Tagungsbericht 30/1995

"Einhüllende Algebren und Darstellungstheorie"

30. Juli bis 5. August 1995

Dies war die achte Oberwolfacher Tagung über Einhüllende Algebren von Lie-Algebren nach den vorangegangenen in den Jahren 1973, 1975, 1978, 1982, 1985, 1987 und 1990.

Die Tagung fand unter der Leitung von W. Borho (Wuppertal), M. Duflo (Paris 7), A. Joseph (Paris 6 und Rehovot) und R. Rentschler (Paris 6) statt.

Die Tagung "Einhüllende Algebren und Darstellungstheorie" war dazu bestimmt, die jüngsten Ergebnisse und laufende Entwicklungen aufzuzeigen. Nicht wiedergegeben wurden, da nunmehr bereits allgemein zugänglich, Ergebnisse wie die Bistetigkeit der Dixmierabbildung (1991) oder die Quantisierung verzweigter Überlagerungen (co)-adjungierter Orbitabschlüsse.

Bis auf eine Ausnahme - Beweis einer Vermutung von Dixmier aus dem Jahre 1978 im auflösbaren Fall - betrafen alle Vorträge die halbeinfache Situation.

Die Tagung war geprägt durch neue Fragestellungen und Resultate, durch den Ausbau und die Verfeinerung bestehender Methoden, sowie durch das Auftreten neuer Methoden. Die Tagung stand insbesondere im Schnittpunkt der folgenden Gebiete und Methoden, die infolge ihrer Verbundenheit mit Untersuchungen Einhüllender Algebren und ihrer Darstellungen in enger Wechselwirkung miteinander stehen :

1) Differentialoperatoren ($\mathcal{D}$-Moduln) und K-Theorie
2) Quantendeformationen Einhüllender Algebren
3) Die kanonische Basis
4) Darstellungen und Methoden in Charakteristik p
5) Hecke Algebren sowie affine Hecke Algebren
6) Orbiten und geometrische Methoden im reellen Fall
7) Multipizitäten in verschiedenen Situationen

Der innere Zusammenhang lässt sich wie folgt skizzieren :

Ursprünglicher Ausgangspunkt waren primitive Ideale in Einhüllenden Algebren, Multiplizitäten in Moduln mit einem höchsten Gewicht, geometrische Fragen (co)-

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adjungierter Orbiten, sowie überraschende Zusammenhänge. Antworten führten verhältnismässig oft auf Informationen, die letzten Endes auf Daten der Weyl-Gruppen, auf ihren Darstellungen und ihrer Kombinatorik beruhen.

Kazhdan-Lusztig Polynome (1978) und ihr Zusammenhang mit Multiplizitäten unterstrichen frühzeitig die Bedeutung von Hecke-Algebren (Deformationen von Gruppenalgebren der Weyl-Gruppen).

Seit der Bernstein-Beilinson Lokalisierung (1981) sind Differentialoperatoren auf Fahnenmannigfaltigkeiten von herausragender Bedeutung.

Ein neuer starker Impuls, der immer weiter trägt, kommt von den Quantendeformationen Einhüllender Algebren (Quantengruppen von Drinfeld-Jimbo, 1985). Hierbei ist anzumerken, dass ein enger Zusammenhang besteht zwischen ihren Einheitswurzelspezialisierungen und den Darstellungen in positiver Charakteristik der entprechenden algebraischen Gruppe. Besonders eindrucksvoll sind auch die Theorie der kanonischen Basis im Plus-Teil einer Einhüllenden Algebra bezw. einer Quantengruppe, die damit zusammenhängenden Modulbasen, verschiedene kombinatorische Beschreibungen, sowie Konsequenzen der Anwendungen.

Fast alle Vorträge fügen sich unmittelbar ein in die geschilderten Zusammenhänge. Zu erwähnen sind auch die klassischen Verbindungen zur Invariantentheorie und zur Theorie der unitärer Darstellungen.

Zu den Inhalten der Vorträge im Einzelnen sollen, soweit nicht bereits implizit angesprochen, hier nur einige kurze Hinweise in teilweiser Anlehnung an die obige Gliederung gegeben werden.

1) Für die Quotienten Einhüllender Algebren nach einem minimalen primitiven Ideal ist nunmehr die K-Theorie bestimmbar ; ausserdem, im regulären Fall, auch die Hochschildcohomologie. In beiden Fällen kommt die Bernstein-Beilinson Lokalisierung zur Anwendung.

2) Die über hundert Jahre alten Capelli-Identitäten erscheinen in neuem Licht in zwei Vorträgen :

i) Mit Hilfe gewisser Quantendeformationen Einhüllender Algebren (Yangians) lassen sich in Einhüllenden Algebren gewisse maximale kommutative Unteralgebren konstruieren, welche zu verallgemeinerten Capelli-Identitäten führen.

ii) Andere verallgemeinerte Capelli-Identitäten treten auf im Zusammenhang mit Differentialoperatoren bei endlichdimensionalen Darstellungen algebraischer Gruppen im Falle eines multiplizitätenfreien Koordinatenringes.

3) Die kanonische Basis ermöglicht den Beweis eines q-Analogons des Kempf'schen Verschwindungssatzes.

4) Für die Darstellung algebraischer Gruppen in positiver Charakteristik kommen die Bedeutung von Kippmoduln und konkrete Resultate über die Zelegung ihrer Tensorprodukte zum Tragen. Analoges gilt für Einheitswurzelspezialisierungen von Quantengruppen. Für Darstellungen der symmetrischen Gruppe in Charakteristik p gibt es explizite Resultate.

5) Verschiedene Vorträge präsentieren neue Resultate über Darstellungen von Hecke-Algebren bezw. affinen Hecke-Algebren.

6) Die Kostant-Sekeguchi Korresepondenz für nilpotente Orbiten lässt sich durch die Konstruktion eines gewissen äquivarianten Diffeomorphismus erklären. Kirillov's orbitbezogene Charakterformel lässt sich in bestimmten Fällen mit einem Feyn-man'schen Wegeintegral in Beziehung setzen.

7) Mehrere Vorträge waren der Bestimmung oder dem Zusammenhang von Multiplizitäten gewidmet, so etwa der frappierende Vergleich (unter Bedingungen) von $\mathfrak{g}$-Multiplizitäten in dem Koordinatenring von $\mathfrak{g}$ ($\mathfrak{g}$ eine einfache Lie-Algebra) mit gewissen W-(Modul)-Multiziplitäten in dem Koordinatenring von $\mathfrak{h}$ ($\mathfrak{h}$ = Cartanalgebra, W = Weyl-Gruppe) (für die triviale $\mathfrak{g}$-Darstellung erhält man den Harish-Chandra Isomorphismus) , oder Kazhdan-Lusztig Theorie im Fall von Super-Lie-Algebren. In diesen Zusammenhang seien auch die Untersuchungen des Invariantenrings einer Einhüllenden Algebra unter einer endlichen Untergruppe der adjungierten Gruppe und seiner Darstellungen erwähnt.

Insgesamt spiegelten die Vorträge die reiche Entwicklung und die Lebendigkeit des Themas sowie das Zusammenspiel des verschiedenen Methoden wieder.

Vortragsauszüge

H. K. Andersen :

Tilting modules and fusion rules

Let G be a semi-simple algebraic group over a field of characteristic $p > 0$. The Weyl module $V(\lambda)$ with highest weight λ has simple quotient $L(\lambda)$ and sits inside the indecomposable tilting module $P(\lambda)$ with highest weight λ. We proved that knowledge of the characters of the $P(\lambda)$'s gives the irreducible characters. It will also give the irreducible characters of the symmetric group (by taking G of type A, see Mathieu's talk). Now, knowing the tilting characters is equivalent to knowing the fusion rules

$$P(\lambda) \otimes P(\mu) = \sum_{\nu}^{\oplus} P(\nu)^{c^{\nu}_{\lambda,\mu}}.$$

If λ is in the bottom alcove C then $P(\lambda) = V(\lambda)$ and we have ([Georgiev-Mathieu] or [Andersen-Paradowski])

$$\lambda, \mu, \nu \in C \Longrightarrow c^{\nu}_{\lambda,\nu} = \sum_{\substack{w \in W_{\text{aff}} \\ w \cdot \nu \text{ dominant}}} (-1)^{\ell(w)} [V(\lambda) \otimes V(\mu) : V(\nu)]$$

All the above results have analogues for quantum groups at roots of 1.

M. Van Den Bergh :

Differential operators and invariant theory

Let $R = k \oplus R_1 \oplus \ldots$ be a graded ring, char $k = p > 0$. We say that R has FFRT if there exists a finite number of indecomposable modules $M_1, \ldots, M_n$ such that for any ℓ, ${}_{F^\ell}R$ can be written as a direct sum of copies of these modules. There ${}_{F^\ell}R$ is R, twisted by the ℓ'th power of the Frobenius. FFRT is preserved under taking direct summands. The main theorem is that if R has FFRT, is F-split and D-simple then $D(R)$ is simple. $D(R)$ is the ring of differential operators of R. This shows that if R is a direct summand of a polynomial ring then $D(R)$ is simple.

P. Caldero :

Finite group action on the enveloping algebra of a semisimple Lie algebra

This is joint work with Gadi Perets. Let G be a simply connected semisimple group, U the enveloping algebra of the Lie algebra of G, and H a finite subgroup of G. Then, H acts on U by the adjoint action. The study of the skew group ring $H * U$ enables us, by a Morita context, to describe the finite dimensional irreducible modules of U^H, as well as its finite codimensional primitive ideals. Another proof for the KRAFT-SMALL decomposition theorem is given. We have an interpretation of the multiplicity of an irreducible representation of H in an irreducible representation of G, as the dimension of an irreducible U^H-module.

By a result of Howe, we prove that we can recover the structure of $\mathbb{C}[H_{ad}]$ from the representation theory of U^H. When $G = SL_2$, we calculate, with help of a Molien serie, the number of U^H-modules of given dimension. A classification permits us to recover the group H from the algebra U^H in this case.

J.Y. Charbonnel :

On the orbit method for solvable Lie algebras

Let $\mathfrak{g}$ be a solvable Lie algebra over an algebraically closed field k of characteristic zero. We denote by $\widehat{A}(\mathfrak{g})$ the completion $\widehat{S}(\mathfrak{g}^*) \otimes_{S(\mathfrak{g}^*)} A(\mathfrak{g})$ of the Weyl algebra $A(\mathfrak{g})$ on $\mathfrak{g}$.

There are natural maps $\widetilde{L}_{\mathfrak{g}}$ and $\widetilde{R}_{\mathfrak{g}}$ from the enveloping algebra $U(\mathfrak{g})$ to $\widehat{A}(\mathfrak{g})$. These maps are injective, $\widetilde{L}_{\mathfrak{g}}$ is a homomorphism and $\widetilde{R}_{\mathfrak{g}}$ is an antihomomorphism. If Q is a $\mathfrak{g}$-invariant prime ideal in $S(\mathfrak{g})$, there is an interesting left ideal L_Q in $A(\mathfrak{g}^*)$. When

$k = \mathbb{C}$, L_Q is the set of differential operators which annihilate invariant distributions supported by orbits of maximal dimension in the nullvariety $\mathcal{V}(Q)$ of Q.

Let $\lambda_\mathfrak{g}$ the isomorphism $A(\mathfrak{g}^*) \xrightarrow{\sim} A(\mathfrak{g})$ such that $\lambda_\mathfrak{g}(v) = v$ and $\lambda_\mathfrak{g}(v') = -v'$ for v in $\mathfrak{g}$ and v' in $\mathfrak{g}^*$.

Theorem. Let $\Lambda_\mathfrak{g}(Q)$ be the left ideal in $\widehat{A}(\mathfrak{g})$, generated by $\lambda_\mathfrak{g}(L_Q)$. Then there exists a $\mathfrak{g}$-invariant, invertible element p in $\widehat{S}(\mathfrak{g}^*)$, such that $\widetilde{L}_\mathfrak{g}^{-1}[p\Lambda_\mathfrak{g}(Q)p^{-1}] = \widetilde{R}_\mathfrak{g}[p\Lambda_\mathfrak{g}(Q)p^{-1}]$ is the image of Q by the Dixmier's map.

The element p comes from the theory of characters for solvable Lie groups. This result was suggested by Dixmier in 1978.

I. Cherednik :

What Young did not manage to complete

The resolutions of the "semi-simple" irreducible representations of affine Hecke algebras for GL_N in terms of induced representations will be constructed, which also gives the corresponding resolutions of finite dimensional representations of $U_q(\mathfrak{gl}_N)$ and p-adic representations of principal series. If is directly related to the Schur-Frobenius character formula for irreducible representations of the symmetric group and Young's approach to its proof.

G. Heckman :

Unipotent representations of reductive p-adic groups

The Plancherel formula for the affine Hecke algebra $\mathcal{H}(W,q)$ for $q > 1$ is known from the work of Macdonald and Matsumoto. The spectrum is purely continuous. For $0 < q < 1$, lower dimensional spectra have to be added whose Plancherel measures can be computed using residue calculation. Via the antilinear involution $q \mapsto q^{-1}$, $T_i \to -q^{-1}T_i$ this can be transfered to the computation of the Plancherel measures of the antispherical part of the spectrum of $\mathcal{H}(W,q)$ for $q > 1$. Using this one can verify a conjecture of Lusztig and Reeder how to fill up the L-packets of unipotent representations (the missing ones in the Kazhdan-Lusztig classification) of $G(\mathbb{Q}_p)$

M.P. Holland :

K-theory of twisted differential operators on flag varieties

This talk represents joint work with Patrick Polo. Let $\mathfrak{g}$ be a semisimple Lie algebra over k, an algebraically closed field of characteristic zero and let $\mathfrak{h} \subset \mathfrak{b}$ be a Cartan subalgebra inside a Borel subalgebra of $\mathfrak{g}$. Let U be the enveloping algebra of $\mathfrak{g}$. For $\mu \in \mathfrak{h}^*$ let $M(\mu)$ denote the corresponding Verma module and let $U_\mu = U/\operatorname{Ann} M(\mu)$. Let W be the Weyl group and let $W_\mu^0 = \operatorname{Stab}_W(\mu)$.

Then we prove that $K_0(U_\mu)$ is free of rank $|W/W_\mu^0|$ affirming a conjecture of Tim Hodges.

F. Knop :

Generalized Capelli identities

The classical Capelli identity is an equality of two differential operators on the space of matrices, one coming from the enveloping algebra of GL_n the other defined by invariant theoretic means. We describe how to obtain similar identities for any representation V of a reductive group G such that the coordinate ring $\mathbb{C}[V]$ is multiplicity free as G-module. In particular, we define a basis of the ring of invariant differential operators (Capelli operators) and characterize their eigenvalues in terms of certain polynomials P_λ (joint work with S. Sahi). In the most important cases, I am able to characterize P_λ also as eigenvector of commuting difference operators.

S. König :

An algorithm for computing decomposition numbers of general linear groups in defining characteristic

Computing decomposition numbers of general linear groups (over infinite fields, in defining characteristic) is equivalent to computing decomposition numbers of (or formal characters of simple modules over) Schur algebras. An algorithm is described which does this job for any given Schur algebra, say A. It is based on an explicit triangular decomposition

$$A \simeq C \otimes_S C^{op}$$

and on a bilinear form (for each primitive idempotent $e_i \in S$)

$$e_iC^{op} \times Ce_i \xrightarrow{mult} e_iC \otimes_S C^{op}e_i \xrightarrow{proj} e_i\left(\frac{C \otimes C^{op}}{C \otimes \mathrm{rad}C^{op}}\right)e_i = k.$$

B. Kostant :

Harmonics and an equality of generalized exponents

Let $\mathfrak{h}$ be a cartan subalgebra of a complex simple Lie algebra $\mathfrak{g}$. Let H and G be corresponding Lie groups and let W be the Weyl group of $(\mathfrak{h}, \mathfrak{g})$. Let $\mathcal{P}(\mathfrak{h})$ and $\mathcal{P}(\mathfrak{g})$ be the algebras of polynomial functions on $\mathfrak{h}$ and $\mathfrak{g}$ respectively so that $\mathcal{P}(\mathfrak{h})$ is a W-module and $\mathcal{P}(\mathfrak{g})$ is a G-module.

Let $D \subseteq \mathfrak{h}^*$ denote the monoid of dominant integral weights in the root lattice and for each $\lambda \in D$ let V_λ be the corresponding irreducible G-module with highest weight λ. The 0-weight space V_λ^H is a W-module.

Proposition. Restriction defines an injective map

$$(*) \qquad \mathrm{Hom}_G(V_\lambda, \mathcal{P}(\mathfrak{g})) \to \mathrm{Hom}_W(V_\lambda^H, \mathcal{P}(\mathfrak{h}))$$

For any irreducible representation τ of W let d_τ be the dimension of τ and let $p_1^\tau, \ldots, p_{d_\tau}^\tau$ be the fake degrees of τ and for any $\lambda \in D$ let $m_1(\lambda), \ldots, m_{\ell(\lambda)}(\lambda)$ be the generalized exponents of V_λ.

Theorem. If V_λ^H is an irreducible W-module τ and if twice a root is not a weight of V_λ then the fake degrees p_i^τ are the same as the generalized exponents $m_i(\lambda)$. Also $(*)$ is an isomorphism.

G. Letzter :

Subalgebras which appear in quantum Iwasawa decompositions

The well-studied pairing of a semisimple complex Lie algebra and a subalgebra of fixed elements under an involution is extended to quantized enveloping algebras. The choice of subalgebra is already more delicate in the quantum case because the associative fixed rings of such enveloping algebras under an arbitrary involution tends to be immanageably large. However, for an involution θ induced by the negative of a diagram automorphism, one can find a subalgebra B_θ which is "practically" invariant and "classical" after specialization. Moreover, such subalgebras B_θ can be used to form an Iwasawa-type tensor product decomposition. We also discuss their role in a theory of Harish Chandra modules.

W. McGovern :

Cells and blocks of Harish-Chandra modules for real classical groups

Let G be a real reductive Lie group with complexified Lie algebra $\mathfrak{g}$, maximal compact subgroup K, and complex Weyl group W. It is well known that the Grothendieck group $K_0(\mathfrak{g}, K)$ of Harish-Chandra modules of finite length carries a natural W-module structure and moreover breaks up into cells that are W-stable. We compute the module structure of these cells for many groups G of classical type.

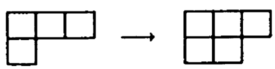
O. Mathieu :

On modular representations of the symmetric groups

For each p-regular Young diagram $\underline{m}$, denote by $E_p(\underline{m})$ the associated simple representation of the symmetric group Σ_N over $\mathbb{F}_p$ (p : prime number, N : total number of boxes in $\underline{m}$). Set : $\mathcal{Y}_\ell = \{\underline{m} : m_1 \geq \cdots \geq m_e,\ m_1 - m_e \leq p - \ell\}$.

Theorem. $\dim E_p(\underline{m})$ = number of paths from $\emptyset$ to $\underline{m}$ entirely contained in $\mathcal{Y}_\ell$.

Here, paths are well defined because $\mathcal{Y}_\ell$ has a structure of oriented graph; e.g. there is an arrow



M. Nazarov :

Twisted Yangians and Capelli Identities

For every classical Lie algebra $\mathfrak{a}$ we construct a family of maximal commutative subalgebras in the universal enveloping $U(\mathfrak{a})$.

These subalgebras correspond to the maximal involutive subalgebras in the Poisson algebra $S(\mathfrak{a})$ obtained by the shift of argument method (Kostant, 1979).

Our commutative subalgebras appear as the images of maximal commutative subalgebras in the Yangian $Y(\mathfrak{a})$ with respect to the evaluation homomorphism $Y(\mathfrak{a}) \to U(\mathfrak{a})$.

We apply our construction to the classical invariant theory : we obtain a generalization of the Capelli identities (1887,1890) for $\mathfrak{a} = \mathfrak{gl}_N$ to the other classical Lie algebras $\mathfrak{so}_N$ and $\mathfrak{sp}_N$.

E. Opdam :

Symmetries for fake degrees and rationality of Hecke algebra characters

If W is a Weyl group and $\tau \in \widehat{W}$ one defines the fake degree $f_\tau(T)$ of τ by $f_\tau(T) = \sum_{i=1}^{d_\tau} T^{p_\tau^i}$, where the p_i^τ are the embedding degrees of τ in the coinvariant algebra and d_τ is the degree of τ.

By checking all the cases Beynon and Lusztig discovered (in 1978) that $f_{\tau\otimes det}(T) = T^{N_\tau} f_{\tau'}(T)$, where τ' is the conjugate representation of τ (obtained by replacing $\sqrt{q}$ by $-\sqrt{q}$ in the representation corresponding to τ of the Hecke algebra $H_w(q)$). In this talk we present a conceptual proof of this theorem, and we extend the theorem to formulas for $f_{\tau\otimes\epsilon}(T)$ (ϵ arbitrary linear character) and for the case of a complex reflection group. The proof is based on the monodromy representation of equations based on Dunkl operators.

bsP. Polo :

Primitive factors of enveloping algebras of semisimple Lie algebras : Morita equivalence and isomorphism results

Let $\mathfrak{g}$ be a complex semisimple Lie algebra, $U = U(\mathfrak{g})$ its enveloping algebra, and Z the centre of U. For every central character χ, let $U_\chi = U/(\mathrm{Ker}\chi)U$. Using the theory of primitive ideals, we show that if χ is regular integral then the Dynkin diagram of $\mathfrak{g}$ is uniquely determined by the algebra structure of U_χ, and is in fact a Morita invariant. From this we deduce that if $\mathfrak{r}, \mathfrak{r}'$ are reductive complex Lie algebras such that $U(\mathfrak{r})$ and $U(\mathfrak{r}')$ are Morita equivalent, then $\mathfrak{r} \simeq \mathfrak{r}'$. We also prove that if $U_\chi \simeq U_{\chi'}$ and χ is regular integral then χ and χ' are conjugate under the group of diagram automorphisms (we conjecture that the assumption on χ is unnecessary). A corollary of this is that every automorphism of U acts on Z as a diagram automorphism. Finally, we conjecture that in the first mentioned result the integrality assumption can be relaxed. Using a result of Soergel, we make a step towards this conjecture by proving that for any regular χ the Weyl group of $\mathfrak{g}$ can be extracted from the algebra structure of U_χ, and is in fact a Morita invariant.

A. Pressley :

Quantum affine algebras and Dorey's condition

Let $\mathfrak{g}$ be a finite-dimensional complex simple Lie algebra, $\{\alpha_1, \ldots, \alpha_n\}$ = the set of simple roots, $s_1, \ldots, s_n$ = simple reflections, $\gamma = s_1 s_2 \ldots s_n$, $\phi_i = s_n s_{n-1} \ldots s_{i+1}(\alpha_i)$, $R_i = \gamma$-orbit of ϕ_i. Following Patrick Dorey's work in affine Toda field theories, say $i_1, i_2, \ldots i_p$ satisfy (D_p) $(p \geq 2)$ if $0 \in R_{i_1} + R_{i_2} + \cdots + R_{i_p}$.

Let $U_\varepsilon(\hat{\mathfrak{g}})$ be the quantum affine algebra ($\varepsilon \in \mathbb{C}^*$ not a root of 1). The finite-dimensional irreducible representations of $U_\varepsilon(\hat{\mathfrak{g}})$ are parametrized by n-tuples $(P_1, P_2, \ldots, P_n)$, where $P_i \in \mathbb{C}[u]$, $P(0) = 1$.

If $a \in \mathbb{C}^*$, let $V(\lambda_i, a)$ be the representation corresponding to the n-tuple $(1, \ldots, 1, 1 - a^{-1}u, 1, \ldots, 1)$, with $1 - a^{-1}u$ in the i'th place.

Theorem. (V. Chari and A. Pressley). Let $\mathfrak{g}$ be of type A or D. Then, i_1, i_2, i_3 satisfy (D_3) iff $\mathrm{Hom}_{U_\varepsilon(\hat{\mathfrak{g}})}(V(\lambda_{i_1}, a_1) \otimes V(\lambda_{i_2}, a_2) \otimes V(\lambda_{i_3}, a_3), \mathbb{C}) \neq 0$ for some $a_1, a_2, a_3 \in \mathbb{C}^*$.

If $\mathfrak{g}$ is of type B or C, there is a similar result involving twisted Coxeter elements.

M. Reineke :

On the coloured graph structure of the canonical basis

For a quantized enveloping algebra of finite type, we study the crystal graph $\mathcal{G}$ of its positive part and the crystal graphs $\mathcal{G}_1$ of its finite dimensional irreducible representations. An explicit description is given

a) of the graph $\mathcal{G}$ for all types except E_8,

b) of the embedding $\mathcal{G}_1 \subset \mathcal{G}$ for all classical types.

This is done using Ringel's Hall algebra approach which provides a parametrization of Lusztig's canonical basis by isomorphism classes of finite dimensional representations of quivers. The description is then given in terms of certain posets and functions on them which are defined in terms of the category of representations of the quiver.

W. Rossmann :

Path integrals and representations of semisimple groups

Feynman's quantization procedure by his path integral leads to an expression for the kernels of the representation operators of the type

$$\text{(I)} \qquad K(\exp X, z'', z') = \int_{z.} \exp\left[\int_{z'}^{z''} \{\alpha(\dot{z}_t) - H_X(z_t)\} dt\right] \Pi \varpi^n(dz.);$$

the "integral" is over all paths $z. = \{z_t\}$ from z' to z''. This formula should be compared to Kirillov's character formula

$$\text{(II)} \qquad \mathrm{tr}\, K(\exp X) = \int_z \exp[-H_X(z)]\, j^{-1/2}(X) \varpi^n(dz).$$

The form ϖ in (I) and (II) is a symplectic form on a real manifold, the phase space, from which the representation is constructed by a method of quantization, possibly in cohomology; the function H_X is the Hamiltonian for X, α is a 1-form satisfying $d\alpha = \varpi$, and j is a universal function.

Physicists seem to believe that the formula (I), or some version of it, holds very generally. It is known that a version of formula (II) holds for all irreducible representations of semisimple Lie groups. Yet, as far as I know, there is no satisfactory explanation of the relation of (I) and (II), and I cannot offer one in any generality. In some examples, however, the relation can be worked out, and this is what I want to discuss.

S. Ryom-Hansen :

A q-analogue of Kempf's vanishing theorem

The modular Kempf's vanishing theorem is the statement that the sheaf cohomology of the line bundles associated to dominant weights vanish.

We consider the induction functor H_q^0 introduced by Andersen, Wen and Polo in the setup of quantum groups and show that it also satisfies a Kempf vanishing theorem. A main ingredient is the crystal basis of Kashiwara.

Y. Sanderson :

Real characters of Demazure modules for rank two affine Lie algebras

Using Littelmann's path model, we obtain explicit, closed form, combinatorial expressions for a certain specialization of the characters of Demazure modules for rank two affine Lie algebras ($A_1^{(1)}$ and $A_2^{(2)}$). This specialization is obtained by collecting together all weight spaces whose weight differs only by an imaginary root.

V. Serganova :

Kazhdan-Lusztig theory for Lie super algebras

We study the category of finite dimensional representation of the Lie superalgebra $\mathfrak{gl}(m|n)$ by category $\mathcal{O}$ methods. Kazhdan-Lusztig polynomials in this case are defined and evaluated at $q = -1$. As a corollary we obtain the character formula for any finite dimensional irreducible $\mathfrak{gl}(m|n)$ module and the multiplicities of irreducible modules in Kac modules (super counterpart of Verma module).

W. Soergel :

Determination of some Hochschild cohomology, and $\mathfrak{n}$-cohomology of limits of discrete series

By the Hochschild cohomology ring $\mathrm{HH}^\bullet(A)$ of a k-algebra A I mean the ring of selfextensions of A considered as an $A \otimes_k A^\circ$-module. Let $\mathfrak{g}$ be a semisimple Lie algebra, $U \supset Z$ the enveloping algebra of $\mathfrak{g}$ with its center, $\chi \subset Z$ a regular maximal ideal. In the first part of the talk, I prove

Theorem. The Hochschild cohomology ring $HH^\bullet(U/U\chi)$ is isomorphic to the coinvariant algebra of the Weyl group.

In the second part of the talk I explain how to compute the $\mathfrak{n}$-cohomology of a limit of discrete series, when $\mathfrak{n}$ is the nilradical of a Borel containing a compact Cartan. I show how this $\mathfrak{n}$-cohomology can be identified with the cohomology of a very explicit "combinatorial" complex of finite dimensional vector spaces with bases indexed by suitable subsets of the Weyl group, the differentials being given in these bases by explicit matrices of zeroes, ones and minus ones.

T. Springer :

A description of B-orbits on symmetric varieties

Let G be a connected reductive group over $\mathbb{C}$, and let θ be an involutorial automorphism of G, with fixed point group K. The quotient $X = G/K$ is a symmetric variety. A Borel group B acts on X with finitely many orbits. The set V of these orbits has several features, for example it carries an action of the Weyl group W.

In the talk a combinatorial description is given of the set of all possible orbits, for fixed G and varying θ. For simplicity assume G to be adjoint. Let R be the root system, Q the root lattice and $P^\vee$ its dual. The description involves pairs (θ, ξ), where θ is an involutorial automorphism of R, and $\xi \in \mathrm{Ker}(\theta - 1, P^\vee)/\mathrm{Im}(\theta + 1, P^\vee)$. It is shown how to pick out the various sets V. The features of V (e.g. the Weyl group action) can be described in terms of the parameters (θ, ξ).

E. Vasserot :

Quantum groups and Langlands correspondence on an algebraic surface

A Langlands-type correspondence on algebraic surfaces is conjectured. In this correspondence some toroidal extensions of affine quantum groups of type $A^{(1)}$, $D^{(1)}$, $E^{(1)}$, should play the role of the affine Hecke algebra in the standard correspondence over an algebraic curve. We show how these algebras act by elementary modifications on functions on the set of isomorphism classes of vector bundles over a surface. The particular case of an elliptic surface suggests particular connections with physicists' elliptic algebras. Moreover we show that the toroidal quantum group of type A is the Weyl-Schur dual of Cherednik double affine Hecke algebras.

M. Vergne :

Kostant-Sekiguchi correspondence for nilpotent orbits

Let $\mathfrak{g}$ be a compact Lie algebra, with 2 commuting involutions. It gives rise to a quaternionic decomposition of $\mathfrak{g}$:

$$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3.$$

Let $\mathfrak{h}_j = (\mathfrak{g}_0 \oplus i\mathfrak{g}_j)$, $\mathfrak{q}_1 = \mathfrak{g}_2 \oplus i\mathfrak{g}_3$, etc... by circular permutations. The subgroup H_i of $G_{\mathbb{C}}$ of Lie algebra $\mathfrak{h}_i$ acts on $\mathfrak{q}_i$. Let $\mathcal{O}$ be a nilpotent orbit in $\mathfrak{g}_{\mathbb{C}}$. Sekiguchi showed that there are natural maps $S_{2,1}, S_{3,2}, S_{1,3}$

$$\mathcal{O} \cap \mathfrak{q}_1/H_1 \xrightarrow{S_{2,1}} \mathcal{O} \cap \mathfrak{q}_2/H_2 \xrightarrow{S_{3,2}} \mathcal{O} \cap \mathfrak{q}_3/H_3 \rightarrow \ldots$$

We show here that it follows from Kronheimer's description of $\mathcal{O}$ by instantons that in fact there is a G_0-invariant diffeomorphism of $\theta \cap \mathfrak{g}_1$ and $\theta \cap \mathfrak{g}_2$ which carries H_1-orbits to H_2-orbits.

In particular, if $\mathfrak{s} = \mathfrak{k} \oplus \mathfrak{p}$ is a real semisimple Lie algebra, a real nilpotent orbit is diffeomorphic to the nilpotent orbit of $K_{\mathbb{C}}$ in $\mathfrak{p}_{\mathbb{C}}$ associated to $\mathcal{O}$ by Kostant correspondence.

A. Zelevinsky :

Parametrizing canonical bases and totally positive matrices

Let N be the maximal unipotent subgroup of a semisimple group of simply-laced type; let $m = \dim N$. Recently, Lusztig discovered a remarkable parallelism between

(1) labellings (by m-triples of non-negative integers) of the canonical basis B of the quantum group corresponding to N;

(2) parametrizations of the variety $N_{>0}$ of totally positive elements in N by m-triples of positive real numbers.

In each case, there is a natural family of parametrizations, one for each reduced word of the longest element of the Weyl group.

In a joint work in progress with A. Berenstein and S. Fomin, the following problem is solved for type A_r : for any two reduced words, find an explicit formula for the transition map that relates corresponding parametrizations of B or $N_{>0}$.

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