MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 30/1994

Freie Randwertprobleme

10. - 16.07.1994

The meeting has been organized by Hans Wilhelm Alt (Bonn) and Avner Friedman (Minneapolis). Some of the subjects treated in the talks are the following:

a) Elliptic free boundary problems
b) Stefan problems with Gibbs-Thomson law
c) Asymptotic limits of phase field models
d) Flow through porous media
e) Trijunction point (in 2D)
f) Codimension two free boundary problems

The interest of the audience showed up in lively discussions after the talks. The conference ended with an evening talk of E. DiBenedetto.

VORTRAGSAUSZÜGE

M. BERTSCH:

Nonnegative solutions of a fourth order parabolic equation

The nonlinear parabolic equation

$$u_t + (u^n u_{xxx})_x = c \qquad (n > 0)$$

describes for example the dynamcis of thin liquid films on a surface. Bernis and Friedman have shown that, although no maximum principle can be applied, the degeneracy of the equation at points where $u = 0$ implies the existence of nonnegative solutions. In this talk several aspects of these solutions and their free boundaries were discussed, such as regularity, large time behaviour, positivity properties and nonuniqueness. Several open problems were presented.

L. BRONSARD:

A three layered minimizer in $\mathbf{R}^2$ for a variational problem with a symmetric three well potential

We study stationary solutions for a vector-valued Allen-Cahn equation with a symmetric potential having exactly three wells. In particular , we prove the existence of a "three - layered" solution by variational methods. The role played by this solution in studying the asymptotic behaviour for the vector-valued Allen-Cahn equation should be the same as the role played by the stationary wave solution in the scalar Allen-Cahn model.

G. CAGINALP:

The phase field approach to alloys

This work was done in collaborations with J. Jones and W. Xie. We derive a system of equations describing a binary mixture or alloy with thermal properties and a phase transition. The system involves the concentration, c, the temperature, T, and a phase or order parameter, φ , and is the generalization of the phase field model for a pure material. Transition layers of width ϵ are exhibited by φ, c , and $\nabla T \cdot \hat{n}$. The model identifies all macroscopic parameters. In the limit as $\epsilon \to 0$, with all other parameters held fixed, one attains a sharp interface problem.
The simplest equations (not too far from equilibrium) have the form

$$c_v T_t + \frac{1}{2}\varphi_t = \nabla \cdot (K_1 \nabla T), \qquad c_t = \nabla \cdot (K_2(\varphi) c(1-c)[N\varphi + RT ln \frac{c}{1-c}]),$$

$$\alpha\epsilon^2 \varphi_t = \epsilon^2 \Delta\varphi + \frac{1}{2}(\varphi - \varphi^3) + \frac{\epsilon}{\epsilon\sigma}[s]_E \{T - T_A c - T_B(1-c)\}$$

where $N := \frac{1}{2}[s]_E(T_A - T_B)$ and T_A, T_B are the two melting temperatures while $[s]_E$ is the entropy jump. In the limit as $\epsilon \to 0$, the sharp interface problem includes the extension of the Gibbs-Thomson relation:

$$[ln \frac{c}{1-c}]^{+}_{-} = -2N,$$

$$-\sigma(\alpha v + k) = [s]_E \{T - T_B - \frac{T_A - T_B}{2N} ln \frac{1-c^+}{1-c^-}\}.$$

These reduce to well known conditions as c is small.

E. DIBENEDETTO and V. VESPRI:

On the singular equation $\beta(u)_t - \nabla u \ni 0$

It is shown that a solution $u \in L^\infty_{loc}(0,T; L^2_{loc}(\Omega)) \cap L^2_{loc}(0,T; W^{1,2}_{loc}(\Omega))$ of this partial differential equation is continuous in its domain of definition for any coercive maximal monotone graph $\beta(\cdot) \subset \mathbf{R} \times \mathbf{R}$. This, for example, might exhibit several jumps on $\beta(0)$ and/or singularities on $\beta'(\cdot)$. The result extends the classical continuity theorem of the early '80s for $\beta(\cdot)$ a graph of the Stefan-type .

M. FLUCHER:

Concentration of low energy solutions of elliptic free-boundary problems

For low energy solutions of elliptic free-boundary problems we prove concentration at a harmonic center of the underlying domain. Examples are Bernoulli's free-boundary problem, the plasma problem, and certain obstacle problems with movable obstacle. In the planar case the proof is based on the concentration-compactness alternative of P.L. Lions and renormalization of the energy in order to detect its dependence on the point of concentration. It turns out that in the limit the energy of the minimizers only depends on the value of the harmonic radius at this point. Thus the point of concentration is a harmonic center. This is a general feature of variational integrals that reveals precise information on the asymptotics of the minimizers. In higher dimensions it is more difficult to prove concentration at a single point. This requires a new concentration-compactness alternative for variational solutions.

For the numerical solution of Bernoulli's free-boundary problem we derive a second order trial method which in combination with the boundary element method permits fast approximation of the free boundary and works equally well in the elliptic (high energy) and the hyperbolic (low energy) regime.

H. GARCKE:

On the Cahn- Hilliard equation with degenerate mobility and its asymptotic limit

An existence result for the Cahn-Hilliard equation with a concentration dependent mobility is presented. In particular the mobility is allowed to vanish in the pure phase. This leads to a degenerate parabolic equation of fourth order.

Furthermore I discuss how the Cahn-Hilliard equation is related to a sharp interface model. Formal asymptotic results suggest that one gets in the limit (for an appropriate scaling) the motion by surface diffusion

$$V = -D\Delta_S \kappa$$

which has a long history in material science. We present a local existence result in a two-dimensional situation for motion by surface diffusion and two related geometrical evolution laws. All there laws have the common property that they decrease perimeter and preserve area. This fact is used to prove results global in time for small perturbations of a circle.

D. HILHORST:

Singular limit for some nonlocal reaction-diffusion equations

We consider some nonlocal reaction-diffusion equations which arise in pattern formation in population dynamics, in chemical reactions and in the microphase separation of block copolymers and prove that their solutions converge to the solutions of some moving boundary problems involving motion by mean curvature as a small parameter tends to zero.

The mathematical method either involves the construction of sub- and super-solutions or the use of matched asymptotic expansions according to the precise problem that we consider. This is joint work with X. Chen and E. Logak.

D. HÖMBERG:

Irreversible phase transitions in carbon steel

We present a mathematical model for the austenite-pearlite and austenite-martensite phase transitions in eutectoid carbon steels.

While the pearlitic transformation is described using Scheil's Additivity Rule, for the formation of martensite we use a rate law, which takes into account the irreversibility of this phase change.

After stating a well-posedness result for the three-dimensional model, we use this model for a numerical simulation of the Jominy end quench test applied to two different plain carbon steels.

V. ISAKOV:

Uniqueness in the inverse conductivity problem

We are looking for a domain $D \subset \Omega$ entering the following elliptic boundary value problem

$$div((1 + \chi(D))\nabla u) = f \quad \text{in} \quad \Omega, \quad u = g \quad \text{on} \quad \partial\Omega$$

from the Neumann data $\partial u/\partial \nu = h$ on $\partial\Omega$. Here f, g, h are given functions and Ω is a domain in $\mathbf{R}^n$. We expose the results of the speaker as well as of Alessandrini, Bellout, Friedman, and Powell. When $f \geq 0$ and $g = 0$ we prove global uniqueness for convex D. When $f = 0$ we can obtain only local uniqueness results in the plane case. We discuss methods of the proofs and difficulties. By writing the diffraction conditions on

∂D and using uniqueness of the harmonic continuation we can reduce this problem to the following free boundary problem for harmonic functions

$$\Delta u = 0 \quad \text{in} \quad D, \quad u = u^e, \partial u/\partial \nu = 2\partial u^e/\partial \nu \quad \text{on} \quad \partial D$$

where u^e is a given function.

P. KNABNER:

Special solutions for a free boundary problem from crystal dissolutions in porous media flow

We consider a diffusion-reaction system consisting of two parabolic and one ordinary differential equation, containing a set-valued nonlinearity. This system is a model for solute transport in porous media with precipitation or dissolution of an immobile crystalline phase, the reaction being in non-equilibrium. A conserved quantity, the electric charge c, allows for a reduction in unknowns. Travelling wave solutions exist only for constant c and as dissolution waves, exhibiting a sharp aissolution front, the free boundary. Also the convergence to the singular limits for vanishing dispersion D or rate parameter to ∞ can be quantified. For nonconstant c the solution profiles are more complicated. For $D = 0$ the Riemann problem is investigated leading to an integral equation for the free boundary.

Y. LIU:

Free boundary problems in plasma physics

In this talk I presented a two phase free boundary problem arising in a magnetohydrodynamic system which models the plasma confinement in the Tokamak machine. The mathematical formulation is given as follows:

Let Ω be boundet domain in $\mathbf{R}^n$, and let γ, μ^2 be given positive constants. The plasma problem is to find a function u,a closed surface Γ_p lying in Ω and a positive constant λ such that

$$\begin{aligned} \Delta u = 0 \quad &\text{in} \quad \Omega_v = \{u > 0\}, \\ \Delta u + \lambda u = 0 \quad &\text{in} \quad \Omega_p = int\{u \le 0\}, \\ U = 0 \quad &\text{on} \quad \Gamma_p, \\ |\nabla u^+|^2 - |\nabla u^-|^2 = \mu^2 \quad &\text{on} \quad \Gamma_p, \\ u = \gamma \quad &\text{on} \quad \Gamma = \partial\Omega, \end{aligned}$$

and

$$\int_\Omega (u^-)^2 \quad \text{is prescribed.}$$

This is the joint work with Avner Friedman. Our approach is based on the variational method developed by Alt, Caffarelli and Friedman, and on the Harnack inequality method of Caffarelli. We proved that the plasma problem has a solution u which is Lipschitz continuous and, in the case where $n = 2$, the free boundary Γ_p is analytic so that the pair (u, Γ_p) is a classical solution to the plasma problem. Some properties regarding the plasma region Ω_p are given.

J. OCKENDON:

Codimension-two free boundary problems

There are many free boundary problems where the free boundary lies so close to a fixed boundary that the perimeter of their intersection is the only geometric feature of interest. Prototypes are the problem of percolation in a gently sloping sandbank and the early stage of penetration of an inviscid liquid by a rigid obstacle. In both these cases the theory of mixed boundary value problems can often be used to provide an explicite solution. However, the following general questions remain open:

(i) the accuracy with which codimension-two models approximate their codimension-one progenitor

(ii) the existence, uniqueness and regularity of the codimension-two model when formulated as a mixed boundary value problem

(iii) the construction of weak formulations and associated numerical algorithms.

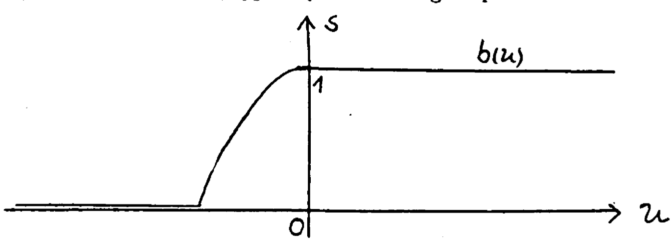
F. OTTO:

Saturated-unsaturated flow through porous media

The following equation of elliptic-parabolic type

$$\partial_t s + \operatorname{div}[a(\nabla u + k(s)e)] = 0, \quad s = b(u)$$

describes nonstationary saturated-unsaturated flow in a porous medium: $s \in [0, 1]$ denotes saturation and $u \in \mathbf{R}$ the (transformed) pressure. e is the gravity vector and b a continuous, monotone function, typically of following shape:



There exist a solution (s, u) to given initial data for s and boundary conditions (Alt, Luckhaus & Visintin 1984), namely (depending on the segment)

- homogeneous Neumann boundary conditions for the flux $a(\nabla u + k(s)e)$ (contact with an impervious layer)

- positive Dirichlet boundary conditons for u (contact with water)
- unilateral boundary condition (contact with air, permits for overflow)

I dealt with uniqueness and the behaviour of solutions (s, u) when b tends to the heavyside function. I can prove uniqueness and strong convergence for heterogeneous and anisotropic permeability $a = a(x) \in \mathbf{R}^n$ to a well posed limit problem (the so-called dam problem, elliptic/scalar conservation law type). The difficulty are the boundary conditions; we improve techniques of Alt & Luckhaus and Carrillo (1993).

M. PRIMICERIO:

Zones of coexistence of phases in crystallization processes

We study the crystallization process in a molton polymer and we characterize it by the nucleation rate, growth rate and by a hindering factor of such rates, to take into account the effect of impingement amoung growing crystals. This scheme - resulting in an integro-differential equation for the crystal fraction - is an alternative approach to the Avrami-Kolmogorov model. We consider the coupling with the temperature field through the latent heat term and study in particular isokinetic processes. A solution in form of a travelling wave is found and analyzed in a model problem and asymptotic cases are discussed in some detail.

J.F. RODRIGUES:

On the Stefan problem with prescribed convection

The general Stefan problem with a convective term, given by a L^2- solinoidal velocity field $\vec{v}$ (without regularity assumptions), corresponds essentially to the singular parabolic equation for the couple $\{\eta, \vartheta\}$ (enthalpy and temperature)

$$(\partial_t + \vec{v} \cdot \nabla)\eta - \Delta\vartheta + f(\vartheta) = 0, \quad \eta \in \gamma(\vartheta) \quad \text{in} \quad \Omega \times (0,T)$$

where γ is a strictly increasing maximal monotone graph. Existence results were presented for weak (bounded) solutions, with mixed boundary conditions

$$\vartheta = \vartheta_D \quad \text{on} \quad \partial_D\Omega \times (0,T) \quad \text{and} \quad -\frac{\partial\vartheta}{\partial n} = g(x,t,\vartheta) \quad \text{on} \quad \partial_N\Omega \times (0,T)$$

and initial condition $\eta(0) = \eta_0$ in Ω, where the nonlinearities, although very general, are compatible with the weak maximum principle. Its uniqueness and its continuous dependence were also obtained for general velocities and some partial results on the steady-state problem and on the asymptotic behaviour as $t \to \infty$ were also discussd.

R. SCHÄTZLE:

A counterexample for an approximation for the Gibbs-Thomson law

Luckhaus and Modica have shown that solutions of a semilinear-elliptic equation arising from Landau-Ginzburg theory satisfy in the limit the Gibbs-Thomson law, if the mass is preserved. The same problem appears in connection with the quasistationary and the ordinary phase-field equations but without preservation of mass. In the radial-symmetric case, one can renounce the preservation of mass, but in the general case even in dimension two there is a counterexample.

A. SCHMIDT:

The computation of threedimensional dendrites with finite elements

Starting from an initial seed crystal inside of an undercooled liquid, the solid phase begins to grow rapidly and develops instable growth patterns. Some growth directions are preferred because of anisotropic parameters in the physical model. This results in the development of dendrites. The physical model includes the heat equation for both the liquid and solid phases. The Gibbs-Thomson law couples the velocity of the interphase, its curvature and the temperature.

We describe a numerical method that enables us to compute dendritic growth of crystals in two and three spaces dimensions.

The method consists of two coupled finite element algorithms. The first one solves the heat equation in the container; the other one operates on a discretization of the free boundary and computes the evolution of this moving interface. The two methods work with totally independent grids. By using timedependent, locally refined and coarsened adaptive meshes in both methods, we are able to reach a spatial resolution necessary to compute dendritic growth in two and three space dimensions.

H. M. SONER:

Convergence of phase field equations to Mullins-Sekerka problem with kinetic undercooling

Phase field equations for solid-liquid phase transitions are

$$\theta^\epsilon_t - \Delta\theta^\epsilon + g(\varphi^\epsilon)\varphi^\epsilon_t = 0 \quad \text{in} \quad (0,\infty)\times \mathbf{R}^d$$

$$\varphi^\epsilon_t - \Delta\varphi^\epsilon + \frac{1}{\epsilon^2}W'(\varphi^\epsilon) - \frac{1}{\epsilon}g(\varphi^\epsilon)\theta^\epsilon = 0 \quad \text{in} \quad (0,\infty)\times \mathbf{R}^d,$$

where θ^ϵ is the temperature deviation, φ^ϵ is the phase field and

$$W(\varphi) = \frac{1}{2}(1-\varphi^2)^2 \quad , \quad g(\varphi) = \sqrt{2W}.$$

Then as $\epsilon \downarrow 0$ we prove that $\theta^\epsilon \to \theta, \varphi^\epsilon \to \varphi$ and $|\varphi| = 1$. Moreover $(\theta, \Omega(t) = \{\varphi = -1\})$ solves

$$\theta_t - \Delta\theta - g(\varphi)\varphi_t = 0 \quad \text{in} \quad (0,\infty)\times \mathbf{R}^d$$

$$V = -\kappa - \theta \quad \text{on} \quad \partial\Omega(t),$$

where V is the normal velocity and κ is the mean curvature of $\partial\Omega(t)$.

B. STOTH:

On the convergence of the nonlocal Allen-Cahn equation to volume preserving mean curvature flow

We present a convergence proof for solutions of the nonlocal Allen-Cahn equation

$$\begin{aligned} \epsilon\partial_t\varphi_\epsilon - \epsilon\Delta\varphi_\epsilon + \frac{1}{\epsilon}W'(\varphi_\epsilon) &= \lambda_\epsilon \quad \text{in} \quad \Omega_t, \\ \partial_\nu\varphi_\epsilon &= 0 \quad \text{on} \quad (\partial\Omega)T, \\ \lambda_\epsilon &= \frac{1}{\epsilon}\int_\Omega W'(\varphi_\epsilon)dx \end{aligned}$$

as the parameter ϵ tends to zero. Formal asymptotic analysis by Rubinstein-Sternberg suggests that φ_ϵ will develop a transition layer of width ϵ. The limit of those solutions will consequently have an interface which solves the volume preserving mean curvature flow

$$V + K = \fint_\Gamma K$$

where V is the normal velocity and K the sum of principal curvatures of the interface Γ. Thus the parabolic evolution equation degenerates into a purely geometric evolution equation for an interface.

Our proof applies in a radial setting, assuming several concentric spheres initally. We use energy type estimates to obtain an estimate between the solution and the well known stationary well solution.

J.J.L. VELAZQUEZ:

Singularities for Stefan problems

It is well known that the velocities of interfaces in the classical Stefan problem may became singular at particular points and times. We describe some detailed mechanisms of formation of singularities for the Stefan problem with and without undercooling. It turns out that some of the previous mechanisms of formation of singularities are related to similar mechanisms of formation of singularities for the Keller-Segal model in biology.

A. VISINTIN:

Two-scale Stefan problem

In the classical Stefan problem, surface tension can be described by the Gibbs-Thomson law, which prescribes that the mean curvature of the interface is proportional to the temperature.

Phase transition can occur either continuously (by motion of the interface), or discontinuously (by necleation or annihilation). A problem accounting for adiabatic nucleation was formulated by Luckhaus in the framework of Sobolev space (cf. Euro. J. Appl. Math. 1990)

A modified model is here proposed, which accounts for nonadiabatic nucleation under small undercooling or superheating (what seems to be a more realistic description). This model is based on an average procedure, which can be regarded as a sort of scale transformation, from mesoscopic to macroscopic variables. The known existence proof can be extended to this modified model.

G. WEISS:

A free boundary problem for non-linear elliptic equations arising in elasticity

The regularity of the free boundary of local minima of the functional $v \mapsto \int_\Omega (f(\nabla v) + Q^2 \chi_{\{v>0\}})$, which has been proved in 1984 by H.W. Alt, L.A. Caffarelli and A. Friedman for radial symmetric f, is extended to the non-radial-symmetric case, under the restriction of $p \mapsto p \cdot \nabla f(p) - f(p)$ being strictly convex.

Functionals of the above type can be used to model the peeling of a membrane from a surface in the context of nonlinear elasticity.

Berichterstatter: H.W. Alt

Tagungsteilnehmer

Prof.Dr. Hans Wilhelm Alt
Institut für Angewandte Mathematik
Abteilung Funktionalanalysis und
Numerische Mathematik
Wegelerstr. 6

D-53115 Bonn

Dr. Martin Flucher
Forschungsinstitut für Mathematik
ETH-Zentrum, HG G 38.1

CH-8092 Zürich

Prof.Dr. Michiel Bertsch
Dipartimento di Matematica
Universita degli Studi di Roma
Tor Vergata
Via della Ricerca Scientifica

I-00133 Roma

Dr. Harald Garcke
Institut für Angew. Mathematik
Universität Bonn
Wegelerstraße 6

D-53115 Bonn

Dr. Lia Bronsard
Dept. of Mathematics
Mc Master University

Hamilton, Ontario L8S 4K1
CANADA

Günther Grün
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

D-53115 Bonn

Prof.Dr. Gunduz Caginalp
Dept. of Mathematics
and Statistics
University of Pittsburgh
507 Thackeray Hall

Pittsburgh, PA 15260
USA

Prof.Dr. Miguel Herrero
Departamento de Matematica Aplicada
Universidad Complutense de Madrid

E-28040 Madrid

Prof.Dr. Emmanuele DiBenedetto
Dept. of Mathematics
Northwestern University

Evanston ,Ill 60208-2730
USA

Prof.Dr. Danielle Hilhorst
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Batiment 425

F-91405 Orsay Cedex

Dr. Dietmar Hömberg
Institut für Angewandte Analysis
und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39

D-10117 Berlin

Barbara Niethammer
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

D-53115 Bonn

Prof.Dr. Victor Isakov
Department of Mathematics
and Statistics
Wichita State University

Wichita , KS 67260-0033
USA

Dr. John R. Ockendon
Mathematical Institute
Oxford University
24 - 29, St. Giles

GB-Oxford , OX1 3LB

Prof.Dr. Peter Knabner
Institut für Angewandte Mathematik
Universität Erlangen
Martensstr. 3

D-91058 Erlangen

Dr. Felix Otto
Institut für Angewandte Mathematik
Wegelerstr. 6

D-53115 Bonn

Prof.Dr. Yong Liu
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.

Minneapolis , MN 55455
USA

Prof.Dr. Mario Primicerio
Dipt. di Matemàtica "U.Dini"
Università di Firenze
Viale Morgagni 67/A

I-50134 Firenze

Prof.Dr. Stephan Luckhaus
Institut für Angewandte Mathematik
Abteilung Funktionalanalysis und
Numerische Mathematik
Wegelerstr. 6

D-53115 Bonn

Prof.Dr. Jose-Francisco Rodrigues
CMAF & University of Lisboa
2, Av. Prof. Gama Pinto
Centro de Matematica e
Aplicacões Fundamentais

P-1699 Lisboa Codex

Reiner Schätzle
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

D-53115 Bonn

Prof.Dr. Augusto Visintin
Dipartimento di Matematica
Universita di Trento
Via Sommarive 14

I-38050 Povo (Trento)

Alfred Schmidt
Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Str. 10

D-79104 Freiburg

Prof.Dr. Georg Weiß
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

D-53115 Bonn

Prof.Dr.Mete Soner
Department of Mathematics
Carnegie Mellon University

Pittsburgh , PA 15213-3890
USA

Barbara Stoth
Institut für Angew. Mathematik
Universität Bonn
Wegelerstraße 6

D-53115 Bonn

Prof.Dr. Juan J. L. Velazquez
Departamento de Matematica Aplicada
Universidad Complutense de Madrid

E-28040 Madrid

Alt, Hans Wilhelm	willi @sfb256.iam.uni-bonn.de
	FAX: +49-228-73-7864
Bertsch, Michel	bertsch @ mat.utovrm.it
Bronsard, Lia	bronsard @ icarus.math.mcmaster.ca
Flucher, Martin	flucher @ math.ethz.ch
Garcke, Harald	harald @ sfb256.iam.uni-bonn.de
Grün, Günther	gg @ sfb256.iam.uni-bonn.de
Hilhorst, Danielle	hilhorst @ ianors.matups.fr
	FAX: +33-1-69416718
Hömberg, Dietmar	hoemberg @ iaas-berlin.d400.de
Niethammer, Barbara	igel @ sfb256.iam.uni-bonn.de
Otto, Felix	felix @ sfb256.iam.uni-bonn.de
Primicerio, Mario	primice @ udini.math.unifi.it
Mete, Soner	hsow @ andrew.cmu.edu
	mete @ fermat.math.cmu.edu
Stoth, Barbara	bstoth @ sfb256.iam.uni-bonn.de
	FAX +49-228-73-7864
Schätzle, Reiner	reiner @ sfb256.iam.uni-bonn.de
	mpfa7 @ central.sussex.ac.uk
Schmidt, Alfred	alfred.schmidt@mathematik.uni-freiburg.de
Visintin, Augusto	visintin @ science.unitn.it
Weiß, Georg	georgw @ sfb256.iam.uni-bonn.de