# Tagungsbericht 27/1995
# Schrödingeroperatoren
09.07. – 15.07.1995

The meeting was organized by M. Demuth (Clausthal) and V. Enß (Aachen), the 38 participants came from 14 countries.

Both deterministic and random Schrödinger operators were studied in particular those with magnetic fields or a periodic background.

Special emphasis was on detailed properties of the point spectrum generated in spectral gaps by deterministic potentials, by boundary conditions, or by random perturbations. Other topics included estimates of semigroups generated by Schrödinger and more general higher order operators, models for bulk matter and stability of matter, direct and inverse scattering theory.

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# Vortragsauszüge

## The Spectral Function and Principal Eigenvalues for Schrödinger Operators

**Wolfgang Arendt**

In the first part of the talk the spectral function $\lambda \mapsto s(\Delta - \lambda V)$ is studied, where $V \in L^1_{loc}(\mathbb{R}^N)$, $V^- \in$ Kato's class and $s(\Delta - \lambda V) = \sup \sigma(\Delta - \lambda V)$ denotes the spectral bound (i.e. top of the spectrum). It is a report on joint work with C. Batty (Potential Analysis, to appear). The aim is to find $\lambda_1 > 0$ such that $s(\Delta - \lambda_1 V) = 0$ and $u > 0$ such that $(\Delta - \lambda_1 V)u = 0$ (assuming $V^- \neq 0$). A new interesting case considered in the second part is $V \in L^{N/2}$. It can be treated by considering $\frac{1}{w}(\Delta - \lambda V)$ instead of $\Delta - \lambda V$ on a weighted $L^2$-space. We conclude by an open problem: Let $0 \leq V \in L^\infty(\mathbb{R}^N)$ such that $s(\Delta - V) < 0$. Is $\lambda \mapsto s(\Delta - \lambda V)$ strictly decreasing?

## How to Construct Self-adjoint Extensions with Prescribed Spectral Properties

**J.F. Brasche**

Let $H$ be a symmetric operator in some separable complex Hilbert space with infinite deficiency indices and a gap $J$ . Let $S_{ac}$, $S_p$ and $F$ be the support of some absolutely continuous measure, some countable set and the closure of some open set, respectively. I give a method how to construct a self-adjoint extension $\tilde{H}$ of $H$ such that

$$\sigma_p(\tilde{H}) \cap J = S_p \cap J \quad , \quad \sigma_{sc}(\tilde{H}) \cap J = F \cap J \quad , \quad \sigma_{ac}(\tilde{H}) \cap J = \emptyset.$$

Under an additional assumption about $H$ I give a method how to construct a self-adjoint extension $\tilde{H}$ of $H$ such that

$$\sigma_p(\tilde{H}) \cap J = S_p \cap J \quad , \quad \sigma_{sc}(\tilde{H}) \cap J = F \cap J \quad , \quad \sigma_{ac}(\tilde{H}) \cap J = S_{ac} \cap J.$$

All the mentioned assumptions are satisfied, e.g. by the minimal Laplacian on a non-empty bounded open domain in $\mathbb{R}^d$, $d > 1$, and by the minimal Laplacian on an infinite disjoint union of non-empty intervals.

## Stochastic Spectral Theory

**Jan van Casteren**

The following topics were discussed:

1. Notation, generalities, pinned Markov process
2. Feynman–Kac semigroups: 0–order regular perturbations
3. Dirichlet semigroups: 0–order singuar perturbations, harmonic functions
4. Sets of finite capacity, wave operators, and related results
5. Some (abstract) problems related to Neumann semigroups: first order perturbations
6. Operator valued Cauchy semigroups

As a sample theorem we quote:

Let $V, W$ be Kato–Feller potentials with respect to some Hamiltonian $K_0$. If $W - V$ belongs to $L^1(E, m)$, then the scattering system $(K_0 + V, K_0 + W)$ is complete. Here $K_0$ generates a Feller semigroup in $L^2(E, m)$. A similar result is true for singular perturbations of order 0.

## Some Mathematical Aspects of "Quantum Chaos"

**Monique Combescure**

This is joint work with D. Robert. "Quantum chaos" is a field of research where one tries to see where, in quantum mechanics, is hidden the "seed" of the chaotic behavior which seems to be somehow the rule in classical mechanics. Here I present one of the mathematical aspects of this field, namely the asymptotics when $\hbar$ (Planck constant) goes to zero, of various quantum quantities, in a situation where the underlying classical dynamics is ergodic (or mixing) on the energy surface at energy $E$. Assuming that the spectrum of our Schrödinger operator is purely discrete in an interval containing the classical energy $E$, with corresponding eigenstates $\varphi_j(\hbar)$, we show that under the mixing assumption, the matrix elements of a suitable observable $A$: $\langle \varphi_j(\hbar), A\varphi_k(\hbar)\rangle$ go to zero as $\hbar \to 0$ for "almost every $j$ and $k$" ($j \neq k$) in a suitable sense.

## Heat Kernels and $L^p$ Spectral Theory for Higher Order Elliptic Operators

**E.B. Davies**

We consider self-adjoint, elliptic operators acting on $L^2(\mathbb{R}^N)$ with the divergence form

$$Hf(x) = \sum_{\substack{|\alpha|\leq m \\ |\beta|\leq m}} (-1)^{|\alpha|} D^\alpha \left\{ a_{\alpha\beta}(x) D^\beta f \right\}$$

under the assumption of complex, bounded, measurable and uniformly elliptic coefficients. If $m = 1$ the theory is well developed, but for $m > 1$ it depends heavily upon whether

$N < 2m$ or not. If $N < 2m$ then

$$e^{-Ht}f(x) = \int_{\mathbb{R}^N} K(t,x,y)f(y)d^Ny$$

for all $t > 0$, where $K$ may be complex but

$$|K(t,x,y)| \leq c_1 t^{-\frac{N}{2m}} \exp\left[-c_2 \frac{|x-y|^{\frac{2m}{2m-1}}}{t^{\frac{1}{2m-1}}} + c_3 t\right]$$

for certain positive constants $c_i$. This bound may be used to prove that $e^{-Ht}$ defines a strongly continuous semigroup on $L^p(\mathbb{R}^N)$ for all $1 \leq p < \infty$. If $H_p$ is its generator then one obtains resolvent bounds of the type

$$\|(z - H_p)^{-1}\| \leq c \frac{(1+|z|^2)^{\alpha/2}}{|\mathrm{Im}\, z|^{1+\alpha}}$$

for some $\alpha \geq 0$ and all $z \notin \mathbb{R}$. The final conclusion is that the spectrum of $H_p$ is independent of $p$. The above requires $N < 2m$, and for $N > 2m$ the situation is entirely different, unless the coefficients have some local regularity properties.

## Gaussian Decay of the Magnetic Eigenfunctions

**László Erdős**

We investigate whether the eigenfunctions of the two-dimensional magnetic Schrödinger operator have a Gaussian decay of type $\exp(-Cx^2)$ at infinity (the magnetic field is rotationally symmetric). We establish this decay if the energy ($E$) of the eigenfunction is below the bottom of the essential spectrum ($B$), and if the angular Fourier components of the external potential decay exponentially (real analyticity in the angle variable). We also demonstrate that almost the same decay is necessary to ensure Gaussian decay. The behavior of $C$ in the strong field limit and in the small gap ($B - E$) limit is also studied. The method is probabilistic and it relies on a generalized Feynman-Kac formula in the angular momentum sector representation. This involves a Bessel process (due to the radial Laplacians acting in the diagonals) *and* a Poisson jump process which is responsible for the transitions between the sectors (due to the nonzero offdiagonal elements of the potential). The formula combines the usual Feynman-Kac formula and the well-known representation of the exponential of a finite matrix using Poisson processes. Since our "matrix" is infinite, the jump rates should be renormalized.

## Wannier–Stark Systems with Singular Interactions

**Pavel Exner**

We consider a one–dimensional quantum system with an equidistant array of $\delta'$–interactions and a potential unbounded below satisfying certain regularity assumptions. A typical example is the $\delta'$ Kronig–Penney model with an electric field, $H = -\frac{d^2}{dx^2} + \beta \sum_{n\in\mathbb{Z}} \delta'(x-na) - Fx$. The absolutely continuous spectrum of this Hamiltonian is shown to be empty. We also formulate a conjecture concerning the essential spectrum and discuss various generalizations. Among them are rectangular–graph–lattice KP models with $\delta$ and $\delta'$ coupling; we show that their band spectra in absence of the external field depend on number–theoretic properties of the parameters.

## On the Regularity of the Boundary Values of a Resolvent Family

**Vladimir Georgescu**

Let $H, A$ be self–adjoint operators in a Hilbert space $\mathcal{H}$ and set $R(z) = (H-z)^{-1}$ for $z \in \mathbb{C}_+$ (i.e. $\{\operatorname{Im} z > 0\}$). We say that $H$ is of class $C^\alpha(A)$ for some real $\alpha > 0$ if the map $\tau \mapsto e^{-i\tau A} R(z) e^{i\tau A} \in B(\mathcal{H})$ is of Besov class $\Lambda^\alpha \equiv B_\infty^{\alpha,\infty}$. If $\alpha > 0$ then one can define the open real set $\mu(H)$ as the set of real $\lambda$ which have a compact neighbourhood $\mathcal{T}$ such that $E(\mathcal{T})[H, iA]E(\mathcal{T}) \geq aE(\mathcal{T})$ for some real $a > 0$ ($E$ = spectral measure of $H$). We set $\Omega = \mathbb{C}^+ \cup \mu(H)$, $\mathcal{H}_s = D(|A|^s)$ for $s > 0$, and we identify $\mathcal{H}_s \subset \mathcal{H} = \mathcal{H}^* \subset \mathcal{H}_s^* \equiv \mathcal{H}_{-s}$; in particular $B(\mathcal{H}) \subset B(\mathcal{H}_s; \mathcal{H}_{-s})$. Let $\alpha, s$ be real numbers such that $0 < \alpha < s - \frac{1}{2}$ and assume that $H$ has a spectral gap and $H \in C^{s+1/2}(A)$. Then the holomorphic function $R(\cdot) : \mathbb{C}_+ \to B(\mathcal{H}_s; \mathcal{H}_{-s})$ extends to a function locally of class $\Lambda^{s-1/2}$ on $\Omega$. Moreover, if $P_-$ is the spectral projection of $A$ associated to the set $(-\infty, 0]$, then $P_- R(\cdot) : \mathbb{C}_+ \to B(\mathcal{H}_s; \mathcal{H}_{s-1-\alpha})$ extends to a function locally of class $\Lambda^\alpha$ on $\Omega$. These results have been obtained in collaboration with Anne Boutet de Monvel and J. Sahbani.

## Isospectral Deformations

**Fritz Gesztesy**

A construction of isospectral potentials for one–dimensional Schrödinger operators $H = -\frac{d^2}{dx^2} + V$ in $L^2(\mathbb{R})$ is presented. The resulting isospectral deformations $\tilde{H} = -\frac{d^2}{dx^2} + \tilde{V}$ are unitarily equivalent to $H$ and result by deforming a given Dirichlet datum $(\mu, \sigma)$ of $H$ (e.g. corresponding to a Dirichlet boundary condition at $x = 0$) to a new Dirichlet datum $(\tilde{\mu}, \tilde{\sigma})$ of $\tilde{H}$. Here $\mu, \tilde{\mu} \in (a, b)$, $(a, b) \subset \mathbb{R}\backslash \operatorname{spec}(H)$, $\sigma, \tilde{\sigma} \in \{+, -\}$ and $(\mu, +)$ (resp. $(\mu, -)$) denotes the right (resp. left) Dirichlet eigenvalue of $H$ on the half–line $(0, \infty)$ (resp. $(-\infty, 0)$). The principal tools involved are factorizations of $H$ into products of first–order differential expressions and Weyl m–function techniques.

## Molecular Propagation through Electron Energy Level Crossings and Avoided Crossings

**George E. Hagedorn**

This seminar concerns the quantum mechanical propagation of molecular systems. We begin by discussing the Time-Dependent Born-Oppenheimer Approximation that describes molecular dynamics and chemical reactions. This standard approximation breaks down if electron energy levels cross one another or approach close to one another at an "avoided crossing". We describe the quantum mechanical phenomena that occur when a molecular system propagates through such a crossing or avoided crossing.

## Atoms and Molecules in Electric Fields

**Ira Herbst**

Joint work with Jacob Møller and Eric Skibsted is presented on the usual model of charged non-relativistic particles interacting via Coulomb potentials in the background of a constant external electric field. It is shown that there are no bound states and that the Hamiltonian has purely absolutely continuous spectrum. In fact the scattering theory of this system is asymptotically complete. The Coulomb potential is just barely Kato-bounded in the sense that for any stronger singularity, $H_0$-boundedness does not hold. This produces special difficulties over the smooth case in contrast to most problems in non-relativistic quantum mechanics. The methods used are based on an idea of finding observables which at fixed energy are (approximately) increasing in the time (for large $|x|$). For example $A = E \cdot p$ or more generally $E \cdot p + \beta |x|^{-1} x \cdot p$ ($\beta > 0$) where $E$ is the electric field.

## Localization in Spectral Gaps for Random Operators

**P.D. Hislop**

In joint work with J.M. Barbaroux and J.M. Combes, we prove exponential localization in spectral gaps for various families of random, self-adjoint operators. These examples include additive and multiplicative perturbations of background operators $H_0$. These unperturbed operators are assumed to have a gap in their spectrum $(B^-, B^+)$. The perturbed, random families $(H_\omega(g))$ are continuous in $g$ and $H_\omega(0) = H_0$. The Anderson model $H_\omega(g) = -\Delta + gV_\omega$ provides a known example with $H_0 = -\Delta$. Taking $H_0 = -\Delta - V_{per}$, for a periodic potential $V_{per}$, we obtain an example, $H_\omega(g) = H_0 + gV_\omega$, of interest in solid state physics. We assume that for $g$ small, the almost sure spectrum $\Sigma(g)$ of $H_\omega(g)$ has a gap satisfying $B^- \leq B^-(g) < B^+(g) \leq B^+$. Under some assumptions on $H_0$, we prove the existence of energies $E^\pm(g)$ satisfying $B^- \leq E^-(g) < B^-(g) < B^+(g) < E^+(g) \leq B^+$, so that $\Sigma(g) \cap [E^-(g), E^+(g)]$ is pure point with exponentially decaying eigenfunctions and $|B^\pm(g) - E^\pm(g)| \geq Cg^2$. In addition to the existence of a gap $(B^-, B^+)$ for $\sigma(H_0)$, we assume $H_0(\alpha) = e^{i\alpha\|x\|}H_0e^{-i\alpha\|x\|}$ is analytic in a strip, that $\sigma(H_0) \subset [-K_0, \infty)$ for some $0 < K_0$, and that the spectral projection for $H_0$ and $[-K_0, B^-]$ has a kernel with good off-diagonal decay properties.

## On Some Properties of Zero Sets and Critical Sets of Solutions of Elliptic Equations

**Maria Hoffmann–Ostenhof**

This is joint work with T. Hoffmann–Ostenhof and N. Nadirashvili. Let $u \not\equiv$ const satisfy an elliptic equation

$$L_0 u = \sum a_{ij} D_{ij} u + \sum b_j D_j u = 0$$

with smooth coefficients in a domain in $\mathbb{R}^3$. It is shown that the critical set $|\nabla u|^{-1}\{0\}$ has locally finite 1–dimensional Hausdorff measure. This implies in particular that for a solution $u \not\equiv 0$ of $(L_0 + c)u = 0$ with $c \in C^\infty$, the critical zero set $u^{-1}\{0\} \cap |\nabla u|^{-1}\{0\}$ has locally finite 1–dimensional Hausdorff measure.

## Random Schrödinger Operators, Asymptotics of the Integrated Density of States for a Poisson Potential

**Dirk Hundertmark**

This is joint work with Kurt Broderix, Werner Kirsch and Hajo Leschke. Random Schrödinger operators are supposed to model effects in solid state physics when the disorder of the matter cannot be neglected, e.g. amorphous semiconductors. In these models the so-called integrated density of states (IDOS) is of special interest, since it determines the thermostatic properties of the model. In the middle 70's the leading asymptotic behaviour of the IDOS for low energies was established rigorously by Pastur and Donsker–Varadhan for a random Poisson potential with single-site potential decaying algebraically at infinity with rate $\alpha$. They found completely different behaviour, depending on whether $d < \alpha < d + 2$ ("classical case") or $\alpha > d + 2$ ("quantum case"), respectively. Here $d$ is the space dimension. For $d = 2$ we investigate the effect of a nonzero constant magnetic field on the leading asymptotics of the IDOS. We show that there is no transition in the asymptotics as soon as the magnetic field is nonzero. The asymptotic behaviour for low energies is always governed by the "classical result" for all $\alpha > 2$. This result immediately generalizes to even dimensions when the magnetic field tensor has full rank.

## Stability of Matter in Magnetic Fields

**Michael Loss**

In the presence of arbitrarily large magnetic fields, matter composed of electrons and nuclei was known to be unstable if the fine structure constant $\alpha$ or the nuclear charge $Z$ is too large. Joint work with Elliott Lieb and Jan Philip Solovej is presented, in which it is shown that matter is stable if $\alpha < 0.06$ and $Z\alpha^2 < 0.04$. A new technique, called running energy scale, is introduced in which the estimates are performed on all energy scales. As another application of this idea a new Lieb–Thirring inequality for the sums of the negative eigenvalues of $\tau - U$ is obtained. Here $\tau$ is the Pauli operator and $-U$ is a potential.

## Dirichlet Forms and Spectral Theory

**Ivor McGillivray**

Let $H$ be the non–negative definite self–adjoint operator associated to a regular irreducible Dirichlet form on $L^2(X, m)$. Assume that $H$ has discrete spectrum. We study perturbations of the operator which arise through the imposition of Dirichlet boundary conditions on a compact subset of $X$. The eigenvalues of the perturbed operator are of course shifted to the right. Under an "ultracontractivity" condition, we show that the magnitude of the shift can be estimated by the capacity. We also obtain a capacity lower bound for the ground state shift under suitable conditions. An application to the "crushed ice problem" was described.

## Spectrum of Schrödinger Operators With Strong Magnetic Field

**Shu Nakamura**

This is joint work with Ira Herbst. We consider a magnetic Schrödinger operator

$$H(\lambda) = (p - \lambda A(x))^2 \text{ on } L^2(\mathbb{R}^n) \quad , \quad p = -i\partial_x,$$

and consider the asymptotic behavior of $\sigma(H(\lambda))$. Let $B(x) = \operatorname{rot} A(x)$ be the corresponding magnetic field, and $M = \{x \in \mathbb{R}^n \mid B(x) = 0\}$. We show that the spectrum converges to either a periodic or a quasi–periodic orbit in $\lambda$, in the space of the subsets of $\mathbb{C}$. The behavior depends on the geometric properties of $M$. In particular, if $n = 2$ and $\partial M$ is smooth, $\sigma(H(\lambda))$ converges to a fixed set if and only if $M$ is simply connected. If $H^1(M; \mathbb{R}) = \mathbb{R}$ (the first homology group), then $\sigma(H(\lambda))$ converges to a periodic orbit.

## A Proof of the Strong Scott Conjecture

**Heinz Siedentop**

The Schrödinger operator of an atom is

$$H_{N,Z} = \sum_{\nu=1}^{N}(-\Delta_\nu - \frac{Z}{|x_\nu|}) + \sum_{\substack{\mu,\nu=1 \\ \mu<\nu}}^{N} \frac{1}{|x_\mu - x_\nu|}$$

self-adjointly realized in $\bigwedge_{\nu=1}^{N}(L^2(\mathbb{R}^3) \otimes \mathbb{C}^q)$. Here

$N$ is the number of electrons,

$Z$ is the nuclear charge and
$q$ is the number of spin states per electron.
The strong Scott conjecture was proven by A. Iantchenko, E. H. Lieb, and the speaker in the following sense: Let $N = Z$, $U$ a bounded integrable function on $\mathbb{S}^2$, $a > 0$, $\hat{\rho}_Z$ a sequence of ground-states densities of $H_{Z,Z}$, and $\rho_Z(x) = \hat{\rho}(x/Z)/Z^3$. Then

$$\int_{\mathbb{S}^2} U(\omega)\rho_Z(a\omega)\, d\omega \overset{Z\to\infty}{\longrightarrow} \int_{\mathbb{S}^2} U(\omega)\, d\omega\, \rho^H(a)$$

where $\rho^H(|x|) = q \sum_{\nu, E_\nu \leq 0}^{\infty} |\psi_\nu^H(x)|^2$ and $\psi_\nu^H$ eigenfunctions defined by $(-\Delta - \frac{1}{|x|})\psi_\nu^H = E_\nu \psi_\nu^H$.

## On the Lieb–Thirring Estimates for the Pauli Operator

**Alexander V. Sobolev**

We establish Lieb–Thirring type estimates for the sums $\sum_k |\lambda_k|^\gamma$ of the negative eigenvalues $\lambda_k$ of the Pauli operator with a non-homogeneous magnetic field perturbed by a decreasing electric potential.

## Results and Problems Related to Localization with Periodic Background Potentials

**Peter Stollmann**

In this talk we present joint work with G. Stolz to the effect that for a model of the form

$$-\Delta + V_{per} + V_w = H_w$$

with a $\mathbb{Z}^d$-periodic $V_{per}$ and an Anderson-type $V_w$ the following holds: Near the lower band edges of the spectrum of the unperturbed operator
$-\Delta + V_{per}$ the operator exhibits point spectrum with exponentially localized eigenfunctions almost surely. We show how the necessary modifications to the Combes–Hislop proof of localization at the bottom of the spetrum is related to the shift of bands of a periodic operator forced by an additional periodic multiplication operator.

## Point Spectrum for a Wannier–Stark Hamiltonian

**Günter Stolz**

The Wannier-Stark Hamiltonian $-\frac{d^2}{dx^2} - Fx + W$ with $F \neq 0$ and $W$ periodic has purely absolutely continuous spectrum if $W \in C^2$. Completely different spectral characteristics are found for

$$H = -\frac{d^2}{dx^2} - Fx + \sum_n \lambda\delta'(x - na) \quad , \quad F > 0, \lambda \neq 0, a > 0.$$

Avron, Last and Exner have shown that $\sigma_{ac} = \emptyset$. Here we show that $Hu = Eu$ has superexponentially decaying solutions at $+\infty$ for almost every $E \in \mathbb{R}$, the same being true at $-\infty$ for every $E \in \mathbb{R}$. Spectral averaging techniques can then be used to study the operators $H_\mu = H + \mu W$, where $\mu \in \mathbb{R}$, $W \geq 0, \not\equiv 0$, compactly supported and integrable. It follows that $H_\mu$ has pure point spectrum for almost every $\mu$. An interesting open problem is to decide on (non-)discreteness of the spectrum, which is expected to depend sensitively on the parameters $F, \lambda$ and $a$. Almost nothing is known for the Wannier–Stark Hamiltonian with periodic $\delta$–interactions (instead of $\delta'$).

## Universal Estimates for Eigenvalues of Schrödinger Operators

**Joachim Stubbe**

We prove and exploit an identity for the spectra of self–adjoint operators $H$ modeled on the Dirichlet Laplacian or, more generally, on Schrödinger operators of the form

$$H = (\vec{p} - \vec{A}(\vec{x}))^2 + V(\vec{x}).$$

We derive inequalities involving sums, differences and ratios of eigenvalues which extend and improve previous results by Hile and Protter. For the Dirichlet Laplacian these inequalities are sharp for high lying eigenvalues in the sense that they are saturated by Weyl's asymptotic expression.

## Optimal $L^p$–$L^q$ Estimates for Schrödinger Semigroups with Magnetic Fields in Two Dimensions

**Bernd Thaller**

For a Schrödinger operator $H$ with a magnetic field $B$ in two dimensions, i.e.,

$$H = \frac{1}{2}(\nabla + iA)^2 \quad , \quad \operatorname{curl} A = B \geq B_0 > 0,$$

we prove

$$\sup_{u \in L^p} \frac{\|e^{tH}u\|_q}{\|u\|_p} \leq K(q,p,t,B_0) \qquad (*)$$

with an optimal constant $K$. For certain combinations of $q, p, t$, the estimate is optimal in the sense that equality holds in $(*)$ for $B(x) = B_0$ (constant field) and $u$ a centered Gaussian function. The techniques used involve logarithmic Sobolev–inequalities. The results can be used to obtain pointwise bounds on the integral kernel of $e^{tH}$.

## Inverse N–Body Scattering

**Ricardo Weder**

In this talk I present recent results on the inverse N–body problem in quantum mechanics that were obtained with a simple time–dependent geometric method. We prove that in the short–range case the high velocity limit of the scattering operator determines uniquely the potential. We also prove that for a given long–range potential the high velocity limit of the modified Dollard scattering operator determines uniquely the short–range potential. Moreover we prove that any one of the Dollard scattering operators determines uniquely the total potential.

The results of this talk are contained in the following papers:

1) Enss, V. and Weder, R., "Inverse Potential Scattering: A Geometrical Approach" included in "Mathematical Quantum Theory II: Schrödinger Operators", Proceedings of the Summer School in Mathematical Quantum Theory, August 1993, Vancouver, B.C., J. Feldman, R. Froese and L. Rosen editors, CRM Proceedings and Lecture Notes 8, AMS Providence (1995)

2) Enss, V. and Weder, R., "Uniqueness and Reconstruction Formulae for Inverse N–Particle Scattering", to appear in: "Differential Equations and Mathematical Physics", Proceedings of the International Conference, Univ. of Alabama in Birmingham, March 1994, I. Knowles editor, International Press Boston (ca. 1995)

3) Enss, V. and Weder, R., "The geometrical Approach to Multidimensional Inverse Scattering", to appear in J. Math. Phys. (1995)

## On the Lieb–Thirring Constants $L_{\gamma,1}$ for $\gamma \geq 1/2$

**Timo Weidl**

Let $\{E_i\}$ denote the sequence of negative eigenvalues of the operator

$$(1) \qquad H = -\frac{d^2}{dx^2} - V(x),\ V(x) \geq 0 \text{ on } L_2(\mathbb{R}).$$

We prove, that $0 \leq V \in L_1(\mathbb{R})$ implies

$$(2) \qquad \tfrac{1}{4}\int V(x)\,dx \leq \textstyle\sum_i |E_i|^{1/2} \leq L_{1/2,1}\int V(x)\,dx,$$

where the best constant on the r. h. s. can be estimated by $1/2 \leq L_{1/2,1} < 1.005$. This solves the "limit" case in the scale of Lieb–Thirring inequalities

$$(3) \qquad \textstyle\sum_i |E_i|^{\gamma} \leq L_{\gamma,1}\int V^{1/2+\gamma}dx,\ \gamma \geq 1/2.$$

We apply (2) to improve the upper bound on $L_{\gamma,1}$ for $1/2 < \gamma < 3/2$. In particular we find $L_{1,1} < 0.854$. Moreover (2) implies a new bound on the transmission coefficient of the problem (1). If one considers (1) as a Neumann problem on $L_2(\mathbb{R}_+)$, then (2) holds as well. For the respective constant $L^N_{1/2,1}$ on the r. h. s. of (2) we find $1 \leq L^N_{1/2,1} < 1.005$.

## New Channels of Scattering for Three–Body Quantum Sytems with Long–Range Potentials

**D. Yafaev**

We consider a system of three one–dimensional particles with one of the pair potentials $V^\alpha(x^\alpha)$ decaying at infnity as $|x^\alpha|^{-\rho}$, $0 < \rho < 1/2$. It is shown that such a system can possess channels of scattering not included in the usual list of channels called the asymptotic completeness.

Reported by Silke Arians, Arnd v.d. Heyden

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