Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 28/1995
Dynamische Systeme
16.–22.07.1995

Die Tagung fand unter Leitung von H. Hofer, J. Moser und E. Zehnder (ETH Zürich) statt.

S. ANGENENT

Curve shortening and closed geodesics on S^2.

Let (M^2, g) be a surface with closed geodesics $\gamma_1, \dots, \gamma_n$. Define $\Pi'(\gamma_1, \dots, \gamma_n) = \{C^1\text{–immersed } g \subset M | g \text{ transversal to } \gamma_1, \dots, \gamma_n \text{ and } g \text{ has no self tangencies}\}$. By definition, two γ, $\gamma' \in \Pi'(\gamma_1, \dots, \gamma_n)$ have the same "knot-type", if they lie in the same component of $\Pi'(\gamma_1, \dots, \gamma_n)$. To each such component B we associate a Conley-index $h(B)$ (associated with the gradient flow), which together with information about the Jacobi- fields of the $\gamma_1, \dots, \gamma_n$ determines how many geodesics B must (at least) contain. In particular, one can show that if ε is a simple closed geodesic with rotation number $r > 1$, there are geodesics of type (p, q) for any $p/q \in (1, r)$, $(p, q) \equiv 1$.

V. BALADI

Generalized Fredholm determinants and dynamical zeta functions.

Weighted dynamical zeta functions or suitably defined Fredholm-type determinants have been successfully used by many authors to discribe the discrete spectrum of transfer operators associated with expanding or hyperbolic maps and smooth weights. By making use of a continuous version of the Milnor and Thurston kneading matrix, it is possible to prove similar results for transfer operators acting on functions of bounded variation on the real line without any hyperbolicity assumption (joint with D. Ruelle, "Sharp determinants", IHES preprint 1994). We also discuss the corresponding one-dimensional complex theory (joint with A. Kitaev, D. Ruelle, S. Semmes, "Sharp determinants and kneading operators for holomorphic maps", IHES preprint, 1995).

V. BANGERT

Homoclinic orbits by variational methods (Report on part of a forthcoming thesis by D. Petroll, MFI Oberwolfach).

A simple closed geodesic γ on an orientable surface M (with Riemannian metric) is called a 1-sided proper local minimum if γ cannot be approximated from (at least) one side by curves with lengths $\leq \text{length}(\gamma)$. Suppose that M is homeomorphic to $S^1 \times [0, 1]$ and that $S^1 \times \{0\}$ and that $S^1 \times \{1\}$ are closed geodesics, γ_0, γ_1.

Theorem. *Assume that γ_0 is a 1-sided proper local minimum and* $\text{length}(\gamma_0) > \text{length}(\gamma_1)$. *If there is no non-contractible closed geodesic in interior* $(M) \cong$

$S^1 \times (0, 1)$ *which is a 1-sided proper local minimum then there exists a geodesic in* M *homoclinic to* γ_0.

The homoclinic geodesic is obtained as a limit of closed geodesics of minimax (saddle-point) type. Examples of surfaces of revolution show the necessity of the assumption length $(\gamma_0) >$ length(γ_1). The same examples show that one has to assume that there does not exist a non-contractible simple closed geodesics g in interior (M) with length $(\gamma) =$ length(γ_0). There is hope that the assumptions of the theorem can be reduced to this weaker assumption.

M. BIALY

Hopf type rigidity for Newton equations.

In 1943 E. Hopf proved that the only Riemannian metrics on the two-torus without conjugate points are flat ones. Recently this result was generalized by Burago & Ivanov for the n-dimensional case. It is a very interesting problem to understand for which class of convex Hamiltonian systems this Hopf type rigidity holds. The purpose of the talk is to establish the Hopf theorem for the Hamiltonian

$$H(p, y, t) = \frac{1}{2}|p|^2 + U(q, t)$$

where g is a Riemannian metric on the n-dimensional torus and U is a time and space-periodic pontential. Our main result states that if H has no conjugate points then g is flat and U does not depend on q. This is joint work with L. Polterovich.

S. BOLOTIN

Variational criterion for quasirandom behavior in a Hamiltionian system with two degrees of freedom.

We consider a classical Hamiltonian system whose configuration space in a 2-dimensional sphere. Variational methods yield the existence of a homolinic orbit Γ to the maximum point of the pontential energy. Even if Γ is transversal, this does not imply quasirandom behavior (Devaney). We consider the case where Γ is tangent to the strong stable and strong unstable directions of the equilibrium. If ind $(\Gamma) = 0$, then the system has a quasirandom behavior. This result is used to prove the quasirandom behavior for a rigid body with a fixed point.

M. CHAPERON

Invariant manifolds, conjugacies and blow-up.

A diffeomorphism germ $\gamma : M_0, p_0 \rightarrow M_1, T_1$ is a conjugacy between $h_0; M_0, p_0$, and$h_1; M_1, p_1$ if and only if graph (γ) is invariant by $h_0 \times h_1 : (x, y) \rightarrow (h_0(x), h_1(y))$. We stated the invariant manifold theorems implying Sternberg's local conjugacy results for hyperbolic diffeomorphism germs, and related them to the stable and pseudo-stable manifold theorems using a new blow-up technique.

A. CHENCINER

Reduction of homothetics in the n-body problem.

(Joint work with A. Albouy) We put the results of Sundman, Mc Gehee, Saari, etc ... on total collision and completely parabolic motion in the simple conceptual framework of the reduction of homothetics which is allowed by the homogeneity of the Newton potential. The symmetry flow (already known to E. Cartan) scales configurations by $\exp(t)$ and velocities by $\exp\left(-\frac{1}{2}t\right)$. It is not Hamiltonian, leaves the energy and angular momentum integrals invariant only when these are zero (a fact already exployted by Euler for 3 bodies) and does not commute with Newton's flow, defining only with it a 2 dimensional foliation in phase space. To restore commutation, one has to change the law of time, multiplying for example the Newton vector-field by $I^{\frac{3}{4}}$,where I is the moment of inertia with respect to the center of mass. Singularities of the quotient vector-field are zero energy homethetic motions, they belong to the "Mc Gehee manifold" quotient of the zero energy and zero angular momentum manifold. Their invariant manifolds correspond exactly to total collision ($I \to 0$) or to completely parabolic motion ($K \to 0$) where K is the kinetic energy w.r. to the center of mass and are respectively contained in zero momentum manifold (Weierstrass) or zero energy manifold. The main tool is Sundman's function which happens to be invariant under the symmetry plane.

L. CHIERCHIA

Instabilities in nearly-integrable Hamiltonian systems.

Consider the phase space $R^4 \times T^4$ with coordinates $I = (I_1, \ldots, I_4) \in R^4, \varphi = (\varphi_1, \ldots, \varphi_4) \in T^4$ and with the standard symplectic structure $dI \wedge d\varphi$. Let

$$H(I, \varphi; \varepsilon) = \frac{1}{2}\sum_{j=1}^{4} I_j^2 + \varepsilon \cos\varphi_4 + \varepsilon^{10}\sum_{j=1}^{3}\cos(\varphi_j + \varphi_4)$$

and denote by φ^t the Hamiltonian flow generated by H. The following holds:

Theorem. *Fix $0 < E_1 < E_2$, then there exist ε_0, $d > 0$ such that for all $E \in [E_1, E_2]$, for all $0 < \varepsilon < \varepsilon_0$ one can find $(I_0, \varphi_0) \in H^{-1}(E)$ such that*

$$\sup_{t>0} |I(t) - I_0| > d$$

where $(I(t), \varphi(t)) = \varphi^t(I_0, \varphi_0)$.

This is an example of a real-analytic one-parameter family (analytic also with respect to the parameter) exhibiting an unstable behaviour à la Arnold. The above constructed unstable orbit $\varphi^t(I_0, \varphi_0)$ stays for long time near a double resonance of the unperturbed system ($I_3 = I_4 = 0$) and moves "shadowing long chains heteroclinic connections" which are obtained as transversal intersections of lower dimensional "whiskered tori" (the transversality being not exponentially small with ε).

K. CIELIEBAK

Symplectic boundaries: creating and destroying closed characteristics.

Let $(M^{2n},\omega), n \geq 2$, be a symplectic manifold with boundary ∂M, and $L \subset \partial M$ be an embedded loop. Modifying V. Ginzburg's symplectic plug we construct a symplectic form ω' on M such that

- $(\partial M,\omega')$ has precisely 2 new closed characteristics $L^{\pm}$, isotopic to $\mp$ L.
- All characteristics of $(\partial M,\omega)$ intersecting L are trapped by $L^{\mp}$ in $(\partial M,\omega')$.
- $(\overset{\circ}{M},\omega')$ is symplectomorphic to $(\overset{\circ}{M},\omega)$.

Thus we obtain symplectic manifolds (M,ω) and $(M;\omega')$ with $(\overset{\circ}{M},\omega) \cong (\overset{\circ}{M},\omega')$ and such that e. g.

- $(\partial M,\omega)$ is of contact type and $(\partial M,\omega')$ is not.
- The action spectra of $(\partial M,\omega)$ and $(\partial M,\omega')$ are disjoint
- $(\partial M,\omega)$ is an ellipsoid in R^4, hence completely integrable, and $(\partial M,\omega')$ possesses no C^1 - embedded invariant 2-torus.Moreover, we find
- a vector field X on S^3 satisfying $i_X d\lambda = 0$ and $\lambda(X) \geq 0$ for some confoliation 1-form λ and having only knotted periodic orbits.
- compact hypersurfaces in $\mathbb{R}^{2n}, n \geq 2$,with any prescribed number $k \geq 2$ of closed characteristics.

L. H. ELIASSON

Discrete one-dimensional quasiperiodic Schrödinger operator with pure point spectrum.

We discuss the following result and its proof: Let $L_\vartheta : l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$ be defined by

$$(L_\vartheta u)_n = -\varepsilon(u_{n+1} + u_{n-1}) + E(\vartheta + n\omega)u_n.$$

Where ω is a real number and $E : \mathbb{R}/_{2\pi\mathbb{Z}} \to \mathbb{R}$ is a smooth function satisfying

$$|E|_{C^\nu} \leq C(\nu!)^2 K^\nu (\text{for all} v \geq 0)$$
$$[K^\nu(\nu!)^2]^{-1}|\partial_x^\nu(x(E(\vartheta + x) - E(\vartheta))| \geq \alpha > 0$$
$$[K^\nu(\nu!)^2]^{-1}|\partial_x^\nu(x(E(\vartheta + x) - E(\vartheta))| \geq \alpha||x||(\text{ for all } \vartheta \text{ and } x, 0 \leq \nu \leq s)$$

Assume also a Diophantine condition

$$||n \cdot \omega|| \geq \kappa|n|^{-\tau}, \text{ for all } n\varepsilon\mathbb{Z}, 0$$

some $\tau, \kappa > 0$. Under these conditions the following theorem holds.

Theorem. $\exists$ $\varepsilon_0 = \varepsilon_0(C,\kappa,s,\alpha,\tau) > 0$ *such that for all* $|\varepsilon| < \varepsilon_0$ *the operator* L_ϑ *has pure point spectrum for a.e.*ϑ. *Moreover, the eigenvectors decay exponentially and the resolvent set of* L_ϑ *goes to* 0 *with* ε.

This theorem generalizes results of Fröhlich-Spencer and of Sinai.

G. FORNI

Global Analysis for Area-Preserving Vector Fields on Compact Riemann Surfaces.

We study the existence of smooth solutions for the cohomolgy equation $Xu = f$, where X is a smooth area-preserving vector field on a compact orientable surface M of genus $g \geq 2$. The singularities of X, whose existence is forced by the topology of the surface, are assumed to be of saddle-type (possibly degenerate). Our results extend to higher genus the solvability properties which hold for constant coefficients vector fields on the two-dimensional torus, whose rotation numbers satisfy a Diophantine condition. In particular it is proved that, for "almost all" vectorfields X on a compact surface M of genus $g \geq 2$, the linear differential equation $Xu = f$ has a finitely smooth solution u provided that the complex-valued function f is sufficiently smooth, is supported away from the singularities of the vector field X and satisfies a finite number of compatibility conditions. The compatibility conditions to be satisfied if one requires the solution u to be infinitely differentiable are a countable number. This last result establishes a significant difference with the case of the torus and shows that for "almost all" vector fields the vector space of invariant distributions has countable dimension, while the cone of invariant measures not supported at the singularities is just 1-dimensional, as it has been proved by several authors (Masur, Veech, Rees, Boshernitzan, Kerckhoff) in the last decade. The results just described can be applied, we believe, to prove KAM stability results with finite codimension for such vector fields or, equivalently, for Thurston's measured foliations. In fact, the results are obtained via a priori estimates in Sobolev spaces (which are important for the Nash-Moser iteration to work) and the loss of derivatives in solving the equation is finite (in fact it can be estimated by 10).

V. GINZBURG

A Hamiltonian flow without closed trajectories on $S^{2n+1}, 2n+1 \geq 7$.

One of the old problems of Hamiltonian mechanics is whether the Hamilton equation for a given smooth proper function $H : \mathbb{R}^{2n} \to \mathbb{R}$ has a periodic solution on a prescribed level $\{H = \text{const.}\}$. For example, such a solution is known to exist when the level has contact type (C. Viterbo) or for almost all values of H (Hofer-Zehnder, Struwe).

In this talk we give an example of a function $H : \mathbb{R}^{2n} \to \mathbb{R}, 2n \geq 8$, proper and smooth, such that the level $\{H = 1\}$ is non-singular and contains no periodic solutions. This result can be thought of as a counterexample to the higher dimensional Hamiltonian Seifert conjecture.

M. HERMAN

Examples of compact energy hypersurfaces in $\mathbb{R}^{2p}, 2p \geq 6$, without closed characteristics.

Let (M^{2p}, w) be a C^∞ metrisable, connected symplectic manifold and $N_0 \subset M^{2p}$ a C^∞ codimension 1, compact orientables submanifold (a hypersurface). We consider the characteristic foliation $\ker(j^*_{N_0} w)$.

Theorem. *We suppose that on N_0 there are only a finite number of closed characteristics. We suppose that $2p \geq 8$(resp. $2p = 6$). Then there exists a hypersurface N_1 diffeomorphic to N_0, of class C^∞ (resp. of class $C^{3-\varepsilon}$, for all $\varepsilon > 0$) with no closed characteristics.*

Examples.

$$\mathbb{C}^p \approx T^*\mathbb{R}^p, \qquad H_\alpha(z) = \sum_j^n \alpha_j |z_j|^2, \qquad \alpha_j > 0, \text{ the } (\alpha_1, \ldots, \alpha_p)$$

are independent over $\mathbb{Q}$, $N^0 = H_\alpha^{-1}(c)$, $c > 0$, *then there are only p characteristics on N_0.*

We stetched the proof of the above theorem. We also announced the existence of C^∞ volume preserving plugs in dimension ≥ 4, to destroy a finite number of periodic orbits of a C^∞ volume preserving flow on a compact manifold. We also announced that the above theorem applies under the more general condition: the closed characteristics of N_0 are included in a finite union of compact hypersurfaces of N_0.

H. HOFER

Dynamics of Reeb vectorfields.

A contact form on a three-manifold M is a 1-form λ such that $\lambda \wedge d\lambda$ is a volume form. Such a contact form determines the so-called Reeb verctorfield X by $i_X d\lambda = 0, i_X\lambda = 1$. We introduce the notion of dynamical convexity requiring that the Conley-Zehnder index satisfies for every contractible periodic orbit that $\mu \geq 3$. In a joint work with Wyscocki and Zehnder we show that an energy surface in $\mathbb{R}^4$ has this property for a suitable contact form λ compatible with the symplectic form. The main result is

Theorem. *(Hofer - Wysocki - Zehnder). Let λ be a dynamically convex contact form on the compact three- manifold M. Assume there exists an embedded disc D with the boundary being a periodic orbit for the Reeb-vectorfield X. If $sl(\partial D, D) = -1$ then M is diffeomorphic to S^3 and λ is a tight contact form.*

A. KATOK

Invariant measures for commuting diffeomorphisms of the torus.

Consider a C^2 action of $\mathbb{Z}^2$ on the 3- dimensional torus, let α_* be the induced action on $H^1(\mathbb{T}^3)$.

Main Theorem. Suppose α_* is effective and not unipotent then α preserves an absolutely continuous measure μ. Furthermore α preserves a measurable flat affine connection. There exists a measurable map $H \to GL(3, \mathbb{R})$ such that for the derivative action $D\alpha : \mathbb{T}^3 \times \mathbb{R}^3 \to \mathbb{T}^3 \times \mathbb{R}^3$

$$D\alpha = \alpha_* \circ H \circ \alpha^{-1}.$$

The conditions on $\alpha_{\bullet}$ are equivalent to hyperbolicity: for $(m,n) \neq (0,0)$, the mathrix $\alpha_{\bullet}(m,n)$ is hyperbolic with real eigenvalues.

This theorem follows by rather elementary considerations from the semi-conjugacy with the linear action α_0 and the following rigidity result for invariant measures.

Rigidity Theorem for measures: Let M^3 be a compact three-dimensional manifold, α a C^2 action by diffeomorphism, μ- an ergodic α-invariant measure such that every non-zero element of the suspension action $\hat{\alpha}$ has positive entropy with respect to the lift $\hat{\mu}$ of the measure μ. Then μ is absolutely continuous and preserves a measurable flat affine connection.

The main ingredients in the proof are

- (i) A cocycle trivialization theorem for Hölder cocycles in the Lyapunov metric associated with the Lyapunov exponent structure for a measure satisfying the totally non-symplectic condition. This works in arbitrary dimensions.
- (ii) construction of invariant affine structure on one-dimensional Lyupanov manifolds in the multiplicity-free case. This is a non uniform and non-stationary version of the Sternberg linearization theorem.
- (iii) Rigidity of conditional measure along one-dimensional stable foliation generalizing an earlier joint result with Spatzier for the linear case.

A. KNAUF

Number Theory, Geodesic Motion and Statistical Mechanics.

Length differences of scattering geodesics in the modular domain are interpreted as energy values of an infinite spin chain. That chain is ferromagnetic and has a phase transition. The Riemann zeta function can be well numerically approximated in the critical strip using this statistical mechanics ansatz. That behaviour can possibly be interpreted in terms of spectral properties of Markov transition matrices.

S. B. KUKSIN

A phase-portrait of a non-linear Schrödinger equation.

We study classical Schrödinger equation with space-variable s in the n-cube, $n = 1,2,3$, under Dirchlet boundary conditions. We prove that the function phase-space of the equation contains a non trivial recursion subset with a good control for the recursion-time. This result is used to estimate oscillations of solutions in terms of a quantity similar to the Reynolds number of the classical hydro-dynamics.

K. M. KUPERBERG

A flow on $\mathbb{R}^3$ with no minimal set.

The following questions was brought to my attention by P. A. Schweitzer: Does there exist a flow on $\mathbb{R}^3$ with no minimal set?

An affirmative answer to this question can be obtained as follows:

1. Construct a noncompact plug (having the matched ends condition and a trapped orbit) with no minimal set.

2. Modify the plug to a content stopping plug P.

3. Insert a sequence of copies of P into a constant vector-field on $\mathbb{R}^3$ so that all orbits are trapped.

M. LEVI

Topology of stability zones of Hills equations.

Topological analysis of stability zones in the (a, b)- plane of Hills eequations $x + (a + bp(t))x = 0$, $p(t+1) = p(t)$ is given in special case of p (t) being in a sufficiently small neighborhood of sgn $(\sin 2\pi t)$. It is shown that the boundaries of the n^{th} and the $(n+1)$ stability zones meet in at least n points. This phenomen is explained by a geometrical picture of a map from the (a, b) - plane into the symplectic group. An indication of why the above statement is expected to hold for any periodic $p(t) \neq$ const is given. This will appear in Archive fo Rat. Mech. Anal.

M. LYUBICH

Complex rigidity and the Feigenbaum universality.

We construct the unstable manifold for the fixed point of a Feigenbaum renormalization transformation of bounded type. The construction is based on a Rigidity Theorem for maps with a priori bounds and a small orbits Lamma for local holomorphic maps of Banach spaces. As an application, we prove Milnor's Hairiness Conjecture for the Mandelbrot set near infinitely renormalized points of bounded type (subject of a priori bounds).

R. S. MACKAY

Self-localised vibrations in Hamiltonian networks of oscillators.

Aubry and I proved that a wide class of weakly coupled networks of oscillators possess time-periodic spatially localisedd solutions (Nonlinearity 1(1994)1623–43). I sketched this result and indicated directions for further work and open problems.

J. MATHER

Variational Construction of Connecting Orbits in Lagrangion Systems.

Let $L : TM \times \mathbb{R} \to \mathbb{R}$ be a Lagrangian on a compact manifold M. Suppose that L is time periodic and positive definite with super linear growth. Suppose that the Euler-Lagrange flow of L is complete. To $c \in H1(M, \mathbb{R})$, there is associated a set $\sum_c \subset TM \times \mathbb{R}$, the support of the set of c-minimal measures. In Annales de l'Institut Fourier (1993), I gave a sufficient condition for finding an orbit connecting $\sum_c$ *and* $\sum_{c'}$. In this talk, I proposed an application of my Annales result to systems of the form $T + U$, where T is a Riemannion metric and U is a periodic potential on the 2 torus, as well as an application to a small perturbation of an integrable system with positive definite normal torsion.

G. POPOV

Nekhoroshev estimates for billiard ball maps.

Let Ω be a strictly convex bounded domain in $\mathbb{R}^n, n \geq 2$, with an analytic boundary. The main result is that any billiard trajectery with initial data which are δ-close to the glancing manifold remains δ-close to the glancing manifold in a time interval which is exponentially large with respect to $1/\delta$. The proof is based on normal forms for the billiard ball map in Gevrey classes.

J. PÖSCHEL

Almost periodic solutions for a nonlinear Schrödinger equation.

We consider the nonlinear Schrödinger equation

(∗)
$$iu_t = u_{xx} - V(x)u - \Psi(f(|\Psi u|^2)\Psi u), 0 \leq x \leq 2\pi$$

with periodic boundary conditions:

(∗∗)
$$u(t, x + 2\pi) = u(t, x), \quad u(t, -x) = -u(t, x).$$

Here, V is a potential in $L^2(0, 2\pi)$ having positive Dirichlet spectrum, f is real analytic of order one and Ψ is a smoothing operator of order $s > 1/2$:

$$\Psi u = u * \psi : H^s \to H^{s+\sigma}, \quad ||\Psi||_{s+\sigma} \leq C_s ||u||_s$$

where Ψ is even. We show that for almost all such potentials V (w.r.t. every measure whose projection on any finite dimensional subspace is absolutely continuous. w.r.t. Lebesgue) (∗) and and (∗∗) possess real analytic almost periodic solutions with ∞ many independent frequencies on compact invariant tori of maximal dimension, which, when projected onto any finite dimensional subspace are of density 1 at $u \equiv 0$. The proof consists in iterating a KAM theorem about finite dimensional invariant tori for such equations due to Kuksin and Pöschel.

P. RABINOWITZ

Multibump solutions of differential equations.

The notions of 1-bump and multibumps solutions of differential equations are defined. Some detailed examples are presented and their proofs indicated. Finally a sketch is given of the basic ideas involved in multibump constructions.

E. SERE

A global condition for quasi-random behavior in a class of conservative systems.

We present here a joint work with Boris Buffoni, from the University of Bath (same title as the talk, to appear in C.P.A.M). As shown by Devaney, an autonomous Hamiltonian system dimension 4, with an orbit homoclinic to a saddle-focus equilibrium, admits a chaotic behavior as soon as the homoclinic orbit is transverse. We present here a variational method which gives such a behavior, without checking transversality. Transversality is replaced by the assumption that the stable an unstable manifolds W^u, W^s of the equilibrium do not coincide. We apply this method to gyroscopic systems and to a fourth-order equation arising in water-wave theory. In both cases, the assumption on W^u, W^s is easy to check, and quasi-random behavior is shown.

D. SZA'SZ

Ergodic properties of the Chernov-Sinai pencase.

The Chernov-Sinai pencase is a system of large hard balls on a n-dimensional torus ($n \geq 2$) elongated in one spatial dimension, the cyclic order of the balls remains invariant. This system is shown to have the K-property if $n \geq 4$, and have open ergodic components if $n \geq 3$. The results are obtained by strengthening those for hard ball systems with restricted graphs on interaction (in these only pairs of balls $\{i, j\}$ interact for which $\{i, j\} \in E$ where E is the set of edges of a graph whose vertices are the balls). Our theorems for these systems say that if E is a (spanning) tree and D denotes the edge-degree of E, then the system is K, if $n \geq D + 2$, and has open ergodic components if $n = D + 1$. The result are joint with Nánder Simányi.

I.A. TAIMANOV

Periodic trajectories in magnetic fields.

We talk about variational problems related to existence of periodic trajectories of the motion of a "particle" in magnetic fields on Riemannian manifolds.

As it was shown by Novikov these problems give rise to multivalued or not bounded from below functionals the on spaces of closed curves. For such functionals Palais-Smale condition does not hold.

In some cases:

i) for strong magnetic fields on two-dimensional manifolds;

ii) for such systems with "positive Ricci curvature"

the existence of periodic trajectories is proved by methods of calculus of variations in the large (but not of symplectic geometry). Some of these results were obtained by myself and some in collaboration with Novikov and Bahri.

A. VESELOV

Multivalued dynamics ang nonlinear representations of multivalued groups.

A group-theoretical approach in dynamics of multivalued mappings (or correspondences) based on the theory of multivalued groups is discussed. It is shown that in various cases the correspondences with intergrable dynamics determine the algebraic representations of certain multivalued groups. The talk is based on the joint paper with V.M. Buchstaber.

C.E. WAYNE

Invariant Manifolds and the asymptotics of a class of parabolic PDE's.

Finite dimensional invariant manifolds for nonlinear parabolic partial differential equations of the form:

$$\frac{\partial u}{dt} = \Delta u + F(u); \quad u = u(x,t), \quad x \in \mathbb{R}^d, \quad t \geq 1$$

are constructed. Such results are somewhat surprising because of the continuous spectrum of the linearized equation. These manifolds can be used to construct systematic expansions of the long-time asymptotics of $u(x,t)$ in inverse powers of t. They also give a new perspective on the change in the behavior of solutions of the equation with nonlinear term $F(u) = -|u|^{p-1}u$ when p passes through the critical value $p_c = 1 + 2/d$.

Z.J. XIA

Elliptic periodic points in symplectic diffeomorphisms.

We show the following two C^1-generic properties for symplectic diffeomorphisms. 1. for any hyperbolic periodic point, its homoclinic points are dense in both its stable and unstable manifold. 2. C^1-generically, a symplectic diffeomorphism is either partially hyperbolic or the elliptic periodic points are dense.

J.C. YOCCOZ

Intersection of regular Cantor sets.

The subject of the talk was joint work with Carlos Gustavo Moreira. Let I_0, I_1 be disjoint subintervals of $I = [0,1]$ with $0 \in I_0$, $1 \in I_1$, and let $g\colon I_0 \cup I_1 \to I$ an expanding map such that g/I_α is a diffeomorphism onto I. The maximal invariant set $K = \bigcup_{n \geq 0} g^{-n}(I_0 I_1)$ is a *regular Cantor set.*

The study of homoclinic bifurcations of surface diffeomorphisms suggests to investigate how frequently two regular Cantor sets intersect. Define, for g, g' as above

$$J(g,g') = \{(s,t) \in \mathbb{R}^2, K_g \cap (e^s K_{g'} + t) \neq \emptyset\}.$$

A point $(s_0, t_0) \in J(g_0, g_0')$ is a *position of stable intersection* if $(s,t) \in J_{g,g'}$, whenever s, t, g, g' are close to s_0, t_0, g_0, g_0'. Our main result is the following

Theorem. *Let g, g' such that $HD(K_g) + HD(K_g') \geq 1$, and $(s,t) \in J(g,g')$. Then one can find arbitrarily small perturbations (s_1,t_1), (g_1,g_1') such that (s_1,t_1) is a position of stable intersection for (g_1,g_1').*

Zürich, den 24. Juli 1996.
E. Zehnder

Tagungsteilnehmer

Prof.Dr. Sigurd B. Angenent
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive

Madison WI, 53706-1388
USA

Prof.Dr. Marc Chaperon
U.F.R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Viviane Baladi
Departement de Mathematique
Universite de Geneve
Case Postale 240
2-4 rue du Lievre

CH-1211 Geneve 24

Prof.Dr. Alain Chenciner
U. F. R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Victor Bangert
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

79104 Freiburg

Prof.Dr. Luigi Chierchia
Dipartimento di Matematica
Terza Universita di Roma
Via Corrado Serge 2

I-00146 Roma

Prof.Dr. Misha L. Bialy
Tel Aviv University, Dept. of Maths
Raymond and Beverly Sackler
Faculty of Exact Sciences
Ramat-Aviv

Tel Aviv 69978
ISRAEL

Prof.Dr. Kai Cieliebak
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Sergej Bolotin
Dept. of Mathematics
Moscow State University

119899 Moscow
RUSSIA

Prof.Dr. Nicole Desolneux-Moulis
Dept. de Mathematiques et Informat.
Universite Claude Bernard de Lyon I
43, Bd. du 11 Novembre 1918

F-69622 Villeurbanne Cedex

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Prof.Dr. Hakan Eliasson
Dept. of Mathematics
Royal Institute of Technology
Lindstedtsvägen 30

S-100 44 Stockholm

Prof.Dr. Anatole B. Katok
Department of Mathematics
Pennsylvania State University
303 McAllister Building

University Park , PA 16802
USA

Prof.Dr. Giovanni Forni
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB-Cambridge , CB2 1SB

Dr. Andreas Knauf
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136

10623 Berlin

Prof.Dr. Victor L. Ginzburg
Department of Mathematics
Stanford University

Stanford , CA 94305-2125
USA

Dr. Tyll Krüger
Physik
BIBOs Forschungszentrum
Universität Bielefeld
Wellenberg 1

33615 Bielefeld

Prof.Dr. Michel R. Herman
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Sergei B. Kuksin
Djalilya St. 23156, kv 267

115 580 Moscow
RUSSIA

Prof.Dr. Helmut W. Hofer
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Krystyna Kuperberg
Dept. of Mathematics
Auburn University
Parker Hall

Auburn , AL 36849-5310
USA

Prof.Dr. Mark Levi
Dept. of Mathematics
Rensselaer Polytechnic Institute

Troy , NY 12180-3590
USA

Prof.Dr. Jürgen Moser
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Torsten Linnemann
Department für Mathematik
Raum HGE 18.4
ETH-Zürich

CH-8092 Zürich

Prof.Dr. Leonid V. Polterovich
Tel Aviv University, Dept. of Maths
Raymond and Beverly Sackler
Faculty of Exact Sciences
Ramat-Aviv

Tel Aviv 69978
ISRAEL

Prof.Dr. Michael Lyubich
Institute of Mathematical Science
SUNY at Stony Brook

Stony Brook , NY 11794-3660
USA

Prof.Dr. Georgi Popov
Universite de Nantes
Departement de Mathematiques
2, rue de la Houssiniere

F-44072 Nantes Cedex 03

Prof.Dr. Robert S. MacKay
Dept. of Applied Mathematics and
Theoretical Physics
University of Cambridge
Silver Street

GB-Cambridge , CB3 9EW

Prof.Dr. Jürgen Pöschel
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57

70550 Stuttgart

Prof.Dr. John N. Mather
Department of Mathematics
Princeton University
Fine Hall
Washington Road.

Princeton , NJ 08544-1000
USA

Prof.Dr. Paul H. Rabinowitz
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive

Madison WI, 53706-1388
USA

Matthias Schwarz
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Iskander A. Taimanov
Institute of Mathematics
Siberian Branch of the Academy of Sciences
Universitetskiy Prospect N4

630090 Novosibirsk
RUSSIA

Prof.Dr. Eric Sere
Departement de Mathematiques
Universite Saint-Martin
Batiment Saint-Martin
2, avenue Adolphe Chauvin

F-95302 Cergy-Pontoise Cedex

Prof.Dr. Alexander P. Veselov
Dept. of Mathematics and Mechanics
Moscow State University
Lenin Hills

119899 Moscow
RUSSIA

Karl Friedrich Siburg
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Clarence Eugene Wayne
Department of Mathematics
Pennsylvania State University
218 McAllister Building

University Park , PA 16802
USA

Prof.Dr. Jean-Claude Sikorav
Universite Paul Sabatier
U.F.R. M.I.G.
118, route de Narbonne

F-31062 Toulouse Cedex

Joachim Weber
Fachbereich Mathematik
Sekr. MA 7-2
Technische Universität Berlin
Straße des 17. Juni 136

10623 Berlin

Prof.Dr. Domokos Szasz
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1364 Budapest

Krzysztof Wysocki
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Zhihong Xia
Department of Mathematics
North Western University
2033 Sheridan Road

Evanston , IL 60208-4070
USA

Prof.Dr. Eduard Zehnder
Mathematik Department
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof.Dr. Jean-Christophe Yoccoz
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Dr. Anton Zorich
Independent University of Moscow
College of Mathematics
POB 230

117463 Moscow
RUSSIA