MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Singuläre Integral- und Pseudodifferential-Operatoren und ihre Anwendungen

23. - 29. Januar 1994

Die Tagung fand unter der Leitung von E. Meister (TH Darmstadt), S. Prößdorf (IAAS Berlin) und B. Silbermann (TU Chemnitz-Zwickau) statt.

Singular integral and pseudodifferential operators represent a central domain of modern analysis with various applications in other fields of mathematics and, moreover, in mathematical physics and engineering. The main intention of this conference was to bring together researchers from both the areas of operator theory and applications to discuss problems and to stimulate the transfer of results, methods and applications between these fields.

46 researchers, including two young mathematicians supported by special grants, who are active in or interested in various aspects of this fascinating field, participated in the conference. The resulting 36 presentations are abstracted below under the following headings: (i) initial value problems and boundary value problems on manifolds with singularities; (ii) theory of linear and nonlinear singular integral equations; (iii) algebras of pseudodifferential operators; (iv) Wiener-Hopf equations, Mellin convolution equations, Toeplitz matrices and determinants; (v) Banach algebra techniques in operator theory and numerical analysis; (vi) approximation and numerical methods for integral and pseudodifferential equations; (vii) applications of integral and pseudodifferential equations. In addition to numerous public and private discussions, a much appreciated highlight was a special session on continuing activities on a framework of a European research program. The great interest of the participants suggests to organize a subsequent conference on this subject in about three years. No proceedings of the conference are planned.

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Abstracts

MICHAEL BEALS:

L^∞ estimates for the wave equation with a potential

Let $u(t,x)$ be the solution to the initial value problem $(\Box + V)u = 0$, $u(0,x) = 0$, $\partial_t u(0,x) = f(x)$. If $n \geq 3$, $V \in \mathcal{S}(\mathbb{R}^n)$, and V is either sufficiently small or nonnegative, the decay of the $L^\infty(\mathbb{R}^n)$ norm of $u(t)$ as $t \to \infty$ is the same as in the case $V \equiv 0$: if $\langle D \rangle = \sqrt{(1-\Delta)}$ and $\lambda > (n-1)/2$, then $||u(t)||_\infty \leq Ct^{-(n-1)/2}||\langle D\rangle^\lambda f||_1$. (If the Hardy space $\mathfrak{H}^1(\mathbb{R}^n)$ takes the place of $L^1(\mathbb{R}^n)$, then $\lambda = (n-1)/2$ is allowed.) Such estimates are useful in the analysis of the existence and scattering properties of solutions to certain semilinear wave equations for which the linearized equation is $(\Box + V)u = 0$. The proof involves a contraction argument and an iteration of Duhamel's formula. If $u(t) = T(t)f$ defines the solution operator when $V \equiv 0$, precise estimates on the iterated operators

$$\int \cdots \int \chi(0 \leq s_{k-1} \leq \ldots \leq s_1 \leq t)$$
$$T(t-s_1)V \cdots VT(s_{k-2}-s_{k-1})VT(s_{k-1})\langle D\rangle^{-\lambda} f\, ds_{k-1}\cdots ds_1$$

are established. For instance, if $k + \lambda > (n+1)/2$, then the associated kernel is bounded by $Ct^{-(n-1)/2}$, and if x and y are in a bounded set the better estimate $Ct^{-(n-1)/2}(1 + \big|t - |x-y|\big|)^{-(n-1)/2}$ holds. In the case of large non-negative V, an additional direct estimate of a small-frequency term is involved.

ALBRECHT BÖTTCHER:

Continuous analogues of the Fisher–Hartwig formula

The Fisher–Hartwig formula describes the asymptotic behavior of large Toeplitz determinants generated by piecewise continuous functions. The talk is devoted to a continuous analogue of this formula, that is, to the description of the asymptotics of large truncated Wiener–Hopf integral operators with piecewise continuous symbols. Such a continuous analogue is established and the main ingredients of the proof, including the "discretization" of Wiener–Hopf integral operators through Toeplitz operators with operator–valued symbols and also including compactness criteria for Hankel operators on several spaces, are outlined. The talk is based on joint work with B. Silbermann and H. Widom.

ROLAND DUDUCHAVA and SIEGFRIED PRÖSSDORF:

On the approximation of singular integral equations by equations with smooth kernels

We consider a singular integral equation

$$A\varphi = a\varphi + bS_\Gamma\varphi + T\varphi\,, \quad S_\Gamma\varphi(t) = \frac{1}{\pi i}\int_\Gamma \frac{\varphi(\tau)d\tau}{\tau - t}$$

with matrix piecewise–continuous coefficients on a smooth curve Γ containing open arcs. T is a compact operator.

Necessary and sufficient conditions for the stability of the approximating sequence of operators

$$A_\varepsilon\varphi = a\varphi + bS^\varepsilon_{\Gamma,n}\varphi + T\varphi\,,\ S^\varepsilon_{\Gamma,n}\varphi(t) = \frac{1}{\pi i}\int_\Gamma \frac{(\tau - t)\varphi(\tau)d\tau}{(\tau - t)^2 - n^2(t)\varepsilon^2}\,,\ \varepsilon \to 0$$

where $n(t)$ denotes a vector field non-tangential to Γ, are obtained in $L^N_p(\Gamma, \varrho)$ spaces with $1 < p < \infty$ and a Khvedelidze weight $\varrho(t)$.

DAVID ELLIOTT and SIEGFRIED PRÖSSDORF:

An algorithm for the approximate solution of integral equations of Mellin type

The cruciform crack problem of elasticity gives rise to a class of integral equations of the second kind on $[0,1]$ whose kernel has a fixed singularity at $(0,0)$. We introduce a transformation of $[0,1]$ onto intself such that an arbitrary number of derivatives vanish at the end points 0 and 1. If the transformed kernel is dominated near the origin by a Mellin kernel then we give conditions under which the use of a modified Euler–Maclaurin quadrature rule and the Nyström method give an approximate solution which uniformly converges to the exact solution of the original equation. The method is illustrated with a numerical example.

JOHANNES ELSCHNER:

Collocation methods for Symm's integral equation on polygons

This is a joint work with Ivan Graham (University of Bath). We consider the collocation method for the integral equation

$$-\pi^{-1}\int_\Gamma \log|x - y|u(y)d\Gamma(y) = f(x)\,,\ x \in \Gamma\,,$$

where Γ is a closed polygon in $\mathbb{R}^2$ enclosing a bounded domain. Before discretization the problem is reformulated using a nonlinear parametrization of the polygon which varies more slowly than arc–length near each corner. This produces a transformed integral equation with a smooth solution. An analysis based on properties of Mellin convolution operators then shows the transformed equation is well–posed in appropriate Sobolev spaces. Using these facts we are able to show that for any k, the collocation method using splines of degree k on a uniform mesh of size h converges with optimal order $O(h^{k+1})$. The collocation points are the mid–points of subintervals when k is even and the break–points when k is odd, and stability is shown under the assumption that the method may be modified slightly. The numerical solutions to the transformed equation yield superconvergent approximations to interior potentials, such as those used to solve harmonic boundary value problems by the boundary integral method.

ISRAEL GOHBERG:

Extension theorems for symbols

The possibility of extensions of invertibility symbols from a subalgebra to its closure is analysed. It is proved that such an extension is possible if and only if the closure does not contain any Kakutani elements. The other difficulty in the problem of extension of a symbol from a closed subalgebra is connected with the fact that some elements of the subalgebra may have inverses which do not belong to the subalgebra. A method of solving this problem is also proposed. Applications to algebras generated by singular integral operators are presented.

The talk is based on joint results with N.J. Krupnik (Integral Equations and Operator Theory V. 15 (1992), pp. 990–1010; V. 16 (1993), pp. 515–529).

HARALD HEIDLER:

Algebras of shift operators

The talk is devoted to the description of the characteristic and essentially characteristic polynomials of composition operators. The knowledge of these polynomials is of interest for classifying the algebras generated by composition operators and their Calkin images. The question what polynomials can play the role of a characteristic or essentially characteristic polynomial of some composition operator on $C(X)$ is intimately tied in the topological nature of X. We show, in particular, how the connectivity of X can be determined by the knowledge of these polynomials.

NIELS JACOB:

Global properties of Feller semigroups generated by pseudodifferential operators (partly joint with W. Hok)

For a class of pseudodifferential operators

$$-p(x,D)u(x) = -(2\pi)^{n/2}\int_{\mathbb{R}^n} e^{ix\xi}p(x,\xi)\hat{u}(\xi)d\xi\,,$$

where $p : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is a continuous function such that $p(x,\cdot) : \mathbb{R}^n \to \mathbb{R}$ is negative definite in the sense of Bochner–Beurling–Oeny, it is proved that they generate a Feller semigroup $(T_t)_{t\geq 0}$. Moreover, we give some sufficient conditions in order that the semigroup $(T_t)_{t\geq 0}$ admits a density, is conservative and admits many excessive functions. From this it follows that the probabilistic solution $E^x(h(X_{\tau_\Omega}))$ of the "boundary value" problem $p(x,D)u = 0$ in Ω, $\Omega \subset\subset \mathbb{R}^n$, $u|_{\Omega^c} = h|_{\Omega^c}$, $h : \mathbb{R}^n \to \mathbb{R}$ suitable, could be identified with the analytic solution obtained by balayage theory.

LOTHAR JENTSCH:

On some new integral operators arising from the contact tensor method in elasticity

The contact tensor method is a useful tool for solving boundary value problems with interfaces. The contact tensor is the sum of the fundamental solution tensor and a compensatrix such that the transmission conditions (contact conditions) are satisfied. The advantage of the method consists in the fact that a potential Ansatz with the contact tensor a priori satisfies the differential equations and the contact conditions and that the boundary integral equations (BIEs) only live on the surface where boundary conditions are given. For constructing contact tensors the pure contact problem has to be solved.

The contact tensors of elastostatics are calculated for three different contact conditions and for straight line and plane interfaces.

For the method it is typical that the BIEs have fixed singularities at interface corners and edges. In the plane case the boundary integral operators (BIOs) are of Mellin convolution type and the question of Fredholm property can be decided with the aid of a Mellin symbol. In the three–dimensional case the local BIOs for a point of the interface edge are investigated in L_p–spaces with weight on the tangential half–plane. For the simplified model of stationary heat conduction the doctoral thesis of D. Mirschinka contains a full theory for such BIEs with fixed singularities along interface edges.

MARINUS A. KAASHOEK:

Time–varying generalizations of invertibility and Fredholm theorems for Toeplitz operators

The main theorems in the theory of block Toeplitz operators dealing with invertibility, Fredholm properties and index, and with factorization of the symbol, are generalized to a new class of non–Toeplitz operators acting on l_2^m. The operators in this new class appear as input–output operators of time–varying linear systems, and they are characterized by the following two properties: (1) the entries a_{jk} in their canonical block matrix representation decay exponentially as functions of $|j-k|$ and (2) the Kronecker rank is finite. In the description and the study of the systems involved dichotomy of difference equations plays an important role. The talk is based on joint work with A. Ben–Artzi and I. Gohberg.

YURI I. KARLOVICH:

Local methods of studying algebras of operators with shifts and their applications

In this talk we present local methods for the investigation of the Fredholm property and invertibility of nonlocal operators with shifts in Banach and Hilbert spaces. We consider some applications of these methods to the study of algebras of one–dimensional and multidimensional convolution type operators with discrete groups of shifts and discontinuous coefficients. These applications include algebras of singular integral operators with shifts in Lebesgue spaces, C^*–algebras of singular integral operators with coefficients admitting discontinuities of semi–periodic or semi–almost–periodic type, Riemann and Haseman boundary value problems with such matrix coefficients in weighted Lebesgue spaces, convolution type operators with oscillating presymbols, an approximation approach to the problem of factorization of almost periodic matrices.

A.I. KOMECH:

The exact solution of boundary value problems in wedges and its applications

The exact solution of a general BVP with constant coefficients in wedges is necessary for dealing with different problems: Fredholm property of BVP in regions with corners, diffraction on wedges, high order approximations, Ursell's problem concerning guided waves moving along sloping beach, elasticity problems in wedges, and others. In 1986

A. Sommerfeld found the solution of the Dirichlet and Neumann BVP for the Helmholtz equation in a wedge, and gave the corresponding scattering amplitude formula. In 1958 G.D. Malujinetz extended Sommerfeld's result to the third BVP. V.G. Maz'ya and B.A. Plamenevskij found in 1971 the solution of the oblique derivative problem for Laplace's operator in wedges, and in 1975 they extended their method to general BVP with real constant coefficients in wedges of arbitrary angle magnitude $< 2\pi$. In 1973 the author found a new method for solving general BVP for second order elliptic operators with constant complex coefficients in wedges of angle magnitude $< \pi$. The complex coefficient case is important for diffraction problems. The method uses Fourier transformation and Paley–Wiener theory to reduce the problem to a functional equation for analytic functions on a riemannian surface of complex characteristics of the operator. We reduce the equation to a Riemann–Hilbert problem on the surface by using Malyshev's automorphic function method, and then obtain all solutions in an explicit form (Mat. Sbornik, Vol. 92, 1973, pp. 89–134 (in Russian)). In 1992 A.E. Merzon and the author extended the method to the angle magnitude $> \pi$ (Operator Theory: Advances and Applications, Vol. 57, 1992, pp. 171–183, Birkhäuser). Using this method, A.E. Merzon established in 1976 the limit absorbing principle to general BVP for the Helmholtz equation in plan angles of magnitude $< \pi$. Recently he obtained the solution of Ursell's problem.

Remark. The limit amplitude principle leading to Sommerfeld's scattering amplitude formula was found in 1986 and has not been proved yet. It seems to be possible to establish the limit amplitude and limit absorbing principles for general BVP for the Helmholtz equation in wedges by our method and to extend them to elasticity and to the Maxwell systems.

VLADIMIR A. KOZLOV:

A strong zero theorem for elliptic boundary value problems in an angle

Let K_ω be the plane angle of opening $\omega \in (0, 2\pi]$ and with the sides $\Gamma_\pm$. Consider the boundary value problem

$$\mathcal{A}(\partial_x, \partial_y)u = f \quad \text{on } K_\omega\,,$$
$$\mathcal{B}_\pm(\partial_x, \partial_y)u = g_\pm \quad \text{on}\Gamma_\pm\,,$$

where $\mathcal{A}$ is an elliptic $N \times N$ system of differential operators with constant coefficients of order $2m$, $\mathcal{B}_\pm$ are $N \times mN$ systems of differential operators with constant coefficients also, whose rows have the orders $b_j^\pm$, $j = 1, \ldots, mN$. We assume that the boundary operators of the problem satisfy the ellipticity condition on $\Gamma_\pm$.

Now suppose that the right–hand sides equal zero in a neighborhood of the vertex and consider a smooth solution subjected to

$$|u(x,y)| \le c_N(x^2 + y^2)^N$$

for small $|x|$, $|y|$ and for all N. If $\omega = \pi$ or $\omega = 2\pi$ this implies that $u = 0$ for small $|x|$, $|y|$. The same is valid if $\det A(1, z)$ has only two roots of multiplicities mN, where A is the leading part of $\mathcal{A}$.

In the general case the ellipticity of the problem is not enough for u to vanish in a neighborhood of 0. Nevertheless under an additional algebraic condition on the leading parts of the operators of the problem this property still holds. This condition is similar to the ellipticity condition for the boundary operators but involves another factorization of A.

RAINER KRESS:

Inverse scattering from an open arc

The mathematical treatment of the scattering of time–harmonic acoustic or electromagnetic waves from thin infinitely long cylindrical obstacles leads to a Dirichlet problem for the Helmholtz equation in the exterior of an arc $\Gamma \subset \mathbb{R}^2$. We consider the corresponding inverse problem to determine the shape of the arc Γ from a knowledge of the far field pattern for the scattering of plane waves. Using a single–layer approach for the solution of the direct problem via a weakly singular integral equation of the first kind, Fréchet differentiability with respect to the boundary is established for the far field operator, which for a fixed incident wave maps the boundary arc onto the far field pattern of the scattered wave. Based on this result and the explicit form of the Fréchet derivate a Newton method is presented for the approximate solution of the inverse problem.

NAUM KRUPNIK:

On the norms of singular integral operators

Some methods of calculation the norms and essential norms of one–dimensional linear singular integral operators (SIO) are proposed. The norms and essential norms of SIO on the contour with cusps are calculated, some new estimates for the norms of SIO on the ellipse are presented.

These results are based on joint work with I. Feldman, R. Duduchava and I. Spitkovsky.

HANS–GERD LEOPOLD:

Pseudodifferential operators and function spaces

The definition of classical function spaces of Sobolev and Besov type is connected with the Laplace operator and its symbol $|\xi|^2$. To relate function spaces and pseudodifferential operators (ΨDO) it is useful to replace the Laplace operator in the definition by suitable hypoelliptic ΨDO's. Function spaces of Sobolev type, defined in such a way, were considered in several papers in 1965–75 and later by R. Beals in 1981. Using the Fourier–analytic approach to Besov spaces, it is possible to define function spaces of variable order of differentiation also of Besov type in a natural way. Variable order of differentiation means that locally in different points x we can get different smoothness demands on a function $u(x)$ belonging to such a space. Therefore it seems useful to consider, for example, degenerate elliptic operators or some hypoelliptic ΨDO's in these generalized function spaces. The definition is done by decompositions which are now induced by the symbol $a(x,\xi)$ of an appropriate ΨDO, instead of $|\xi|^2$. Here we presented the definition and some basic properties of these spaces. For example, we can get them again by real interpolation of function spaces of variable order of differentiation of Sobolev type and it is possible to describe equivalent norms by "variable" differences. Finally if the symbol $a(x,\xi)$ is independent of x, there are connections to function spaces of generalized smoothness too.

GIUSEPPE MASTROIANNI:

Lagrange error estimates in Sobolev weighted norms

Let us denote by $L^s_{u,p}$, s being a non negative integer, $1 < p < \infty$ and u a weight function, the set of functions f such that $||f^{(i)}\varphi^i u||_p^p := \int_{-1}^1 |(f^{(i)}\varphi^i u)(x)|^p dx < \infty$, $i = 0, \dots, s$, $\varphi(x) = \sqrt{1-x^2}$. Let $||f||_{u,p,s} = \sum_{i=0}^s ||f^{(i)}\varphi^i u||_p$ be the norm in $L^r_{u,p}$. By $L_m(w,f)$ we denote the Lagrange polynomial which interpolates a given function f at the zeros of $p_m(w)$, where $\{p_m(w)\}$ is the sequence of orthonormal polynomials, with positive leading coefficients, associated with a weight function w. Further, let us assume that $w \in GJ$ and $u \in GDT$ (for instance, we may have $w(x) = (1-x)^\alpha(1+x)^\beta|x|^\gamma$ and $u(x) = (1-x)^\alpha(1+x)^\beta \left(\log^\gamma \frac{e}{1-x^2}\right) |x|^\Gamma \log^\delta \frac{e}{|x|}$, $\alpha, \beta, \gamma, \Gamma, \delta > -s$), then we show that, for any $f \in L^s_{u,p}$ $s \geq 1$, the bound

$$||L_m(w,f)||_{u,p,s} \leq c||f||_{u,p,s} \quad s \geq 1$$

holds, where c is a positive constant independent of m and f. When $w(x)$ is a Jacobi weight and $u = \sqrt{w}$, the proof is particularly simple and it is presented in the talk.

GIOVANNI MONEGATO:

The numerical evaluation of hypersingular integrals in the Galerkin BEM. 2D problems on polygonal domains

The formulation of boundary value problems in terms of hypersingular integral equations is currently gaining increasing interest. In this talk we consider such type of equations on polygonal boundary and assume to have to solve them by the Galerkin BEM. In particular, given any local (polynomial) basis, we show how to compute efficiently and using a very low number of points all integrals required by the method. These 2D integrals have kernels of the form $\ln r$, r^{-1} and r^{-2}.

Some of the formulas we present can be effectively used to compute also the integrals required by the collocation method.

V.S. RABINOVICH:

The limit operators method and its applications

The well-known local principles of Simonenko, Gohberg–Krupnik, Silbermann and other authors reduce the problem of Fredholmness of singular integral operators, pseudodifferential operators, convolution and Toeplitz operators to the problem of their local invertibility. As a rule this problem is investigated by means of the "freezing" method. But there are many situations where the investigated operators are of local type, however the "freezing" method does not apply.

The lecture is devoted to the limit operators method which could be applied in many situations when the "freezing" method does not apply.

The applications of this method to singular integral operators with symbols having second kind discontinuities and to boundary value problems for differential operators with coefficients having second kind discontinuities are presented.

ALEXANDER G. RAMM:

A new class of multidimensional integral equations basic in estimation theory

A class of multidimensional integral equations basic in estimation theory is introduced and studied. The equations are of the form $\int_D R(x,y)h(y)dy := Rh = f$ (1) in $D \subset R^r$, $r \geq 1$, where D is a bounded region with a smooth boundary. The operator R is a positive rational function of an arbitrary selfadjoint elliptic operator $\mathcal{L}$ in $L^2(\mathbb{R}^r)$. We describe the range of R and the space of the solutions to (1). In general, the solution to (1) is a distribution. Formulas for the solution are given. The singular perturbation problem $\varepsilon h_\varepsilon + Rh_\varepsilon = f$ in D, is discussed.

References:
A.G. Ramm,
1) Random Fields Estimation Theory, Longman, NY., 1990.
2) J. Math. Anal. Appl., 110, N2 (1985), 384–390; 178, N2 (1993), 322–343.

ANDREAS RATHSFELD:

Spline collocation and wavelets for the numerical solution of the double layer equation

The double layer potential equation over the boundary of a polygonal is solved numerically. We apply a collocation method, where the trial functions are spline functions over a uniform partition of a parameter domain. The parametrization is defined with the help of the exponential mapping. Therefore, the partitions over the boundary curve have a geometric mesh refinement near the corner points. The method leads to almost optimal convergence rates and admits a wavelet compression algorithm for the solution of the matrix equation.

STEFFEN ROCH:

Local inclusion theorems for Banach algebras

Let $\mathfrak{A}$ be a Banach algebra with identity e, $\mathfrak{B}$ be a central subalgebra of $\mathfrak{A}$ containing e, and $M(\mathfrak{B})$ be the set of maximal ideals of $\mathfrak{B}$. Given $x \in M(\mathfrak{B})$ let I_x denote the smallest closed two–sided ideal of $\mathfrak{A}$ containing x, and write $\phi_x(a)$ for the coset $a + I_x$. The pair $(\mathfrak{A}, \mathfrak{B})$ is said to be KMS if $\mathfrak{B}$ is a C^*–algebra and if there is a constant $C > 0$ such that

$$||a \sum_{i=1}^{s} b_i|| \leq C \max_{1 \leq i \leq s} \{||ab_i||\}$$

for all $s \geq 2$, $a \in \mathfrak{A}$, and $b_1, \ldots, b_s \in \mathfrak{B}$ having pairwise disjoint supports. The following local inclusion theorems (essentially motivated by the Glicksberg and the Shilov–Bishop theorem) hold:

1. Let $(\mathfrak{A}, \mathfrak{B})$ be a KMS pair and $\mathfrak{C}$ be a subalgebra of $\mathfrak{A}$ containing $\mathfrak{B}$. Then, for all $a \in \mathfrak{A}$,

$$\text{dist}_{ex}(a, \mathfrak{C}) \leq \text{dist}\,(a, \mathfrak{C}) \leq C\,\text{dist}_{ex}(a, \mathfrak{C})$$

(where $\text{dist}_{ex}(a, \mathfrak{C}) = \sup \text{dist}\,(\phi_x(a), \phi_x(\mathfrak{C}))$).
2. If, moreover, $\mathfrak{C}$ is closed, then

$$a \in \mathfrak{C} \Longleftrightarrow \phi_x(a) \in \phi_x(\mathfrak{C}) \quad \text{for all } x \in M(\mathfrak{B}).$$

The proof bases on work by Böttcher, Krupnik, Silbermann (IEOT, 1988) who introduced the notion of KMS and showed that, whenever $(\mathfrak{A}, \mathfrak{B})$ is KMS, then $||a||_{ex} \leq ||a|| \leq C||a||_{ex}$ with $||a||_{ex} := \sup_{x \in M(\mathfrak{B})} ||\phi_x(a)||$. Several applications are given for operators of local type, approximation sequences of local type, and for the finite section method for singular integral operators by weighted Chebyshev polynomials.

ANTONIO DOS SANTOS:

A non–linear method for the generalized factorization of matrix symbols

A non–linear method for investigating the existence of canonical generalized factorization of matrix–valued functions is proposed. The method involves the solution of a homogeneous scalar Riemann–Hilbert problem that is obtained from the usual Riemann–Hilbert problem associated with the factorization by means of a non–linear technique. If the factorization is canonical the factors are obtained from another Riemann–Hilbert problem.

The method proposed applies to several classes of functions that appear in applications (joint work with M.C. Cămara).

JUKKA SARANEN and GENNADI VAINIKKO:

Trigonometric collocation methods with product integration for boundary integral equations on closed curves

We consider equations of the form

$$\sum_{k=0}^{m} \int_0^1 \kappa_k(t-s) a_k(t,s) u(s) ds + \int_0^1 a_{m+1}(t,s) u(s) ds = f(t)$$

with 1–biperiodic functions a_k and 1–periodic functions u, f, κ_k, whereby $|\hat{\kappa}_k(n)| \leq c|n|^{-\alpha_k}$, $n \neq 0$, $\alpha_0 < \alpha_1 \leq \ldots \leq \alpha_m$. Let Q_N be the trigonometric interpolation projection to the linear span of $e^{ip2\pi t}$, $-\frac{N}{2} < p \leq \frac{N}{2}$. We study the method

$$Q_N \sum_{k=0}^{m} \int_0^1 \kappa_k(t-s) Q_{N,s}[a_k(t,s) u_N(s)] ds + Q_N \int_0^1 Q_{N,s}[a_{m+1}(t,s) u_N(s)] ds = Q_N f$$

for the trigonometric polynomial u_N. This yields a very simple matrix form. Under some auxiliary assumptions on the main part we obtain optimal order error estimates in Sobolev norms. Moreover, if the solution u and functions $a_k(t, \cdot)$ have analytic extension out of the real line estimates with exponential rate are shown.

REINHOLD SCHNEIDER:

Multiscale methods for the numerical solution of pseudodifferential equations

We consider Petrov–Galerkin methods for the numerical solution of a pseudodifferential equation $Au = f$, $A \in \Psi^m$. The Petrov–Galerkin scheme is supposed to be subordinated to nested sequences of trial and test spaces $V^0 \subset V^1 \subset ...V^i \subset V^{i+1} \subset ...$, generated by local bases $\{\varphi^j_k\}$. The multiscale bases are based on a direct decomposition $V^{j+1} = V^0 + \sum_{l=0}^{j} W^l$ into different scales l. This decomposition is performed through a, on each scale, local basis $\{\psi^l_\mu\}$. On uniform grids, this kind of basis is refered to as biorthogonal wavelets and provides an unconditional Schauder basis in a wide range of function spaces, e.g. Besov spaces. It is shown that, by using this basis performing the Petrov–Galerkin method, the arising stiffness matrices have uniformly bounded condition numbers. Additionally, most of the coefficients in the stiffness matrix are appropriately small. This results in compressed $N \times N$ matrices with only $O(N)$ or $O(N(\log N)^b)$ nonzero coefficients, preserving a given accuracy ε or preserving an optimal order of convergence.

This talk was based on several joint papers together with W. Dahmen and S. Prössdorf.

ELMAR SCHROHE:

Boundary value problems in Boutet de Monvel's algebra for manifolds with conical singularities

In joint work with B.–W. Schulze a pseudodifferential calculus for boundary value problems on manifolds with finitely many conical singularities is constructed [2]. The idea is to combine Boutet de Monvel's concept for smooth manifolds with boundary with the calculus of B.–W. Schulze for singular manifolds without boundary.

On the smooth part of the manifold, the operators we are considering are standard elements in Boutet de Monvel's algebra. Near one of the singularities the manifold looks like the cone $X \times \overline{\mathbf{R}}_+/X \times \{0\}$, where X is a smooth compact manifold with boundary. All the analysis is then performed on the cylinder $X\times \mathbf{R}_+$. Choosing coordinates (x,t) in $X\times \mathbf{R}_+$, we introduce Mellin symbols with values in Boutet de Monvel's algebra: the action is of Mellin type with respect to the t–direction, while it is pseudodifferential (in the sense of Boutet de Monvel) on the cross-section X. The operators correspondingly act on Sobolev spaces involving the Mellin transform. They coincide with the standard L^2-Sobolev space outside the singularities; close to $\{t = 0\}$ we additionally use weight functions $\sim t^\gamma, \gamma \in \mathbf{R}$ and the Mellin action with respect to t combined with the pseudodifferential action in x. The construction of both, the operators and these Sobolev spaces require the introduction of a parameter–dependent version of Boutet de Monvel's algebra. Here, the parameter plays the role of an additional covariable. Instead of relying on the theory proposed e.g. by G. Grubb, we present a new approach to Boutet de Monvel's calculus based on operator–valued symbols and group actions. This allows a considerably faster access. Moreover, it makes some of the constructions in Boutet de Monvel's algebra more transparent and brings the concept of ('singular') Green, potential and trace operators closer to the usual pseudodifferential theory, cf. also [1]. In order to handle the asymptotics of solutions near the singularities, discrete asymptotics types play an important role.

1. Schrohe, E.: A Characterization of the Singular Green Operators in Boutet de Monvel's Calculus via Wedge Sobolev Spaces, preprint MPI/93-52, MPI für Mathematik, Bonn 1993.
2. Schrohe, E., and B.-W. Schulze: Boundary Value Problems in Boutet de Monvel's Algebra for Manifolds with Conical Singularities I, to appear in Advances in Partial Differential Equations, Akademie-Verlag, Berlin 1994.

BERND SILBERMANN:

On the limiting set of singular values of Toeplitz matrices

Let f be a bounded complex-valued function on the unit circle $\mathbb{T} \subset \mathbb{C}$. Form the Toeplitz matrices

$$T_n(f) := (\hat{f}_{i-j})_{i,j=0}^{n-1},$$

where $\hat{f}_i$, $i \in \mathbb{Z}$, are the Fourier coefficients of f. It will be pointed out that for f locally normal over QC the limiting set of the singular values of the sequence $\{T_n(f)\}$ can be described. This result can be extended to some classes of Toeplitz-like matrices.

IAN H. SLOAN:

Qualocation methods for Symm's integral equation on a polygon

The qualocation method has so far only been applied to boundary integral operators on smooth curves. In this context it has been successful in generating spline approximations that converge faster than the corresponding collocation approximations. In this joint work (with J. Elschner and S. Prössdorf from Berlin), the aim is to see if the recent Elschner-Graham work on the collocation method for polygons can be extended to the higher order qualocation methods. Interesting questions remain.

FRANK-OLME SPECK:

Operator matrix factorization through Bessel potential spaces

Various applications in diffraction theory lead us to the question to find a generalized factorization in $L^2(\mathbb{R})^n$ of a matrix function $G \in \mathcal{G}C^\beta(\dot{\mathbb{R}})^{n\times n} \subset L^\infty(\mathbb{R})^{n\times n}$ which involves unbounded factors in general. So the corresponding factorization of the operator matrix $A = F^{-1}G \cdot F$ is also unbounded in $X = L^2(\mathbb{R})^n$, but can be interpreted as a bounded operator factorization with respect to an intermediate space $Z = \text{im}\, A_+$ in a sense: $A = ACA_+ : X \leftarrow \check{Z} \leftarrow Z \leftarrow X$, more precisely as a cross factorization. It can be shown that Z consists of Bessel potential spaces $\mathop{X}\limits_{j=1}^{n} H^{-\eta_j}(\mathbb{R})$, $|Re\eta_j| < \frac{1}{2}$ assuming that the jumps of G at ∞ are diagonalizable and β is not too small. Generalizations and applications are discussed. See F. Penzel and F.-O. Speck: Asymptotic expansion of singular operators on Sobolev spaces, Asymptotic Analysis 7 (1993), 287-300; and L. Castro: The intermediate Space Problem, MSc thesis, I.S.T., Lisboa, to appear.

ILYA SPITKOVSKY:

Singular integral operators with piecewise continuous coefficients. General contour and weight

A symbol calculus is constructed for operators from the algebra generated by elements of the form $aP_+ + bP_-$, where $P_\pm = \frac{1}{2}(I \pm S)$, S is a canonical singular integral operator with the Cauchy kernel, and the (matrix) coefficients a, b are piecewise continuous.

These operators are considered on spaces $L^p(\Gamma, \varrho)$ with general Hunt–Muckenhaupt–Wheeden weights ϱ, and a contour Γ consisting of simple arcs which may have common endpoints. The talk is based on joint work with I. Gohberg and N. Krupnik.

FRANCISCO S. TEIXEIRA:

Singular integral operators with Carleman shift and unbounded coefficients

A criterion for the Fredholmness of singular integral operators with Carleman shift in $L_p(\Gamma)$ is obtained, where Γ is either the unit circle or the real line. The approach allows to consider unbounded coefficients in a class related to that of quasicontinuous functions. Applications to Wiener–Hopf–Hankel type operators and operators with linear fractional Carleman shift on $\mathbb{R}$ are included.

E. HENNEBACH, PETER JUNGHANNS and GENNADI VAINIKKO:

Radiation transfer problems and weakly singular integral equations with operator–valued kernels

The standard radiation transfer problem in a bounded region $G \subset \mathbb{R}^n$ with non–isotropic scattering is reformulated as a weakly singular integral equation with an unknown function $u : G \to C^m(S^{n-1})$ and a kernel $K : G \times G \to \mathcal{L}(C^m(S^{n-1}))$ which is m times continuously differentiable with respect to the operator strong convergence topology. This observation is taken into the basis of an abstract treatment of weakly singular integral equations with $\mathcal{L}(E)$–valued kernels, where E is a Banach space. We characterize the smoothness of the solutions by proving that they belong to special weighted spaces of smooth functions. On the way, realizing the proof techniques, we established the compactness of the integral operator or its square in $L_p(G, E)$, $BC(G, E)$, and other spaces of interest in the numerical analysis as well in the weighted spaces of smooth functions. The results about the smoothness of the solution are specified for the radiation transfer problem and for the corresponding eigenvalue problem.

LEONID VOLEVICH:

Wiener–Hopf equations in distributions

The lecture will be devoted to the following problems.

1. Abstract definition of convolution operators in $\mathbb{R}^n$ and in the half–space R^n_+. Definition of spaces of convolutors. Description of the algebras of convolutors for Schwartz's spaces.

2. Abstract definition of Wiener–Hopf operators. Algebras of Wiener–Hopf convolutors and their description.
3. Wiener–Hopf operators and convolutors with transmission property. Their description in the case of Schwartz's spaces on the line.
4. Wiener–Hopf equation on the half-line in the case of convolutors with transmission property. Equivalence of
   (i) invertibility of the kernel in the space of convolutors;
   (ii) existence of the canonical factorization of the kernel;
   (iii) the property of the Wiener–Hopf operator to be Fredholm.

Connections of the presented results with the classical theory of M. Krein are discussed.

ELIAS WEGERT:

A weighted norm estimate and nonlinear singular integral equations

The lecture gives an application of a norm estimate for singular integral operators in pairs of weighted Lebesgue spaces to linear and nonlinear integral equations.

In particular we present existence and uniqueness results for nonlinear equations on the complex unit circle $\mathbb{T}$

$$u = HF(.,u)$$
$$v = F(.,Hv)\,,$$

involving the singular integral operator H with the Hilbert cotangent kernel, where the function $F : \mathbb{T} \times \mathbb{R} \to \mathbb{R}$ is assumed to have uniformly bounded continuous derivatives. The integral equations are reduced to fixed point equations. The norm estimates for H guarantee that Schauder's fixed point theorem applies.

HAROLD WIDOM:

Fredholm determinants and differential equations

We discuss determinants $\det(I - K)$, where K is an integral operator with kernel of a certain class, and differential equations associated with them. In the simplest case the kernel is $\lambda \sin(x-y)/\pi(x-y)$, the "sine kernel", and the operator acts on $L_2(-t,t)$. This is a finite Wiener–Hopf operator with symbol $\lambda\chi_{[-1,1]}(\xi)$. In 1980 Jimbo, Miwa, Môri and Sato proved that the logarithmic derivative with respect to t of the determinant satisfies a 2nd order nonlinear differential equation which after transformation becomes a Painléve V equation. We describe joint work with C. Tracy in which we find a (relatively) simple derivation of the equation which extends to more general kernels of the form $[\varphi(x)\psi(y) - \psi(x)\varphi(y)]/(x-y)$ where φ and ψ are related by certain types of differentiation formulae. We also discuss, for the sine kernel, the question of the asymptotics of the determinant as $t \to \infty$, which depend fundamentally on whether $\lambda > 1$, $\lambda = 1$ or $\lambda > 1$. Results obtained (formally) from the differential equation are compared to known results.

LOTHAR VON WOLFERSDORF:

A class of nonlinear Hilbert problems

The lecture reports on the investigation of a general class of nonlinear Hilbert problems for analytic functions. The Hilbert problem has the conjugacy condition

$$\Phi^+(t) = F(\Phi^-(t), t) \tag{1}$$

on the line of discontinuity L with a given function $F(w,t)$, $w \in \mathbb{C}$, $t \in L$, analytic in the first argument w. Via the solution of the problem with the boundary condition differentiated along L the problem (1) is reduced to a fixed point equation for $\Phi^-(t)$ and by means of Schauder's fixed point theorem an existence theorem for the problem is proved. Further the case of a nonanalytic function F with sufficiently small $|F_w|$ is briefly discussed.

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