MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35/1994

Nonlinear Evolution Equations

14.8. - 20.8.1994

Die Tagung fand unter der Leitung von Herrn S. Klainerman (Princeton), und Herrn M. Struwe (Zürich) statt. Die Teilnehmer kamen aus der Bundesrepublik Deutschland, den USA, Russland, China, Frankreich und anderen Ländern. Sie vertraten einen breiten Themenkreis aus dem Gebiet der nichtlinearen Evolutionsgleichungen.

Die Ergebnisse wurden in interessanter und verständlicher Weise vorgetragen. Sicherlich gaben auch die fruchtbaren Diskussionen vielerlei Anregungen.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge:

Generalized Strichartz Inequalities for the Wave Equation

by J. Ginibre (Orsay)

Generalized Strichartz inequalities for the wave equation $\Box u = f$ are estimates of the solution u of the Cauchy problem for that equation, in the form of space time integral norms, in terms of similar norms of the inhomogenity f, and of suitable norms of the initial data. These inequalities are essential tools in the study of the Cauchy problem for nonlinear wave equations, and in particular play an important role in recent studies of that problem for critical non linearities and/or low regularity initial data. We review the available inequalities of this type, most of which were obtained in the mid eighties after the original result of Strichartz in 77, including some recently obtained limiting cases. The proof uses (i) Palay-Littelwood dyadic decompsitions, (ii) stationary phase estimates, (iii) the Hardy-Littelwood and Young inequality in the time variable, and (iv) abstract duality and interpolation arguments. All these ingredients except (ii) are of a general nature and not specific to the wave equation.

On the Critical Wave Equation outside Convex Obstacles

by Christohper D. Sogge (UCLA)

Let $\Omega = \mathbb{R}^3 \setminus O$, where O is a compact smooth obstacle. Then if $u_0, u_1 \in C^\infty(\Omega)$ satisfy necessary compatibility conditions, there is a smooth solution of the critical Dirichlet-wave equation $\Box u = -u^5$, $u(0,x) = u_0$, $\partial_t u(0,x) = u_1(x)$, $u(t,x) = 0$, $x \in \partial\Omega$. This generalizes work in the Euclidean case by Struwe and Grillakis. Our work relies on techniques developed by Grillakis and Shatah and Struwe. The main new ingredient is that estimates of Strichartz and Pecher for the linear wave equation extend to the obstacle case. This is joint work with Hart Smith.

Global Existence of Nonlinear Waves

by Paul Godin (Bruxelles)

We consider the equation

$$(1) \qquad \Box z = \sum_{0 \leq i,j \leq N} f^{ij}(z,z')\partial^2_{ij} z + f(z,z'),$$

where N is odd and ≥ 3. In (1), $\Box := \partial_0^2 - \sum_{1 \leq j \leq N} \partial_j^2$, $z' = (\partial_0 z, \ldots, \partial_N z)$; $f^{ij} = f^{ji}$, f^{ij} and f are C^∞ in a neighbourhood of $(0,0)$, and $f^{ij}(0,0) = \partial^\alpha f(0,0) = 0$ if $|\alpha| \leq 1$. If $N = 3$, we always assume that Klainerman's null condition holds. If $\bar{z}_0$, $\bar{z}_1$ are small $C_0^\infty(\mathbb{R}^n)$ functions, it is a consequence of the work of Klainerman and Christodoulou that (1) has a global solution when $x_0 > 0$, $(x_1, \ldots x_N) \in \mathbb{R}^N$, such that $\partial_0^j z = \bar{z}_j$ if $x_0 = 0$. In this talk we use conformal inversion to obtain global oscillatory solutions of (1). In the radial case and if $N = 3$, we also prove that exterior Cauchy-Dirichlet problems with small initial data have a global solution.

Minimal Compact Global Attractor for a Damped Semilinear Wave Equation

by Lev Kapitanski (Manhattan, St. Petersburg)

We deal with semilinear damped wave equations of the form

$$u_{tt}(t,x) + u_t(t,x) - \Delta u(t,x) + u(t,x) + f(u(t,x)) = h(x), \quad x \in M,$$

where M is a closed Riemannian manifold of dimension $n \geq 3$, h is a given function in $L^2(M)$ and $f(u)$ is the scalar nonlinearity which behaves like $|u|^\sigma u$ for large $|u|$. For the case $\sigma < \frac{4}{n-2}$, I prove that the semi-dynamical system generated by the above equation in the energy space $H^1(M) \times L^2(M)$ has compact global attractor.

Local Solutions of Semilinear Wave Equations

by Hartmut Pecher (Wuppertal)

Consider the Cauchy problem

$$u_{tt} - \Delta u = f(u), \quad u(0) = \varphi, \quad u_t(0) = \psi,$$

where $\varphi \in H^{s+1,2}(\mathbb{R}^n)$, $\psi \in H^{s,2}(\mathbb{R}^n)$ and $f(u) = c|u|^\sigma u$.

If $0 < s < \frac{n}{2} - 1$ and $\sigma \leq \frac{4}{n-2-2s}$, local solutions in the class $C^0([0,T], H^{s+1,2}(\mathbb{R}^n)) \cap C^1([0,T], H^{s,2}(\mathbb{R}^n))$ are shown to exist, if $n \leq 8$. In arbitrary dimension similar results can be proven under some additional assumptions on s and/or σ. Uniqueness holds in a closely related class. Related results were also given by Kapitanski and Lindblad-Sogge.

Local Existence Theorem for First-Order Hyperbolic Systems with Compatible Non-linearities

by Pedro Schirmer (Bonn)

We prove a local existence theorem for symmetric hyperbolic systems involving non-linearities of compatible type under weak regularity assumptions on the initial data. The proof consists in obtaining estimates of Klainerman-Machedon type and is accomplished by estimating some Fourier-integral operators arising from the parametrix representation of the solutions. This is joint work with V. Geogiev.

Nonlinear Perturbations of the Kirchhoff Equation

by P. D'Ancona (Pisa)

The results cited in this talk are contained in a series of joint papers with S. Spagnolo (Pisa). The main result is the following:

We consider the Cauchy problem

$$u_{tt} - m\left(\int |\nabla u|^2 dx\right) \Delta u = F(u, u_t, \nabla u), \quad x \in \mathbb{R}^n, t \geq 0$$

$$u(0,x) = \varepsilon u_0, \quad u_t(0,x) = \varepsilon u_1(x),$$

where $u_0, u_1 \in C_0^\infty$, F is a C^∞-function with $F(\lambda) = O(|\lambda|^{\nu+1})$ near $\lambda = 0$, and $m \geq \nu_0 > 0$ is a C^1-function.

We prove that the above Cauchy problem has a global solution (in time), provided ν is greater than a suitable $\nu_0(n)$ depending on time and provided ε is small enough. More precisely, we have $n \geq 2$ and

$$\nu_0(2) = 10, \quad \nu_0(3) = 6, \quad \nu_0(n) = 5 \quad for \quad n \geq 4.$$

Moreover, when F does not depend on u, we can improve $\nu_0(2)$ to $\nu_0(2) = 9$.

A String of Variable Length

by Herbert Koch (Bonn)

We study the time T-map which maps initial data (u_0, u_1) to the solution and its derivative $(u(T), u_t(T))$ of the homogeneous wave equation in a domain with time T-periodic boundary. The spectrum and the type of the spectrum can be completely analysed. The spectrum is the unit circle if a certain rotation number is irrational. It is a full annulus if this rotation number is rational and a weak additional assumption is satisfied. This is joint work with J. Cooper.

Almost Global Existence for nonrelativistic Wave Equations in 3D

by Thomas C. Sideris (Santa Barbara)

The Lorentz invariance of the d'Alembertian is important for both existence and regularity of nonlinear waves. Nonrelativistic theories, such as the equations of motion for isotropic, homogeneous elastic materials, have a smaller symmetry group. A new proof of almost global existence of small solutions to quadratically nonlinear scalar wave equations in 3D can be given which uses only the classical invariance of the d'Alembertian under translations, rotations and dilations. This argument generalizes to the case of classical elasticity, giving an easy proof of a result of Fritz John. This is joint work with S. Klainerman (Princeton).

Counterexamples to local Existence for Quasi-linear Wave Equations

by Hans Lindblad (Princeton)

We show that the problem

$$\Box u = D^l u D^{k-l} u, \quad (x,t) \in [0,T] \times \mathbb{R}^3$$

$$u(0,x) = f(x) \in \dot{H}^k, \quad u_t(0,x) = g(x) \in \dot{H}^{k-1},$$

where $D := (\partial_{x_1} - \partial_t)$, $0 \leq l \leq k \leq 2$, $l = 0,1$, is ill posed. In fact we show that there are data $(f,g) \in \dot{H}^k \times \dot{H}^{k-1}$ of compact support such that we do not have any solution u for any $T > 0$.

The Ricci Flow

by Richard Hamilton (La Jolla)

The Ricci flow is an evolution equation for a Riemannian metric g on a compact manifold, where the right-hand side is determined from the Ricci curvature:

$$\frac{\partial}{\partial t} g(X,Y) = -2\,\mathrm{Rc}(X,Y).$$

Here, we choose the Ricci curvature because it is a two-tensor like the Riemannian metric. We choose the negative sign because in this way, we obtain a (weakly) parabolic system, which should be thought of as a heat equation for the metric. We have short-time existence of a smooth solution, but in the long run singularities may develop, like for example a neck pinch or a degenerate neck pinch. The curvature tensor satisfies a reaction-diffusion equation:

$$\frac{\partial}{\partial t}\mathrm{Rm} = \Delta\,\mathrm{Rm} + \mathrm{Rm}^2,$$

where Rm^2 denotes an expression which is quadratic in the curvature tensor. The main tool for the study of these geometric equations is the maximum principle. There is also a Harnack inequality. The Ricci flow can be used to prove theorems in differential geometry like for example the following

Theorem: Let M be an oriented 4-manifold, such that $\pi_1(M)$ has no finite elements. Then M is diffeomorphic to S^4 or $S^3 \times S^1$ or $S^3 \times S^1 \sharp S^3 \times S^1 \sharp S^3 \times S^1 \sharp \ldots$.

Regularity and Uniqueness for the Weak Flow of Harmonic Maps

by Yunmei Chen (Gainesville)

We prove that the weak flow from an m-dimensional Riemannian manifold into a sphere satisfying the monotonicity inequality and the energy inequality is smooth off a set which is closed and has m-dimensional Hausdorff measure zero. This is joint work with Fang-Hua Lin and Jiayu Li.

We also show that the weak flow constructed by Chen-Struwe coincides with the smooth flow before the first time of blow up for the smooth flow, if the latter exists. This is joint work with Fang-Hua Lin.

Conformal p-harmonic Flow

by Norbert Hungerbühler (Zürich)

Given two smooth compact Riemannian manifolds M (with metric γ) and N (with metric g) without boundaries, the p-energy of a map $f : M \to N$ is defined by

$$E_p(f) := \frac{1}{p}\int_M |\nabla f|^p dM$$

where $|\nabla f|^p = (\gamma^{\alpha\beta} g_{ij} f^i_\alpha f^j_\beta)^{\frac{p}{2}}$ in local coordinates. The heat flow of the p-energy is given by

$$\left.\begin{array}{rcl} \partial_t - \Delta_p f & \perp & T_f N \\ f\big|_{t=0} & = & f_0 \end{array}\right\} \qquad (F)$$

if N is isometrically embedded in some $\mathbb{R}^k$ and where the p-Laplace operator Δ_p is

$$\Delta_p f = \frac{1}{\sqrt{\gamma}}\partial_\beta(\sqrt{\gamma}|\nabla f|^{p-2}\gamma^{\alpha\beta} f_\alpha).$$

We show that in the conformal situation, i.e. when $p = \dim(M)$, (F) has a global partially regular solution. Only finitely many singular times $T_1, \dots, T_l$ may occur and l is a priori bounded by $l \leq \frac{E_0}{\epsilon}$ (E_0 is the initial energy and $\epsilon > 0$ is a constant only depending on M and N. So, if the initial p-energy is small enough, the solution is regular globally). Outside the singular set ∇f is Hölder-continuous (in space-time). In the class $L^\infty(0,T; W^{1,p}(M,N))$ the solution is unique. The proof involves a priori estimates for $\|\nabla f\|_{L^\infty}$ combined with results of DiBenedetto, analysis of a regularized flow and its linearized version together with Hamilton's technique of a totally geodesic embedding of N.

$L^p - L^q$-estimates for Anisotropic Elastic Media

by Markus Stoth (Bonn)

Let $u = (u^1, u^2, u^3) = u(x,t)$, $t > 0$, $x \in \mathbb{R}^n$ be a solution of the linear equations of elasticity with constant coefficients

$$u^i_{tt} - c_{imjk}\partial_m\partial_k u^j = 0, \quad u(0) = G, \quad u_t(0) = H\,,$$

with material coefficients $c_{imjk} \in \mathbb{R}$, $1 \leq i,m,j,k \leq 3$.

For $G \equiv 0$ and $H \in C_0^\infty(\mathbb{R}^3)$, we prove the following $L^p - L^q$ estimate with rate $p > 0$:

$$\exists c > 0 \quad \forall t > 0 \quad \|u_t(t)\|_q \leq c(1+t)^{-p(1/p-1/q)}\|H\|_{N_{p,p}},$$

$$2 \leq q < \infty, \quad \frac{1}{p} + \frac{1}{q} = 1, \quad N_p \in \mathbb{N}.$$

The optimal rate for isotropic media is $p = 1$, which is well known. For the hexagonal symmetric materials Zink and Beryllium the optimal rates are $p = \frac{1}{2}$ and $p = \frac{5}{6}$ respectively.

We use the method of stationary phase to derive the result. The rate p depends on the curvature of the characteristic manifold.

Stability and Instability in Kinetic Theory

by Walter Strauss (Providence)

Nonlinear stability is what physicists usually want to know about P.D.E.'s. It is usually studied via the linearized equation. For the

Bolzmann equation, the maxwellian is asymptotically stable, due to the increase of entropy. For the Vlasov-Poisson system, the equilibrium $\mu(|v|)$ is stable if μ is decreasing and unstable under the Penrose condition. For certain BGK modes $\mu(\frac{1}{2}v^2 - \phi(x))$ there is a similar result. For the 2-dimensional Euler equations of fluids, the flow $\binom{\sin(my)}{0}$ for $m^2 \neq m_1^2 + m_2^2$ is unstable.

A priori Estimates and Existence for Nonlinear Schroedinger and KdV Equations on a Circle

by Manoussos Grillakis (Ann Arbor)

Consider the following problem

$$(1) \qquad \begin{cases} iu_t + u_{xx} + |u|^\alpha u &= 0, \qquad (t,x) \in \mathbb{R} \times T \\ u(0,x) &= \varphi(x), \quad x \in T, \end{cases}$$

where T is the unit circle. Since the L^2-norm of the solution remains constant, one would like to show that there exist global weak unique solutions for (1) with $\varphi \in L^2$. The present work builds on recent work by J. Bourgain and utilizes two estimates. Consider a function $f(t,x)$ with $(t,x) \in T^2$ and denote by $\hat{f}(\tau,\xi)$ its Fourier transform with $(\tau,\xi) \in \mathbb{Z}^2$ the integer lattice.

$$(2) \qquad \left(\sum_{\xi \in \mathbb{Z}} |\hat{f}(\xi^2,\xi)|^2 \right)^{1/2} \leq C\|f\|_{L^{4/3}(T^2)}$$

$$(3) \qquad \|D_x^{\alpha/4} f\|_{L^2(T^2)} \leq C(\alpha)\|(1+|\tau-\xi^2|^{(3+\alpha)/8}\hat{f}(\tau,\xi)\|_{L^2(\mathbb{Z}^2)}$$

with $0 \leq \alpha < 1$. This is joint work with Y. F. Fang.

The Compressible Euler Equations with Geometrical Structure

by Gui-Qiang Chen (Chicago)

We are concerned with global solutions and corresponding approximation methods for the Euler equations of compressible gas dynamics with geometric structure. An existence theory for global entropy solutions with large L^∞ initial data is introduced by developing compensated compactness frameworks. Corresponding approximation methods are proposed to compute such global entropy solutions. Then this theory is applied to many physical flows, including transonic nozzle flow, spherically symmetric flow and cylindrically symmetric flow.

Sability of Corotational Solitary Wave Maps under Equivariant Perturbations

by A. Shadi Tahvildar-Zadeh (Princeton)

A map $\vec{U} : S^2 \times \mathbb{R} \to S^2$ is equivariant, if $\vec{U}(\alpha,\beta,t) = e^{Al\beta}\vec{u}(\alpha,t)$ for

$$A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$l \in \mathbb{Z}$ and $\vec{u} \in S^2$. It is corotational if

$$\vec{u} = (0, \sin\varphi(\alpha,t), \cos\varphi(\alpha,t)).$$

We study a class of special solutions to the wave map equation for U :

$$\partial_t^2 U - \Delta_{S^2} U + \left(|U_t|^2 - |\nabla_{S^2} U|^2\right) U = 0,$$

satisfying $U(\alpha,\beta,t) = e^{(\omega t + l\beta)A}(0, \sin\varphi(\alpha), \cos\varphi(\alpha))$. The function φ then satisfies an ODE which can be regarded as the Euler-Lagrange equation for a functional $H_{l,\omega}(\varphi)$ which is closely related to the conserved wave map energy $E(U) = \frac{1}{2}\int_{S^2} |\partial_t U|^2 + |\nabla U|^2$.

We prove, using direct variational methods, that the minima of $H_{l,\omega}$ are attained at smooth functions $\varphi^0_{l,\omega}$. We then show the stability of these co-rotational solitary waves under small-energy equivariant perturbations of their initial data. This is done by first using the conservation of energy to prove stability as long as the solution is regular. We then observe that the energy of the solution cannot concentrate in a small cone and thus, using the result that small energy implies regularity for equivariant wave maps, we obtain the desired global result. This is joint work with Jalal Shatah (Courant Institute).

Cosmic Censorship and the Einstein Equations

by James Isenberg (Eugene)

Einstein's equation $G_{\mu\nu} = 0$ for the gravitational field in a spacetime is hyperbolic, and hence has a well-posed Cauchy problem. Roughly three things may occur in the far future of a set of initial data:

1) The evolution may proceed for infinite proper time.

2) Curvature blow-up may occur in finite time.

3) A Cauchy horizon may develop (with consequent loss of determinism).

The cosmic censorship conjecture of Penrose suggests that for generic initial data the third possibility—extension across a Cauchy horizon—does not occur. This conjecture is very much an open question, but we have proven some results in recent years (with Chrusciel and Moncrief) which support the conjecture. We discuss some of these results, such as the proof of cosmic censorship in polarized Gowdy spacetimes. The focus is on the method of proof, which we believe should work for larger classes of spacetime solutions.

The Goursat Problem and the Scattering Operator of Nonlinear Wave Equations

by Zhengfang Zhou (Michigan State University)

The Goursat problem, in which a datum is given on the light cone, has a unique global solution in the positive energy, Sobolev-controllable case. Such equations include those of the form $\Box\phi + H'(\phi) = 0$ where H denotes an interaction Hamiltonian that is a fourth-order polynomial, bounded from below in $\mathbb{R} \times S^3$. The local existence is established from one light cone to any sufficiently close light cone by studying the evolution equation involving Goursat data. The method is shown to establish the existence and continuity of the wave and scattering operators for nonlinear wave equations on $\mathbb{R}^1 \times \mathbb{R}^n$ in finite Einstein energy space.

Stability for Nonlinear Weakly Hyperbolic Systems

by Sergio Spagnolo (Pisa)

We report a jointly paper with P. D'Ancona concerning the $N \times N$ system

$$\begin{cases} u_t &= f(t, u, u_{x_1}, \dots, u_{x_n}), \quad t \geq 0, \; x \in \mathbb{R}^n \\ u(0,x) &= \epsilon\varphi(x), \end{cases}$$

where $f : \Omega^+ \times U \to \mathbb{C}^N$ (U = neighborhood of $(0, \dots, 0)$ in $\mathbb{C}^N$) is continuous in t and analytic in the other variables and

$$f(t, 0, \dots, 0) = 0,$$

while $\varphi : \mathbb{R}^n \to \mathbb{C}^N$ is uniformly analytic on $\mathbb{R}^n$. Thus we have, by Cauchy-Kovalewski, that there exists a local solution $u : [0, T_\epsilon[\times \mathbb{R}^n \to \mathbb{C}^N$ on some strip, and we ask when it occurs that

$$\lim_{\epsilon \to 0} T_\epsilon = +\infty,$$

as in the case of the O.D.E.'s ($u' = f(t,u)$).

Theorem: Assume that the above system is weakly hyperbolic at $u = 0$, i.e. the matrix

$$\sum_{h=1}^{n} \zeta_h \frac{\partial f}{\partial u_{x_h}}(t, 0, \dots, 0), \qquad (\zeta \in \mathbb{R}^n)$$

has only real eigenvalues, and that $\varphi \in L^1(\mathbb{R}^n)$. Then $T_\epsilon \to +\infty$. When $f \equiv f(u, \nabla u)$ does not depend on t, one proves the estimate

$$T_\epsilon \geq c \left(\log \frac{1}{\epsilon}\right)^{1/N}, \qquad (c > 0)$$

which is sharp.

The Critical Power Yang-Mills-Higgs Equations in $\mathbb{R}^{3+1}$

by Markus Keel (Princeton)

I prove two global existence results for the Yang-Mills-Higgs equations with critical power Higgs self-interaction: In $\mathbb{R}^{3+1}$, a unique global solution exists for both smooth and finite energy data.

Global Spherically Symmetric Solutions to the Equations of a Viscous Polytropic Ideal Gas in an Exterior Domain

by Song Jiang (Bonn)

We consider the equations of a viscous polytropic ideal gas in the domain exterior to a ball in $\mathbb{R}^n$ $(n \geq 2)$ and prove the global existence of spherically symmetric solutions for (large) initial data with spherical symmetry. To prove the existence we first study an approximate problem in a bounded annular domain and then obtain *a priory* estimates independent of the boundedness of the domain. Letting the bounded annular domain tend to infinity, we get a global spherically symmetric solution as the limit.

Generalized Fourier Transforms and Global Small Solutions to Kirchhoff Equations

by Reinhard Racke (Konstanz)

It is proved that the inverse of the generalized Fourier transform associated to $-\Delta + V$, V an appropriate compactly supported potential, maps $C_0^\infty(\mathbb{R}^n \setminus \{0\})$ into the space of rapidly decreasing functions. This is used for the study of wave equations with non-local nonlinearities of the type

$$u_{tt} + \left(1 + \int_{\Omega_1} |\nabla u|^2 + \int_{\Omega_1} V|u|^2\right)(-\Delta + V)\,u = 0,$$

for $\Omega_1 = \mathbb{R}^n$, or $\Omega_1 = \Omega$ being an exterior domain in $\mathbb{R}^3$ with $V = 0$, assuming Dirichlet boundary conditions for u. For a class of smooth data we obtain global existence of small solutions, as well as partial characterization of the asymptotic behaviour as $t \to \infty$.

Energy Conservation in Blow-ups of Harmonic Map Heat Flow and Yang-Mills Flow

by Rugang Ye (Santa Barbara)

Blow-ups occur in many geometric evolution equations. An important problem here is whether energy is preserved along the evolution equation provided that the energy of the blow-up limits is counted. This is crucial for the purpose of establishing Morse theory via the evolution flow. We show that for Yang-Mills flow in dimension 4, energy is preserved. We conjecture that the same holds for the harmonic map heat flow. This conjecture indeed holds for the case that the target is a round sphere.

Maximal Regularity for a Free Boundary Problem

by Joachim Escher (Basel)

The equations of the flow of an incompressible fluid through a porous medium can be reduced to a nonlinear evolution equation for the interface of the form:

$$\partial_t f + \Phi(f) = 0, \quad f(0) = f_0 \,. \tag{1}$$

The operator Φ, the so-called Dirichlet-Neumann operator, is a nonlinear, non-local pseudo-differential operator of first order. We show, using the Mihlin-Hörmander multiplier theorem, representation formulas for Poisson- and singular Green operators, and the theory of maximal regularity, that problem (1) generates a smooth local semiflow on an appropriate phase space. From that result we then get classical solutions of the original problem.

Blow-up for some Degenerate Parabolic Equations

by Michael Wiegner (Bayreuth)

We study the problem $u_t = u^p(\Delta u + u)$, $p > 1$, on a bounded domain

$\Omega \subset \mathbb{R}^n$. Assuming for the initial value

$$0 < c_0 \leq u(x,0)\,\mathrm{dist}(x,\partial\Omega)^{-1} \leq c_1\,,$$

we show that in contrast to the standard problem $u_t = \Delta u + u^{p+1}$, where the solution exists globally for c_1 small and blows up for c_0 large, here the *size* of the domain plays the crucial role: Blow-up occurs precisely, if λ_1, the first eigenvalue of $-\Delta$, is smaller than 1. The life span T_0 of the solution may be estimated by $\tilde{c}c_1^{-p} \leq T_0 \leq cc_0^{-p}$. In one space-dimension, we give further some refined estimates for the behaviour of the solution near the blow-up time.

Space-time Means and Asymptotic Properties of Nonlinear Klein-Gordon Equations

by Philip Brenner (Göteborg)

Properties of the solution u of the nonlinear Klein-Gordon equation

$$(NLKG)\quad \begin{cases} \partial_t^2 u - \sum \partial_{x_j}^2 u + m^2 u + f(u) &= 0,\ x \in \mathbb{R}^n,\ t \geq 0 \\ u\big|_0 &= \phi \\ \partial_t u\big|_0 &= \psi \end{cases}$$

is compared to those of the solution u_0 of the (linear) Klein-Gordon equation

$$(KG)\quad \begin{cases} \partial_t^2 u_0 - \sum \partial_{x_j}^2 u_0 + m^2 u_0 &= 0, \qquad x \in \mathbb{R}^n,\ t \geq 0 \\ u_0\big|_0 &= \phi \\ \partial_t u_0\big|_0 &= \psi \end{cases}$$

with the same initial data (assumed to be in at least $H^1 \times L^2$). Here $f(u)$ is a C^2-function which is bounded by $|u|^{\varrho_0}$ at 0 and by $|u|^{\varrho_1}$ at ∞. It also generates a positive definite energy, that is $F(u) = \int_0^u f(v)dv \geq 0$, $f(\mathbb{R}) \subset \mathbb{R}$. In addition, to avoid the appearance of bound states (or corner of energy), we assume (following Morawetz) that $uf(u) - 2f(u) \geq \alpha f(u)$ (some $\alpha > 0$).

We give conditions under which boundedness of space-time integrals ψ implies the boundedness of the same integrals of the solutions of $(NLKG)$, that is ψ for the solution of the (KG) when (in the above notation) $u_0 \in L_r(L^{s'}_{q'})$ implies that $u \in L_r(L^{s'}_{q'})$. Some improvements of previous results are given, proofs are discussed and applications to the existence of scattering operators in $H^{s+1} \times H^s$, $s \geq 0$, and to decay-results for the $(NLKG)$ are given.

Wellposedness in the Energy Space for the Van Karman Model for Plates

by Daniel Tataru (Evanston)

The Van Karman model for plates is a semilinear plate equation of the form

$$u_{tt} + \Delta^2 u = G(u),$$

$$u(0) = u_0, \quad u_t(0) = u_1$$

in a bounded domain $\Omega \subset \mathbb{R}^2$. This equation admits a natural coercive energy functional, defined for $(u, u_t) \in H^2(\Omega) \times L^2(\Omega)$, which is preserved along the the trajectories.

Then the structure of the nonlinearity allows the use of some compensated compactness arguments to slightly improve the regularity of the nonlinear term, and finally lead to global well-posedness for the problem in the energy space $H^2(\Omega) \times L^2(\Omega)$.

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