MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 45/1994

Wahrscheinlichkeitsmaße auf Gruppen und verwandten Strukturen

23.10. bis 29.10.1994

Die Tagung fand unter Leitung der Herren H. Heyer (Tübingen) und A. Mukherjea (Tampa) statt.

Von den 44 Teilnehmern aus 10 europäischen Ländern, Australien, Indien, Japan, Tunesien und den USA wurden insgesamt 42 Vorträge gehalten. Dabei kamen vielfältige wahrscheinlichkeitstheoretische Fragestellungen auf algebraisch-topologisch-geometrischen Strukturen wie Gruppen, Hypergruppen, Halbgruppen, Banachräumen und Bäumen zur Sprache. So wurden z.B. neue Grenzwertsätze und Eigenschaften unendlich teilbarer Wahrscheinlichkeitsmaße vorgestellt, man diskutierte Momentenprobleme, Fragen über Irrfahrten und stabile Prozesse. Eigenes Gewicht erhielt die nichtkommutative Wahrscheinlichkeitstheorie. Schließlich ergaben sich neue Ansätze für die Anwendung der strukturellen Wahrscheinlichkeitstheorie in der statistischen Modellbildung und -analyse.

Am Anfang des täglichen Programms stand jeweils ein einstündiger Hauptvortrag. Hierbei handelte es sich um Übersichtsvorträge zu den folgenden Themen: Zentrale Grenzwertsätze auf lokalkompakten abelschen Gruppen; Hypergruppen und Darstellungstheorie von Liegruppen; Faltungshalbgruppen in der Quantenstochastik; neue Ergebnisse bei Momentenproblemen; Darstellungen und Wahrscheinlichkeitstheorie.

Neben den übrigen zwanzigminütigen Vorträgen gab es noch Abendsitzungen über spezielle Aspekte der Theorie der Hypergruppen bzw. Pseudodifferentialoperatoren mit negativ definiten Symbolen.

Die strukturelle Wahrscheinlichkeitstheorie erfährt weiterhin bemerkenswerte Impulse von verschiedenen Teilgebieten der Mathematik, weist aber auch ihrerseits in Richtungen praktischer Forschung in Nachbargebieten wie etwa der mathematischen Physik. Reger Austausch von Ideen und starke Publikationstätigkeit halten an.

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Abstracts:

O.E. BARNDORFF-NIELSEN:
On the role of continuouse groups in statistics

A brief survey was given of some main aspects of the use of continuous groups in statistics.

The concept of transformation models was discribed and exemplified, with focus on exponential transformation models.

Furthermore, the relation to the conditionality principle and the p^*-formula for the conditional distribution of the maximum likelihood estimator was indicated.

Ch. BERG:
Recent results about moment problems

In the talk we gave a survey of recent results concerning the moment problem on $\mathbb{R}^d$ and in particular about the Hamburger and Stieltjes moment problems. Two main questions are in focus:

(a) How to decide determinancy / indeterminancy?

(b) For which $p \in [1, \infty[$ and measures $\mu \in \mathcal{M}^*(\mathbb{R}^d)$ (the measures with moments of any order) is the set of polynomials $\mathbb{C}[x_1, \ldots, x_d]$ dense in $L^p(\mu)$?

In contrast to dimension $d = 1$ where $\mathbb{C}[x]$ is dense in $L^2(\mu)$ if μ is determinate (Riesz, 1923) it turns out that there exist determinate measures $\mu \in \mathcal{M}^*(\mathbb{R}^d)$ when $d > 1$ such that $\mathbb{C}[x_1, \ldots, x_d]$ is not dense in $L^2(\mu)$.

There are now several examples calculated in the indeterminate case, i.e. the entire functions A, B, C, D and extremal measures are known explicitly. This is true for the case of the q^{-1}-Hermite polynomials as well as rate birth and death polynomials (Berg, Valent, 1994).

Ph. BIANE:
Noncommutative Brownian motion and Borel process

Let A_t and A_t^+ be the annihilation and creation process on the Fock space (cf. the book by K.R. Parthasarathy *Quantum Stochastic Calculus*). We study the noncommutative stochastic process $A_t A_t^+$ and prove that it is a dilation of a commutative semigroup on a space which is the Gelfand spectrum of the Gelfand pair $(G(1) \times \mathbb{H}, U(1))$ when $\mathbb{H}$ is the Heisenberg group and $U(1)$ is the unitary group . It turns out that this stochastic process can be expressed in terms of branching processes, and has a very interesting structure.

M.S. BINGHAM:
Central limit theorems on locally compact abelian groups

The purpose of the talk was to review the development of central limit theory for random variables with values in locally compact abelian groups, from its beginning up to the present time. The classic theory can be found in the monograph of Gnedenko and Kolmogorov (1960), the generalization of the theory for locally compact second countable abelian groups is contained in Parthasarathy, Rango Rao and Varadhan (1963), much of it is reproduced in Parthasarathy (1967).

Byczkowski (1967) proved an invariance principle in which the limit process is a group-valued Wiener process, and Heyer (1977) reproved many of the results without assuming second countability.

In the meantime martingale central limit theory was being developed for real-valued random variables and the monograph of Hall and Heyde appeared on this topic in 1980. Subsequently, a number of martingale-like central limit theorems for triangular arrays of random variables with values in locally compact second countable abelian groups were proved. The limit distribution in these can be mixtures of Gaussian distributions, or, more generally, mixtures of infinitely divisible distributions. Here is also included a function central limit theorem in which the limit is a mixture of Wiener measures.

W.R. BLOOM:
Negative definite functions on commutative hypergroups

Corresponding to the definitions of positive definite functions there are various approches to defining negative definite functions on hypergroups. These range from the obvious *pointwise* definition through to approximation via the Schoenberg duality. We give an overview of how the various approches fit together, both on the hypergroup and its dual. One open problem is to decide whether the following spaces are equal:

$$\begin{aligned} N_{S_0}^{(w)}(K) &= \{f \in C(K) \ : \ f(e) \geq 0,\, f^- = \overline{f} \ \text{ and } \int_K f\,d\nu \leq 0 \ \forall \nu \in M_p(K) \\ &\qquad \text{with } \hat{\nu} \geq 0 \ \text{ and } \hat{\nu}(\mathbb{1}) = 0\} \\ N_{S_0}^{w'}(K) &= \{f \in C(K) \ : \ f(e) \geq 0,\, f^- = \overline{f} \text{ and } \int_K f\,d\nu * \nu^- \leq 0 \ \forall \nu \in M_p(K) \\ &\qquad \text{with } \hat{\nu}(\mathbb{1}) = 0\} \end{aligned}$$

where $S_0 = M_p(K)$ is the space of all finitely supported measures on K. Corresponing questions for functions on the dual are also disussed.

Ph. BOUGEROL:
Random walk on the affine group: critical case

The affine group $G = \mathbb{R}^d \times \mathbb{R}_*^+$ acts on the homogeneous space $G/\mathbb{R}_*^+ = \mathbb{R}^d$ by $(b,a)\cdot x = ax + b$. If $g_n = (b_n, a_n)$ is a sequence of i.i.d. random elements of G then $X_n^x = g_n \ldots g_1 x$ is a Markov chain on $\mathbb{R}^d$ given by $X_{n+1}^x = a_{n+1}X_n^x + b_{n+1}$, $n \in \mathbb{N}$. It occures in many applications. By using the properties of the random walk $g_n \ldots g_1 g_0$ on G the following holds:
Theorem: If $\mathbb{E}\log a_1 = 0$, $\mathbb{E}|\log a_1| < \infty$, $\mathbb{E}\log^+|b_1| < \infty$, $a_1 \neq 1$ the markov chian X_n^x has a unique invariant measure m. Moreover: $\frac{1}{M_n}\sum_{k=1}^n f(X_k^x)$ converges almost

surely to $\int f\,dm$ for any $f : \mathbb{R}^d \to \mathbb{R}$, continuous with compact support where $M_n = \max_{1\leq k\leq n}(-\log a_k \dots a_1)$.

Corollary: $\frac{1}{\sqrt{n}} \sum_{k=1}^{n} f(X_k^x) \overset{\text{law}}{\to} \sigma \int f\,dm|X|$ where $X \sim \mathcal{N}(0,1)$.

G. BUDSBAN:

Convolution products of non-identical distributions on finite semigroups

Let $(\mu_n) \subset P(S)$, the probability measures on S, where S is a locally compact, Hausdorff, second-countable semigroup. Convolution products of the form

$$\mu_{k,n} = \mu_{k+1} * \dots * \mu_n$$

are considered from three different, but related, perspectives.

(i) What is the semigroup of the tail limits of the products?
(ii) What are the supports of the tail limit measures?
(iii) What kind of conditions are sufficient for weak convergence of the products?

Some conditions are generated for finite groups and are applied to examples of non-homogeneous Markov chains.

T. BYCZKOWSKI:

Zeros of the densities of infinitely divisible measures on $\mathbb{R}^n$

Let μ be an infinitely divisible probability measure on $\mathbb{R}^n$ without Gaussian component and let ν be its Lévy measure. Suppose that μ is absolutely continuous with respect to the Lebesgue measure λ. The structure of the set of admissible translates of μ is investigated. This yields a unified presentation of previously known results. We also show that if $\lambda(S) > 0$ then μ is equivalent to λ, under the assumption that $\text{supp}\mu = \mathbb{R}^n$, where $S = \text{Sem}[\text{supp}\nu]^-$ is the closure of the semigroup generated be $\text{supp}\nu$.

H. CARNAL:

Phase retrieval for probabillity measures on Abelian groups

Mass distributions μ on $\mathbb{R}^d$,$\mathbb{R}^d/\mathbb{Z}^d$ or more generally, on an abelian group can be studied using physical devices which afford the observation of $|\hat{\mu}(y)|$, $\hat{\mu}$ being the Fourier transform (or characteristic function) of μ on the dual group $\hat{G}$.

The problem is then to find the argument of the complex function $\hat{\mu}$ in order to identify μ. This is only possible up to a translation $\mu \to \mu * \delta_x$ or a central symmetry $\mu(A) \to \overline{\mu}(A) = \mu(-A)$. We call μ and ν eqivalent if $|\hat{\mu}| \equiv |\hat{\nu}|$ on $\hat{G}$ and say that μ has a trivial equivalence class if $\nu \sim \mu \iff \nu = \mu * \delta_x$ or $\nu = \overline{\mu} * \delta_x$.

Examples of measures with trivial equivalence class are given by normal distributions on $\mathbb{R}^d$ (using Lévy Cramer's theorem) or uniform distributions on products of intervals $I_1 \times I_2 \times \dots \times I_d$ (the proof uses Plancherel's theorem and Schwartz's inequality). On compact groups like $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ or $\mathbb{Z}_n = \mathbb{Z}/n\mathbb{Z}$, there exist intervals with non-trivial equivalence class, namely those with Haar measure > 1/2. On the opposite, intervals covering at most half of the group have trivial equivalence classes.

In some special cases (e.g. on groups like $\mathbb{Z}_2^I$), the direct product of measures with trivial equivalence class has again the same property.

W.C. CONNETT:

Measure algebras with positive convolutions

We are concerned with the case $H \subset \mathbb{R}$, and we have an algebracally complete family of real polynomials which have a *product formula*, i.e. for each $s, t \in H$ exist $\mu_{s,t}$ such that $p_n(s)p_n(t) = \int p_n \, d\mu_{s,t}$. We give three examples of families of polynomials that have a bounded, positive product formula with an identity: Jacoby, Generalized Chebychev and continuous q-ultraspherical polynomials. In all three cases we obtain a Banach algebra on H, but only in the Jacoby case do we obtain a hypergroup. This leads a number of theorems that give sufficient conditions for a product formula to become a hypergroup, and these classifications allow us to weaken the theorem we reported in 1990, for that if $(H, *)$ is a hypergroup with polynomial characters, then this characters must be the Jacobi polynomials. First version: $H = [-1, 1]$, second version: H is a compact set with e an accumulation point with some technical control on the support, third version: H is infinite and compact or the $p_n(x)$ are orthogonal with respect to a measure μ, supp$(\mu) \subset H$. This is joint work with A. Schwartz.

D.P. DOKKEN:

Probability measures on groups, compactifications and limit theorems

In probability and dynamical systems theory, one is often interested in the asymptotic behaviour of a sequence or net of elements from a group. In probability theory given a measure on a group one might ask, in the associated random walk, which sequences converge to points in the group. For those which don't we could perhaps attach limit points at infinity so sequences would in some sense converge. Another approach (the one adopted here), motivated by problems in topological dynamics and ergodic theory, is to compactify the group initially. This is done so that the points in the compactification inherit some of the groups structure. For example, there will be a semi-group structure on the compactification. We make statements about the limits of the matrix coefficients as the nets of the group go to infinity. It will be also be shown how certain results in probability theory can be stated in terms of these compactifications. This work is motivated by attempts to prove a certain of A. Raugi's boundary theory results. Ultimately we would like to prove certain results of his using properties and structure of these compactifications.

S.N. EVANS:

Local field Brownian motion and stochastic analysis

We define a natural notion of Gaussian measures on a Banach space over the p-adic numbers. We then define a corresponding notion of Brownian motion indexed by a p-adic vector space and taking values in another p-adic vector space. The potential theory of such a process is developed. In certain cases local time exist, and we investigate its properties. Finally, we define an appropriate notion of white noise and multiple stochastic integral, and develop a Wiener chaos expansion for functionals of the white noise.

Ph. FEINSILVER:
A probability distribution associated to the Euclidean group

Consider E_2, the Euclidean group in two dimensions. Using a complex form, one constructs a Hilbert space so that every element of the Lie algebra is self-adjoint. A basis is found in terms of Bessel functions J_m, $m \in \mathbb{Z}$. A distribution on $\mathbb{Z}$ is found such that for $t > 0$, $P(X_t = m) = J_m(t)^2$. Two limit theorems are given: One to inverse cosine, the second to Airy-function squared. Applications are indicated to electronic music and to quantum mechanics on a lattice with uniform electric field.

J. GLOVER:
Inversions and reflecting Brownian motion

For a class of domains $E \subset \mathbb{R}^2$, we construct a function Φ and a time charge ϱ_t of two-dimensional Brownian motion b_t such that $\Phi(b(\varrho_t))$ is a Brownian motion with normal reflection at the boundary of E. The construction is based on a study of the noncommutative group generated by a finite or infinite collection of inversions in the plane. The talk is based on work done with Murali Rao.

P. GŁOWACKI:
Pseudodifferential operators with negative definite symbols

Let F be a C^∞ real negative definite function on $\mathbb{R}^n \times \mathbb{R}^n$. If

a) $F(w) \to \infty$, as $|w| \to \infty$,
b) $|\partial^\alpha F(w)| \leq C_\alpha$, $|\alpha| > 0$,
c) $|\nabla_w F(w)| \geq C > 0$ for large $\lambda > 0$,

then the operator

$$Af(x) = \iint e^{2\pi i\xi(x-y)} F(\frac{x-y}{2}, \xi)\hat{f}(\xi)\, d\xi dy$$

is selfadjoint, positive, has discrete spectrum, and the following Weyl formula holds

$$\left| \frac{\mathcal{N}_A(\lambda)}{\iint_{F(w)\leq\lambda} dw} - 1 \right| \leq Ce^{-\lambda}$$

for large $\lambda > 0$, where $\mathcal{N}_A(\lambda)$ is equal to the number of the eigenvalues of A less than or equal to λ.

P. GRACZYK:

Central limit theorems on symmetric spaces

Let $X = G/K$ be a Riemanian symmetric space with G-semisimple finite center, non-conpact and K a maximal compact subgroup of G. In this talk we consider K-invariant probability measures on X and we present some central limit theorem for such measures.

We presented in the talk a certain kind of limit theorems where one studies the asymptotic behaviour of μ^{*m} comparing it with the asymptotic behaviour of Ω^{*m} where Ω is a Gaussian measure on X with same mean and covariance as μ. Results of this kind were initiated by Tutubalin (1961-63), and developed by Virtser, Bougerol, Raugi and Guivarc'h who proved in 1985 some completely general theorems of this kind. We show how to prove such results when X is complex or of rank 1 by methods of spherical Fourier analysis. Some interesting analytic results concerning the Abel transform and spherical functions have been proved at this occasion.

W. HAZOD:

Limit theorems for products of a random number of group valued random variables

1. Let G be a simply connected nilpotent Lie group, let $(X_{nk})_{n,k\geq 1}$ be an array of G-valued random variables, such that for all n $(X_{n,k})_{k\geq 1}$ are i.i.d., $X_{n0} = e$, $X_{nk}(P) = \nu_n$. Let $S_m^{(n)} = X_{n0} \dots X_{nm}$ be the random walk (for any row). Further more let $N_n : \Omega \to \mathbb{Z}_+$ be discrete random variables (*random times*). We assume (X_{nm}, N_n) to be independent. Put $Z_n := S_{N_n}^{(n)} = \prod_0^{N_n} X_{nk}$. Let $\xi_n = N_n(P) = \sum_k p_{nk}\varepsilon_k$. Then the distribution of Z_n is given by $\nu^{\xi_n} := \sum_k p_{nk}\nu_n^k$.

2. Let Y_t be a Lévy process, such that $Y_t(P) = \mu_t, t \geq 0$, is a continuous convolution semigroup. Let $T : \Omega \to \mathbb{R}_+$ be a random variable, independent of $(Y_t, t \geq 0)$. Put $Z := Y_T$, then the distribution of Z is given by $\mu_\varrho := \int_0^\infty \mu_t \, d\varrho(t)$ where $\varrho = T(P)$.

The subordination maps $\xi \mapsto \mu_\varrho$ have "nice properties" which allows to prove a generalized

Transfer theorem: Let $\nu_{n_k} \in M^1(G)$, $k_n \uparrow \infty$ and assume a *deterministic limit* $\nu_n^{[k_n t]} \to \mu_t$, $t \geq 0$. Let $\tau_{1/k_n} : x \mapsto \frac{1}{k_n}x$, and assume $\tau_{1/k_n}(\xi_n) \to \varrho \in M^1(\mathbb{R}_+)$. Then a *random limit theorem* $\nu_n^{\xi_n} \to \mu_\varrho$ holds.

Moreover an inverse transfer theorem can be proved, which allows for simply connected nilpotent Lie groups to study the concepts of geometric convolution, geometric divisibility, geometric stability etc.

Furthermore, using two important results of E. Siebert, it can be shown that in this situation limit theorems for groups hold iff the corresponding theorems hold for the tangent space.

G. HÖGNÄS:
A mixed random walk on nonnegative matrices

Consider a semigroup S of $d \times d$ nonnegative matrices, generated b a probability measure μ. Assume that the convolution sequence $(\mu^n)_{n\in\mathbb{N}}$ is tight. The Cesáro mean $w-\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\mu^k := \beta$ always exists and is supported by the completely simple kernel consisting of all matrices of minimal rank in S.

Let $X_1, X_2, \ldots$ be a sequence of μ-distributed random variables. While the left and write random walks in S in general do not have unique invariant (stationary) probability measures (β is always one) a randomized walk, called mixed random walk may be better behaved. The mixed random walk is defined by $S_n = S_{n-1}X_n$ with probability α, $S_n = X_n S_{n-1}$ with probability $1-\alpha$, the choice of left and right being independent of the previous X's and S's. Under some algebraic conditions on S and μ we prove that the mixed random walk S_n indeed has β as its only invariant probability measure. In this case we also obtain a law of large numbers

$$\frac{1}{n}\sum_{k=1}^{n} f(S_k) \longrightarrow \int f\,d\beta, \quad f \in C_0(S), \quad \text{a.s.}$$

Z.J. JUREK:
A filtration of the class ID of all infinitely divisible distributions

The convolution semigroup ID of all infinitely divisible measures (on a Banach space) is divided into subsemigroups $\mathcal{U}_\beta$, $\beta \geq -2$, in such a way that for $-2 \leq \beta_1 \leq 0 \leq \beta_2 \leq \beta_3 \leq \ldots$ we get

$$\{\text{Gaussian}\} = \mathcal{U}_{-2} \subsetneq \mathcal{U}_{\beta_1} \subseteq \mathcal{U}_0 = L \subseteq \mathcal{U}_{\beta_2} \subseteq \mathcal{U}_{\beta_1} \subseteq \mathcal{U}_1 \subseteq \mathcal{U}_{\beta_3} \subsetneq \ldots \subseteq \text{ID}, \quad \text{and}$$

$$\text{ID} = \overline{\bigcup_\beta \mathcal{U}_\beta} \quad \text{(closure in weak topology).}$$

In the above inclusions we marked the class $L = \mathcal{U}_0$ which coincides with the Lévy class of selfdecomposable measures, the class $\mathcal{U}_{-2}$ of all Gaussian measures and the class $\mathcal{U}_1$ of s-selfdecomposable measures.

The classes $\mathcal{U}_\beta$ are defined as classes of limiting distributions of the following sequences

$$n^{-1}(\xi_1 + \xi_2 + \ldots + \xi_n)(n^{-\beta}), \; n = 1, 2, \ldots,$$

where $\xi_j' s$ are independent Lévy processes evaluated at time $n^{-\beta}$. Further, each class $\mathcal{U}_\beta$, with $\beta > -2$, is characterized via random integrals (with respect to some Lévy processes) that induce random integral mappings between corresponding semigroups of measures. Algebraic and topological properties are then studied.

V.A. KAIMANOVICH:

The Poisson boundery of polycyclic groups

Let G be a group and μ a probability measure on it. A function f on G is μ-harmonic if $f\mu = f$. There exists a measure G-space (Γ, ν) such that the Poisson formula $f(g) =< \hat{f}, g\nu >$ is an isometry of the space of bounded μ-harmonic functions and $L^\infty(\Gamma, \nu)$. This space is unique and is called the Poisson boudery of (Q, μ). The talk is devoted to a description of the Poisson boundery in terms of the internal structure of polycyclic groups. Roughly speaking, these are semidirect products $\mathcal{A} \times \mathcal{N}$, where $\mathcal{A}$ is abelian and $\mathcal{N}$ nilpotent. It turns out, that if the measure μ has a finite first moment then the Poisson boundery coincides with the contracting subgroup $\mathcal{N}_- \subset \mathcal{N}$. The proof is based on considering the behavior of the random walk with respect to the *neutral* subgroup $\mathcal{N}_0 \subset \mathcal{N}$ and a general geometric *strip criterion* obtained as an application of the entropy theory. The same idea applies to Lie solvable groups (to the affine group "$ax + b$" of the real line $\mathcal{N}_-$ coincides with the real line in the contracting case $\int \log a(g)\, d\mu(g) < 0$ and is trivial otherwise.

Yu.S. KHOKHLOV:

Stable distributions and their generalizations: Structure and limit theorems

Let $X_1, X_2, \ldots$ be real i.i.d. random variables. Consider $S_n = X_1 + \ldots + X_n$ and $S_n^* = A_n^{-1}(S_n) + B_n$ where $A_n > 0$, $B_n \in \mathbb{R}^1$.

If μ is an accumulation point of the sequence $\{\mathcal{L}(S_n^*)\}$ then μ is a *semistable distribution* and $\mathcal{L}(X_1)$ is in the *domain of semistable attraction of* μ. In this case we write $\nu = \mathcal{L}(X_1) \in \mathcal{DSOA}(\mu)$.

Krapov showed that μ is semistable iff μ is Gaussian, or μ has no Gaussian component and its spectral function is of the form

$$H(x) = \begin{cases} |x|^{-\alpha} c_1(\ln |x|), & x < 0, \\ |x|^{-\alpha} c_2(\ln |x|), & x > 0. \end{cases}$$

where c_1 and c_2 are periodic with common period.

Let $\{a_n\}$ be an increasing sequence of positive numbers and consider $x_n = x \cdot a_n$. Theorem: $\nu \in \mathcal{DSOA}(\mu)$, where μ has spherical function H as above iff

(1) $F(-x_n) = x_n^{-\alpha} L(x_n)[c_1(\ln x) + h_1(x, n)]$,

(2) $1 - F(x_n) = x_n^{-\alpha} L(x_n)[c_2(\ln x) + h_2(x, n)]$ where L is a slowly varying function and $\lim_{n\to\infty} h_i(x, n) = 0$.

We consider also a law of iterated logarithm of Chover's type and generalizations of these results in the case of $\mathbb{R}^d$, simply connected nilpotent Lie groups and other situations.

R. LASSER:

Modified moment problem carried by a compact intervall

In order to investigate the covariance structure of stochastic processes $(X_n)_{n \in \mathbb{N}_0}$ we study the following modified moment problem:

Let $(P_n(x))_{n\in\mathbb{N}_0}$ be a sequence of polynomials with real coefficents with degree $(P_n) = n$, $P_0(x) = 1$. Denote

$$P_nP_m = \sum_{k=0}^{n+m} g(n,m,k)P_k$$

and define for any sequence $(c_n)_n$ of complex numbers

$$c_{n,m} = \sum_{k=0}^{n+m} g(n,m,k)c_k.$$

We say $(c_n)_n$ is P_n-positive definite if $\sum_{i,j=0}^{N} \lambda_i\lambda_j c_{i,j} \geq 0$ for all $n \in \mathbb{N}_0$ and $\lambda_0, \dots, \lambda_N \in \mathbb{R}$. Further denote

$c_n^{(\pm 1)} = c_n \pm (\alpha_{n,n+1}c_{n+1} + \dots + \alpha_{n,0}c_0)$, when $xP_n(x) = \alpha_{n,n+1}P_{n+1}(x) + \dots + \alpha_{n,0}P_0(x)$

and

$c_n^{(2)} = c_n - (\beta_{n,n+2}c_{n+2} + \dots + \beta_{n,0}c_0)$, when $x^2P_n(x) = \beta_{n,n+2}P_{n+2}(x) + \dots + \beta_{n,0}P_0(x)$.

Theorem: The following conditions are equivalent:

(a) $c_n = \int_{-1}^{1} P_n(x)\,d\mu(x)$ for $\mu \in \mathcal{M}_+([-1,1])$,

(b) $(c_n)_n$, $(c_n^{(1)})_n$, $(c_n^{(-1)})_n$, $(c_n^{(2)})_n$ are P_n-positive definite.

G. LETAC:
Wishart distributions on symmetric cones

Let E be a Euclidean Jordan algebra, let E_+ be the symmetric cone of square elements of E which are invertible and let F be the natural exponential family on E generated by the Lebesgue measure on E_+. The Wishart distributions on E_+ are the powers F_p of the elements of F. The variance function of the natural exponential family F_p is the quadratic application $m \mapsto \frac{1}{p}\mathbb{P}(m)$ of the Jordan algebra E, restricted to the domain of the means $M_{F_p} = E_+$. We denote by G the connected component containing the identity of the group of automorphisms of E_+.

A division algorithm is a map $E_+ \to G$, $x \mapsto q(x)$ such that $q(x)(x) = e$, where e is the neutral element of E. We prove the following analogue of a famous characterization of the Gamma distribution given by Lukács: Given two independent random variables X and Y in $\overline{E_+}$ such that $X + Y \in E_+$, and given a divison algorithm q, then we have: $Z = q(X+Y)(X)$ and $(X+Y)$ are independent and the law of Z is invariant by $G \cap \mathbb{O}(E)$ if and only if X and Y are Wishart distributed with the same scale paramater.

M. McCRUDDEN:
Siebert's Gaussian semigroup on the affine group

On the connected affine group

$$G = \left\{ \begin{pmatrix} \beta & \alpha \\ 0 & 1 \end{pmatrix} \; : \; \beta > 0, \alpha \in \mathbb{R} \right\}$$

we consider the Gaussian semigroup (μ_t) whose infinitesimal generator is $X + Y^2$, where $X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Eberhard Siebert studied this semigroup in his second paper *Abs. continuity, singularity and supports of Gauss semigroups on a Lie group* (Monat. Math. 1982). We report the result of D. Kelly-Lyth, obtained in his doctoral thesis (University of Manchester 1993) that the densities f_t for the Siebert semigroup above are given by

$$f_t(\alpha, \beta) = \begin{cases} \frac{1}{2\pi^2\alpha} e^{-(1+\beta)/\alpha} \int_0^\infty \tau \sinh(\pi\tau) K_{i\tau}(2\sqrt{\beta}/\alpha)\, e^{-t\tau^2/4}\, d\tau, & \alpha \geq 0, \\ 0, \quad \alpha \leq 0, \end{cases}$$

where K_w is the adopted Bessel function of the third kind.

D. NEUENSCHWANDER:
Brownian motion and robust limit theorems on the Heisenberg group

Let $\{\mu_t\}_{t\geq 0}$ be a continuous convolution semigroup on the Heisenberg group $\mathbb{H}$ which is (not necessarily shiftly) stable in the sense of Hazod with respect to the one-parameter automorphism group $\{t^A\}_{t>0}$ Assume $\{X_k\}_{k\geq 1}$ are i.i.d. $\mathbb{H}$-valued random variables which lie in the domain of attraction of $\{\mu_t\}_{t\geq 0}$ with norming sequence $\{(t_n^A, x_n)\}_{n\geq 1} \subset$ Aut($\mathbb{H}$) $\times$ $\mathbb{H}$. Then a suitable *intermediate* trimming procedure as suggested by Kuelbs, Ledoux for Banach spaces yields that the trimmed products of the X_k, after suitable renorming, converge to a non-trivial Gaussian limit. For *light* trimming (i.e. a fixed number of the most extreme observations is deleted) the limit can be discribed by some generalized Lévy construction of first kind.

Finally we introduce the following: For X, Y i.i.d. $\mathbb{H}$-valued random variables, $\mathcal{L}(X)$ is called a *Gauss measures in the sense of Bernstein* if $(X \cdot Y, Y \cdot X)$ and $(X \cdot (-Y), (-Y) \cdot X)$ are independent. Disapointingly enough, this definition yields, of all Gaussian measures on $\mathbb{H}$, only uninteresting ones from the point of view of the group structure (i.e. concentrated on a 2-dimensional subspace of $\mathbb{H}$ containing the center of $\mathbb{H}$).

N. OBATA:
Integral kernel operators on Fock space and applications to harmonic analysis on Gaussian space

Let $E = \mathcal{S}(\mathbb{R}) \subset H = L^2(\mathbb{R}, dt) \subset E^* = \mathcal{S}'(\mathbb{R})$ and let μ be the Gaussian measure on E^* of which characteristic function is $e^{-|\xi|^2/2}$. The probability space (E^*, μ) is called the Gaussian space and is an infinite dimensional analogue of Euclidean space with Lebesgue measure. Let $(E) \subset L^2(E^*, \mu) \subset (E)^*$ be the triplet of white noise test, L^2, and generalized functions. The main features are:

(a) $t \mapsto \Phi_t = <\cdot, \delta_t> \in (E)^*$, which is called the white noise, becomes a continuous flow.

(b) The annihilation and creation operators, denoted by ∂_t and ∂_t^* respectively, are defined for each point, and $t \mapsto \partial_t, \partial_t^*$ are continuous flows.

The integral kernel operator is a superposition of normally ordered products $\partial_{s_1}^* \dots \partial_{s_l}^* \partial_{t_1} \dots \partial_{t_m}$ with distribution weights. Furthermore, a generalization is possible to use operator-valued distributions. The important consequences are:

A Fubini-type theorem garantees the iterated integration.

Expansion theorem: every operator is decomposed into an infinite series of integral kernel operators.

With these theorems we can discuss characterizations of operators by certain invariance properties and applications to quantum stochastic calculus.

G. PAP:
The accuracy of Gaussian approximations in nilpotent Lie groups

We give refinements of the following version of the CLT. Let $|.|$ denote a homogeneous norm on a nilpotent Lie group G, $\delta_t \in \mathrm{Aut}(G)$, $t > 0$, denote a semigroup of standard dilation, and let μ be a centered probability measure on G. For $t > 0$ we define the measure $\delta_{1/t}\mu(A) = \mu(\delta_t A)$, where $A \in \mathcal{B}(G)$. Denote by μ^n the n-fold convolution of μ. Then the condition $\int |x|^2 \mu(dx) < \infty$ implies that $\mu_n = \delta_{1/\sqrt{n}}\mu^n \to \nu$ converge weakly to a Gaussian measure, say ν.

Under the second moment assumption a Berry-Esséen type bound is given for $\int f(x)(\mu_n - \nu)(dx)$ provided $f : G \to \mathbb{R}$ is a sufficiently smooth function.

Moreover, we obtain the edgeworth expansion

$$\int f(x)(\mu_n - \nu)(dx) = \alpha n^{-1/2} + O(n^{-1}),$$

where $\alpha = \alpha(G, f, \mu)$ depends on G, several lower order moments of μ and derivatives of f only. The term α has certain new features specific only for the non-commutative case.

C. RENTZSCH:
Lévy Khinchin-formula on products of some hypergroups

In the talk we considered a finite product of noncompact Sturm-Liouville hypergroups. For this convolution structure we stated a representation of the infinitesimal generator of a continuous convolution semigroup on a certain subset of the space of test functions. We gave three sufficient conditions for these test functions to be contained in the domain of the generator and hinted how those conditions can be proved in this case.

A similar proof holds for the representation of the generator if there are other factors as for example the euclidean space in the hypergroup product.

M. RÖSLER:
Quotient hypergroup deformations and signed hypergroups

Let K be a second countable, commutative hypergroup and G a closed subgroup of K. Besides the natural quotient hypergroup structure on K/G, there often occur further convolution structures on K/G which can be described as deformations of the original convolution by so-called *partial characters*. It is shown that such a deformation of a hermitian quotient hypergroup K/G, though being no hypergoup again always carries the structure of a signed hypergoup.

An important class of examples is obtained from the orbit hypergroups $H_n^{U(n)}$, where H_n is the n-dimensional Heisenberg group: The Bessel-Kingman hypergroup of order $n-1$ can be interpreted as a quotient of $H_n^{U(n)}$, and deformation by a suitable partial character leads to the signed Laguerre-hypergroup of order $n-1$ on $\mathbb{R}_+$.

I. RUZSA:
An arithmetic applications of measures on semigroups

A density of a set A of integers is defined as

$$D(A) = \lim \alpha_k(A), \quad \text{where } \alpha_k = \Big(\sum_{a\in A} w_k(a)\Big) \Big/ \Big(\sum_{n=1}^{\infty} w_k(n)\Big)$$

is a measure defined by some functions $w_k(n) \geq 0$. If the functions w_k are homomorphisms into $(\mathbb{R}^+, \cdot)$, and $w_i(n) \leq w_{i+1}(n)$, then there are measures μ_k such that $\alpha_k = \mu_1 * \ldots * \mu_k$. This connects the existence problem of D to a convergence problem of convolutions. Now if $f : (\mathbb{N}) \to H$ is a homomorphism into a commutative discrete semigroup H and A is of the form $A = f^{-1}(g)$, $g \in H$, then we get a convolution problem for $f(\mu_1) * \ldots * f(\mu_k)$ at a single point g. A previous result provides convergence if the individual factors $\mu_i' = f(\mu_i)$ satisfy $\liminf \mu_i'(e) > 0$ for an idempotent $e \in H$. This is the main tool in getting the following result.

Theorem: If each w_i is a homomorphism, $w_i(n) \to W(n)$ (not necessarily monotonically) and $w_j(n) \leq W(n)$ for all j and N, then the density $D(A)$ exists for any set of the form $A = f^{-1}(g)$. Moreover, it depends only on W and not on the defining functions w_i.

An important special case that falls into this cathegory is the logarithmic density. The existence of a logarithmic density for ideals is a classical result of Erdös and Davenport from 1986.

L. SALOFF-COSTE:
Random walks on finite groups

Let S be a symmetric generating set in a finite group G. Consider the probability distribution $P = \frac{1}{|S|}\mathbb{1}_S$ (uniform on S). If we assume that id $\in S$, $P^n = P * \ldots * P$ converges to $\frac{1}{|G|}$ (the uniform measure on G) as n tends to infinity. Let β_i be the eigenvalues of the symmetric matrice associated to P with $\beta_0 = 1$ and $\beta = \sup\{|\beta_i|; i \neq 0\}$. Define the time to equilibrum T by

$$T = \inf\{n \ : \|P^n - \frac{1}{|G|}\| \leq 1/10\}.$$

A universal bound for T is given by

$$T \lesssim \frac{c}{1-\beta} \log |G|.$$

In special examples T can be studied precisely. For instance if $G = S_n$ is the symmetric group and S is the set of all transpositions, Diaconis and Shashahani proved that $T \sim \frac{1}{2} n \log n$. More recently, we gave results for nilpotent groups: If G is a finite nilpotent group of class $\leq A$, generated by a set S such that $|S| \leq A$, then there exists $c(A) > 0, C(A)$ such that

$$c(A)\Omega^2 \leq T \leq C(A)\Omega^2$$

when Ω is the diameter of the Cayley graph of (G,S).

R. SCHOTT:
Representations and probability theory

The purpose of this talk was to review the development of this field over the last decades and to present some new trends developed by P. Feinsilver and myself.

We first presented some results concerning probability measures on groups whose proof involves representation theory. The second part of the talk was devoted to the computation of infinite dimensional representations of locally compact Lie groups via operator calculus. We presented a general technique for obtaining the matrix elements for representations of a Lie group that arise from the representation of the corresponding Lie algebra acting by multiplication on its enveloping algebra and quotients there of. Examples concerning abelian groups and nilpotent groups were presented.

Using the probabilistic interpretation of Appell polynomials as systems of moments, we have shown how to define them in the noncommutative case. The method is based on certain infinite-dimensional representations of local Lie groups. For processes, limit theorems play an essential role in the construction. Polynomial matrix representations of convolution semigroups are a principal feature. The corresponding Appell systems are obtained as solutions of evolution equations. This operator calculus approach is well-suited for symbolic computation on Lie groups.

M. SCHÜRMANN:
Convolution semigroups in quantum stochastics

The three basic possibilities of forming associative direct sums of alterning tensor products correspond to three notions of independence in non-commutative probability theory and to three types of *Fock spaces* on each of which there is a non-commutative stochastic calculus. For two of these independences a stochastic calculus has already been established in the literature. White noise on a Voiculescu dual semigroup with respect to a non-commutative independence can be realized as the solution of a non-commutative stochastic differential equation on the corresponding Fock space. This generalizes the result for *tensor independence* to all three types of independence. The coefficients of the equation are given through the GNI construction for the generator of the convolution semigroup associated with the white noise. Dual semigroup and independence give rise to a (commutative) $*$-bialgebra structure on the symmetric tensor algebra of the vector space underlying the dual semigroup. The white noise convolution semigroup is the $*$-bialgebra convolution exponential of the derivation given by its generator.

A.L. SCHWARTZ:
Which hypergroups have multiveriable polynomial characters?

When $n = 1$, a complete characterization can be given. Some partial results have been obtained for $n > 1$: When $n = 2$, and H can be noralized to be a rectangle or a disk, then the polynomial characters can be described explicitly. The geometric conditions on H can be considerably weakened. Other examples of hypergroups with polynomial characters are given where H is a triangle or a region bounded by a parabolic arc and one or two line segments. This is joint work in part with W.C. Connett and in part with T.H. Koornwinder.

R.D. SHAH:
Concentration functions of probability measures on Lie groups

We show that if G is a Lie goup (not nescessarily connected) and μ is a probability measure on G for which the concentration functions do not converge to zero then the following hold: There exist normal subgroups H and N of $G(\mu)$, the closed subgroup generated by the support of μ, and x in $G(\mu)$ satisfying the following conditions: N is nilpotent, $N \subset H$ and H/N is compact, H and x generate $G(\mu)$, the conjugation action of x on N is contracting and the support of μ is contained in xH.

Conversely, given subgroups H and N of G and an element x normalising H and N for which those of the above conditions which do not involve μ are satisfied then for any probability measure μ with compact support contained in xH, the concentration functions of μ do not converge to zero. This is joint work with S.K. Dani.

K. TRIMÈCHE:
Inversion of the spherical mean operator and it's dual using generalized wavelets

In this work we define and study generalized wavelets and generalized continuous wavelet transform associated with the spherical mean operator R. Next using the operator R and it's dual ${}^tR$, we give relations between this generalized continuous wavelet transform and the classical continuous wavelet transform on $[0,+\infty[\times\mathbb{R}^n$, and we deduce new expressions of the inverse operators of R and ${}^tR$ using generalized wavelets.

M. VOIT:

Two limit theorems for series of homogeneous spaces

Diaconis et al. proved the following convergence result to the uniform distribution U_N on the hypercube $\mathbb{Z}_2^N$: If μ is the law of the nearest neighbour random walk $(X_l^N)_{l\geq 0}$ on $\mathbb{Z}_2^N$, and if $c\in\mathbb{R}$ and $K=K(n,c):=[\frac{1}{4}N(\ln N+c)]$, then $\|\mu^{(k)}-U_N\|\sim \operatorname{erf}(e^{-c/2}/\sqrt{8})$ $(*)$. We reprove this result by shifting it to the associated orbit hypergroup $(\mathbb{Z}_2^N)^{S_N}\simeq\{0,1,\dots N\}$. We then find measures $\nu_{N,l}\in M^1(\mathbb{Z}_2^N)$ with $\|\mu^{(k)}-\nu_{N,l}\|=O(1/N)$ and $\|\nu_{N,l}-U_N\|=\operatorname{erf}(e^{-c/2}/\sqrt{8})$. This yelds $(*)$ as well as a central limit theorem for $|X^N_{K(N,c)}|$ for $N\to\infty$.

The second part of the talk was devoted to a continuous analogue of this CLT. For $d\in\mathbb{N}$, consider

$$S^d\simeq SO(d+1)/SO(d)\longrightarrow[-1,1]\simeq SO(d+1)//SO(d)$$
$$z\mapsto\cos\angle(z,z_0^d),\qquad z_0\in S^d \text{ the } \textit{north pole}.$$

Then the random walks $(X_k^d)_{k\geq 0}$ on S^d starting at z_0 and with $\cos\angle(X_{k+1}^d,X_k^d)=r(d)$ for all $k\geq 0$ have the following property:
Theorem: If $r(d)\to 1$ and $(1-r(d))k(d)-(\ln d)/2\to c$ for some $c\in\mathbb{R}\cup\{\infty\}$, then $\sqrt{d}\cos\angle(z_0,X^d_{k(d)})\xrightarrow{d}N(e^{-c},1)$ for $d\to\infty$ ($N(e^{-c},1)$ being the normal distribution).

A. WERON:

Misra-Prigogine-Courbage semigroups in statistical physics

The aim of this talk was to discuss a stochastic reformulation of the Prigogine theory of irreversibility for classical and quantum systems. The basic question: What types of unitary groups U_t can be intertwined with markov semigroups M_t, $t\geq 0$, through nonunitary transformation Λ in the sense that

$$M_t\Lambda=\Lambda U_t,\qquad t\geq 0\quad ?$$

We demonstrate that the action of the MPC-semigroup M_t is highly delocalizing (Z. Suchanecki, I. Antoniou and S. Tasaki, J. Stat. Phys. 15 (1994) 919-928). We illustrated this by some computer experiments showing the action of M_t for the baker transformation $B(t,s)$ on $[0,1]^2$.

N.J. WILDBERGER:

Hypergroups and representation theory of Lie groups

We reviewed some basic material on the representation theory of a compact Lie group, and introduced the hypergroups $K(G)$ of conjugacy classes, $K(\mathcal{G}:G)$ of adjoint orbits, $K(\mathcal{G}^+ : G)$ of coadjoint orbits and $K(\hat{G})$ of representations. We then showed how a *Wrapping Map* Φ from the distributions of $\mathcal{G}$ to the distributions of G, which satisfies the fundamental identity $\Phi(\mu*\nu) = \Phi(\mu)*\Phi(\nu)$ for distributions μ, ν of compact support on $\mathcal{G}$, and the convolution being on $\mathcal{G}$ and G respectively, can be used to *redo* the representation theory of G, and leads both to the classification of $\hat{G}$ by co-adjoint orbits and the Kirillov theory. This work is joint with A.H. Dooley.

We described explicitly the structure of the hypergroup $K(\mathcal{G} : G)$ in terms of piecewise-polynomial measures. This motivates the definition of a *convex* hypergroup. This is joint with J. Rapka and A.D. Dooley.

A possible extension of the program is sketched for the Heisenberg group, where a non-Hausdorff hypergroup is defined using a theory of means of almost periodic functions. The case of general nilpotent groups will require more sophisticated methods.

G.A. WILLIS:
Totally disconected groups and concentration functions

Let μ be a probability measure on a locally compact group G with corresponding sequence of concentration functions $(f_n)_{n\in\mathbb{N}}$. It was conjectured by K.H. Hofmann and A. Mukherjea that, if the closed semigroup generated by supp(μ) is equal to G, then

$$\lim_{n\to\infty} f_n(K) = 0 \quad \text{for each compact subset } K \text{ of } G.$$

They reduced this conjecture to a question about the structure of totally disconnected, locally compact groups.

This question stimulated the development of a structure theory (as yet incomplet) for totaly disconnected groups. The principal feature of this structure theory is the *scale function* $s : G \to \mathbb{Z}^+$ which may be shown to exist on each totally disconnected group G.

The scale function and related strutures in G may be used to prove the conjecture of Hofmann and Mukherjea and also to prove other structure theorems for totally disconnected groups, for example, it may be shown that, if G is compactly generated, nilpotent ant totally disconnected, then G is the extension of a discrete group by a compact one.

W. WOESS:
Random walks on the affine group of homogeneous trees and local fields

Let $T = T_q$ be the homogeneous tree of degree $q+1$. Choose and fix a boundary point ω of T. The *affine group* of T is

$$\mathrm{AFF}(T) = \{\Omega \in \mathrm{AUT}(T) \;:\; \Omega\omega = \omega\}$$

(here, AUT(T) is the isometry group of T. Its action extends naturally to the boundary ∂T.) AFF(T) acts transitively on T, it is amenable and non-unimodular. If $q = p$, a prime, then the affine group over $\mathbb{Q}_p$ embeds naturally into AFF(T). We study random walks $R_n = X_1 \ldots X_n$ and $L_n = X_n \ldots X_1$ (right and left), where the X_n

are i.i.d. AFF(T)-valued. The results obtained concern convergence to the boundary, law of large numbers, central limit theorem, Dirichlet problem and Poisson boundary. These apply, in particular, to products of random affine transformations over $\mathbb{Q}_p$ and to AFF(T)-invariant random walks on T. The spirit is that of studying R_n and L_n in terms of their action on T. [This would correspond to studying random walks on $\mathrm{AFF}_0(\mathbb{R})$ in terms of their action on the hyperbolic upper half plane.]

This is joint work with D.I. Cartwright and V.A. Kaimanovich, to appear in Ann. Inst. Fourier.

Hrn. ZEUNER:

Domains of attraction on one-dimensional hypergroups of polynomial growth

Let $(\mathbb{R}, *)$ be a Sturm-Liouville hypergroup and let A, the Lebesgue density of the Haar measure, satisfy

$$\frac{A'(x)}{A(x)} \searrow 0, \ x\frac{A'(x)}{A(x)} \to 2\alpha + 1 \text{ as } x \to \infty.$$

Then we say that a probability ν is in the domain of attraction $\mathcal{DOA}_*(\mu)$ of a measure $\mu \neq \varepsilon_0$ if for the random walk S_n with $P_{S_n} = \nu^{*n}$we have $c_n S_n \xrightarrow{\mathcal{D}} \mu$ for some $c_n > 0$.

It is shown that :

- if $1-\mathcal{F}_*\nu$ is regulary varying at 0 with index $\kappa \in]0,2]$ then $\nu \in \mathcal{DOA}_*(\mu)$ for the $\mu \in \mathcal{M}^1(\mathbb{R}_+)$ with Hankel transform $\mathcal{F}_\alpha\mu(\lambda) = \int j_\alpha(\lambda x)\, d\mu(x) = \exp(-\lambda^k)$,
- if $\nu \in \mathcal{DOA}_*(\mu)$ then $1 - \mathcal{F}_*\nu$ is regularly varying,
- if $\kappa \in]0,2[$ and the tail probability $\nu[x,\infty[$ is regularly varying at ∞ with index $-\kappa$ then $1 - \mathcal{F}_*\nu$ is regularly varying at 0,
- for $\kappa = 1/2$, $\nu \in \mathcal{DONA}_*(\varrho_\alpha)$ iff $\int x^2\, d\nu(x) < \infty$; where ϱ_α is the Rayleigh distribution,
- for $\kappa \in]0,2[$, $\lim_{x\to\infty} x^\kappa \nu[x,\infty[> 0$ and $< \infty$ implies $\nu \in \mathcal{DONA}_*(\mu)$ with μ as above.

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