# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 36/1994

## Gruppen und Geometrien

21.08. bis 27.08.1994

Die Tagung "Gruppen und Geometrien" fand unter der Leitung der Herren M. Aschbacher, Caltech, W.M. Kantor, Eugene und F. Timmesfeld, Gießen statt. Im Mittelpunkt des Interesses standen die Anwendung der Klassifikation der endlichen einfachen Gruppen, gebäudeverwandte geometrische Strukturen und die Ausdehnung von Methoden der endlichen Gruppentheorie und Geometrie auf beliebige Gruppen und verwandte geometrische Strukturen.
Zum ersten Themenkreis zählte der Vortrag von M. Aschbacher, der ausgehend von einer "Preisfrage" von Hirzebruch, eine 8-dimensionale differenzierbare Mannigfaltigkeit konstruierte, auf der das Bimonster operiert. Diese Verbindung von endlicher Gruppentheorie und Topologie scheint für die Zukunft sehr fruchtbar zu werden. Zu letzterem Themenkreis zählten die Vorträge von A. Steinbach, F. Timmesfeld und R. Weiss. Hier scheint es in Zukunft möglich, mit Methoden der endlichen Gruppentheorie Fragen zu beantworten, die man bisher nur für algebraische Gruppen (über algebraisch abgeschlossenem Körper) lösen konnte.

## Vortragsauszüge

### M. ASCHBACHER: Representing groups on manifolds

Prize Question: (Hirzebruch) Does there exist a 24-dimensional, compact, orientable, differential manifold $X$ (admitting the action of the Monster) with $p_1(X) = w_2(X) = 0$, $\hat{A}(X) = 1$, and $\hat{A}(X, T_{\mathbf{C}}) = 0$?

Motivated in part by this question, I consider rigid actions of groups $G$ on homology $n$-manifolds preserving a regular cell structure with $G$ transitive on $n$-cells. These conditions translate into a group theoretic setting. The stabilizers of all cells are forced to be

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spherical Coxeter groups; this is highly restrictive. The Conway-Norton-Soicher involutions can be used to construct such representations for the BiMonster. I described one such representation of dimension 8.

E. BANNAI:
On Broué-Enguehard theory (Weight enumerators and invariants of finite groups)

This is a joint work with Michio Ozeki at Yamagata University in Japan.
We obtain some extensions of Gleason's theorem and Broué-Enguehard theorem.

Theorem 1. Let $R = \mathbb{C}[x_1, y_1, x_2, y_2]$, and let $G = \langle \sigma_1, \sigma_2 \rangle$ with

$$\sigma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & & 0 \\ 1 & -1 & & \\ & & 1 & 1 \\ 0 & & 1 & -1 \end{pmatrix} \quad \text{and} \quad \sigma_2 = \begin{pmatrix} \sqrt{-1} & & & 0 \\ & 1 & & \\ & & \sqrt{-1} & \\ 0 & & & 1 \end{pmatrix}.$$

($|G| = 192$ and $G$ is a finite u.g.g.r, i.e. No 9 in Shephard-Todd's list.) Then the invariant ring $R^G$ has the Molien series

$$\Phi_G(t) = \frac{1 + 8t^8 + 21t^{16} + 58t^{24} + 47t^{32} + 35t^{40} + 21t^{48} + t^{56}}{(1 - t^8)^2(1 - t^{24})^2}.$$

(This expression also gives a homogeneous system of parameters of $R^G$.)
Here, $R^G$ is identified with the space of certain multivariable weight enumerators of binary self-dual doubly even codes.

Theorem 2. Let $f(x_1, y_1, x_2, y_2)$ be a homogeneous polynomial of degree $n$ in $R^G$ (in Theorem 1). Then $f(\Theta_3(2\tau, 0), \Theta_2(2\tau, 0), \Theta_3(2\tau, 2z), \Theta_2(2\tau, 2z))$ is a Jacobi form (on $SL(2, \mathbf{Z})$) of weight $k = \frac{n}{2}$ and index = the total degree of $f$ in $x_2$ and $y_2$.

We obtain many new and old Jacobi forms by applying Theorem 2. Further extensions of above theorems were also mentioned in the talk.

B. BAUMEISTER:
On flag-transitive $C_2$.c-geometries

Let $H$ be a primitive permutation group with point stabilizer $H_0$. If $H$ admits a factorisation $H = H_0 H_0^{\alpha}$ for some $\alpha \in \mathrm{Aut}(H)$, then $H$ is almost simple and $Soc(H) \simeq \Omega_7(q)$, $M_{12}$, $Sp_4(q)'$, $q$ even, or $H$ is affine and $H \simeq 2^3 L_3(2)$ or $H$ is of product action type, in the sense of Liebeck, Praeger and Saxl, ON THE O'NAN-SCOTT THEOREM.

We apply this proposition to the classification of flag-transitive $C_2.c$-geometries (geometries with diagram $\circ\!=\!=\!=\!\circ\overset{c}{-\!\!-}\circ$ , which consist of points, lines and quads, quads being $n \times n$-grids). We obtain

Theorem. Let $G$ act flag-transitively on a $C_2.c$-geometry. Suppose for $P$ a point, $K_P = 1$, and for $Q$ a quad, primitive and faithful action of $G_Q/K_Q$ on one parallel class of lines of $Q$. If $G_Q/K_Q$ is not of product action type, then $G \simeq J_3$ or $3J_3$.

A.E. BROUWER:
(A) $Sp(2d, F)$-modules

Let $F$ be a field, $V$ an $n$-dimensional vector space over $F$, $f$ a non-degenerate symplectic form on $V$ (so that $n = 2d$), $G = Sp(V, f)$. Let $F_m$ be the span in $\Lambda^m V$ of all t.i. $m$-spaces.

Proposition 1. The $FG$-module $F_m$ is the Weyl module $W(\lambda_m)$ of $G$ for the fundamental weight $\lambda_m$. We have $\dim F_m = \binom{n}{m} - \binom{n}{m-2}$.

(This is due to Bourbaki in char 0, to Premet and Suprunenko in odd characteristic.)

Proposition 2. $F_m$ only has composition factors $M_j$ $(0 \le j \le m)$, where $M_j := F_j/N_j$ is the irreducible $G$-module with highest weight $\lambda_j$. If $M_j$ occurs, then with multiplicity 1. It occurs if and only if $m-j$ is even and $d-j+1$ contains $\frac{1}{2}(m-j)$ to base $p$. $\{b$ contains $a$ to base $p$ when $b > 2a$ and if $a = \sum a_i p^i$, $b = \sum b_i p^i$ then $a_i = 0$ or $a_i = b_i$ for all $i.\}$

(Again, this is due to Premet and Suprunenko in odd characteristic.) This proposition allows one to compute $\dim M_j$ by induction on $j$.

(B) Spectrum of adjacency matrices

In an association scheme $(X, R)$ with adjacency matrices $A_i$ and intersection matrices $L_i$ (defined by $(L_i)_{jk} = p^j_{ik}$) the map $A_i \mapsto L_i$ is an isomorphism from the Bose-Mesner algebra to the algebra spanned by the $L_i$. Thus, the eigenvalues of $A_i$ are the same as those of $L_i$. It is well-known how to compute the mulitiplicities when all eigenvalues of $L_i$ are distinct. In general one has:

Lemma. Put $L = L_i$, $A = A_i$. If $\Theta$ is an eigenvalue of $L$, then there is a unique real eigenvector $u$ of $L$ such that $Lu = \Theta u$, $u_0 = 1$ and $u^T\Delta_k u$ minimal. Now the multiplicity of $\Theta$ as an eigenvalue of $A$ equals $v/u^T\Delta_k u$.

F. BUEKENHOUT:
Atlas of group-geometry pairs

This is a report on joint work with Michel Dehon, Dimitri Leemans, Philippe Cara and CAYLEY. About 30 groups, including $M_{11}$, the $G$ with $PSL(2,q) \le G \le P\Gamma L(2,q)$,

$q \leq 19$ and some primitive groups of affine type are presented to an exhaustive search of all flag-transitive, residually connected geometries with restrictions on the maximal parabolics and their intersections. Hundreds of geometries are displayed with a diagram, boolean lattice of parabolics containing a given Borel subgroup, "maps" of their incidence graph, collinearity graph, correlation groups, etc.

B. COOPERSTEIN:

Frames of Lie Incidence Geometries

Let $V$ be a vector space of dimension $n+1$ over $GF(q)$ and denote by $A_{n,k}$ the incidence geometry $\Gamma = (P, L)$ whose points are the $k$-dimensional subspaces of $V$, lines the partial flags consisting of an incident pair of $k-1$, $k+1$ dimensional subspaces. A frame for $\Gamma$ is a subset $F$ of $P$ such that the induced subgraph of the collinearity graph is isomorphic to the Johnson graph $J_{(n+1,k)}$. We will investigate the conjugacy of frames under $\mathrm{Aut}(\Gamma)$ and the geometric span of such frames. We will further discuss extensions of this concept and analogous results to other geometries, e.g. $D_{n,n}$, $B_{n,n}$, $E_{6,1}$, $E_{7,1}$.

H. CUYPERS:

Extended near hexagons and line systems

In their paper "Near $n$-gons and line systems", Geom.Ded. 9 (1980), 1-72, Shult and Yanushka study systems of lines in $\mathbb{R}^n$ passing through the origin and having the property that two lines are either perpendicular or make an angle $\alpha$ with $\cos\alpha = \frac{1}{3}$. An example of such a line system is the set of 4 lines passing through the vertices of a regular tetrahedron centered at the origin. We will call any such line system tetrahedrally closed if and only if we have the following: if 3 of the 4 lines through the vertices of a tetrahedron are in the system then so is the 4th.

Under some weak assumptions Shult and Yanushka showed that the lines and tetrahedra of a line system form an extended near hexagon. Here we classify such extended hexagons under some regularity conditions, and show that associated to each of them there is indeed a unique line system. All the line systems found in this way turn out to be subsystems of a system of 2300 lines in $\mathbb{R}^{23}$ with automorphism group $2 \times Co_2$ or of 2048 lines in $\mathbb{R}^{24}$ with group $2^{12} : M_{24}$.

D. FON-DER-FLAASS:
A combinatorial construction for twin trees

A twin tree of valency $k$ is a triple $\mathcal{T} = (T^+, T^-, \mathrm{cod})$ where $T^\pm$ are trees of valency $k$ and cod: $T^+ \times T^- \cup T^- \times T^+ \to \mathbb{N}$ satisfies

$$\begin{array}{ll} (T1) & \mathrm{cod}(x,y) = \mathrm{cod}(y,x) \\ (T2) & |\mathrm{cod}(x',y) - \mathrm{cod}(x,y)| = 1 \quad \text{for} \quad x - x' \\ (T3) & \{\mathrm{cod}(x',y) \mid x' - x\} = \{n+1, n-1 \cdots n-1\} \quad \text{if} \quad \mathrm{cod}(x,y) = n > 0. \end{array}$$

A pairing is an isomorphism $\Pi : T^+ \leftrightarrow T^-$ such that $\mathrm{cod}(x, \Pi(x)) = 0$. We present an inductive construction of a twin tree with a pairing. This gives all twin trees (uncountably many for each valency $k \geq 3$).
When valency $k = 3$ and Aut$\mathcal{T}$ is transitive on pairs of opposite directed edges, we obtain that

$$\mathrm{Aut}\mathcal{T} = \langle \alpha, \beta, \gamma \rangle \text{ with involutions } \alpha, \beta, \gamma \text{ such that}$$
$$\langle \alpha, \beta \rangle \simeq S_3,\ \langle \alpha\beta, \gamma \rangle \simeq L_2(\mathbf{Z}) \simeq \mathbf{Z}_2 * \mathbf{Z}_3.$$

D. FROHARDT:
Fixed Point Ratios in Buildings

Let $\Delta$ be a (thick) building, and let $g$ be a type-preserving non-trivial automorphism of $\Delta$. Assume further that $g$ fixes a chamber in $\Delta$, and that $\Delta$ is irreducible and spherical. Let $\Delta_0$ be the fixed subcomplex of $g$. Then $\Delta_0$ is either contractible and convex, or a non-degenerate (and possibly thin) subbuilding. In the former case $\Delta_0$ is contained in the union of half apartments centered at some simplex of $\Delta$ (a vertex except in case $A_n$). In the latter case $\Delta_0$ corresponds to the subbuilding of one of the subgroups of the Weyl group generated by reflections, which is maximal within the Weyl group. This enables one to compute the largest possible fixed point ratio that a non-trivial element of an exceptional group of Lie type can have when it acts on the cosets of a maximal parabolic subgroup.

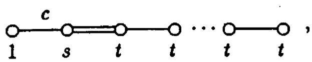
C. HUYBRECHTS:
An observation on locally projective spaces

By a theorem of Doyen and Hubaut, the $n$-dimensional locally projective spaces of orders $(a,b)$ are known except when $n = 3$ and either $b = (a+1)^2$, $a > 1$ or $b = (a+1)^3 + (a+1)$, $a \geq 1$.
The existence of such spaces has become a famous unsolved problem. Some restrictions on the values of the parameter $a$ in the second family can be made by using non-existence

results of affine planes of some given order. I observe that $a = 2$ cannot occur in the first family of the Doyen-Hubaut theorem in view of the relationship between maximal arcs and locally projective spaces and the following result:
"There is no maximal $\{21, 3\}$-arc in the projective planes of order 9" (Cossu, Penttila, Royle).

A.A. IVANOV:
<u>On extended dual polar spaces</u>

I consider flag-transitive geometries belonging to the diagrams




where the residue of an element of the leftmost type is a classical dual polar space and I call them extended dual polar spaces ($EDPS's$). There are 13 rank 3 $EDPS's$. I discuss the classification problem of $EDPS's$ of rank 4 and more.
The results recently proved include simple connectedness proofs for a number of $EDPS's$ associated with Fischer's groups and a description in terms of universal representation groups of the universal covers of $EDPS's$ from two infinite families. In these families the residual dual polar spaces are those related to $Sp_{2n}(2)$ and $U_{2n}(2)$, respectively. If $n = 2$ the universal representation groups are abelian, if $n = 3$ then the commutants are of order 2. I know nothing on what happens when $n \geq 4$.

P.M. JOHNSON:
<u>Embeddings and geometric hyperplanes of certain geometries</u>

We use standard terminology for point-line geometries $\mathcal{S} = (\mathcal{P}, \mathcal{L})$, with lines being sets of points. The aim is to show that, for many parapolar spaces, there is a universal embedding space $U$ whose hyperplanes correspond naturally to the full collection of geometric hyperplanes of $\mathcal{S}$. First, observe that the singular subspaces are projective spaces, all over some fixed division ring $K$. Inclusions between them are of the form $PG(X) \hookrightarrow PG(Y)$, so give rise to semilinear maps $X \to Y$, at least when $\dim X \geq 3$ (and usually when $\dim X = 2$). This gives a diagram in which the maps commute only up to scalars. The problem is to adjust the vector spaces (via semilinear automorphisms) to get linear maps that commute. This can usually be done for the subdiagrams consisting of spaces lying above some point $p$. Taking a limit, we get a universal embedding space $U$ for the "residue" at each point $p$. Geometric hyperplanes of $\mathcal{S}$ not through $p$ give non-zero linear functionals on $U$ and show that the embedding is faithful.

One shows that the geometric hyperplanes form a projective space via a criterion of Teirlinck, which is verified locally (above each $p$), then shown to be globally consistent. This has been done for most polar spaces and should extend easily to many related geometries.

R.A. LIEBLER:

Antipodal Distance Transitive Covers of Complete Bipartite Graphs

The generic examples of such graphs have as vertices the points and hyperplanes of a finite projective space that are off a given incident point hyperplane pair. Certain projective planes coordinatized by twisted fields also fall in the class. There are two almost simple examples arising from the outer automorphisms of $S_6$ and of $M_{12}$. One example arises from the non-abelian Singer group on $PG(2,4)$. Finally there is an infinite family having the solvable automorphism group $q^b K_{q^a q^a}$, where $(q^b - 1)gcd(b, q-1)$ divides $2a(q-1)$. Joint work with A.A. Ivanov, T. Penttila and C.E. Praeger shows this to be a complete list.

K. MAGAARD:

Subgroups of $F_4(q)$

Represent $F_4(q)$ as Aut$(J)$ where $J$ is a quadratic Jordan algebra. Work of McCrimmon, Racine and Jacobson allows a classification of subalgebras of $J$. This in turn allows a classification of non almost simple maximal subgroups.

G.E. MOORHOUSE:

Some existence and nonexistence results for ovoids

An *ovoid* (resp., *cap*) in a polar space $\mathcal{P}$ is a collection $\mathcal{O}$ of points $\mathcal{P}$ such that every maximal subspace of $\mathcal{P}$ contains a unique point (resp., at most one point) of $\mathcal{O}$. Recently we have constructed ovoids in polar spaces of type $O_8^+(p)$ for every prime $p$, generalising a construction of Conway, Kleidman and Wilson (1988), using the $E_8$ root lattice. In recent joint work with Blokhuis, we have shown that if $\mathcal{O}$ is a cap in $\mathcal{P}$, then $|S| \le \binom{p+n-1}{n}^e + 1$ where $PG(n, p^e)$ is the ambient space for the natural embedding of $\mathcal{P}$. In particular, there are no ovoids in $O_7(2^e), O_{10}^+(2^e), O_{10}^+(3^e), O_9(5^e), O_{12}^+(5^e), O_{12}^+(7^e)$, etc. Slightly stronger bounds are obtained by showing that the $p$-rank of the incidence matrix of points of $\mathcal{P}$ versus hyperplanes of the ambient space, equals $\left[\binom{p+n-1}{n} - \binom{p+n-3}{n}\right]^e + 1$ in the case of an orthogonal polar space embedded in $PG(n, p^e)$ (for $n \ge 3$), or $\left[\binom{p+n-1}{n}^2 - \binom{p+n-2}{n}^2\right]^e + 1$ in the case of a unitary polar space embedded in $PG(n, p^{2e})$ (for $n \ge 2$). The latter computations of $p$-ranks depend upon versions of the Nullstellensatz for varieties over finite fields.

B. MÜHLHERR:
Twin buildings

This is a joint work with M. Ronan.
In 1974 J. Tits proved that the local structure in a spherical building determines its global structure. He conjectured the validity of this theorem also for twin buildings of locally finite type.
We have proved this conjecture for all those twin buildings having no rank-2-residues corresponding to $Sp_4(2), G_2(2), G_2(3)$ or ${}^2F_4(2)$.

P. MÜLLER:
Monodromy Groups and Number Theory

Let $K$ be a number field, and $O_K$ its ring of integers. Let $f(x) \in K[x]$ be a polynomial of degree $n > 0$. Fix an integer $0 \leq i \leq n$. Let $\mathcal{R}$ (resp. $\mathcal{V}$) be the set of those $a \in O_K$, such that $f(x) - ax^i$ is reducible (resp. has a root in $K$). Obviously $\mathcal{V} \subseteq \mathcal{R}$. M. Fried started to investigate when $\mathcal{R}\backslash\mathcal{V}$ is finite. This is generically the case, the only interesting situation left open was $i = 1$. We show how to formulate this problem as a question about finite groups, and how to find constraints which finally yield the following: $\mathcal{R}\backslash\mathcal{V}$ is finite (if $i = 1$) if $K = \mathbb{Q}$ or $n \notin \{4,6,8,9,12,16\}$. Conversely, there are counter examples for all the excluded degrees.

S. NORTON:
Transpositions and footballs

We examine properties of the Conway variant of the Griess algebra in the least degree representation of the Monster. Results on invariant multilinear forms on this algebra are used to derive relations among algebra products of vectors corresponding to particular elements of the Monster. Results of particular interest include formulae for the traces of products of transpositions with elements of certain other classes; a "football invariant" on sets of three transpositions invariant under braiding; and the existence of "outer automorphisms", which may correspond to the Atkin-Lehrer involutions of Monstrous Moonshine, for the sub-algebras stabilized by particular abelian subgroups of the Monster.

## CH. PARKER:
## Symplectic Amalgams

Suppose that $G$ is a group and $p$ is a prime. We call $G$ a $p$-constrained amalgam if

(A1) $G = \langle P_\alpha, P_\beta \rangle$, $P_\alpha$ and $P_\beta$ distinct finite subgroups of $G$.

(A2) $P_\alpha \cap P_\beta$ contains no non-trivial normal subgroup of $G$.

(A3) $P_\alpha \cap P_\beta = N_{P_\alpha}(S_{\alpha\beta}) = N_{P_\beta}(S_{\alpha\beta})$ where $S_{\alpha\beta} \in \mathrm{Syl}_p(P_\alpha \cap P_\beta)$.

(A4) $F^*(P_\gamma) = O_p(P_\gamma)$ for $\gamma \in \{\alpha, \beta\}$.

Now for $\gamma \in \{\alpha, \beta\}$ set $L_\gamma = O^{p'}(P_\gamma)$, $Q_\gamma = O_p(L_\gamma)$ and $Z_\gamma = \langle \Omega_1(Z(S_{\alpha\beta}))^{L_\gamma} \rangle$. Then $G$ is a Symplectic Amalgam over $GF(p)$ if

(SA1) $L_\alpha/Q_\alpha \simeq SL_2(p)$.

(SA2) $O^p(L_\beta/Q_\beta) = \langle 1 \neq AQ_\beta/Q_\beta{}^{L_\beta} \rangle$ where $AQ_\beta/Q_\beta$ is any normal subgroup of $S_{\alpha\beta}/Q_\beta$.

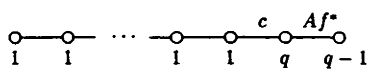
(SA3) $Z_\beta = \Omega_1(Z(L_\beta))$.

(SA4) $Z_\alpha \leq Q_\beta$ and there is $x \in L_\beta$ such that $Z_\alpha{}^x \not\leq Q_\alpha$.

In my talk I discussed my joint work with P. Rowley which will result in a complete classification of such structures.

## A. PASINI:
## Flag-transitive $c.Af^*$-geometries

(Joint work with A. del Fra and C. Huybrechts)
We classify all flag-transitive geometries belonging to the following diagram:




## L. PYBER:
## On random generation and maximal subgroups of finite and profinite groups

In the talk I considered various probabilistic results for the symmetric groups including the following:

Theorem I (joint with T. Luczak)
A random element of $\mathrm{Sym}(n)$ is not contained in any transitive subgroup other than $\mathrm{Sym}(n)$ or $\mathrm{Alt}(n)$.

Theorem II Let $G_1$ and $G_2$ be (non-trivial) finite groups not both of them isomorphic to $C_2$. Then a random pair of subgroups of $\mathrm{Sym}(n)$ isomorphic to $G_1$ resp. $G_2$ generates $\mathrm{Sym}(n)$ or $\mathrm{Alt}(n)$.

These results were conjectured by Cameron resp. Lubotzky.

Let $C_t$ denote the class of finite groups not having an alternating section of degree $\geq t$.

Theorem III (joint work with A. Borovik and A. Shalev)
Suppose that $G$ is a finitely generated pro-$C_t$-group. There exists an integer $k = k(t)$ and an $\epsilon > 0$ such that a random subset of $k$ elements generates $G$ with probability $> \epsilon$.

P. ROWLEY:

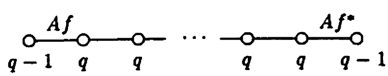
## The maximal 2–local $J_4$–geometry

The sporadic group $J_4$ acts flag-transitively on a rank 3 string geometry $\Gamma$ in which the residue of a point is the geometry of trios and sextets of the Steiner system $S(24,5,8)$ and the residue of a plane is the geometry of duads and hexads of the Steiner system $S(22,3,6)$. In fact, $\Gamma$ is a subgeometry of the maximal 2-local geometry introduced by Ronan and Smith. The point-line collinearity graph of $\Gamma$, $\mathcal{G}$, has 173,067,389 vertices. This talk outlined joint work with Louise Walker in which a detailed description of $\mathcal{G}$ is obtained in terms of geometric configurations in $\Gamma$.

S. SHPECTOROV:

## Geometries with bi-affine diagrams

We announce the following result due to A. del Fra, A. Pasini and the author.
Theorem If $\Gamma$ is a geometry belonging to




rank $\geq 4$, $q \geq 2$,

then $\Gamma$ is a natural quotient of two bi-affine geometries over $GF(q)$.
A bi-affine geometry is obtained from a projective space over $GF(q)$ by removing all the elements incident to either a fixed point, or a fixed hyperplane.

E. SHULT:

**The Cohen-Cooperstein Theorems Revisited**

A. Cohen and B. Cooperstein proved two theorems which characterized all spherical buildings (and some of their homomorphic images) as parapolar spaces satisfying certain conditions $(CC)_0$ and $(CC)'_1$.

In their first theorem, the assumptions of that $\Gamma$ is a strong parapolar space, and $\Gamma$ has constant symplectic rank can each be dropped at the expense of increasing the list of conclusion geometries. In their second theorem, there is evidence that the assumption of finite singular rank may be relaxed.
A natural situation which forces the condition $(CC)_1$ in parapolar spaces, make their theorem more applicable.

A. STEINBACH:

**Subgroups of classical groups generated by transvections resp. Siegel transvections**

Let $G$ be a subgroup generated by (Siegel-) transvections of a finite dimensional classical group $Y$ over an arbitrary commutative field. Under suitable assumptions (especially $G$ is defined over a field) $G/Z(G)$ is known by results of Timmesfeld on groups generated by "abstract root subgroups".
In my talk I presented the results for the embeddings $G \leq Y$ with $G/Z(G) \simeq PSL(W)$, $PSp(W)$, $PSU(W,f)$, $P\Omega(W,f)$ where $W$ is a finite dimensional vector space over a commutative field. The situation is the same as in Kantor's paper on subgroups of finite classical groups generated by long root elements.
In the second part of my talk I showed how to handle embeddings of unitary groups over quaternion division rings of char $\neq 2$ in orthogonal groups.

B. STELLMACHER:

**A $ZJ$-Theorem for $\Sigma_4$-free groups**

The basic ideas for a proof of the following theorem have been described:
Let $S$ be a finite 2-group. Then there exists a characteristic subgroup $W(S)$ of $S$, which contains $\Omega_1(Z(S))$, such that for every finite group $H$ satisfying

(i) $S \in \mathrm{Syl}_2(H)$,

(ii) $C_H(O_2(H)) \leq O_2(H)$, and

(iii) $H$ is $\Sigma_4$-free

$W(S)$ is normal in $H$.

This theorem can be viewed as an analogue of Glauberman's $ZJ$-Theorem for the prime 2.

G. STROTH:
Some remarks on the uniqueness problem

In the revised version of the classification of the finite simple groups (of even type) one has a subdivision in the small groups (2-local $p$-rank at most three, or $N_G(S)$ is in exactly one maximal 2-local of $G$ for $S \in \mathrm{Syl}_2(G)$) and the groups in which the signalizer functor method works very well. The uniqueness problem is to show that this really divides the simple groups (of even type) into two parts, i.e. it deals with groups $G$ such that for any maximal 2-local $M$ (uniqueness group) of $G$ such that the $p$-rank is at least four for some odd prime $p$, $N_G(P) \leq M$ for $p$-groups $1 \neq P \leq M$, $m_p(P) \geq 2$ or $m_p(\mathbb{C}_M(P)) \geq 3$. The following theorem is stated and parts of the proof are sketched.
Theorem: If $G$ is in the uniqueness case and $M$ is some uniqueness subgroup, $S \in \mathrm{Syl}_2(M)$, $H$ a 2-local of $G$ with $N_G(S) \leq H$, then $H \leq M$.

P.H. TIEP:
Globally irreducible representations of finite groups and integral lattices

In 1976 J.G. Thompson initiated the consideration of all pairs $(G, \Lambda)$ where $G$ is a finite group and $\Lambda$ a $\mathbb{Z}G$-module such that the $F_pG$-module $\Lambda/p\Lambda$ is irreducible for every prime $p$. A generalization of this condition - the global irreducibility of rational representations - is suggested by B.H. Gross, in order to explain new lattices, constructed by N. Elkies and T. Shioda by means of Mordell-Weil lattices of certain elliptic curves. In this talk we first give a necessary condition for $\chi \in \mathrm{Irr}(G)$ to be globally irreducible. Then we classify all globally irreducible representations ($GIR's$) coming from projective representations of most of 26 sporadic simple groups, and of finite groups of Lie type of rank 1,2. We also classify all $GIR's$ coming from Weil representations of finite classical groups, and from basic spin representations of $A_n$ and $S_n$. In particular, we get new series of even unimodular Euclidean lattices.

F.G. TIMMESFELD:
Quadratic action and abstract root-subgroups

Let $k$ be a field, Char $k \neq 2$ and $k \neq GF(3)$ and $M$ a finite dimensional $k$-vector space. $\sigma \in GL(M)$ is quadratic on $M$ if $\sigma = \mathrm{id} + \alpha$, $\alpha^2 = 0$. If $c \in k$, then $c \circ \sigma = \mathrm{id} + c\alpha$ and

$k \circ \sigma = \{c \circ \sigma \mid c \in k\} \simeq k^+$. If $G \subseteq GL(M)$, then the pair $(G, M)$ is called a quadratic pair (over $k$), if

1. $M$ is an irreducible $kG$-module,
2. $G$ is generated by quadratic elements $\sigma$ with $k \circ \sigma \subseteq G$.

If $k = \mathbb{Z}_p$, $p \geq 5$ then Thompson has classified in a famous unpublished paper the groups $G$, when $(G, M)$ is a quadratic pair over $\mathbb{Z}_p$. This classification is generalized to arbitrary fields $k$ as above. The most important ingredient of the proof is the notion of abstract root-subgroups. Indeed quasisimple groups generated by a class of abstract root-subgroups were classified under certain weak additional conditions.

V. TROFIMOV:
Graphs with projective suborbits

Let $\Gamma$ be an undirected connected graph and let $G$ be a vertex-transitive group of automorphisms of $\Gamma$. For a vertex $x$ of $\Gamma$, let $\Gamma_x$ be the set of vertices adjacent to $x$ in $\Gamma$ and $G_x$ be the stabilizer in $G$ of $x$. We determine $G_x$ in the case $G_x$ is finite and $G_x^{\Gamma_x}$ contains a normal subgroup isomorphic to $PSL_n(q)$, $n \geq 3$, in its natural action on the points of $PG_{n-1}(q)$.

N. VAVILOV:
Long root tori

This is a joint work with A.A. Semenov. We describe Bruhat decomposition of long root tori in a Chevalley group over a field. It turns out that this problem is equivalent to the classification of triples of long root subgroups, two of which are opposite and thus the general case reduces to $D_4$. All elements of a given torus, apart from at most three of them lie in the same Borel double coset. Using these results V.V. Nesterov determined orbits of Chevalley groups acting by conjugation on pairs of long root tori.

H. VÖLKLEIN:
Finite quotients of braid groups, and rigidity

A new criterion for the realization of groups as Galois groups is given. Let $S$ be a finite Coxeter group with Coxeter system $s_1, \cdots, s_\ell$. Let $m_{ij}$ be the order of $s_i s_j$. Define an $S$-system in a finite group $G$ to be a system of generators $g_1, \cdots, g_\ell$ satisfying $g_i g_j g_i \cdots = g_j g_i g_j \cdots$ for each $1 \leq i, j \leq \ell$ (where we have $m_{ij}$ factors on each side).

The $S$-system is called rigid if for all $S$-systems $g_1', \cdots, g_\ell'$ in $G$ with $g_i'$ conjugate $g_i$ for all $i$ there exists $g \in G$ with $g_i' = g_i^g$ for all $i$.
Theorem If $G$ has a rigid $S$-system then $G/Z(G)$ is a Galois group over $\mathbb{Q}_{ab}$ (= the maximal cyclotomic field).

R.M. WEISS:
Moufang Trees

Let $\Gamma$ be a tree and $G \leq \mathrm{aut}(\Gamma)$. $\Gamma$ is called $(G,n)$-Moufang (for some $n \geq 3$) if for every $n$-path $(x_0, \cdots, x_n)$

(i) $G^{(1)}_{x_1,\cdots,x_{n-1}}$ acts transitively on $\Gamma_{x_n} \backslash \{x_{n-1}\}$ and

(ii) $G^{(1)}_{x_0,x_1} \cap G_{x_0,\cdots,x_n} = 1$.

It is known that thick $(G,n)$-Moufang trees exist only for $n = 3, 4, 6$ and 8.

Conjecture: Let $\Gamma$ be a thick $(G,n)$-Moufang tree such that $G$ is generated by the subgroups $G^{(1)}_{x_1,\cdots,x_{n-1}}$ for all $(n-2)$-paths $(x_1, \cdots, x_{n-1})$. Then there exists a $G$-invariant equivalence relation $\approx$ on $\Gamma$ such that $\Gamma/\approx$ is a Moufang generalized $n$-gon and the natural map from $\Gamma$ to $\Gamma/\approx$ is a local isomorphism.

This conjecture rests on a result of Delgado-Stellmacher who proved something similar in the case that $|\Gamma_x| < \infty$ for all $x$. Our efforts to prove this conjecture ( still in progress) are heavily dependent on Tits' work on Moufang polygons.

S. YOSHIARA:
Flag-transitive $C_3$-geometries

The following theorem is announced together with a brief sketch of some important steps.

Theorem. A flag-transitive locally finite thick $C_3$-geometry is either a building or the sporadic $A_7$-geometry.

Berichterstatterin: Anja Steinbach

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