MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 31/1994

Algebraische Zahlentheorie

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Die Tagung fand unter der Leitung von Herrn Prof. C. Deninger (Münster), Herrn Prof. G. Frey (Essen) und Herrn Prof. P. Schneider (Köln) statt. Die Vortragenden berichteten über aktuelle Forschungsergebnisse, die überwiegend aus dem Bereich der Algebraischen Zahlentheorie und der Arithmetischen Geometrie stammten. Zwei Vorträge behandelten Fragen der konstruktiven Galoistheorie.

Vortragsauszüge

The fundamental group of arithmetic surfaces
Yasutaka Ihara

In this talk it was shown that several arithmetic surfaces have trivial geometric fundamental groups. The method is to find a suitable divisor D on the arithmetic surface X in question, such that $\pi_1(D) \to \pi_1(X)$ is surjective and that $\pi_1^{geom}(D)$ is small. By using some verical prime divisor Yasutaka showed that $\pi_1^{truly\ geom}(X) = 1$ if X is the normalization of $\mathbb{P}^1/\mathcal{O}$ in a pro-p extension (finite) of the function field of $\mathbb{P}^1/k$ unramified outside $0, 1, \infty$ (k: any number field, $\mathcal{O}$: the ring of integers). By using some horizontal prime divisors and a theoren of Belyi, he showed that every curve over a number field k having a k-rational point has an arithmetic surface model over $\mathcal{O}$ which is normal and has trivial geometric fundamental group. Other series of examples are related to regular models and to applications of D ="mixed type divisor".

Artin-Verdier duality for arithmetic surfaces
Michael Spieß

Let X be a 2-dimensional proper $\mathbf{Z}$-scheme. Spieß constructs a complex $\mathcal{K}$ on X and proves the following duality Theorem for the étale cohomology of constructible sheaves $\mathcal{F}$ on X: The Yoneda-pairing combined with a natural trace map $H^6(X,\mathcal{K}) \to \mathbf{Q}/\mathbf{Z}$ gives a non-degenerate pairing of finite groups

$$H^i(X,\mathcal{F}) \times Ext_X^{6-i}(\mathcal{F},\mathcal{K}) \to \mathbf{Q}/\mathbf{Z}.$$

The construction of $\mathcal{K}$ is analogous to Deninger's construction of the dualizing complex for one-dimensional schemes and uses Lichtenbaum's compex $\Gamma(2)$ for the generic points of X. Using the duality theorem and results of Bloch and Jannsen, Spieß gets a new proof of a result of Kato/Saito concerning 2-dimensional Class field theory.

An analog of Tate's conjecture on exterior Galois representations in pro-l fundamental groups of algebraic curves
Hiroaki Nakamura

Let X be a hyperbolic algebraic variety defined over a number field k, and put $G_k = Gal(\bar{k}/k)$, $\bar{X} = X \otimes \bar{k}$. Then there is a natural exact sequence

$$1 \to \pi_1(\bar{X}) \to \pi_1(X) \to G_k \to 1$$

of profinite fundamental gorups. This induces also a natural exterior Galois representation $\varphi_X : G_k \to Out(\pi_1(\bar{X}))$. When $\pi_1(\bar{X})$ has trivial center, the automorphism group of $\pi_1(X)$ over G_k modulo inner automorphisms by $\pi_1(X)$ is isomorphic to the centralizer of the Galois image $\varphi_X(G_k)$ in $Out(\pi_1(\bar{X}))$ denoted by $Z(\varphi_X)$. A problem raised by A. Grothendieck is to find a class of X such that the natural mapping $\Phi_X : AutX \to Z(\varphi_X)$ gives a bijection (an aspect of "anabelian" algebraic varieties). Nakamura explained the pro-l version of the problem for X = hyperbolic curves $/k$. In particular he showed the result (Math. Z. 206) that $Aut_k(\mathbf{P}^1\backslash\{0,1,\infty\}) \xrightarrow{\cong} Z(\varphi^{pro-l}_{\mathbf{P}^1\backslash\{0,1,\infty\}})$, in this case due to Anderson, Coleman, Ihara-Kaneko-Yuhinari. He also described how this result can be extended to the case of higher genus curves (cf. Crelle 441).

A p-adic analgue of Beilison's conjectures for Hecke characters of imaginary quadratic fields

Thomas Geisser

Let φ be a Hecke character of an imaginary quadratic field K of weight $w > 0$ and infinity type (a, b). Then one can attach a motive M_φ to φ in the category $M_{\mathbb{Z}_p}(K)$ of Chow motives over K with coefficients in $\mathbb{Z}_p$ and prove the following

Theorem Let $p \gg 0$ be split in K, $l \geq 0$, $p - 1 \nmid b + l + 1$, and $b + l \neq 0$. Then there exists a submodule $V \subseteq K_{2l+w+1}(M_\varphi, \mathbb{Z}_p)^{(l+w+1)}$ such that the length as an $\mathcal{O}_\varphi$-module of the cokernel of a regulator map R restricted to V equals the valuation of the p-adic L-function $G(\bar{\varphi}^{-1}\kappa^{-l}, u^{-b-l} - 1, u^{-a-l} - 1)$.

Since the p-adic L-series in question is a p-adic analogue of $L(\bar{\varphi}, -l)$, this fits neatly into the Beilinson conjectere framework. The module V arises in connection with elliptic units, and the proof uses the fact that that the quotient of local and elliptic units is related to p-adic L-functions.

On Beilinson's conjecture for Hilbert-modular surfaces

Guido Kings

Let $F/\mathbb{Q}$ be real qudratic, $G := \mathbb{G}_m \times_{Res_{F/\mathbb{Q}}\mathbb{G}_m} Res_{F/\mathbb{Q}} GL_2$ and $S/\mathbb{Q}$ be a canonical model of the Shimura variety associated to G. Let W be the standard representation of GL_2 and $V^{p,q} := Sym^p \hat{W} \otimes Sym^q \hat{W}$, where $\hat{W}$ is the dual representation. Kings constructed a (relative) motive $\mathcal{V}^{p,q}/S$ such that $H_{et}^{\bullet+p+q}(\mathcal{V}^{p,q} \times \bar{\mathbb{Q}}, \mathbb{Q}_l) \cong H_{et}^{\bullet}(S \times \bar{\mathbb{Q}}, \tilde{V}^{p,q})$. Let $H_{!\mathcal{D}}^{\bullet}(\mathcal{V}^{p,q} \otimes \mathbb{R}, \mathbb{R}(*)) \subset H_{\mathcal{D}}^{\bullet}(\mathcal{V}^{p,q} \otimes \mathbb{R}, \mathbb{R}(*))$ be the elements "coming from a compactification". Then there is the following result (weak form of Beilinson's conjecture on L-values outside the central strip):
Let $p \geq q \geq 1$ and $\pi \subset L^2(G(\mathbb{Q})\backslash G(\mathbb{A}))$ be stable, then

$$dim_{\mathbb{R}\otimes\bar{\mathbb{Q}}} H_{!\mathcal{D}}^{p+q+r}(\mathcal{V}^{p,q} \otimes \mathbb{R}, \mathbb{R}(p+q+3-l))(\pi_f) = 1 \text{ for } l = 1, \cdots, q$$

and there exists a $G(\mathbb{A}_f)$ -module $\mathcal{K}(p,q,l) \subset H_{\mathcal{M}}^{p+q+3}(\mathcal{V}^{p,q}, \mathbb{Q}(p+q+3-l))$ $(l = 1, \cdots, q)$, such that

$$r_{\mathcal{D}}(\mathcal{K}(p,q,l)) \subset H_{!\mathcal{D}}^{p+q+3}(\mathcal{V}^{p,q} \otimes \mathbb{R}, \mathbb{R}(p+q+3-l))$$
$$r_{\mathcal{D}}(\mathcal{K}(p,q,l))(\pi_f) = L(\pi_f, l-3-p-q)^{*} R(p,q,l)(\pi_f)$$

Here $r_{\mathcal{D}}$ is Beilinson's regulator, $L(\pi_f, s)^{*}$ is the leading coefficient of the Taylor series expansion of L in s and $R(p,q,l)(\pi_f)$ is Beilinson's $\bar{\mathbb{Q}}$-structure on $H_{!\mathcal{D}}^{p+q+3}(\mathcal{V}^{p,q} \otimes \mathbb{R}, \mathbb{R}(p+q+3-l))(\pi_f)$.

Derivatives of p-adic L-functions
Ralph Greenberg

In this lecture, Greenberg described various formulas for the derivative of a p-adic L-functions at the so-called "trivial zero". In the case of Kubota-Leopoldt p-adic L-functions, the formula allows one to prove the simplicity of a trivial zero. In the case of the p-adic L-function attached to an elliptic curve $E/\mathbb{Q}$, there is a trivial zero (at $s = 1$) just when the E has split multiplicative reduction at p. The simplicity (when the complex L-function is nonzero at $s = 1$ and assuming E is modular) turns out to amount to a certain conjecture about the Tate period for E. This is a consequence of a formula for the derivative proved by Glenn Stevens and Greenberg. There is a similar formula for the derivative at $s = 2$ for the p-adic L-function associated to the symmetric square of such an elliptic curve, proved by Jaques Tilouine and Greenberg. All of these formulas take a very similar form, involving a factor $\mathcal{L} = log_p(q)/ord_p(q)$ for some $q \in \mathbb{Q}_p^*$. In the lecture , Greenberg indicated a common way to define the p-adic number $\mathcal{L}$.

Uniform boundedness of the torsion of elliptic curves
Loic Merel

The lecture consisted in a partial exposition of the proof of the strong uniform boundedness conjecture:

Theorem The order of a torsion point of an elliptic curve over an number field of degree d is bounded by a constant which depends only on d.

This was proved by Mazur for $d = 1$, by Kamienny for $d = 2$, by Mazur and Kamienny for $3 \leq d \leq 14$ and by Merel in the general case. By the work of Faltings, Frey, Kamienny and Mazur it was known to follow from the existence of a bound for torsion prime. In particular, refining the techniques of Merel, Oestérlé proved: *Torsion primes in degree d are bounded by* $(3^{d/2} + 1)^2$.

Finiteness of torsion in codimension two Chow groups; the case of a product of modular elliptic curves
Andreas Langer

Let E be a modular elliptic curve over $\mathbb{Q}$ without complex multiplication over $\bar{\mathbb{Q}}$, let N be its conductor and p a prime not dividing N. Consider the codimension two Chow group $CH^2(E \times E)$. Then the following theorem holds

Theorem 1 The p-primary torsion $CH^2(E \times E)\{p\}$ is a finite group.

This provides a first finiteness result on torsion of codimension two Chow groups in the case $H^2(X, \mathcal{O}_X) \neq 0$. The result is cosely related to the following

Theorem 2 The cycle map $\rho_p : CH^2(E \times E)\{p\} \to H^4_{cont}(E \times E, \mathbb{Z}_p(2))$ is injective.

Both theorems were obtained in joint work with Shuji Saito and use methods of Mildenhall and Flach and in addition Jannsen's cohomological Hasse principal and local class field theory for curves over $\mathbb{Q}_l$.

Galois module structure of K-groups of rings of integers
Ted Chinburg

This talk was about recent work on the $G = Gal(N|K)$-module structure of the K groups $K_i(\mathcal{O}_N)$ of the ring of integers of a finite Galois extension $N|K$ of number fields. By a theorem of Borel, there is an explicit integral combination Y'_{2n+1} of permutation G-modules for each integer $n \geq 0$ such that the class $\Omega_n(N|K)^{Naive} = (K_{2n+1}(\mathcal{O}_N)) - (K_{2n}(\mathcal{O}_N)) - (Y'_{2n+1})$ has finite order in the Grothendieck group $G_0(\mathbb{Z}G)$ of all finitely generated $\mathbb{Z}G$- modules. A basic problem in number theory has been to construct arithmetically a natural pre-image $\Omega_n(N|K)$ of $\Omega_n(N|K)^{Naive}$ in the Grothendieck group $K_0(\mathbb{Z}G)$ of all finitely generated projective $\mathbb{Z}G$-modules. One would like to relate the image of $\Omega_n(N|K)$ in various quotients of $K_0(\mathbb{Z}G)$ to the leading term $L^*(-n, V)$ in the expansions at $s = n$ of Artin L-series $L(s, V)$ of representations V of G. This should in turn relate $\Omega_n(N|K)$ to Artin root numbers $W(V)$.
By generalizing results of Snaith and Pappas from 1993, Snaith, Pappas, Kolster and Chinburg proved:

Theorem 1) If $n = 1$, there is a canonical preimage $\Omega_1(N|K) \in K_0(\mathbb{Z}G)$ of $\Omega_1(N|K)^{Naive}$ which is defined using work of B. Kahn on the Galois cohomolgy of K-groups.
2) A form of Lichtenbaum's conjecture about $L^*(-1, V)$ implies $\Omega_1(N|K) \equiv W_{N|K}$ mod $D(\mathbb{Z}G)$.
3) The function field counterpart of $\Omega_1(N|K)$ satisfies $\Omega_1(N|K) = W_{N|K}$ if $char(N) \nmid \#G$.

4) Suppose Lichtenbaum's $\Gamma(n+1, X)$ complexes exist for some $n \geq 2$ and regular X of dimension 1. Then one can define a class $\Omega_n(N|K) \in K_0(\mathbf{Z}G)$ for which the counterpart of 1),2),3) above hold up to classes of finite modules supported on $(n+1)!$ (or $(n+1)!p$ in the function field case).

Arithmetic tame covers of schemes

Martin J. Taylor

This was a report on joint work with T. Chinburg, B. Erez and G. Pappas. Let $S = Spec(\mathbf{Z})$, and let $X \xrightarrow{f} S$ be a regular, projective, equidimensional flat scheme, which carries an action by a finite group G. Put $X \xrightarrow{\pi} Y = X/G$ for the the quotient and suppose that Y is regular and that the action $X \times G \to X$ is tame. Then, for any bounded complex $F^\bullet$ of coherent $G-X$ modules, the usual Euler characteristic $f_G(F^\bullet) \in G_0(\mathbf{Z}G)$ can be refined, in a natural way, to a class $f_K(F^\bullet) \in K_0(\mathbf{Z}G)$.
With the above notation and hypothesis, the root number class $W_{X/Y} \in Cl(\mathbf{Z}G)$ and the ramification class $R_{X/Y} \in Cl(\mathbf{Z}G)$ were defined. This then enables Taylor to formulate

Conjecture $\sum_{i=0}^{dim(X)-1}(-1)^i f_K(\lambda^i\Omega_{X/Z}) = W_{X/Y} + R_{X/Y}$ in $Cl(\mathbf{Z}G)$.

The two special cases when X/Y is étale and when X is an arithmetic surface were discussed in detail. The lecture then concluded to show how the Euler characteristics of certain sheaves on geometric surfaces correspond to root numbers.

A Tate sequence for global units

Jürgen Ritter

Let K/k be a finite Galois extension of number fields with group G, E the unit group in K and, for a G-invariant finite set S of primes of K containing all the infinite ones, E_S the S-units and $\Delta S = \langle \mathfrak{p}_1 - \mathfrak{p}_2, \mathfrak{p}_{1,2} \in S\rangle_{\mathbf{Z}}$. From Dirichlet's unit theorem one gets (nonexplicit) isogenies $\varphi : \Delta S \to E_S$ which are injective and have finite cokernel. Each such φ gives rise to a Stark regulator $R_{S,\varphi}(\chi)$ and to a Tate q-index $q_{S,\varphi}(\chi)$, both attached to a complex character χ of G. Define $L^*(\chi)$ to be the first non-vanishing coefficient in the Taylor expansion at $s = 0$ of the L-series $L(s,\chi)$ (with the Euler factors to $\mathfrak{p} \in S$ removed) and $A_{S,\varphi}(\chi)$ to be the quotient $R_{S,\varphi}(\chi)/L^*(\chi)$. It is conjectured that $A_{S,\varphi}(\chi) = q_{S,\varphi}(\chi)$, which has been confirmed by Tate in some cases using a sequence

$$0 \to E_S \to A \to B \to \Delta S \to 0.$$

Here, A and B have finite projective dimension and the set S has to be sufficiently large. The various conjectures of Chinburg concerning his class $\Omega = A - B \in Cl(\mathbb{Z}G)$ were discussed. Then the construction of a Tate sequence for small sets S was given, in particular for $S = \infty : 0 \to E \to A \to B \to \nabla \to 0$, which uniquely arises from class field theory. The number theoretical meaning of ∇ was displayed in detail. Also, it was shown that the new Chinburg class $A - B$ is, in fact, the old Ω. All the results came from joint work with A. Weiss.

Iwasawa-theory of abelian varieties at primes of non-ordinary reduction
Heiko Knospe

Let $A/\mathbb{Q}$ be an abelian variety with Néron-model $\mathcal{A}/\mathbb{Z}$ and good reduction at $p \neq 2$, and let $\mathbb{Q} \subseteq \cdots \subseteq \mathbb{Q}_n \subseteq \cdots \subseteq \mathbb{Q}_\infty$ be the cyclotomic $\mathbb{Z}_p \cong \Gamma$-extension with rings of integers $\mathbb{Z} \subseteq \cdots \subseteq \mathfrak{o}_n \subseteq \cdots \subseteq \mathfrak{o}_\infty$. Let $X = H^1_{et}(\mathfrak{o}_\infty[\frac{1}{p}], \mathcal{A}(p))^*$ and assume that p is a supersingular prime (i.e. $\mathcal{A}(\bar{\mathbb{F}}_p)(p) = 0$). Then Knospe defined a finitely generated $\mathbb{Z}_p[[\Gamma]]$-submodule $\mathcal{L}_0$ of $\mathcal{O}^g$ (where $\mathcal{O}$ is the the ring of $\mathbb{C}_p$-analytic functions converging in the open unit disc) which satisfies

Theorem 1) $\# H^1_{fl}(\mathbb{Z}, \mathcal{A}(p)) = |char(tor(X))(0)|^{-1} \cdot (vol\ \mathcal{L}_0(0))^{-1}$
(vol w.r.t. the Haar-measure on $\mathbb{Z}_p^g$) and ∞ if and only if one of the factors is ∞.
2) $0 \leq corank\ H^1_{fl}(\mathfrak{o}_n, \mathcal{A}(p)) - \sum_{\zeta \in \mu_{p^n}} g - rk_{\mathbb{Q}_p(\zeta_{p^n})} \mathcal{L}_0(\zeta - 1) \leq \sum_{\zeta \in \mu_{p^n}} ord_{T=\zeta-1} char(tor(X))$
The proof uses results and techniques of B. Perrin-Riou and P. Schneider, and there is a generalization of the theorem to cases of 'mixed' reduction at p. The classical Selmer groups coincide up to finite index which is independent of n with the above flat (fpqf) cohomology groups. Furthermore, there is a theorem which supplies a lower bound for the size of the groups $H^1_{fl}(\mathfrak{o}_n, \mathcal{A}(p))$.

Artin-Schreier towers, Galois invariants of Milnor K-groups and higher local class field theory
Ivan Fesenko

Let F be a complete discrete valuation field of dimension n over a perfect residue field k, $char(k) = p > 0$. A totally ramified with respect to the discrete valuation of rank n p-extension L/F is called Artin-Schreier tower if there exists a chain of subfields $L = F_m \supseteq F_m \supseteq F_{m-1} \supseteq \cdots \supseteq F_0 = F$ such that $F_i = F_{i-1}(\alpha_i)$ for some root α_i of Artin-Schreier equation $X^p - X = a_i$, $a_i \in F_{i-1}$. The same extension is called AST, if the previous property holds for any intermediate subchain of extensions of degree p. There is a Neukirch map $Y_{L/F}$ for any Galois totally ramified p-extension L/F

$$Y_{L/F} : Hom^{cont}_{\mathbb{Z}_p}(G(\tilde{F}/F), G(L/F)) \to UK^{top}_n(F)/N_{L/F}UK^{top}_n(L)$$

and a Hazewinkel map $\Psi_{L/F}$ (defined only for extensions such that any intermediate cyclic extension is AST)

$$\Psi_{L/F} : UK^{top}_n(F)/N_{L/F}UK^{top}_n(L) \to Hom^{cont}_{\mathbb{Z}_p}(G(\tilde{F}/F), G(L/F)^{ab}).$$

The properties of AST imply that for abelian p-extensions $Y_{L/F}$ and $\Psi_{L/F}$ are isomorphisms which are inverse to each other.

Non abelian local class field theory
Helmut Koch

Let K be a local field with finite residue class field. The talk reported on joint work with E. de Shalit and presented a generalization of class field theory over K by means of Lubin-Tate extensions. The goal was to describe the Galois group $G_K = Gal(K^{sep}|K)$ and its ramification groups by means of a group which is defined over K. So far one has such a description of the Weil group of the maximal metabelian group. The construction depends on the choice of a prime element π of K. Let $q = \#\mathcal{O}_K/\pi$, let v be the exponential valuation with $v(\pi) = 1$, and let $[a](X)$ be the multiplication by a in the formal group belonging to the polynomial $f_\pi(X) = \pi X - (-1)^p X^q$. Then Koch defined

$$\mathcal{G}_f := \{(a, \xi(X)) | \ a \in K, \ \xi(X) \in \bar{\mathbb{F}}_q[[X]]^*, \varphi^f \xi(X) = \xi(X)\{u\}(X)/X\}$$

where φ denotes the Frobenius automorphism acting on $\bar{\mathbb{F}}_q$, furthermore $u := a\pi^{-v(a)}$ and $\{u\}(X) := [u](X)(\mod \pi)$. There is a natural group structure defined on $\mathcal{G}_f$ and the groups $\mathcal{G}_f$, $f \in \mathbb{N}$, form in a natural way a projective system. The main result was the construction of an isomorphism

$$\phi : \varprojlim \mathcal{G}_f \xrightarrow{\cong} W := W((K^{ab})^{ab})$$

which depends on the choice of π but is beside this unique up to inner automorphisms.

Free pro-p-extensions of algebraic number fields
Kay Wingberg

In this lecture Wingberg presented the following theorem:

Theorem Let k be of CM-type containing the group μ_p of p-th roots of unity, $p \neq 2$, with maximal totally real subfield k^+.
1) Assume that no prime of k^+ above p splits in k and that the Iwasawa μ-invariant of k_∞/k is zero. Then if the p-part of the p-ideal class group $Cl_{S_p}(k_n^+)(p) = 0$ for all $n \gg 0$, there exists a free pro-p extension of k of rank $r_2 + 1$. Here r_2 is the number of complex places of k and k_n^+ denotes the n-th layer of cyclotomic $\mathbf{Z}_p$-extension k_∞^+ of k^+.
2) Assume that Leopoldt's and Greenberg's conjecture are true for k resp. k^+ and p. Then $Cl_{S_p}(k_n^+)(p) = 0$ for all $n \gg 0$, if k has a free pro-p-extension of rank $r_2 + 1$.

Corollary Let $k = \mathbb{Q}(\zeta_p)$, $p \neq 2$. Then Vandiver's conjecture is equivalent to Greenberg's conjecture and the existence of a free pro-p-extension of k of rank $\frac{p+1}{2}$.

An arithmetic site over the ring of integers of number fields
Alexander Schmidt

Let $K/\mathbb{Q}$ be an algebraic number field. Schmidt constructs a Grothendieck topology (called positive topology) over $X = Spec(\mathcal{O}_K)$ which is finer than the étale topology and under which X behaves like a smooth projective curve over a finite field. Let $H^*_{pos}(X, \mathcal{F})$ denote the cohomology of a sheaf on the positive site. The following theorem holds:

Theorem There is an integer $d = d(K)$ such that for every n with $(n, d) = 1$ there is a canonical trace map $tr : H^3_{pos}(X, \mu_n) \xrightarrow{\cong} \mathbf{Z}/n\mathbf{Z}$ and for every locally constant constructible sheaf $\mathcal{F}$ of $\mathbf{Z}/n\mathbf{Z}$-modules on X_{pos} the cup product:

$$H^i_{pos}(X, \mathcal{F}) \times H^{3-i}_{pos}(X, \mathcal{H}om(\mathcal{F}, \mu_n)) \to H^3_{pos}(X, \mu_n) \xrightarrow{\cong} \mathbf{Z}/n\mathbf{Z}$$

induces a perfect pairing of finite groups for all i. If K is an abelian number field then $d(K) = 2$.

Representations of central simple algebras over p-adic fields
Ernst Wilhelm Zink

Let A/F be a central simple algebra of reduced degree N over a p-adic number field F. According to the Abstract Matching Theorem of Deligne, Kazhdan, Vigneras there is a character preserving (up to sign) bijection between the sets of irreducible discrete series representations of different groups A^* for all A/F with N fixed. On the other hand there are explicit constructions of the discrete series in the "extreme" cases $A = M_n(F)$ (G. Bushnell and P. Kutzko, L. Corwin) and $A =$

D_N a division algebra (L. Corwin, E.W. Zink). From the explicit constructions a certain system $\mathcal{T}_N^{\sim}$ of parameters has emerged which is a noncanonical substitute for the indecomposable degree N representations of the Weil-Deligne group. $\mathcal{T}_N^{\sim}$ is essentially described in terms of irreducible polynomials over F using a distinguished exponential distance w_F. In joint work with A. Selberger, Zink dealt with the following problems:

1) Explicit construction of the discrete series representations of A^* for all A/F
2) How do the explicit constructions fit with the Abstract Matching Theorem ?

This report was mainly concerned with the first problem.

A mysterious class of p-adic fields and their applications to abelian varieties

John Coates

The lecture reported on joint work with R. Greenberg, which was motivated by two classical problems about abelian varieties over p-adic fields. Inspired by the arguments in Tate's paper "p-divisible groups", he defined so called *deeply ramified* algebraic extensions of $\mathbb{Q}_p$. The first part of the lecture explained a number of differnent characterizations of such fields; for example, such a field K is characterized by $H^1(K, \bar{\mathfrak{m}}) = 0$, where $\bar{\mathfrak{m}}$ denotes the maximal ideal of the ring of integers of $\bar{\mathbb{Q}}_p$. The remainder of the lecture discussed applications to two problems on abelian varieties. Let $K|F|\mathbb{Q}_p$ be field extensions with F finite and K algebraic over $\mathbb{Q}_p$, A/F an abelian variety and $A[p^\infty]$ the G_F-module of p-power torsion points. Let $\kappa_K : A(K)\otimes\mathbb{Q}_p/\mathbb{Z}_p \hookrightarrow H^1(K, A[p^\infty])$ be the Kummer map, and let $N_{K|F}(A) \subseteq A(F)$ be the universal norm subgroup. When K is deeply ramified, Coates explained the description of $Im(\kappa_K)$ in motivic terms, i.e. in terms of the G_F-module $A[p^\infty]$. Secondly, whenever there is motivic description of $Im(\kappa_K)$, he outlined a general motivic description of the group of universal norms.

p-adic regulators

Jan Nekovář

Let k be a perfect field of $char(k) = p > 0$, $W = W(k)$, $K_0 = W[\frac{1}{p}]$, K/K_0 a finite totally ramified extension. Let $\mathcal{X}$ be a proper smooth scheme over the ring of integers, $X = \mathcal{X} \otimes_{O_K} K$, $\hat{\mathcal{X}} = \varprojlim_i \mathcal{X} \otimes \mathbb{Z}/p^i\mathbb{Z}$. For $n < p$ we have the syntomic cohomology groups $H^*(\hat{\mathcal{X}}_{syn}, s_{\mathbb{Q}_p}(n))$ and the Fontaine-Messing map (for $p \neq 2$)

$$H^\bullet(\hat{\mathcal{X}}_{syn}, s_{\mathbb{Q}_p}(n)) \xrightarrow{\beta} H^\bullet(X_{et}, \mathbb{Q}_p(n)).$$

Theorem Assume either (a) $p > 2dim(X) + 1$; or (b) $p > dim(X)$ and $\mathcal{X} \cong \mathcal{X}_0 \otimes_W \mathcal{O}_K$ with $\mathcal{X}_0$ smooth over W. Then there is a commutative exact diagram

$$\begin{tikzcd}
0 \arrow[r] & H^1_f(K, H^i(\bar{X}_{et}, \mathbb{Q}_p(n))) \arrow[r] \arrow[d, "\beta"] & H^{i+1}(\hat{\mathcal{X}}_{syn}, s_{\mathbb{Q}_p}(n)) \arrow[r] \arrow[d, "\beta"] & H^0(K, H^{i+1}(\bar{X}_{et}, \mathbb{Q}_p(n))) \arrow[r] \arrow[d, "="] & 0 \\
0 \arrow[r] & ker(\gamma) \arrow[r] \arrow[d, "\delta"] & H^{i+1}(X_{et}, \mathbb{Q}_p(n)) \arrow[r, "\gamma"] & H^0(K, H^{i+1}(\bar{X}_{et}, \mathbb{Q}_p(n))) \arrow[r] & 0 \\
& H^1(K, H^i(\bar{X}_{et}, \mathbb{Q}_p(n)) & & &
\end{tikzcd}$$

(δ is induced by the Hochschild-Serre spectral sequence, $\delta \circ \beta$ is injective).

Corollary 1 Assume (a),(b), $K|\mathbb{Q}_p$ finite and $i + 1 < 2n$. If the Chern classes

$$\begin{tikzcd}
K_{2n-i-1}(\mathcal{X}) \arrow[r] \arrow[d] & K_{2n-i-1}(X) \arrow[d] \\
H^{i+1}(\hat{\mathcal{X}}_{syn}, s_{\mathbb{Q}_p}(n)) \arrow[r] & H^{i+1}(X_{et}, \mathbb{Q}_p(n))
\end{tikzcd}$$

are compatible, then ($Im(\gamma) = 0$ by the crystalline Weil conjectures)

$$Im[K_{2n-i-1}(\mathcal{X}) \to H^1(K, H^i(\bar{X}_{et}, \mathbb{Q}_p(n)))] \subseteq H^1_f(K, H^i(\bar{X}_{et}, \mathbb{Q}_p(n)))$$

Corollary 2 Assume (a),(b). Then

$$Im[CH^n(X)_0 \to H^1(K, H^{2n-1}(\bar{X}_{et}, \mathbb{Q}_p(n)))] \subseteq H^1_f(K, H^{2n-1}(\bar{X}_{et}, \mathbb{Q}_p(n)))$$

Parametric solutions of embedding problems

B. Heinrich Matzat

This lecture introduced parametric solutions of embedding problems. In the case of a Hilbertian base field these can be specialized to ordinary solutions, and in the case of geometric embedding problems over function fields even to geometric solutions. In contrast to the geometric solutions of embedding problems introduced earlier, parametric solutions have the advantage that standard reduction theorems like the theorem of Kochendörffer remain true. Then it was shown that besides of the solvable Brauer embedding problems the two fundamental constructions for strong solutions of embedding problems (split embedding problems with abelian kernel, centerless embedding problems with GAR-kernel) always lead to proper parametric solutions. At the end an overview on new GAR-realizations over $\mathbb{Q}$ and $\mathbb{Q}^{ab}$ was given extending the lists in "Der Kenntnisstand in der konstruktiven Galois'schen Theorie", PM 95 (1991).

GAR-realizations of orthogonal groups over $\mathbb{Q}$

Gunter Martin Malle

It is still an open question whether every finite simple (nonabelian) group G possesses a GAR-realization over $\mathbb{Q}^{ab}$ or even $\mathbb{Q}$. Embedding probelms over Hilbertian fields with kernel G^r, where G has a GAR-realization, are solvable; this demonstrates the importance of finding GAR-realizations for finite simple groups. In the talk Malle sketched a proof that for infinitely many natural numbers n there exist infinitely many primes p such that the orthogonal groups $O_{2n+1}(p)$, $O_{2n}^{+}(p)$ and $O_{2n}^{-}(p)$ possess GA-realizations over $\mathbb{Q}$, which are even GAR-realizations for $O_{2n+1}(p)$. For the proof, one shows rigidity for a suitable class vector of G. The question of generation can be handled by using a theorem of Wagner classifying groups generated by reflections. For the calculation of the structure constant, deep results from Lusztig's theory of characters of reductive groups have to be used, as well as extensions of this theory to disconnected groups by Digne and Michel. These allow to give bounds on character values and thus prove that the structure constant of the chosen class vector equals 1.

Semistable hyperelliptic curves over discrete valuation rings

Ivan Kausz

Let R be a discrete valuation ring of $char \neq 2$, $\pi : X \rightarrow S = Spec(R)$ a stable S-curve of genus $g \geq 2$ with smooth hyperelliptic generic fibre X_K over the field of constants $K = Quot(R)$. Assume that the image of the canonical mapping $X_K \rightarrow \mathbf{P}_K^{g-1}$ is $\mathbf{P}_K$. Then there exists a canonical rational section Λ of $(det(\pi_*\omega_{X/S}))^{\otimes(8g+4)}$. Let $s \in S$ be the special point, $char(\kappa(s)) \neq 2$. In this lecture Kausz showed that

$$0 \leq ord_s(\Lambda) \leq g^2\delta_s$$

where δ_s is the number of singular points in the geometric fibre of the minimal regular model of X.

The method of Coleman-Chebauty and Fermat curves

William G. McCallum

Let X be a complete nonsingular curve of genus $g > 1$, defined over a number field K, and let A be the Jacobian of X. The method of Coleman & Chebauty bounds

$X(K)$ when the rank of $A(K)$ is less than g. The key is to construct a logarithm on $A(K_v)$, for some valuation v of K which vanishes on $A(K)$, and whose pullback to X therefore vanishes on $X(K)$. This pullback may be represented as the Coleman integral of a differential ω on X, and its zeroes may therefore be controlled in terms of zeroes of ω.
McCallum illustrated a refinement of this method using the example of curves $y^p = x^s(1-x)$, p an odd prime, $1 \leq s \leq p-2$. The refinement depends on knowing that the closure of $A(\mathbb{Q})$ in $A(\mathbb{Q}_p)$ has positive codimension, which in turn depends on a certain descent hypothesis. This hypothesis is satisfied when (a) p is regular or (b) $\zeta = N_{K_n/K}\epsilon_n \forall n$, where ζ is a primitive p-th root of unity, $K = \mathbb{Q}(\zeta), K_n = K(A[p^n])$ and $\epsilon_n \in \mathcal{O}_{K_n}[\frac{1}{p}]^* \otimes \mathbb{Z}_p$. Under this hypothesis, he showed that the only rational points are $(0,0)$, $(1,\infty)$ and ∞; i.e. that Fermat's last theorem is true for p.

Connectedness of certain linear algebraic groups arising from abelian varieties

Yuri G. Zarhin

Zarhin reported on joint work with A. Silverberg. They proved the following generalizations of Raynaud's semistable reduction criterion and a theorem of Borovoi. Let K be a field, X an abelian variety over K. Let $\lambda : X \to X^t$ be a polarization of X, $e_{\lambda,n} : X_n \times X_n \to \mu_n$ the corresponding skew-symmetric Galois-equivariant Weil-pairing.

Theorem Assume that there exist a maximal isotropic subgroup $\tilde{X}_n \subset X_n$ with respect to $e_{\lambda,n}$ such that $\tilde{X}_n \subset X(K)$ (*). If $n \geq 5$ then X has semistable reduction outside n.

Now, assume that K is a number field and let $V_l(X)$ be the $\mathbb{Q}_l$- Tate module of X. Let $\rho_l : G(K) \to Aut\ V_l(X)$ be the corresponding l-adic representation of the Galois group $G(K)$ of K and let $G_l^{alg} \subset GL(V_l(X))$ be the Zariski closure of the image $Im(\rho_l) \subset Aut\ V_l(X)$.

Theorem Assume (*) as above. If $n \geq 5$ and $\mu_n \subset K$ then the linear algebraic group G_l^{alg} is connected.

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