u(nx) \quad ,$$

where $u$ is a given function. We call these *separable series*, because $u$ is sampled at points whose variation with $n$ and $x$ is separated. Examples of this type were considered by Ramanujan. Non–separable series are analyzed by first approximating them by separable series. Several examples are given.

G. KRISTENSSON

Energy inequalities and constraints for the time–dependent electromagnetic fields

The definition of electromagnetic dissipative (passive) media was introduced, and its consequences on the material parameters in complex (chiral) media were discussed. The motivation of this work is to obtain constraints on the material parameters in the time domain, since Fourier transformation of data is not possible due to time–limited data. The material parameters consist in the time–domain setting of time convolutions between four susceptibility kernels and the electric and magnetic fields. The definition of dissipation implies an inequality of the type

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (f(t), A(t-t')f(t))\, dt dt' \geq 0 \quad \text{for all } f \in C_0^{\infty}$$

where $A(t)$ is a two by two matrix–valued function (or distribution) and $(.,.)$ is the scalar product in $\mathbb{R}^2$. A generalization of a theorem by Mercer connects this



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inequality with matrix-valued functions of positive type, which implies restrictions on the components of the matrix $A(t)$, which contains the material parameters. The first simple inequalities were shown explicitly. The distribution case was also briefly analyzed. The connections to Bochner's theorem were discussed.

S. RITTER

Dipole distributions in equilibrium and the electrostatic integral operator for an ellipsoid

A model for permanent magnetization using dipole distributions in a compact domain $D \subset \mathbb{R}^3$ with boundary $S$ is discussed. The determination of a dipole distribution in equilibrium turns out as an eigenvalue problem for the electrostatic integral operator. The least eigenvalue that is bigger than $-1$ is of interest. The case of an ellipsoid with three different axes is discussed and explicit formulas for the eigenvalues are given. For the case that the far-field is of dipole type, bounds for the axes' ratio are computed.

E. MARTENSEN

On the magnetostatic integral operator

Let $D \subset \mathbb{R}^3$ be a compact domain with continuously curved boundary $S$, then the magnetostatic integral operator $L_m$ with values

$$L_m j = -\frac{1}{2\pi} \int_S [n, [\operatorname{grad} \frac{1}{r}, j']] \, df' \quad ,$$

$n$ being the exterior normal of $S$, is considered in the function space of current distributions on $S$

$$F(S) = \{j \in [C(S)]^3 | (n, j) = 0, \operatorname{Div} j = 0\} \quad .$$

It is shown that if $\lambda \in (-1, 1) \backslash \{0\}$ is an eigenvalue of the electrostatic integral operator, then $-\lambda$ and $\lambda$ both are eigenvalues of $L_m$ but that the converse of this is not valid.

F. Hettlich

The domain derivative of the far–field pattern for a transmission problem

In recent work of A. Kirsch the use of the domain derivative of the far–field pattern for the inverse scattering problem with Dirichlet boundary conditions is discussed. It is shown that a similar result of existence and a characterization can be derived also for penetrable obstacles, which leads to a transmission problem. The characterization of the domain derivative is a first theoretical step in the implementation of Newton type methods for the inverse scattering problem for penetrable obstacles and in establishing sensitivity results.

M. Hanke

Iterative regularization methods for ill–posed problems

In this paper we present our results on the regularizing properties of Hestenes–Stiefel–conjugate gradients and King's minimal error method for linear ill–posed problems with perturbed data. Because of their well–known optimality properties for exact data, these Krylov subspace methods can be expected to achieve good approximations of the exact solution with fewer iterations than, e.g., the minimal residual methods investigated by Nemirovskii and others. Our main result states that the above methods are order–optimal regularization methods provided the iteration is terminated according to a very simple stopping rule. Opposed to this, the discrepancy principle is not regularizing.

J. Gottlieb

Some problems in tomography using potential fields

Tomographical methods using elliptic or parabolic potential fields are related to the identification of spatially distributed coefficients in corresponding partial differential equations. These problems are characterized by a linear input–output operator and a nonlinear relation between data and solution. This structure leads to nonlinear moment problems.

We discuss some aspects of reconstruction, resolution, optimal experimental design and data redundancy.

Finally we give a new uniqueness result for a parabolic problem.

T.S. ANGELL, R.E. KLEINMAN, B.R. VAINBERG

Asymptotic approximation of optimal solutions of an acoustic radiation problem

We have presented, elsewhere, the problem of choosing Neumann data for the exterior Helmholtz equation in order to optimize a functional of the radiated far field. In this paper we use asymptotic methods to determine an approximate optimal solution, for high frequency, whose support is in a prescribed region of the boundary. This solution is given entirely in terms of geometric properties of the boundary.

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