MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 32/1994

Complex Geometry: Moduli Problems

24.7.-30.7.1994

Die Tagung fand unter der Leitung von Arnaud Beauville (Orsay), Fabrizio Catanese (Pisa) und Christian Okonek (Zürich) statt. Im Mittelpunkt des Interesses standen Fragen über Modulräume komplexer Mannigfaltigkeiten und holomorpher Vektorbündel. Schwerpunkte bildeten einerseits die Berechnung von Donaldson-Invarianten algebraischer Flächen mit algebraisch-geometrischen Methoden und andererseits die Untersuchung von Eigenschaften der Modulräume wichtiger Klassen von algebraischen Mannigfaltigkeiten.
Modulräume von Calabi-Yau Mannigfaltigkeiten, Flächen vom allgemeinen Typ, abelschen Flächen, und Kurven wurden besonders untersucht.

Vortragsauszüge

J. Li:
The topology of moduli of vector bundles over surfaces

In this talk I report on the recent progress in understanding the topology of moduli of vector bundles over algebraic surfaces. Let (X, H) be a polarized surface. For any given $(I, d) \in \mathrm{Pic}(X) \times \mathbb{Z}$, there is a coarse moduli space $\mathcal{M}(I, d)$ parametrizing all Mumford-stable locally free sheaves E with $\det E \cong I$ and $c_2(E) = d$. A recent theory shows that the geometry of $\mathcal{M}(I, d)$ reflects the geometry of X in a rather straightforward way for d sufficiently large. The first two Betti numbers of $\mathcal{M}(I, d)$ make no exception. We proved recently the following result: For any surface (X, H) and $I \in \mathrm{Pic}(X)$, there is constant C such that for all $d \geq C$ we have $H_1(\mathcal{M}(I, d); \mathbb{Q}) \cong H_1(X; \mathbb{Q})$ and $H_2(\mathcal{M}(I, d); \mathbb{Q}) \cong H_2(X; \mathbb{Q}) \oplus \Lambda^2 H_1(X; \mathbb{Q})$. The proof of this result is a combination of Gauge theory and algebraic geometry. First of all, at least for $i = 1$ and 2 , there is a canonical homomorphism

$$\tau(d)_i : H_i(\mathcal{M}(I,d);\mathbb{Q}) \longrightarrow H_i(\mathcal{M}(I,d+1);\mathbb{Q}) \qquad (*)$$

C. Taubes has studied this homomorphism extensively and he showed that the limit $\varinjlim H_i(\mathcal{M}(I,d))$ of this direct system is $H_1(X,\mathbb{Q})$ when $i = 1$ and $H_2(X;\mathbb{Q}) \oplus \Lambda^2 H_1(X;\mathbb{Q})$ when $i = 2$. Hence, if someone can show that $(*)$ are isomorphisms for $d \gg 0$, then the homology of $\mathcal{M}(I,d)$ in dimensions 1 and 2 will be exactly the mentioned above spaces for $d \gg 0$. To establish the isomorphism of $(*)$ for $d \gg 0$, we will use algebraic geometric techniques, for instance the Lefschetz-type theorem for singular quasiprojective varieties.

K. O'Grady:
Moduli of vector bundles on projective surfaces: some basic results

Let S be a smooth projective irreducible complex surface, and let H be an ample divisor on S. A "set of sheaf data" consists of a triple $\xi = (r_\xi, \det_\xi, c_2(\xi))$, where $r_\xi \in \mathbb{Z}$ with $r_\xi \geq 2$, $\det_\xi$ is a line bundle on S, and $c_2(\xi) \in \mathbb{Z}$. Set $\Delta_\xi := c_2(\xi) - \frac{r_\xi - 1}{2r_\xi} c_1(\det_\xi)^2$. We let $\mathcal{M}_\xi$ be the coarse moduli space of H-semistable torsion-free coherent sheaves F on S with $\mathrm{rk}(F) = r_\xi$, $\det F = \det_\xi$, $c_2(F) = c_2(\xi)$. If L is a line bundle on S, let $h^0(F, F \otimes L)^0$ be the dimension of the space of traceless maps $\varphi : F \longrightarrow F \otimes L$. Set $W_\xi^L := \{[F] \in \mathcal{M}_\xi | h^0(F, F \otimes L)^0 > 0\}$.

Theorem 1. *Given L there exists $\alpha_2(r_\xi) < 2r_\xi$, $\alpha_1(r_\xi, S, H)$, $\alpha_0(r_\xi, S, H, L)$ such that* $\dim W_\xi^L < \alpha_2 \Delta_\xi + \alpha_1 \sqrt{\Delta_\xi} + \alpha_0$.

The condition $\alpha_2 < 2r_\xi$ in the statement of Theorem 1 makes the theorem non-vacuous:

Corollary. *There exists $\Delta_0(r, s, H)$ such that if $\Delta_\xi > \Delta_0(r_\xi, S, H)$, then $\mathcal{M}_\xi$ is generically smooth, of dimension equal to the expected dimension.*

Theorem 1. for $r_\xi = 2$ has been proved by Donaldson (also by Friedman and Zuo, with a better value for α_2).
We notice that $\alpha_2, \alpha_1, \alpha_0$ (and hence also Δ_0) are effective. As an example assume that K_S is ample and $K_S^2 \gg 0$ ($K^2 > 100$ will do). Then if $H = K$ and $r_\xi = 2$ we can set $\Delta(2, S, K) = 42K^2 + 15\chi(\mathcal{O}_S)$.

Theorem 2. *There exists $\Delta_1(r_\xi, S, H)$ such that if $\Delta_\xi > \Delta_1(r_\xi, S, H)$, then $\mathcal{M}_\xi$ is irreducible.*

The above theorem for $r_\xi = 2$ has been proved by Gieseker and Li. We remark that Δ_1 is not effective.

J. Le Potier:
Systèmes cohérents et Polynômes de Donaldson

Le but de l'exposé est de montrer comment on peut ramener le calcul de certaines formules d'intersection sur l'espace de modules M_n des faisceaux semi-stable de rang 2 et de classe de Chern $(0, n)$ sur le plan projectif au calcul de certaines formules d'intersection sur le schéma de Hilbert $\mathrm{Hilb}^{n+1}(\mathbb{P}_2)$.
On introduit pour ceci l'espace de modules $S_{\alpha,n}$ des systèmes cohérents α-semi-stable (Γ, F): il s'agit des paires formées d'un faisceau cohérent sans torsion F de rang 2 et classes de Chern $(2, n+1)$ et d'un sous-espace $\Gamma \subset H^0(F)$ de dimension 1. Un tel système est dit α-semi-stable ($\alpha \in \mathbb{Q}, \alpha > 0$) si pour tout sous-faisceau $F' \subset F$ de rang 1 on a

$$p_{F'} \leq \begin{cases} p_F + \alpha & \text{si} \quad \Gamma \cap H^0(F') = 0 \\ p_F - \alpha & \text{si} \quad \Gamma \subset H^0(F') \end{cases}$$

On suppose $n \geq 3$. L'etude de la lisseté repose sur le travail de Min He.

Théorème 1. *(1) L'espace de modules $S_{\alpha,n}$ est lisse sauf pour $\alpha + n \equiv 0$ mod 2, $0 < \alpha \leq n$ (valeurs critiques). (2) Si α_+, α_- sont proches d'une valeur critique α, $\alpha_- < \alpha < \alpha_+$, on a des applications birationelles*

$$S_{\alpha_-,n} \xrightarrow{p_-} S_{\alpha,n} \xleftarrow{p_+} S_{\alpha_+,n}$$

(3) Pour $\varepsilon > 0$ petit, on a un morphisme canonique $f : S_{\alpha,n} \longrightarrow M_n$ qui est génériquement injectif pour $n \geq 5$. (4) Si $\Xi \subset \mathrm{Hilb}^{n+1}(\mathbb{P}_2) \times \mathbb{P}_2$ est le sous-schéma universel et $\mathcal{V} := pr_{1*}(\mathcal{O}_\Xi(0,-1))$, alors $S_{n_+,n}$ s'identifie au projectif de Grothendieck $\mathbb{P}(\mathcal{V})$.

Sur chacun des espaces de modules M_n, $S_{\alpha,n}$ on défini un fibré determinant $\mathcal{D}$ resp. $\mathcal{D}_\alpha$ et une intégrale $I_{\alpha,n} := \int_{S_{\alpha,n}} c_1(\mathcal{D}_\alpha)^{3n+2}$

Théorème 2. *L'intégrale $I_{\alpha,n}$ est independente de α.*

Corollaire. *$q_{17}(\mathbb{P}_2) := \int_{M_5} c_1(\mathcal{D}))^{17} = \int_{\mathrm{Hilb}^6(\mathbb{P}_2)} s_{12}(\mathcal{V} \otimes \mathcal{L})$, où $\mathcal{L}$ désigne l'image réciproque du générateur positif de $\mathrm{Pic}(S^6(\mathbb{P}_2))$.*

(Cet énoncé a aussi été obtenu par Tyurin-Tikhomirov et Ellingsrud-Stromme. Ces derniers ont établi un programme qui permet de calculer le second nombre en utilisant la formule de résidus de Bott. Il trouvent $q_{17}(\mathbb{P}_2) = 2540$.)

A. N. Tyurin:
On the diffeomorphisms-group of an algebraic surface of general type

For a simply connected algebraic surface of general type with $p_g > 0$ the standard representation of the diffeomorphisms group in the ortogonal group of the intersection form of $H^2(S, \mathbb{Z})$ is reducible. The algebraic geometrical methods using the spin-polynomials give the invariant sublattice $sV(S)$ containing the canonical class K_{min} of the minimal model of S. The question is how close can be $sV(S)$ to $\mathbb{Z}K_{min}$? Recently E. Witten proposed a new approach to the computation of the shape of the Donaldson polynomials. As a corollary we obtain a new diffeomorphism-invariant sublattice $L_{K-M-W} \subset H^2(S, \mathbb{Z})$, the so called Kronheimer-Mrowka-Witten sublattice, which is closer to $\mathbb{Z}K_{min}$ then $sV(S)$. Even if Witten's arguments are pure physical, we compare these two approaches to the problem. We use the new results and constructions of S. Donaldson and D. Salamon on the Fukai-Floer cohomology ring of the normal neighbourhood of an algebraic curve in S.

L. Göttsche:
Change of Donaldson polynomials under change of polarisation

This is a report on joint work with E. Ellingsrud (Oslo).
Let S be a smooth projective surface with $\pi_1(S) = 0$, $p_g(S) = 0$ and L ample on S. Using the moduli space $M_L(c_1, c_2)$ of $L-$ semistable torsion free sheaves of rank 2 with $c_1(E) = c_1$, $c_2(E) = c_2$ one can determine the Donaldson invariants $\Phi_{g_L} = \Phi_L : \mathrm{Sym}^N H_2(S, \mathbb{Z}) \longrightarrow \mathbb{Q}$ $(N = 4c_2 - c_1^2 - 3$, g_L-Fubini Study metric metric to $L)$. Unlike the case $p_g(S) > 0$ they depend on the metric; according to Kotschick and Morgan only on its chamber.

Definition. *$\xi \in H^2(M, \mathbb{Z})$ defines a wall W^ξ =(ample cone of S) $\cap\, \xi^\perp$ if $\xi = c_1$ mod 2 and $-(4c_2 - c_1^2) \leq \xi^2 < 0$. W^ξ is good if for all η defining W^ξ, $\eta - K_S$ is not effective.*

We study the change of $M_L(c_1, c_2)$ and Φ_L when L passes through a good wall. Qin had studied in the case of locally free sheaves the change of the moduli space. If L_1 and L_2 are only separated by good walls, we show that

$M_{L_2}(c_1, c_2)$ is obtained from $M_{L_1}(c_1, c_2)$ by a sequence of blow ups and blow downs along projective bundles over products $\mathrm{Hilb}^n(S) \times \mathrm{Hilb}^m(S)$ of Hilbert schemes of points on S. We obtain $\Phi_{L_2} - \Phi_{L_1} = \sum_{\xi} \delta(\xi)$, where ξ defines a wall with $L_1\xi < 0 < L_2\xi$ and $\delta(\xi)$ is obtained by integrating natural classes over $\mathrm{Hilb}^{d(\xi)}(S \coprod S) = \coprod_{n+m=d} \mathrm{Hilb}^n(S) \times \mathrm{Hilb}^m(S)$ $(d(\xi) = \frac{1}{4}(4c_2 - c_1^2 + \xi^2))$. We partially carry out this integration to obtain

Theorem. *$\delta(\xi) = \sum_{i=0}^{3} a_i q_S^{d(\xi)-i} \langle \xi, \cdot \rangle^{N-2d(\xi)+2i} \bmod \langle \xi, \cdot \rangle^{N-2d+7}$, where a_0, a_1, a_2 are explicit expressions in (N, d, ξ^2, K^2).*

The coefficient $a_3 \in \mathbb{Q}$ we cannot determine. (q_S is the quadratic form on $H^2(S, \mathbb{Z})$ and $\langle \xi, \cdot \rangle$ the multiplication by ξ.) Kotschick and Morgan have conjectured such a behaviour for all walls and also in the nonalgebraic case.

I. Dolgachev:
Calabi-Yau manifolds, abelian varieties and arrangements of hyperplanes

Let R be one of the four division algebras $(\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O})$ and $\Omega_n(R) = H_n(R) + iH_n(R)^+$ be the corresponding tube domain ($H_n(R)$ denotes the space of Hermitian matrices w.r.t. the natural involution in R, $H_n(R)^+$ is the cone of positive definite matrices). Except the case $n > 3$ and $R = \mathbb{O}$, $\Omega_n(R)$ is a Hermitian bounded domain and hence classifies (abstract) varitions of Hodge structures. In the talk we discuss possible candidates of families of varieties ($K3$ surfaces if $n = 2$ and Calabi-Yau 3-folds if $n = 3$) which give a geometric realization of the variations of Hodge structures. For $R = \mathbb{R}$ or $\mathbb{C}$ they are birational isomorphic to branched covers of $\mathbb{P}_2$ or $\mathbb{P}_3$ ramified along the union of 6 (resp 8) hyperplanes in general positions.

F. O. Schreyer:
Moduli of (1,7)-polarized abelian surfaces and Fano 3-folds of index 1 and genus 12

The "Horrocks-Mumford model" $X(1,7)$ of the moduli space $\mathcal{A}(1,7)$ of abelian surfaces with $(1,7)$ polarization and canonical level structure is a prime Fano 3-fold V_{22} of index 1 and genus 12, by a result of Manolache and myself. In the talk I compare the geometry of $X(1,7)$ with the geometry of a general Fano 3-fold of this type. According to Mukai, every Fano 3-fold of this type has three different geometric realizations :

(a) as the space $\mathbb{G}(3, V, N)$ of isotropic subspaces of a 7-dimensional vector space V with respect to a net N of alternating forms on V.
(b) as the variety $VSP(F, 6)$ of polar hexagons of a plane quartic.
(c) as the variety $H(q)$ of twisted cubics in $\mathbb{P}^3$ whose equations are annihilated by a net of quadrics q in $\check{\mathbb{P}}^3$.
In the talk I described, how the alternating net N, the plane quartic F and the net of quadrics q for a given V_{22} are related to each other. For example the covariant quartic S_F of F coincides with the discriminant S_q of q, and can be identified with the Hilbert scheme of lines on V_{22}. For $X(1,7)$ one has $F = X(7)$ the Klein quartic, $S_F = F$ and the union of lines $B = \bigcup_{L \in S_F} L \subset X(1,7)$ coincides with the boundary of the moduli space. Conics on $X(1,7)$ correspond to Calabi-Yau 3-folds in $\mathbb{P}^6$, which contains a pencil of these abelian surfaces, a result due to Anre and Ranestad.

O. Debarre:
<u>Subvarieties of abelian varieties</u>

As Sommese showed, a large part of the geometry of a <u>smooth</u> subvariety of an abelian variety X depends on "how ample" its normal bundle is. Our aim is to study the geometry of general irreducible subvarieties V of X and for that we introduce following definition: We say that V is k-geometrically non degenerate if for all abelian quotients $p : X \longrightarrow X/K$ one has $\dim p(V) \geq \min(\dim(V) - k, \dim p(X))$. We show that it is a good substitute to k-ampleness of $N_{V/X}$. Given two irreducible subvarieties V, $W \subset X$, we get simple conditions that imply that $V \cap W$ is non-empty, that it is connected or that $\pi_1^{alg}(V) \xrightarrow{\sim} \pi_1^{alg}(X)$. The proofs rely on a connectedness theorem analog to the Fulton-Hansen theorem. This theorem can be also used to prove Zak-type results. If $S \subset V \subset X$ and if say (for simplicity) V is smooth along S, define $T(V,S) = \bigcup_{s \in S} \mathbb{P}T_sV \subset \mathbb{P}T_0X$ (the analog of the tangential variety of a subvariety of a projective space); then $V \setminus S$ (the analog of the secant variety) has dimension $\dim T(V,S) + 1$. This theorem has several consequences on the geometry of the Gauss map of smooth (k-geometrically non degenerate) subvarieties.

Y. Laszlo:

Vectorbundles over curves: a survey

The aim of the talk was to give some old and new results about moduli spaces of semi-stable vector bundles over curves. Essentially here are the points:
1) The topology and even the holomorphic structure of the moduli spaces is due to the theorem of Narasimhan and Seshadri comparing unitary representations of the fundamental group of the curve and the semistable bundles.
2) The symplectic structure of the cotangent bundle of the moduli spaces give examples of algebraically complete integrable systems (cf. Hitchin). Moreover the nilpotent cone of the cotangent bundle is a Lagrangian subvariety of the cotangent bundle.
3) The complex structure of the moduli spaces heavily depends on the complex structure of the curve. In fact Narasimhan and Ramannan proved in the coprime case that the third intermediate Jacobian is the Jacobian of the curve, giving the Torelli theorem.
4) The geometry of the moduli space. Basic facts about this subject were explained. In particular there exist a natural linear system and it is proved by the author that in the rank 2 case and degree 0, it is an embedding for a generic curve.

W. Ebeling:

Differentiable structures on Milnor fibres

Let $B(p,q,r)$ denote the Milnor fibre of the isolated hypersurface singularity in $\mathbf{C}^3$ defined by $x^p + y^q + z^r = 0$. R. Fintushel and R. Stern have shown that for any unordered triple $\{p,q,r\} \neq \{2,3,5\}$ of pairwise coprime positive integers there exist infinitely many distinct smooth 4-manifolds homeomorphic to $B(p,q,r)$. They construct these manifolds by applying a differential topological analogue of a logarithmic transformation in a cusp neighbourhood in $B(p,q,r)$. Let Y_m denote the result of such a transformation of multiplicity $m > 0$. They show that Y_m is homeomorphic to $Y_1 = B(p,q,r)$ for any odd m. In order to prove that these manifolds are all non-diffeomorphic they use deep results by Morgan, Mrowka and Ruberman. We give a simpler proof of this fact in the cases $\{2,3,7\}$ and $\{2,3,11\}$ where the compactification of the Milnor fibre is a K3 surface. We prove the following result: Let $\varphi_0(Y_m)$ denote the Donaldson Floer invariant of Y_m of degree 0. Then $\varphi_0(Y_m) = m\varphi_0(Y_1)$ and $\varphi_0(Y_1) \neq 0$ (for $\{p,q,r\} = \{2,3,7\}, \{2,3,11\}$).

L. Ein:

Adjoint linear systems and effective Matsusaka's theorem

We study linear systems of the form $|K_X + A|$, where A is an ample divisor on a smooth projective n-fold X. In a joint work with R. Lazarsfeld and M. Nakamaye, we found the following result: Let b be a nonnegative number such that $-K_X + bA$ is nef. Then there exists an explicit constant C depending on n and b only, such that if all the subvarieties of X have degrees larger than C with respect to A, then the linear system $|K_X + A|$ will give an embedding of X into a projective space.

E. Looijenga:

Cohomology of moduli space of curves

This talk reported to a large extent on work by M. Pickart whose main result can be explained as follows. If, as usual, $\overline{\mathcal{M}}_g$ stands for the moduli space of stable curves (/$\mathbb{C}$) of genus g, then for any positive integer k there exists for sufficiently large g a finite filtration of $\overline{\mathcal{M}}_g$ by closed subvarieties $\overline{\mathcal{M}}_g = X_0 \supset X^1 \supset X^2 \supset \cdots \supset \emptyset$ which has the following properties:
i) Each succesive difference $S^\alpha = X^\alpha - X^{\alpha+1}$ is an orbifold of constant dimension,
ii) The Gysin maps $H^{i-2\mathrm{codim} S^\alpha}(S^\alpha;\mathbb{Q}) \longrightarrow H^i(\overline{\mathcal{M}}_g \setminus X^{\alpha+1};\mathbb{Q})$ are injective for $i \leq k$.
iii) $S^0 = \mathcal{M}_g$ and each S^α is a union of "strata of constant topological type".
This theorem implies that the stable cohomology $\lim_{g\to\infty} H^k(\mathcal{M}_g;\mathbb{Q})$ (which according to Harer-Ivanov, already stabilises for $g = 2k+1$) has a natural pure Hodge structure of weight k. This in turn, implies that the restriction homeomorphism $H^k(\overline{\mathcal{M}}_g;\mathbb{Q}) \longrightarrow H^k(\mathcal{M}_g;\mathbb{Q})$ is surjective fr $g \geq 2k+1$. A corollary of the proof (rather than the theorem) is that the cohomology of $\overline{\mathcal{M}}_g$ is not all of Tate type for large g; in fact $H^{22,11}(\overline{\mathcal{M}}_g) \neq 0$ for $g \gg 0$.

B. Fantechi:

Deformations of abelian covers and moduli space of surfaces of general type

This is a report on joint work with Rita Pardini. A G abelian cover will be a finite Galois cover $f : X \longrightarrow Y$ of smooth complex projective varieties with abelian Galois group G. It is assumed that $\dim Y \geq 2$, and that the cover is totally ramified. It is possible to define natural deformations of X, which are finite (possibly non-Galois) covers of deformations of Y described by explicit equations.

Theorem 1. *If $f : X \longrightarrow Y$ is a G-abelian cover with sufficiently ample branch divisors, the natural deformations of X are complete (i.e. surjective on the Kuranishi family of X).*

Theorem 2. *If $f : X \longrightarrow Y$ is a G-abelian cover with sufficiently ample and generic branch divisor, then* $\mathrm{Aut}(X) = G$.

The previous result can be applied to construct examples of surfaces of general type. In particular, we can prove:

Theorem 3. *Given $N \in \mathbf{N}$, there exists a surface of general type lying in at least N components of the moduli space.*

The different components M are distinguishable because they have different G_M, the automorphism group of the generic surface of the component. With a similar construction we get:

Theorem 4. *Let G be a finite abelian group. Then there are infinitely many components M of the moduli of surfaces of general type with $G_M = G$.*

W. Barth:
Projective surfaces with many lines, conics or nodes

The talk dealt with classical results on the following questions:
1.What is the maximal number of lines on a smooth projective surface $X \subset \mathbb{P}^3$ of fixed degree n?
2. What is the maximal number of skew lines on such a surface X?
3. What is the maximal number of conics?
4. What is the maximal number of nodes?
In addition, the following new results were discussed:
There are smooth quartic surfaces with 16 skew lines (Barth-Nieto, Naruki).
There are smooth quartic surfaces with 16 skew conics. They contain at all 352 conics, and sometimes even 432 ones.
There are surfaces of degree six with 65 nodes and of degree ten with 345 nodes.

K. Zuo:

Rational representations of π_1 of algebraic varieties

(A part of my talk is joint work with J. Jost) Let X be a smooth algebraic variety/$\mathbb{C}$ (in general we can also consider Kähler varieties or varieties over $\bar{\mathbb{Q}}_p$). Let $\rho : \pi_1(X) \longrightarrow G(\mathbb{C})$ be a Zariski dense representation into a linear algebraic simple $\mathbb{Q}$-group.

Definition. *The Shafarevich map of ρ is a morphism $sh_\rho : X \twoheadrightarrow Sh_\rho(X)$ with the following two properties:*
a) The fibres of sh_ρ are connected and the restriction $\rho|_{sh_\rho^{-1}}$ to the generic fibre is trivial.
b) for any $f : X \twoheadrightarrow Y$ which satisfies a) there exists $g : Y \longrightarrow Sh_\rho(X)$ such that $g \circ f = sh_\rho$.

Theorem 1. (Jost-Zuo) *sh_ρ exists and if ρ is nonrigid nonintegral, then* $\dim Sh_\rho(X) \leq \mathrm{rk}_{\mathbb{C}} G$.

"ρ is integral" means that there exists a number field $K \supset \mathbb{Q}$ so that after a conjugation $\rho\pi_1(X) \subset G(\mathcal{O}_K)$, where $\mathcal{O}_K \subset K$ is the ring of the algebraic integers of K.

Theorem 2. *If $sh_\rho = \mathrm{id}_X$, then X is an algebraic variety of general type. In general $Sh_\rho(X)$ is always an algebraic variety of* log. *general type with respect to the multiple fibres of sh_ρ.*

A. Strømme:

Some Donaldson numbers of $\mathbb{P}^2$

I reported on joint work with Ellingsrud, based on work by and discussions throughout this week with A. Tyurin and J. Le Potier.
The Donaldson numbers are $q_{4n-3}(\mathbb{P}^2) = \int_{M_n} c_1(f^*\mathcal{O}(1))^{4n-3}$, where M_n is the moduli space for Gieseker semistable rank 2 sheaves with $c_1 = 0$, $c_2 = n$ and $f : M_n \longrightarrow |\mathcal{O}_{\mathbb{P}^{2*}}(n)|$ is the Barth map $f(E) =$ jumping line curve of E. It is well known that $q_5 = 1$, $q_9 = 3$ and $q_{13} = 54$. In this talk I indicated how one can compute $q_{17} = 2540$ and $q_{21} = 233208$ using the geometric approximation procedure of Tyurin (which reduces the computation to the evaluation of certain Segre classes of vector bundles on the Hibert schemes $\mathrm{Hilb}^6_{\mathbb{P}^2}$, $\mathrm{Hilb}^7_{\mathbb{P}^2}$) and a residue formula of Bott, which allows us to reduce this

evaluation to the study of the fixed points of the Hilbert scheme under the natural action of a torus in $GL(3)$. The geometric approximation procedure is to replace M_n by the moduli space GAM_n which parametrizes all extensions $0 \longrightarrow \mathcal{O}(-1) \longrightarrow E \longrightarrow I_Z(1) \longrightarrow 0$, where $Z \subset \mathbb{P}^2$ is a closed subscheme of length $n+1$. GAM_n is a projective bundle over $\mathrm{Hilb}^{n+1}(\mathbb{P}^2)$ and we can start computing.

A. Teleman:

Moduli spaces of vector bundles over class VII surfaces

We explain the concept of stability in the general Hermitian case and the Kobayashi Hitchin correspondence. We study the following three problems concerning moduli spaces of stable bundles over surfaces with odd first Betti number:

1. The extension of the complex-space structure to the Donaldson compactification of the moduli spaces.
2. Lifting differential geometric structures (Hermitian metric, quaternionic structure) from the base to the moduli spaces.
3. The behaviour of the locus of reducible stable holomorphic bundles inside the moduli space.

We prove that in the case of SL(2,$\mathbb{C}$) bundles with second Chern class 1, there is a natural extension of the complex structure in a neighbourhood of the virtual connections, which is smooth in the case of class VII surfaces (for a generic Gauduchon metric). We define a Hermitian Petersson-Weil metric on the moduli space and we show that the hypercomplex (quaternionic) structure of some Hopf surfaces can be lifted to the moduli space.

M. Manetti:

Iterated double covers and connected components of the moduli space

Given a surface of general type S, we define the subset $\mathcal{M}_d(S)$ of the moduli space $\mathcal{M}$, $\mathcal{M}_d(S) = \{[S'] \in \mathcal{M} |\ S$ is homeomorphic to S', $r(S) = r(S')\}$, where $r(S)$ is the divisibility of the canonical class in $H^2(S,\mathbb{Z})$. Since K^2, χ are topological invariants, the set $\mathcal{M}_d(S)$ is a quasiprojective variety (Gieseker, '77), in particular has a finite number $\delta(S)$ of connected components.

We prove that in the region $4\chi \le K^2 \le 8\chi$, δ takes in general high values.

More precisely:

Theorem. *Given $\beta \in [4,8]$, there exists a sequence $\{S_n\}$ of 1-connected minimal surfaces of general type such that 1) $\lim_{n\to\infty} \frac{K^2(S_n)}{\chi(S_n)} = \beta$; 2) $\delta \geq (K^2)^{\frac{1}{5}\log K^2}$.*

Corollary. *It cannot exist any polynomial upper bound for δ.*

These result are obtained by studying deformations of a particular class of surfaces called "Simple iterated double covers of $\mathbb{P}^1 \times \mathbb{P}^1$", i.e. iterated flat double covers of $\mathbb{P}^1 \times \mathbb{P}^1$ whose branching divisors are linearly equivalent to pull-back of divisors of $\mathbb{P}^1 \times \mathbb{P}^1$. Under some (not too restrictive) conditions on the Chern classes on the branching divisors, we prove that the family of simple iterated double covers is a connected component of the moduli space.

J. Wahl:

Curves on canonically trivial surfaces

In order to study curves on K3 surfaces, we introduce the notion of a canonically trivial (c.t.) surface X: X is normal and Gorenstein, $\omega_X \simeq \mathcal{O}_X$, and $h^1(\mathcal{O}_X) = 0$. These X have the property that a general hyperplane section is a canonical curve (e.g., X is a K3, or a normal quartic surface in $\mathbb{P}^3$, or a cone over a canonical curve). We say C is extendable if C is $X \cap H$, where X is c.t., and X is not the projective cone over C. A refinement of an old result of ours is

Theorem. *If C is extendable, then the Gaussian map Φ_K is not surjective.*

Further, there is a "moduli space" for the X's that can occur. If C is a complete intersection curve in $\mathbb{P}^r$, or a bielliptic curve, then we can account for all such X, by an explicit construction. This leaves "no room for K3's" and we have

Theorem. *A plane curve of degree ≥ 7, or a bielliptic curve of genus ≥ 10, does not lie on a K3.*

(this has been proved by Green-Lazarsfeld and Reid, using linear-systems methods). If Φ_K is not surjective, we may try to "lift" C to a c.t. X, the relevant obstruction group is $H^1(I_C^2(3))$. We explain why this (very hard to compute) space is frequently (probably generically) 0; we also discussed some of the possible implications to questions about the K3-locus in $\mathcal{M}_g$.

W. K. Seiler:
Moduli of surfaces of general type with a hyperelliptic fibration

Consider nonsingular projective surfaces X admitting a fibration $f : X \longrightarrow C$ onto a nonsingular curve of genus g such that the general fibre is hyperelliptic. Then X has a model at most rational double points as singularities, which is a two-fold covering of a blown up ruled surface W. The surface is then determined by the triple $(W, \mathcal{F}, b)$, where $\mathcal{F} \in \mathrm{Pic}W$ is such that $b \in H^0(W, \mathcal{F}^{\otimes})$ is an equation of the ramification divisor B of X over W. Let $(6, n+3e)$ be the bidegree of B, where $e = C_0^2$ is the self-intersection number of the zero-section of W. Since $B^2 = 12n$, n is invariant under deformations. If W contains no exceptional curves (which I assume from now on), $n = \chi + 1 - g$ for genus two fibrations. For $g \geq 2$, a general W has $6g-6$ moduli, $\mathcal{F}$ has g more; for genus two fibrations and $n > 2g-2$, $\dim|B| = 7n + 6 - 7g$, hence X has $7n$ moduli; all X with small e are parametrized by one irreducible component of this dimension. For the biggest possible value of e , $e = \frac{n}{2}$, B is disconnected, hence such surfaces belong to a different component, also irreducible, of dimension $7n$. The main problem now is to decide wether or not all other surfaces of the type considered are parametrized by the two components obtained so far; it turns out that for $n > 6g$, or $n > 8g-8$, there can be no other component. For $g = 0$ or 1, similar bounds exist.

M. Schneider:
Manifolds with semipositive first Chern class

This is joint work with J. P. Demailly and T. Peternell.
We prove the following results:

Theorem. *Let X be a compact Kähler with $c_1(X) \geq 0$ (i.e. by Aubin-Yau X has a Kähler metric g with* $\mathrm{Ricci}(g) \geq 0$*)*
1) $\pi_1(X)$ is almost abelian, i.e. an extension of an abelian and a finite group.
2) The Albanese map $\alpha : X \longrightarrow$ Alb(X) is a surjective submersion.
3) If $\pi_1(X)$ is finite, the universal cover decomposes $\tilde{X} \simeq \prod X_i$, where X_i are either Calabi-Yau, or symplectic, or satisfy $k_+(X_i) = -\infty$.

Here $k_+(Y) = \max\{k(\det \mathcal{F}) | \mathcal{F} \hookrightarrow \Omega^p_X \text{ for some } p\}$. It is expected that $c_1(Y) \geq 0$ and $k_+(Y) = -\infty$ implies that Y is rationally connected.

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