Tagungsbericht 29/1995

Darstellungstheorie endlichdimensionaler Algebren

23. - 29.07.1995

The meeting was organized by I. Reiten (Trondheim) and C.M. Ringel (Bielefeld). Originally, Maurice Auslander was one of the organizers, until his untimely death on Nov. 18, 1994. Many of the lectures at the conference documented very well his strong influence in representation theory.

The topics treated in the conference ranged from discrete problems, concerning algebras and matrix problems of finite representation type, to geometrical questions on the variety of modules. Combining both discrete and geometrical aspects, tame algebras played a central role. Here, besides general results (e.g. on coverings), the class of strongly simply connected algebras was the main object of interest.
Further important topics were the investigation of Auslander-Reiten components for several classes of algebras (quasi-tilted, selfinjective or perpendicular categories of hereditary algebras), as well as the study of Koszul and quasihereditary algebras. The spectrum of lectures was completed by contributions to the theory of quantum groups, mainly using the Hall algebra approach. Many open problems were posed in the lectures, and there were discussions immediately following them.

The inspiring setting of Oberwolfach was very stimulating. The contacts which were established and fostered certainly will contribute to further progress in the field.

Vortragsauszüge

R. BAUTISTA

Quadratic forms and algorithms in bocses

If $\mathcal{A} = (A, V, \mu, \epsilon)$ is a free triangular bocs, we call it of strongly unbounded representation type if there is an infinite sequence of numbers $d_1 < d_2 < \ldots < d_i < \ldots$ such that for each $i \in \mathbb{N}$ there are infinitely many isoclasses of indecomposable modules M in rep $\mathcal{A}$ such that

$$\frac{\dim_k M}{\dim_k \mathrm{End}(M)/\mathrm{rad}\,\mathrm{End}(M)} = d_i\,.$$

We prove the following : If $\mathcal{A}$ is not of strongly unbounded representation type, the number of isoclasses of indecomposable modules M in rep $\mathcal{A}$ with $\underline{\dim} M = \underline{d}$ and $\dim_k \mathrm{End}(M) = s$ is bounded by

$$\binom{\|\underline{d}\| - 1 + s - q(\underline{d})}{s - q(\underline{d})}$$

where q is the quadratic form of the bocs and $\|\underline{d}\|$ is the norm of $\underline{d}$.
The proof uses properties of the reduction algorithms in terms of the quadratic form.

K. BONGARTZ

Minimal singularities of Kronecker modules

There is an old question by Kraft and Procesi whether orbit closures of finite-dimensional modules are always normal. They proved that statement in characteristic 0 for modules over the polynomial algebra $k[X]$. Furthermore they classified the minimal singularities in that case using the normality. Here a minimal singularity means the singularity of the pointed variety $(\overline{O(m)}, n)$ where n is a minimal degeneration of m, whence belongs to the orbit closure $\overline{O(m)}$.
In my talk I presented several general reduction techniques for the study of minimal singularities. One of them gives an easy direct proof of the Kraft-Procesi result mentioned before. Using all of them one can describe completely the minimal singularities occurring for Kronecker-modules. The most difficult case is when $M = P_1 \oplus P_{n+1} \oplus P_0^{p-1} \oplus P_{n+2}^{q-1}$ degenerates to $N = P_0^p \oplus P_{n+2}^q$. Here P_i is the preprojective indecomposable with dimension vector $(i, i+1)$. Then the occurring singularity is smooth equivalent to the singularity of 0 inside the closure of the orbit of a highest weight vector in the $SL_2 \times GL_p \times GL_q$ module $k[X,Y]_n \otimes k^p \otimes (k^q)^*$. In characteristic 0 all these singularities are normal by a result of Vinberg and Popov. Many open questions connected with the set-theoretic and geometric structure of orbit closures have been mentioned.

S. BRENNER

Irreducible maps between decomposable modules

Let Λ be an artin R-algebra. A map $f : X \to Y$ is said to be strongly irreducible if and only if, for all split monomorphisms $\iota : A \to X$ and all split epimorphisms $\sigma : Y \to B$, with A and B indecomposable, $\iota f \sigma$ is irreducible.

Theorem (Bautista) Let λ be an artin R-algebra. A map f in mod-Λ is irreducible if and only if it is the direct sum of an isomorphism and a strongly irreducible map between non-zero modules.

Let M and N be indecomposable modules such that there is an irreducible map from M to N. We may suppose that R is local. Suppose also that $\mathrm{End}M/\mathrm{rad}\ \mathrm{End}M \simeq \mathrm{End}N/\mathrm{rad}\ \mathrm{End}N \simeq R/\mathrm{rad}R = k$ and that k is a field. We discuss conditions under which there is an irreducible map $f : M^m \to N^n$. This depends on k as well as on $\dim_k \mathrm{irr}(M,N) = r$. In the case where $k = \mathbb{R}$ and $r = n$, this is the same as the condition for the existence of $m-1$ independent vector fields on the $(n-1)$-sphere.

TH. BRÜSTLE

On the growth function of a tame algebra

If A is a tame algebra, there is in each dimension d a finite number of affine lines $L_i \subset \mathrm{mod}_d A$ which cover all but finitely many isoclasses of indecomposable d-dimensional A-modules. By setting $\mu_A(d)$ as the minimal number of those lines, we obtain the growth function μ_A of A.
The aim of this talk is to present the following bound for μ_A :

Proposition. *Let A be a tame algebra (over an algebraically closed field k), and r the dimension of the radical of A. Then*

$$\mu_A(d) \leq r\, e^{4(4d)^{2d}} \qquad \text{for all } d \in \mathbb{N}.$$

For a fixed algebra, this double exponential bound seems to be not yet minimal.
On the other hand, we obtain a linear growth condition if we fix the dimension : For all tame algebras, the number of 1-parametric families of (isoclasses of) indecomposables in dimension d is bounded by $c \cdot r$, where the constant c only depends on d.
The proposition results from a detailed analysis of the proof of the tame-wild dichotomy as it is given by Gabriel,Nazarova,Roiter,Sergeichuk and Vossieck.

M.C.R. BUTLER

Almost split sequences for modules of small support

This talk reported a new theorem, from the Ph.D. Thesis (1995) of D.Stewart, about existence and construction of almost split sequences over an artin algebra Λ (with more than one isomorphism type of simple modules). For any f.g. module $X = X_\Lambda$, define

$\mathcal{L}(X) = \{$ simple S such that $\mathrm{Hom}(S,X) \neq 0$ *or* $\mathrm{Ext}^1(S,X) \neq 0\}$ and
$\mathcal{R}(X) = \{$ simple S such that $\mathrm{Hom}(X,S) \neq 0$ *or* $\mathrm{Ext}^1(X,S) \neq 0\}$.
Also, for any non-zero idempotent e in Λ, define
$\mathcal{E}(X) = \{$ simple S such that $Se \neq 0\}$.
Stewart's Theorem gives the following construction for the almost split sequence (ASS) starting at an indecomposable non-injective module X :

Assume, $\mathcal{L}(X) \subset \mathcal{E}(X)$. There is an ASS

$$E' : 0 \to Xe \to Y' \to Z' \to 0$$

of $e\Lambda e$-modules, $\tau_\Lambda^{-1}(X) \cong Z' \otimes_{e\Lambda e} e\Lambda$, and the pushout of the *right* exact sequence $E' \otimes_{e\Lambda e} e\Lambda$ along the natural map $Xe \otimes_{e\Lambda e} e\Lambda \to X$ is an ASS of right Λ-modules.

If also $\mathcal{R}(X) \subset \mathcal{E}(X)$, then $E' \otimes_{e\Lambda e} e\Lambda$ is (isomorphic to) an ASS for X, and the endomorphism ring of its middle term is isomorphic to $\mathrm{End}_{e\Lambda e}(Y')$, so has the same number of indecomposable (indecomposable projective) summands as does Y'.

By choosing e so that $\mathcal{L}(X) \cup \mathcal{R}(X) = \mathcal{E}(e)$, the theorem allows construction of the ASS for X from that for Xe over the 'smaller' algebra $e\Lambda e$.

C. Cibils

Representation type of half quantum groups

For each symmetric Cartan matrix C, Drinfeld and Jimbo have considered $U_q(C)$, a Hopf algebra (quantum group) which admits a finite dimensional quotient $\overline{U}_q(C)$ in case q is a root of unity.
We study the representation type of $\overline{U}_q^+(C)$, the 'upper triangular' sub-Hopf algebra of $\overline{U}_q(C)$. Actually $\overline{U}_q^+(C)$ is basic, connected iff det C is invertible modulo the order of q and its quiver and relations can be completely described. Moreover, the coalgebra structure is explicit in terms of the quiver and the monoidal structure of the preprojective modules over these 'half-quantum groups' can be studied.

F. U. Coelho

The Auslander-Reiten quiver of a quasitilted algebra

Let A be a quasitilted algebra, that is, an (finite dimensional associative algebra over an algebraically closed field) algebra with global dimension at most two and such that each indecomposable A-module has either the projective dimension at most one or the injective dimension at most one. Tilted, tubular and canonical algebras are examples of quasitilted algebras. Denote by Γ_A the Auslander-Reiten quiver of A. We are mainly interested in the structure of the connected components of Γ_A. We have shown in a joint work with D. Happel that Γ_A has a postprojective and a preinjective component. Also, in a joint work with A. Skowroński we have shown : (i) the components of Γ_A with oriented cycles are semiregular tubes; and (ii) if A is not tilted, then all the components of Γ_A are semiregular. Observe that (ii) generalizes the following result proven by Happel-Reiten-Smalø: representation-finite quasitilted algebras are tilted.

W. CRAWLEY-BOEVEY

On homomorphisms from a fixed representation to a general representation of a quiver

We study the dimension $\hom(X,\beta)$ of the space of homomorphisms from a fixed representation X of a quiver to a general representation of dimension vector β. Our basic result is a formula which relates this number, the general rank of homomorphisms from X to representations of dimension β, and the dimension of an open subset of a Grassmannian of subrepresentations of X. The formula has two corollaries which describe the asymptotic behaviour of $\hom(X,r\beta)$. These results are closely related to work of A. Schofield on general representations of quivers and also to another theorem of Schofield's, that the indecomposable modules for a path algebra which have no self-extensions and have finite length over their endomorphism rings are in 1-1 correspondence with the Schur roots for the quiver.

P. DRÄXLER

Multiple one-point extensions by indecomposable modules of regular length 2

Fix an algebraically closed field k. We want to report on the following joint result with A. Skowroński:

Theorem. *Let A be a connected tame hereditary k-algebra of type $\widetilde{\mathbb{A}}_n$ or $\widetilde{\mathbb{D}}_n$. Suppose T is a tilting A-module, $B = \mathrm{End}_A(T)$ is the associated tilted algebra and $F = \mathrm{Hom}_A(T,-)$ the corresponding functor.*
If $R_1,\dots,R_r$ is a sequence of pairwise orthogonal indecomposable T-torsion A-modules of regular length 2 lying in the non-homogenous tubes in case $\widetilde{\mathbb{A}}_n$ and in one tube of rank $n-2$ in case $\widetilde{\mathbb{D}}_n$, then the multiple one-point extension $B[F(R_1)]\dots[F(R_r)]$ is of tame representation type.

This generalizes results of C.M. Ringel (Tame algebras, SLNM 831, 137-287, 1980) where one-point extensions with modules of regular length 2 are investigated by means of vector space categories. But in contrast to this case the multiple one-point extension situation leads to the multiple vector space category $(B-\mathrm{mod}, \bigoplus_{i=1}^r \mathrm{Hom}_B(F(R_i),-))$. Since $B-\mathrm{mod}$ is embedded in $D^b(A)$, it suffices to prove the tameness of the multiple vector space category $(D^b(A), \bigoplus_{i=1}^r \mathrm{Hom}_{D^b(A)}(R_i,-))$. The tameness of this is shown by proving that the corresponding multiple one-point extension of an algebra of the shape $\begin{pmatrix} A & 0 \\ \mathrm{D}(A) & A \end{pmatrix}$ is tame where possibly n has to be increased. This is possible because this algebra turns out to be a generalized one-point extension whose associated (ordinary) vectorspace category is easily seen to be tame.

Yu. A. Drozd

Coverings of tame algebras and boxes

(joint work with S. A. Ovsienko)
Let A be a locally bounded category, $\pi : \tilde{A} \to A$ its Galois covering with torsion free Galois group G. Then :

1) A is tame if and only if $\tilde{A}$ is tame.
2) If A is tame, then :
   a) $\operatorname{mod} A = \operatorname{mod}_0 \coprod \operatorname{mod}_1$, where $\operatorname{mod}_0 = \operatorname{Im} \Pi_*$ (the direct image functor) and mod_1 is a disjoint union of 1-parameter families
   b) the induced functor $\pi_* : \operatorname{mod} \tilde{A} \to \operatorname{mod}_0$ is also a Galois covering with the same group G.

The proof is based on the techniques of boxes. Namely, the same result (indeed, its technical generalization) is proved for representations of free triangular boxes using the 'reduction algorithm'.

K. Erdmann

The stable category of a selfinjective algebra

This is a report on joint work with O. Kerner. Let Λ be a finite-dimensional self-injective algebra over an algebraically closed field k, and let $\mathcal{C}$ be a stable AR-component which is quasi-serial (of the form $\mathbb{Z}A_\infty$ or $\mathbb{Z}A_\infty/\langle\tau^r\rangle$). We have studied stable homomorphisms between modules in $\mathcal{C}$; some of our results are analogues of properties of wild hereditary algebras. In particular :
Theorem 1 Assume $s \in \mathbb{Z}$ such that it is not the case that $\tau^{s-1} \cong \Omega^{-1} \neq id$ on $\mathcal{C}$; let $\Psi_S(M) = \dim \underline{\operatorname{Hom}}(M, \tau^s M)$. Then Ψ_s is weakly increasing on $\mathcal{C}$ as a function of the quasi-length. Moreover, if $\mathcal{C}$ is a tube then Ψ_s is unbounded. (For group algebras, this has refinements.)
We specialize to the case $s = 0$ and study modules such that $\underline{\operatorname{End}}\ M = k$ which we call stable bricks (although they need not be bricks). One of the results is
Theorem 2 Assume that it is not the case that $\tau^{s-1} \cong \Omega^{-1} \neq id$ on $\mathcal{C}$. Assume M in $\mathcal{C}$ is a stable brick of quasi-length r. Then all modules in $\mathcal{C}$ of quasi-length $\leq r$ are stable bricks. Moreover, if X is the quasi-socle of M then $\underline{\operatorname{Hom}}(X, \mathcal{W}(M/X)) = 0$.

We also have sufficient conditions which guarantee the existence of stable bricks of quasi-length r.

P. Gabriel

Subspace problems of local dimension ≤ 2

Let A be a finite-dimensional algebra over $k = \bar{k}$ and M a finite-dimensional left A-module. The *local dimension* of M is by definition

$$\max_P \dim P \otimes_A M ,$$

where P ranges over the indecomposable projective (right) A-modules.

Problem. Classify up to isomorphism the triples (V, f, X), where V is a finite-dimensional vector space, X a finite-dimensional projective (right) A-module and $f : V \to X \otimes_A M$ a linear map.

Results.

1) (trivial) The local dimension of M is ≤ 3 if M is finitely spaced, i.e. if there are only finitely many isoclasses of triples (V, f, X).
2) (Nazarova/Roiter) If M has local dimension ≤ 2, M is finitely spaced iff some associated poset $C(S)$ is representation-finite.
3) (Guidon/Hassler/Nazarova/Roiter) If M has local dimension ≤ 2 and is finitely spaced, the triples (V, f, X) can be described in terms of $C(S)$.

E.L. Green

Periodic-like behaviour of resolutions of modules

In this talk, I describe recent results of my student, Mike Bardzell. If $\Lambda = k\Gamma/I$ is a monomial algebra then it is shown that the maps in the Λ^e-resolution of Λ have an alternating behaviour of period 2.
Thus, tensoring with Λ-modules gives resolutions with alternating map behaviours. Finally, using the work of Anick and myself, we see that there are resolutions with periodic-like map behaviour for many finite dimensional algebras and modules.

L. Hille

Moduli of thin sincere representations of finite dimensional algebras

Representations of quivers and finite dimensional algebras play an important role in many fields of mathematics. A series of papers show, that there is a closed relation to algebraic geometry and problems of classification of vector bundles. So it is an important question, how to provide the modules of fixed dimension vector with the structure of an algebraic variety. The general construction of modulispaces of representations of finite dimensional algebras was published by A. King, using geometric invariant theory. In this talk we consider thin sincere modules, the simplest nontrivial case, and compute all modulispaces.

B. Keller

Invariance and localization for cyclic homology of DG algebras

We show that two flat differential graded algebras whose derived categories are equivalent by a derived functor have isomorphic cyclic homology. In particular, 'ordinary' algebras over a field which are derived equivalent in the sense of J. Rickard share their cyclic homology, and iterated tilting preserves cyclic homology. This completes results of J. Rickard's and D. Happel's on the preservation of Hochschild (co-)homology under derived equivalence.

It also extends well known results on preservation of cyclic homology under Morita equivalence due to A. Connes, J.-L. Loday-D. Quillen, Chr. Kassel, and R. McCarthy. In fact, we show that cyclic homology is even preserved under what we call a K-theoretic equivalence. For example, a *finite*-dimensional algebra of *finite* global dimension is K-theoretically equivalent to its largest semisimple quotients (both finiteness hypotheses are essential). This yields for example the result, first proved (in characteristic 0) by Th. Goodwillie, that the cyclic homology of such an algebra only depends on the number of isoclasses of simple modules.
We go on to show that under suitable flatness hypotheses, an exact sequence of derived categories of DG algebras yields a long exact sequence in cyclic homology. This is to be viewed as a (comparatively easy) analogue of Thomason-Trobaugh's and Yao's localization theorems in K-theory.

S. KÖNIG

Triangular decompositions of Schur algebras, and decomposition numbers of GL_n in defining characteristic

Computing decomposition numbers of general linear groups (over infinite fields, in defining characteristic) is equivalent to computing decomposition numbers of (or formal characters of simple modules over) Schur algebras (by results of I.Schur and J.A.Green). Subproblems are to determine the simple modules over finite GL_n or over symmetric groups. An elementary algorithm is described which may serve as a low-level substitute of a solution of this problem, since it computes the decomposition numbers of any given Schur algebra.
The algorithm is based on a triangular decomposition $A \simeq C \otimes_S C^{op}$ of the Schur algebra. The subalgebras C and C^{op} are explicitely described, hence one gets a new basis of the Schur algebra, consisting of elements of the form $c \otimes b$. It follows that one can explicitly compute the values of a bilinear form, which in turn determines the simple modules (as modules over the basic semisimple subalgebra S, which gives their formal characters).

H. KRAUSE

Generic modules over artin algebras

The talk is divided into three parts. In the first part I introduce the notion of a (right) dualizing ring and sketch a proof of the following result.

Theorem *For a ring Λ the following conditions are equivalent:*
(1) *Λ is of finite representation type.*
(2) *Λ is right dualizing and every right Λ-module is a direct sum of (finitely presented) indecomposable Λ-modules.*

The second topic is the relation between generic modules and one-parameter families.

Theorem *Let Λ be an artin algebra. If Γ is a stable tube of the AR-quiver, then the Ziegler closure $\overline{\Gamma}$ contains a generic module.*

Call a stable homogenous tube Γ *familiar*, if the generic module in $\overline{\Gamma}$ is unique. Denote for any generic module G by $\operatorname{ind}_G(\Lambda)$ the indecomposables which belong to a familiar tube Γ with $G \in \overline{\Gamma}$.

Theorem *Let Λ be a tame algebra and let $n \in \mathbb{N}$.*
(1) $\operatorname{ind}_G(\Lambda) \neq \emptyset$ *for every G and* $\operatorname{ind}_G(\Lambda) \cap \operatorname{ind}_{G'}(\Lambda) = \emptyset$ *for every pair $G \neq G'$.*
(2) *Almost all indecomposables of length n belong to* $\operatorname{ind}_G(\Lambda)$ *for some G.*

The last part of my talk is devoted to certain criteria for the existence of generic modules over dualizing rings. These results are motivated by the Brauer-Thrall II conjecture.

H. LENZING

Why are the canonical algebras canonical ?

My talk will provide an answer to this question by exhibiting three different, related reasons:

Theorem 1 (joint with de la Peña). *Let Σ be a connected artin algebra with a sincere separating tubular family of stable tubes. Then Σ is concealed-canonical, i.e. isomorphic to the endomorphism algebra of a tilting module T over a canonical algebra (sense of Ringel and Crawley-Boevey), where T consists of indecomposable summands of strictly positive rank.*
Moreover, Σ can be realized as (the endomorphism ring of) a tilting object on a hereditary noetherian category $\mathcal{H}$ without projectives.

Theorem 2 *Let $\mathcal{H}$ be a connected hereditary noetherian k-category without non-zero projectives and admitting a tilting complex, where k is an algebraically closed field.*
Then $\mathcal{H}$ is equivalent to a category of coherent sheaves on a weighted projective line, in particular derived-equivalent to the category of finite dimensional modules over a canonical algebra.

Theorem 3 (S. Mori, D. Kussin). *Let R be a commutative graded factorial algebra of Krull dimension two, which is affine over an algebraically closed field, and which is graded by an abelian group of rank one.*
Then R is isomorphic, as a graded algebra, to the coordinate algebra of a weighted projective line. In particular, the homogeneous components of R describe the morphism spaces between rank one modules over the corresponding canonical algebra.

S. Liu

Strongly simply connected algebras

This is a joint work with Ibrahim Assem. We first show that an algebra A over an algebraically closed field k is strongly simply connected in the sense of Skowronski's provided that there is a presentation $A = kQ/I$ of A such that the fundamental group of each connected full convex bound subquiver of (Q, I) is trivial. Then we give a necessary and sufficient condition on a module M such that the one-point extension $A[M]$ (respectivly, the one-point co-extension $[M]A$) of a strongly simply connected algebra A is again strongly simply connected. In particular this gives rise to an algorithm to construct all Schurian algebras which are strongly simply connected.

N. Marmaridis

Grothendieck groups arising from contravariantly finite subcategories

(joint work with A. Beligiannis)
The subject of the talk is the study of relative homological properties of a contravariantly finite subcategory $\mathcal{X}$ of a given additive category $\mathcal{C}$. It is introduced the notion of the Grothendieck group relative to $\mathcal{X}$ and also that of the Cartan map $c_{\mathcal{X}}$ relative to $\mathcal{X}$. It is proven that the cokernel of $c_{\mathcal{X}}$ is isomorphic to the corresponding stable Grothendieck group of the stable category $\mathcal{C}/\mathcal{I}_{\mathcal{X}}$. It is also showed that if the right $\mathcal{X}$-dimension of $\mathcal{C}$ is finite, then $c_{\mathcal{X}}$ is an isomorphism. If $\mathcal{C}$ is a finite dimensional k-additive Krull-Schmidt category, it is introduced the notion of the $\mathcal{X}$-dimension vector of an object of $\mathcal{C}$. There are given criteria for when an indecomposable object is determined up to isomorphism by its $\mathcal{X}$-dimension vector.

R. Martinez

Applications of Koszul algebras to selfinjective, preprojective and Auslander algebras

During the VII ICRA in Oaxtepec, Mor., 1994, Nagashima asked if certain quiver algebras, appearing in Differential Geometry, were Koszul algebras. These quivers are precisely the quivers corresponding to preprojective algebras. Some of them appear as Yoneda algebras of selfinjective, radical cube zero algebras.
Using the theory of Koszul algebras developed in a previous paper, we prove that the preprojective algebras are Koszul if and only if they are not the preprojective algebras coming from a Dynkin diagram. We obtain a non finite dimensional class of algebras of global dimension two generalizing the preprojective algebras and some information on their modules is obtained.
Another application is given to Auslander algebras obtaining new proofs of known results by Igusa-Todorov.

R. NÖRENBERG

Tame minimal non-polynomial growth strongly simply connected algebras

(joint work with A. Skowroński)

As has been shown by Skowroński, a strongly simply connected tame algebra is of polynomial growth if and only if it does not contain a convex subalgebra which is pg-critical. There the pg-critical algebras are defined to be certain extensions of tame concealed algebras of type $\tilde{\mathbf{D}}_n$ by means of indecomposable modules of quasilength one or two lying in the exceptional tubes of rank $n-2$ of these algebras.
Starting from a description of the modules of quasilength one in the exceptional tubes of rank $n-2$ of a tame concealed algebra of type $\tilde{\mathbf{D}}_n$ in terms of projective resolutions, it is possible to obtain an explicit description of the pg-critical algebras in terms of quivers and relations. In particular this yields a list of 16 'frames' completely describing the strongly simply connected pg-critical algebras in terms of quivers and relations.

S. A. OVSIENKO

Coverings of vector space categories

Let $k = \bar{k}$ be a field and $\mathcal{A} = (K, V)$ a finite dimensional vectorspace category over k with $K = k_0 \times L$ and $Vk_0 = 0 = LV$.
Proposition. If for every $S \subset \mathrm{Ob}\ L$ with $|S| \le 2$ the restriction of $\mathcal{A}$ on S is of finite type, then there exits a Galois covering $\pi : \tilde{\mathcal{A}} \to \mathcal{A}$ such that

1) either the Tits form $q_{\tilde{\mathcal{A}}}$ of $\tilde{\mathcal{A}}$ is not weakly positive or $q_{\tilde{\mathcal{A}}}$ is weakly positive and endomorphism rings of indecomposable representations of $\tilde{\mathcal{A}}$ are trivial; in this case, either $\tilde{\mathcal{A}}$ is locally representation finite and $\mathcal{A}$ is representation finite or else $\mathcal{A}$ contains infinitely many representations in dimension $\le 2\ \dim_k V + 2$.

2) The fundamental group of the covering in the finite case is free .

3) There exist $a, b > 0$ such that for every indecomposable representation M of $\mathcal{A}$ holds
$$\sum_{i \in \mathrm{Ob}\, K} \dim_k M(i) < a\ \dim_k V + b\,;$$

4) There exists a $c > 0$ such that for every indecomposable representation M of $\mathcal{A}$ the following holds :
$$\left| \sum_{i \in \mathrm{Ob}\, L} \dim_k M(i) - 2\ \dim_k M(0) \right| \le c\,.$$

J.A. DE LA PEÑA

Geometrical aspects of polynomial growth algebras

Let $A = kQ/I$ be a finite dimensional connected algebra over an algebraically closed field k. Assume that A is strongly simply connected. Then the first Hochschild cohomology group $H^1(A)$ vanishes. If A is of polynomial growth, $H^3(A) = 0$ and we may calculate $\dim_k H^2(A)$.

Let $v \in \mathbb{N}^{Q_0}$ be a vector and $\underline{\mathrm{mod}}_A(v)$ the scheme of A-modules with dimension vector v. We have,

Theorem. The following are equivalent :

a) A is of polynomial growth,

b) $\dim_k \mathrm{Ext}^1(X,X) \leq \dim_k \mathrm{End}_A(X)$ and $\mathrm{Ext}^r_A(X,X) = 0$ for every $r \geq 2$ and every indecomposable A-module X.

c) For every $v \in \mathbb{N}^{Q_0}$, and every indecomposable $X \in \mathrm{mod}_A(v)$, the scheme is smooth at X. Moreover, the Tits form q_A is weakly non-negative.

Most of the work is joint with A. Skowroński.

M. REINEKE

On extensions by simple modules and Kashiwara's operators on quantized enveloping algebras

We study the action of Kashiwara's operators on the canonical basis of a quantized enveloping algebra of finite type.

This is done by using Ringel's Hall algebra approach which provides a bijection between elements of the canonical basis and isomorphism classes of representations of quivers.

A sufficient criterion leads to the consideration of extension of modules by simple ones which reduces for special choices of the quiver to combinatorics in certain posets.

This allows a verification of the criterion and thus an explicit description of the action of Kashiwara's operators in terms of representation theory in the cases A_n–D_n.

A. SKOWROŃSKI

On tame algebras without higher selfextensions of indecomposable modules

We shall discuss the problem when $\mathrm{Ext}^2_A(X,X) = 0$ (resp. $\mathrm{Ext}^r_A(X,X) = 0$, $r \geq 1$) for all indecomposable finite dimensional modules X over a tame algebra A. We plan to describe all tame quasitilted algebras and tame strongly simply connected algebras with the above property.

S. O. SMALØ

Lengths of endomorphism rings of indecomposable modules

In the spirit of Maurice Auslanders proof of the first Brauer-Thrall conjecture we present the following result :

Theorem : *An artin algebra A is of infinite representation type if and only if the length of the endomorphism rings of indecomposable modules is not bounded.*

This result is obtained in some joint work with S. Venås.

Ø. SOLBERG

The representation-finite symmetric algebras with liftable simple modules

(joint work with G. Michler)

Janusz has showed that a p-block algebra Λ of a finite group G of finite representation type over a splitting field F with characteristic $p > 0$ is uniserial if and only if every simple Λ- module M can uniquely be lifted to a simple $\hat{\Lambda} \otimes_R S$- module, where R is a complete discrete rank one valuation ring with maximal ideal πR, residue class field $F = R/\pi R$, and quotient field S with characteristic zero, and where $\hat{\Lambda}$ is a RG-block such that $\Lambda \simeq \hat{\Lambda} \otimes_R F$. If Λ has at least two simple modules it can be shown that this liftability condition is equivalent to that $\mathrm{Ext}^2_\Lambda(M, M) = (0) = \mathrm{Ext}^1_\Lambda(M, M)$ for every simple Λ-module M.

These Ext-conditions also imply the unique liftability of the simple module M of an arbitrary finite dimensional symmetric algebra Λ over a commutative field K in the sense of Auslander-Ding-S. For artin algebras it is known that $\mathrm{Ext}^2_\Lambda(M, M) = (0)$ implies that $\mathrm{Ext}^1_\Lambda(M, M) = (0)$. So in order to generalize Janusz' theorem to the theory of finite dimensional symmetric algebras Λ over an algebraically closed field K, it therefore suffices to classify all such connected algebra Λ of finite representation type satisfying $\mathrm{Ext}^2_\Lambda(M, M) = (0)$ for all simple Λ-modules M. In particular, we have.

Theorem Let Λ be a connected representation finite, symmetric algebra over an algebraically closed field K with at least two non-isomorphic simple Λ-modules. The the following are equivalent.

(1) $\mathrm{Ext}^2_\Lambda(M, M) = (0)$ for every simple Λ-module M.

(2) Either Λ is uniserial, or Λ has multiplicity 1 and $\rho P/\operatorname{soc}(P)$ is indecomposable for all indecomposable projective Λ- modules P.

A key result in showing this is the following. Let Λ be an artin algebra and suppose P is an indecomposable projective Λ-module with simple socle. Then, if $\mathrm{Ext}^2_\Lambda(P/\rho P, \operatorname{soc}(P) = (0)$ and $\rho P/\operatorname{soc}(P)$ is nonzero, then $\rho P/\operatorname{soc}(P)$ is indecomposable.

The algebras characterized in the Theorem are also given explicitly by quivers and relations using work of Bretscher, Läser and Riedtmann, Roggon and Waschbüsch.

L. UNGER

Perpendicular categories of stones over quiver algebras

This is a report on a joint work with Dieter Happel, Silke Hartlieb and Otto Kerner. Let $H = k\vec{\Delta}$ be a finite dimensional path algebra, k some field. A finite dimensional module $X \in \operatorname{mod} H$ is called a stone if X is indecomposable and if $\operatorname{Ext}^1_H(X,X) = 0$. We call X a brick if $\operatorname{End} X$ is a division ring. The perpendicular category $X^\perp$ of a stone X is defined as

$$X^\perp = \{Y \in \operatorname{mod} H \mid \operatorname{Hom}(X,Y) = 0 = \operatorname{Ext}^1_H(X,X)\}.$$

It is known that $X^\perp$ is equivalent to $\operatorname{mod} A$, $A = k\vec{\Delta}'$ a finite dimensional algebra, and A has one simple module less than H. In general it is a difficult problem to determine A for a given stone X. We outline the proof of the following results :

Theorem 1 : *Let $H = k\vec{\Delta}$ and $A = k\vec{\Delta}'$. Up to a shift by the Auslander-Reiten translation τ there are only finitely many stones X such that $X^\perp \simeq$* mod A.

An equivalent formulation is :

Theorem 2 : *Let $H = k\vec{\Delta}$, $N \in \mathbb{N}_0$ be fixed. There are only finitely many regular components in the Auslander-Reiten quiver Γ_H of H containing bricks Z of quasilength 2 with* $\dim \operatorname{Ext}^1(Z,Z) = N$.

Putting $N = 0$ in theorem 2 we get :

Corollary : There are only finitely many regular components in Γ_H of H containing stones Z of quasilength ≥ 2.

T. WAKAMATSU

Construction of Frobenius algebras

Frobenius algebras appear in mathematics as group algebras or homological dual of Artin-Schelter regular algebras. Some constructions of Frobenius algebras are shown first and then, by using them, some block ideals of group algebras and homological duals of Artin-Schelter regular algebras are described.

C. C. XI

Quasi-hereditary algebras which are twisted double incidence algebras of posets

This is a joint work with Deng.

Given a finite poset X, we may associate it with a family M of matrices corresponding to the meshes in the Hasse diagram of X. Using the entries of the matrices one may define the socalled M-twisted double incidence algebra $\mathcal{A}(X,M)$ obtained from $I(X)$ and its opposite algebra $I(X)^{op}$ by using the twisted relations, where $I(X)$ denotes the incidence algebra of X. Then we consider the following questions:

(1) When is $\mathcal{A}(X,M)$ quasi-hereditary ?

(2) If it is quasi-hereditary, what is its Ringel dual ?

To the above questions we have the following answers.

Theorem. (1) If X is a tree poset then $\mathcal{A}(X, M)$ is quasi-hereditary.
(2) If X is a tree poset and each matrix in M is invertible then the Ringel dual of $\mathcal{A}(X, M)$ is isomorphic to $\mathcal{A}(X^{op}, M)$.

J. XIAO

Hall algebras in root categories

In order to realize the quantum group in a global way, the Hall algebra of a root category (which is an orbit category of the derived category of a hereditary algebra) is constructed. The PBW-basis, filtration structure and integral form are investigated. Their degeneration at $v = 1$ actually provides a realization of the universal enveloping algebra of semisimple Lie algebras.

P. ZHANG

On composition algebras

Let A be a finite dimensional hereditary algebra over a finite field, $\mathcal{H}(\mathcal{A})$ and $\mathcal{C}(\mathcal{A})$ be respectively the Hall algebra and the composition algebra of A over $\mathbb{Q}$. If A is representation-finite, C.M.Ringel has proved that $\mathcal{C}(\mathcal{A}) = \mathcal{H}(\mathcal{A})$; otherwise, $\mathcal{C}(\mathcal{A})$ is a proper subalgebra of $\mathcal{H}(\mathcal{A})$. In these cases we are interested in the following question: Which kinds of isoclasses $[M]$, or their $\mathbb{Q}$-combinations, lie in $\mathcal{C}(\mathcal{A})$?

Define $r_{\mathbf{d}}$ to be the element $\sum[M] \in \mathcal{H}(\mathcal{A})$, where $[M]$ runs over the isoclasses of the regular $A-$modules with dimension vector $\mathbf{d}$.

Theorem (1) The preprojective and preinjective A-modules lie in $\mathcal{C}(\mathcal{A})$.
(2) For $\mathbf{d} \in \mathbb{N}_0^n, r_{\mathbf{d}} \in \mathcal{C}(\mathcal{A})$.
(3) The indecomposables without self-extension all lie in $\mathcal{C}(\mathcal{A})$.
(4) If A is tame and X is an indecomposable regular A-module with quasi-length less than the rank of the tube in which X lies, then $[X] \in \mathcal{C}(\mathcal{A})$.

Let K be the Kronecker algebra, $\mathcal{P}$ (resp. $\mathcal{I}$) the subalgebra of $\mathcal{C}(\mathcal{K})$ generated by the preprojective (resp. preinjective) $K-$modules, and $\mathcal{T}$ the subalgebra generated by $r_{(n,n)}$ for $n \geq 0$, denote by $\mathcal{P} \cdot \mathcal{T} \cdot \mathcal{I}$ the set of elements in $\mathcal{H}(\mathcal{K})$ which are $\mathbb{Q}$-span of $[P] \cdot T \cdot [I]$ with P (resp. I) preprojective (resp. preinjective) and $T \in \mathcal{T}$.

Theorem $\mathcal{C}(\mathcal{K}) = \mathcal{P} \cdot \mathcal{T} \cdot \mathcal{I} \simeq \mathcal{P} \otimes \mathcal{T} \otimes \mathcal{I}$, where $\otimes = \otimes_{\mathbb{Q}}$.

B. ZIMMERMANN-HUISGEN

Algebras of finite uniserial type and approximations by representations with uniserial building blocks

We start with a brief review of our varieties V_S describing the uniserial modules with sequence $S = (S_0, \dots, S_m)$ of consecutive composition factors, where the S_i are simple modules over a split finite dimensional algebra, as well as of the associated canonical maps Φ_S assigning uniserial modules to the points of V_S. In particular, we explain how these varieties differ qualitatively from the obvious open subvarieties of the classical varieties of representations.
These varieties are used to obtain a firm hold on the algebras having only finitely many uniserial modules, up to isomorphism. We will characterize these algebras in terms of quiver and relations and discuss their uniserial representation theory. It turns out that, among the algebras satisfying a certain easily described necessary condition, the algebras of finite uniserial type are characterized by a condition of the following ilk: Whenever the simple module sitting at the top of a uniserial module U occurs with multiplicity larger than 1 in U, a major segment of the sequence of composition factors of U must repeat. As a consequence, the intricacy of the uniserial representation theory of an algebra Λ of finite uniserial type is determined by the patterns of nested oriented cycles in the quiver of Λ.

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