MATHEMATISCHES FORSCHUNGS INSTITUT OBERWOLFACH

Tagungsbericht 31/95

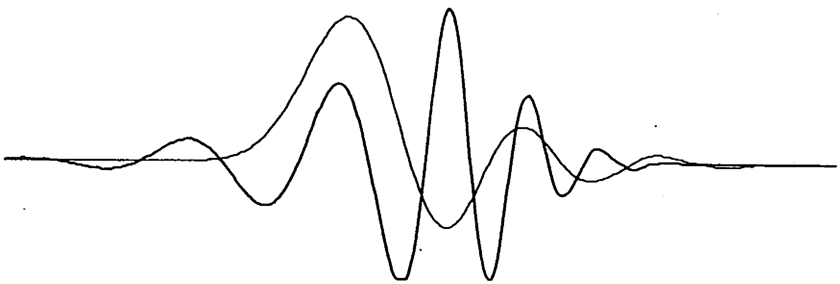
Wavelets & Signalprocessing

06.08 - 12.08.1995

This years meeting was chaired by Ingrid Daubechies (Princeton) and Alfred. K. Louis (Saarbrücken).

The purpose of this conference was to bring together researchers interested in various aspects of the theory and applications of wavelet analysis. The talks that were delivered reflected well the different facettes of this field: there have been talks on the construction of wavelets (wavelets on bounded domains, multi-dimensional wavelets, orthogonal wavelets in H^1), on regularity estimates for wavelets, on non-linear approximation properties of wavelet expansions, on the application of wavelets to PDE- and BEM- algorithms and on signal processing (image compression, fractal data analysis).

The talks have been followed by lively discussions and a useful exchange of ideas.



Reporters: Volker Dicken and Georg Zimmermann.
Potsdam, Wien: September 7, 1995.

1 Abstracts

Directional wavelets in image processing: Some new results

Jean-Pierre Antoine, *Louvain-la-Neuve/B*

The talk reviews a number of topics in the application of the Continuous Wavelet. Transform (CWT) to image processing, more precisely the exploitation of 2-D directional wavelets in the detection and analysis of oriented features. The talk is organized as follows:

(1) Generalities on the D-dimensional CWT and the different representations available for visualization in the case $D = 2$.

(2) Directional and multidirectional wavelets. Example: the Cauchy wavelet.

(3) Scale-angle resolving power, application to the discretization problem.

(4) Application #1: directional filtering.

(5) Application #2: disentangling of a wave train (with the Morlet and the Cauchy wavelet).

(6) Application #3: character recognition.

(1) J-P. Antoine, P. Carrette, R. Murenzi and B. Piette, "Image analysis with two-dimensional continuous wavelet transform", Signal Proc., *Vol. 31 (1993) 241-272*

(2) J-P. Antoine and R. Murenzi, Two-dimensional directional wavelets and the scale-angle representation, preprint UCL-IPT-95-03 (May 1995) (submitted)

Wavelet-based fractal analysis of DNA sequences

Alain Arneodo, *Pessac, France*

The fractal scaling properties of DNA sequences are analyzed using the wavelet transform. Mapping nucleotide sequences onto a "DNA walk" produces fractal landscapes that can then be studied quantitatively by applying the so-called wavelet transform modulus maxima method. This method provides a natural generalization of the classical box-counting techniques to fractal signals, the wavelets playing the role of "generalized oscillating boxes". From the scaling behavior of partition functions that are defined

from the wavelet transform modulus maxima, this method allows us to determine the singularity spectrum of the considered signal and thereby to achieve a complete multifractal analysis. Moreover, by considering analyzing wavelets that make the "wavelet transform microscpoe" blind to "patches" of different nucleotide composition that are ubiquitous to genomic sequences, we demonstrate and quantify the existence of long-range correlations in the noncoding regions. Although the fluctuations in the patchy landscape of the DNA walks reconstructed from both noncoding and (protein) coding regions are found homogeneous with Gaussian statistics, our wavelet-based analysis allows us to discriminate unambiguously between the fluctuations of the former which behave like fractional Brownian motions, from those of the latter which cannot be distinguished from uncorrelated random Brownian walks. We discuss the robustness of these results with respect to various legitimate codings of the DNA sequences. Finally, we comment about the possible understanding of the origin of the observed long-range correlations in noncoding DNA sequences in terms of the nonequilibrium dynamical processes that produce the "isochore structure of the genome". **Keywords** : wavelet transform, fractal scaling, multifractal formalism, fractional Brownian motions, DNA sequences.

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A. Arneodo, E. Bacry, P.V Graves and J.F. Muzy, Phys. Rev. Lett. **74** *(1995) 3293.*

Regular compactly supported wavelets in the Sobolev space $H^1(I\!R)$

Francoise Bastin, *Liege/B*

We study the construction of orthonormal bases of wavelets in $H^1(I\!R)$ for the usual scalar product. We construct a family of functions $\varphi^{(j)}(j \in \mathbb{Z})$ with a fixed compact support and arbitrary high regularity such that the

$$\psi_{jk}(x) = 2^{j/2}\psi^{(j)}(2^j x - k) \qquad (j, k \in \mathbb{Z})$$

are an orthonormal basis for $H^1(I\!R)$. To achieve this, we use a family of trigonometric polynomials $m_0^{(j)}(\xi)(j \in \mathbb{Z})$ of fixed degree and the multiresolution analysis procedure.

Spline-Wavelets with arbitrary knots on a bounded interval

Charles Chui, *College Station, USA*

This is a joint work with Johan M. De Villiere, but although the construction algorithms in the paper are very efficient, I choose a more intuitive way to present the results at the sacrifice of efficiency. Spline-wavelets of arbitrarily given order and arbitrary knot sequence on a bounded interval are constructed and a general scheme for orthogonal decomposition and reconstruction of Hilbert spaces is formulated. Based on this frame-work; dual wavelets are constructed, decomposition matrices are obtained, and the duality principle is established. In addition, efficient algorithms for computing both the decomposition and reconstruction matrices are developed in this paper. As is usually done in practice, the localness property and flexibility of spline functions enable the user to construct spline curves to fit randomly generated data very efficiently and accurately. With the option of moving the knots to adapt to the data set, it is feasible to construct spline-wavelets to adapt to the desirable waveforms, and to achieve certain desirable filter criteria, without depending on a "wavelet dictionary". Further research is required to develop "real-time" (efficient) algorithms for this purpose.

Multiscale Methods on bounded domaines

Albert Cohen, *Paris*

Multiresolution and Wavelets are well understood in the euclidean or periodic setting. We present a construction that allows to deal with bounded domaines $\Omega \subset \mathrm{I\!R}^d$.
The main features of MRA that are preserved in this construction are:

1. Regularity and Polynomial reproduction

2. Locality and stability of the basis

3. Fast and simple decomposition algorithms

This construction allows to characterize classical function spaces $B^s_q(L^p(\Omega))$ and yields stable extension operators

$$E : B^s_q(L^p(\Omega)) \longrightarrow B^s(L^p(\Omega))$$

that are easy to complement numerically.
This work was made jointly with W. Dahmen (Aachen) and R. DeVore (Columbia, S. Carolina)

Besov Regularity for Elliptic Boundary Value Problems

Stephan Dahlke, *Columbia/USA*

We study the regularity of solutions to boundary value problems for Laplace's equation on Lipschitz domains Ω in $\mathbb{R}^d$ and its relationship with adaptive methods for approximate solutions. The smoothness space which determine the efficiency of such nonlinear approximations in $L_p(\Omega)$ are the Besov spaces $B^\alpha_\tau(L_\tau(\Omega)), \tau = (\frac{\alpha}{d} + \frac{1}{p})^{-1}$. Thus, the regularity of the solution in this scale of Besov spaces is investigated with the aim of determining the largest α for which the solution is in $B^\alpha_\tau(L_\tau(\Omega))$.
The regularity theorems given in this talk build upon recent results of Jerrison and Kenig. The proof of the regularity theorem uses characterisations of Besov spaces by wavelet expansions.

Stable completions and some applications

Wolfgang Dahmen, *RWTH Aachen*

Suppose that S is a sequence of closed nested subspaces S_j of some Hilbert space H and that each S_j is spanned by a Riesz basis Φ_j. Given any Riesz basis Ψ_j spanning some complement W_j of S_j in S_{j+1}, it is indicated first how to parametrize then all other Riesz bases spanning some complement of S_j in S_{j+1} in terms of certain block triangular matrices. For instance, this can be exploited to raise the order of moment conditions of a given complement basis or to identify the complements induced by the differences of two successive linear projectors onto the spaces S_j. Several applications of these facts are indicated. In particular, the results can be used to construct biorthogonal Riesz bases on parametrically defined two dimensional surfaces in $\mathbf{R}^3$ in such a way that the exactness of the dual multiresolution and hence the order of moment conditions of the primal wavelets becomes as large as one wishes. These bases satisfy all requirements that are needed to realize optimal convergence and matrix compression rates in connection with the numerical treatment of a wide range of boundary integral equations including also operators of negative order.

Empirical Atomic Decomposition

David L. Donoho, *Stanford/USA*

Recently the time-frequency and time-scale communities have developed a large number of dictionaries of "atoms" $(\varphi_\gamma)_{\gamma\in\Gamma}$ which may be used to represent signals by superposition:
$f = \sum a_\gamma \varphi_\gamma$. Examples include: wavelets, overcomplete wavelets, sinosoids, overcomplete sinosoids, wavelet packets, cosine packets, chirplets, directional wavelets.
Empirical atome decomposition is a method for decomposition of noisy data into a superposition of atoms and residual:

$$f = \sum_{j=1}^{m} a_{\gamma_j} + R.$$

Assuming data $y_i = f(t_i) + oz_i, i = 1, \ldots, n, \quad z_i \sim N(0,1)$ i.i.d. it takes the form $\hat{f}^* = \arg\min \quad ||y - \Phi\alpha||_{l^2}^2 + \lambda||\alpha||_{l^0}$
where Φ is an $n * p$ matrix with columns $(\varphi_{\gamma_j}(t_i))_{i=1}^n$, and $\lambda = const, \cdot 2\log(p), \sigma^2$. Let $\mathcal{R}^0(f) = \arg\min_\alpha ||f - \Phi\alpha||_{l^2}^2 + \lambda||\alpha||_{l^0}$. Then $E||f - \Phi\alpha^*||_{l^2}^2 \leq const_2 \mathcal{R}^0(f) \ \forall f$, where $const_2$ is a universal constant depending on $const_1$.
This inequality can be used to show that there are atomic devomposition estimators achieving rates of convergence on on classes of objects F which are faster than the rates of convergence in a wavelet basis or other orthogonal basis, and which achieve the optimal rate among all nonlinear estimates (modulo log-terms). Hence wavelet bases and other fixed bases can be outperformed by atomic decompositions.
Examples where this is true:

1. Horizons: $F = \{f : f(x,y) = 1\{y \geq \Theta(x)\}$ where $\Theta \in$ Hölder $^\sigma(C)$.
2. Starshaped sets: $F = \{$ indicators of starshaped sets with radius function $\rho(\Theta) \in$

Hölder$^\sigma(C)\}$. In both 1 & 2, we can use the triangle dictionary. Time-frequency examples were also considered. Unfortunately $\hat{f}^*$ is NP-hard to compute. We give examples of a method for solving its convex relaxation $\arg\min_\alpha \; ||y-\Phi\alpha||_{l^2}^2+\lambda||\alpha||_{l^1}$. It performs well in examples.

Wavelet Analysis of Elastic Wave Nondestructive Evaluation and Newton inverse Optics

Brian DeFacio, *Univ. Missouri-Columbia*

The Mallat multiresolution for orthonormal wavelets is applied to the analysis of ultrasonic backscatter of flaurs in stainless steel and to the glancing angle scattering of thermal neutrons from a metallic NiC-Ti sandwich and a cell membrane bilipid crystal. Real, noisy, destorted time domaine short transient pulses are denoised by 29 wavelet families and then inverted using an optimal Wiener-Tikhonov filter: In blind tests sphere radius estimates whithin 5 % were obtained. The neutron inverse optics project detects the edges from the large wavelet coefficients and determined the width of the interdiffusion to be approximately 1 $\mu = 10^{-6}$ m for NiC on Ti and 0.6 μ fot Ti on NiC. Hence the diffusion constant differ depending upon which material is being sputtered onto the other. The wavelet multiresulution improves both reconstructions by denoising.

Are wavelets, and related multiscale techniques, useful for turbulence?

Marie Farge, *Paris/F*

One of the main factor limiting our understanding of turbulent flows is that we have not yet identified the structures responsible for its chaotic and therefore unpredictible behaviour. Based on laboratory and numerical experiments, we think that vortices (or coherent structures) are these elementary objects, from which we may be ask to construct a new statisfical mechanics and define equations appropriate for computing turbulent flows.

The complementary simultaneous space and scale information provided by the wavelet transform makes it an appropriate tool for identifying and analyzing coherent structures in turbulence. We have and the wavelet transform to segment the vorticity field into coherent and incoherent components as the first stage in a conditional sampling algorithm. In particular we have shown, that the wavelet packet representation, associated with maximums entropy statistical method and nonlinear filtering, extracts the coherent structures in a computationally efficient way. There is a non-separability of turbulent motions in the Fourier representation,which a wavelet representation seems able to provide such separability. We have experimental reasons to expect a gap in wavelet coordinates between organized structures, that should be explicitly computed, and random background flow, that could be modulled by an appropriate stochastic process having similar statistics. The difficulty is that both components are multiscale. This composition may be the basis of a new way of numerically simulating turbulent flows, and possibly other kind of intermittend phenomena.

In conclusion, we think that the theory of fully developed turbulence, namely the limit of large Reynolds numbers (nonlinear advection >>linear dissipation), is in a prescientific phase because we do not yet have an equation, or a set of equations, that could be used to efficiently compute turbulent flows. The Navier-Stokes equations are not the right ones for turbulence because their computational complexity becomes intractable for large Reynolds numbers. However, in this limit it should be possible to obtain averaged quantities, or the model of statistical mechanics, and find the corresponding transport equations to comput the evolution of these new quantities, which would be the appropriate variable to describe turbulence. But it is easier to define the appropriate parameter, to go from Boltzmann or Navier-Stokes than from Navier-Stokes to turbulence equations. In the first case only a linear avaraging, namely coarse pairing, is needed while in the second case we have to find a nonlinear procedure. Wavelets are good tools to explore this conditional averaging and to find an atomic decomposition appropriate to describe the turbulent flow dynamics.

Gabor analysis over lca. groups

Hans Georg Feichtinger, *Wien/A*

It is explained, that the setting of locally compact groups is most appropriate for a unified discussion of discrete/continuous, periodic/infinite, 1D or higher dimensional Gabor analysis. Let G be a lca. group, $T_x f(z) = f(z-x)$, and $X \in \hat{G} : X(x+y) = X(x)X(y)$, such as $X_s(x) = e^{2\pi i x s}$ on $G = (R,+)$. For a nice function g on G and (an additive) subgroup $H \triangle G x \hat{G}$ we consider Gabor families $\{\pi(h)g\}_{h\in H}$, where $\pi(x,X) = X \cdot Txg$. If it is a frame, there exists some γ such that $\{g_h\} = \{\pi(h)\gamma\}$ defines the dual frame. There is Janssen's representation for the frame operator $Sf := \sum \langle f, g_h \rangle g_h$, stating that $S = \sum_{h' \in H^\circ} \langle g, \pi(h')g \rangle \pi(h')$, where $H^\circ = \{h' : [\pi(h)\pi(h')] \equiv 0, \forall h \in H\}$. In the discrete case H° is small, therefore the CG-method for solving $S\gamma = g$ will solve for γ after a small number iterations. Feit Gröchenig showed (1994/95): If $g \in S_0(\mathbb{R})$ generates a Gabor frame $\{\pi(h)g\}_{h \in H}$, then the frame operator is invertible on $L^2(\mathbb{R})$, but also on $S^\circ(\mathbb{R})$, or equivalently $\gamma = S'g \in S_0(\mathbb{R})$.

Sharply localized time-frequency/scale representations via "reassignment"

Patric Flandrin, *Lyon/F*

Quadratic signal energy representations based on linear transforms such as the spectrogram or the scalogram, based respectively on the short-time fourier transform and the wavelet transform - are known to be poorly localized on the time-frequency/scale plane. In the use of monocomponent frequency modulated signals, better distributions - such as the Wigner-Ville distribution and its affine generalizations - are known to exist, but their use is limited in the case of multicomponent signals, due to the occurence of spurious cross-terms. A key problem in time-frequency analysis is therefore to find an approach conciling the high resolution properties of Wigner-type methods with the low level of cross-terms observed in spectrograms/ scalograms. A possible solution to this problem is proposed, based on an idea of "reassignment" which consists in moving the computed value of a low resolution distribution to a different point of the plane which is more representative of the local energy distribution. The idea of reassignment had been put forward Kodera, Gendin and de Villeday in the mid *70's, but its revisiting in the light of "modern" time-frequency analysis allows for many generalizations, as well as for efficient algorithmus. The basic ingredient of time-frequency reassignment is the simulthaneous computation of 3 different transforms based on 3 related windows/wavelets (or, more generally, time-frequency- smoothing operators). In the case of noisy signals, a further improvement can be obtained by averaging in a suitable way representations obtained with different windows/wavelets since the coherent part of the observation (signal) can be proved to be reassigned almost indedendent of the length of the window, wheras the incoherent part (noise) is not.

Regularity of Multivariate Scaling Equations

Karlheinz Gröchenig, *Wien/A*

We give a characterization of the smoothness of compactly supported refinable functions $\phi, \phi(x) = \sum_{k \in Z^d} c_k |\det A| \phi(Ax - k)$, where A is a $d \times d$-dilation matrix. The known charactizations of regularity in dimension 1 are all based on the fact that polynomials can be factorized. As this is impossible for (trigonometric) polynomials in several variables, the regularity estimates in higher dimension seem to be difficult to come by. The only results in d-D, due to A. Cohen and I. Daubechies, and to L. Villemoes, deal with very special cases only and do not allow to treat general dilation equations.

In this talk we show to avoid the difficulties due to the absence of factorization in higher dimensions. We derive estimates for the Sobolev regularity of refinable functions. If one accounts for the inhomogenity of the dilation and uses anisotropic Sobolev spaces, a complete characterization of the regularity can be achieved.

This is joint work with Albert Cohen, Paris VI.

Singular Values Of Time-frequency Localization Operators

Christopher Heil, *Georgia Institute of Technology, Atlanta, Georgia 30332-0160*

We will discuss the singular value decomposition of a class of time-frequency localization operators, defined in terms of a Cohen's class joint time-frequency distribution and a region in the time-frequency plane. Effectively, the singular vectors of these operators are ones for which the chosen time-frequency distribution is most concentrated in the given region of the plane. Signals that are extrema for such localization operators define a suitable subspace to project onto for purposes of time-frequency filtering. As noted by Flandrin, each such localization operator is represented under the Weyl correspondence by a symbol function. For the special case of the Wigner distribution, the symbol is the characteristic function of the time-frequency region. The time-frequency localization operators of Daubechies correspond to using the spectrogram as the distribution and localizing on a circular or elliptical disk. We will present a general result which prescribes the asyptotic decay of the singular values of such time-frequency localization operators in terms of the decay of the symbol and its Fourier transform. This result is applicable in principle to any of the Cohen's class of time-frequency distributions, and to any bounded region. We discuss the motivation and implications of this result, and the philosophy of proof, which involves the construction of an approximating operator through the use of Gabor frames.

C. Heil, J. Ramanathan, and P. Topiwala, Asymptotic singular value decay of time-frequency localization operators, *in Wavelet Applications in Signal and Image Processing II, A.F. Laine and M.A. Unser, eds., SPIE Proceedings Series Vol. 2303, Society of Photo-Optical Industrial Engineers, Bellingham, WA (1994), pp. 15–24. Available by anonymous ftp: address ftp.math.gatech.edu*
directory: /pub/users/heil filename: spie94.ps
Preprint in preparation: Singular values of compact pseudodifferential operators, C. Heil, J. Ramanathan, and P. Topiwala.
Should be finished soon, contact me at heil@math.gatech.edu. Those papers are in connection with the main topic of my talk, which was on time-frequency localization operators. I also talked briefly about the Nyquist Phenomenon for Gabor Systems. There is a survey paper available on that:
J. Benedetto, C. Heil, and D. Walnut, Differentiation and the Balian–Low theorem, *J. Fourier Anal. Appl., Vol. 1, No. 4, to appear in 1995.*
This paper is available by anonymous ftp at the same site as above, under the filename blt.ps

A functional calculus using wavelet transforms

Mathias Holschneider, *Marseille*

Let $s \in S_n$, the class of Schwartz and suppon that supp $\hat{s} \subset [-\infty, 0]$. Let A be a closed operator acting in some Banach space B. Suppose that its resolvent satisfies the following growth condtions in $z > 0$ $\quad (z = b + ia)$

$$||R(A;b + ia)|| \leq \mathcal{O}(1)(a + 1/a)^M(1 + |b|)^M$$

for some $M > 0$. Then the following expression exists in a Bochner sense:

$$s(A) = \frac{1}{Cg} \int_0^{+\infty} \frac{da}{a} \int_{-\infty}^{+\infty} db \;\; W_g s(b,a) \;\; R(A;\, b + ia)$$

if $g \in s_0$, the class of Schwartz with all moments vanishing. Here $W_g s(b,a)$ is the wavelet transform of s with respect to g. That can be shown that $s(A)$ does not depend on g. For A self adjoint this functional calculus coincides with the standard functional calculus.

Multifractal functions

Stephane Jaffard, *Noisy Le Grand/F*

A function $f: \mathbb{R}^n \to \mathbb{R}$ has an α-singularity at x_0 if there exists a polynomial P such that

$$|f(x) - P(x - x_0)| \sim |x - x_0|^d$$

Multifractal functions have α-singularities located on fractal sets E_α $(d \in (-n, \infty])$. The spectrum of singularities is $d(\alpha)$ = Hausdorff dimension of E_α. One is interested in determining spectra either of "classical" functions of Mathematics (z.g. Riemann's $f\ t\ \sum \frac{1}{n^2} \sin n^2 x^r$, a Polya's "peano-type" function, ...) or of determining spectra of signals (of particular interest is turbulence which is believed to be multifractal). In both uses the wavelet transform is a useful tool because Hölder singularities are characterized by decay conditions of the wavelet transform.
The Multifractal Formalism asserts that $d(\alpha)$ can be obtained as the legendre transform of the structure function $\eta(p)$.η is defined by :

$$\int |f(x+h) - f(x)|^p dx \sim h^{\eta(p)}$$

This presupposes that the only singularities are "cusp-like" (i.e. something like $|x - x_0|^\alpha$).
In a joint work with Arneodo, Bacry and Muzy we extended the Multifractal Formalism in order to take into account oscillating singularities like (a) $|x - x_0|^\alpha \sin \frac{1}{|x - x_o|^\beta}$. This yields a "two - parameter" spectrum $d(\alpha, \beta)$ which gives the dimension of points where a behavior simular to (a) appears.

Multilevel Preconditioning — Appending Boundary Conditions by Lagrange Multipliers

Angela Kunoth, *Weierstrass-Institute for Applied Analysis and Stochastics Berlin, Germany*
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Solving an elliptic partial differential equation by means of a Galerkin method, a standard technique is to include essential boundary conditions into the solution and approximation spaces and take finite elements as basis functions. However, it may be very difficult to adapt these or different basis functions like wavelets to the boundary conditions. Thus, appending the conditions by means of Lagrange multipliers is a more promising method although this approach leads to saddle point problems where the system matrix is no longer positive definite.

This talk is concerned with multilevel techniques for preconditioning linear systems that arise from Galerkin methods for such saddle point problems to accelerate the convergence of iterative methods. The construction of the preconditioners is based on the characterization of Sobolev spaces on the underlying domain and its boundary in terms of weighted sequence norms related to corresponding multilevel expansions. The results indicate how various ingredients of a typical multilevel framework affect the growth rate of condition numbers and, in particular, how to realize condition numbers that are uniformly bounded independent of the refinement level. The results are applied to a general multivariate framework of refinable shift-invariant spaces inducing various types of wavelets.

A. Kunoth, Multilevel Preconditioning — Appending Boundary Conditions by Lagrange Multipliers, *Advances in Computational Mathematics, Vol. 4, No.I–II, Special Issue on Multiscale Methods, 1995, 145-170.*

Wavelet accelerated iteration schemes for inverse problems

Peter Maaß, *Universität Potsdam*

Discretizations of inverse problems $Af = g$ typically lead to large linear sytems with full matrices. These systems have to be solved by iteration methods, e.g. $f^{k+1} = f^k + \beta A^*(g - Af^k)$, where most of the computational effort is needed to evaluate A^*Af^k.

During the first iterations these schemes reconstruct the coarse details of f,smaller details are added during the later iterations. This can be complemented by iterating with differently discretized operators A_k, where the discretization might change in each iteration step. This can be efficiently implemented using wavelet–compression techniques leading to faster algorithms.

We prove optimal convergence rates for the compressed methods if the approximation levels $||A_k - A||$ are chosen according to an a posteriori criterion. Numerical simulations are shown for an application in hyperhtermia treatment.

Wavelet approximation methods for solving the exterior Dirichlet problem for the Helmholtz equation

Siegfried Proessdorf, *Berlin*

We start with reclusing the boundary value problem to an integral equation over the (plane, smooth) boundary by applying the single-layer ansatz. The latter equation is a periodic pseudodifferential equation of order -1, where the principal part is the logarithmic-kernel integral operator. For the numerical solution of this equation we apply the knot collocation method where piecewize linear continuous functions and biorthogonal wavelets with two vanishing moments as trial functions are used. We apply an a-priori compression criterion (see [DPS]) to the wavelet representation of the collocation stiffness matrix to get sparse matrices. In order to get a fully discrete method we use an adapted quadrature rule with graded quadrature points for integration. This yields a stable method with convergence rate 2 in the L^2 - norm and complexitly $O(N(\log N)^b)$, where N is the number of unknowns and $b \geq 0$ is some real number. For improving the accuracy of the method an improved quadrature for calculating the integrals on coarser scales is proposed. The results are confirmed by numerical experiments for different values of the wave numbers.
Moreover, using this method, the radar cross section for several shapes has been calculated (this is a numerical scheme of order 3).
The talk is based on joint work with W. Dahmen, B. Kleemann and R. Schneider.

Dahmen, W., Prössdorf, S., Schneider, R.: Wavelet approximation methods for pseudodifferential equations I: Stability and convergence. Math. Zeitschrift **215** *(1994), 583–620.*

Dahmen, W., Prössdorf, S., Schneider, R.: Wavelet approximation methods for pseudodifferential equations II: Matrix compression and fast solution. Advances in Computational Mathematics, 2nd Issue, 1 (1993), 259–335.

Dahmen, W., Prössdorf, S., Schneider, R.: Multiscale methods for pseudodifferential equations. In: Recent Advances in Wavelet Analysis (eds. L.L. Schumaker and G. Webb), Academic Press 1994, 191–235.

Dahmen, W., Kleemann, B., Prössdorf, S., Schneider, R.: A multiscale method for the double layer potential equation on a polyhedron. Advances in Computational Mathematics (eds. H.P. Dikshit and C.A. Micchelli), World Scientific, Singapore, New Jersey, London, Hong Kong 1994, pp. 15–57.

These events are more singular than was previously supposed (their "regularity" is closer to 1,5 than 2, indicating that non integer powers of time should be used in representing the time series between the jerks). And finaly it exists a delay of 2 years between the 2 hemispheres.

Multiscale- and Waveletapproximation for the numerical solution of operator equations

Reinhold Schneider, *Darmstadt*

The Galerkin discretization of nonlocal operators by a local single scale basis results a dense system matrix. This is typically the case for boundary element methods. In contrast to the single scale bases, the Petrov-Galerkin using biorthogonal wavelet bases discretization gives rise to numerically sparse matrices.
We characterize the properties of biorthogonal wavelet bases, which are required for fast computations without compromizing the accuracy of the numerical scheme i.e. we fit the convergence rate. It is shown that we need only $\mathcal{O}(N)$ non zero matrix coefficients. We have explicit construction of this bases and can compute the provided matrix coeffizients with totally $\mathcal{O}(N \log^b N)$ function calls.

Applications of wavelets in image classification and data compression

Hans-Georg Stark, *Kaiserslautern*

In this talk we discuss two applications of wavelets to image processing and signal compression.

1. Image classification

Images are considered to be multiscale processes, i.e. composed of structures of verying size and orientation and exhibiting contours, the shape of which also shows characteristic ruggedness properties depending on some resolution levels. We describe methods, to measure these notions:
Multiresolution structure properties are qunatified, computing variances of the detail signals of a dyadic orthonormal tensor wavelet decomposition of the image. From then also resolution dependent numbers characterizing texture anisotropy are derived. These numers are collected in a pattern vector x_w.
Multiresolution properties of object contours are measured by computing certain circumscribing polygons $p(\tau, f)$, depending on a scale parameter τ, and measuring the

On the H^1-orthogonality of wavelet packets

Andreas Rieder, *Saarbrücken*

The H^1-orthogonality of wavelet packet spaces can be measured by the smallest constants in the strong Cauchy inequalities they satisfy with respect to the H^1-inner product.

We discuss some properties of those constants, present two conjectures concerning their decay behavior for which we have both, numerical and analytical evidence, and address an open problem.

Also, we give the link between these strong Cauchy inequalities and the numerical solution of anisotropic elliptic PDEs.

Characterization of secular variation in geomagnetic field using wavelet analysis

Ginette Saracco, *Rennes/F*

Wavelet analysis is applied to detect and characterize singular events such as jerks, in the time series made of the last century monthly mean values of components of the geomagnetic field from differents observatories though the world. After choosing a well adapted wavelet function, the analysis is first performed on synthetic series including the "interval", or main contribution (signal made of smooth variation intervals separated by singular events with different "regularities"); a white Gaussian noise and an "external" contribution (solar effects). This last one is built by the sum of a few humanies of a long period variation (11 years for the fundamental). The signatures of the main, noise and humanic components are studied and compared, and the conditions in which the singular events can be clearly isolated in the composite signal are elucidated. We have defined by the ridge function of the wavelet transform modulus some criteria which define the ridge like a magnetic signature for the "Jerks" and harmonic component . We have applied this method to real geomagnetic series (first montly means of just the Y component from European observatories) then on the X and Y component for a vectorial study from world observatories. We show that only five remarkable events are found in 1901, 1913, 1925, 1989 and 1978 for the northern hemisphere. The characteristics of these singularities (in particular, homogenity of some derived functions of the wavelet transform over a large range timescales), demonstrate that these events have a single source (of course internal).

complexity of these polygons on successive resolution levels τ. The resulting numbers are collected in a pattern vector x_c. We show results of classifying medical MR-images with x_w and/or x_c. With a neural net classification rates of ~ 90 % are reached. Since geometrical analyses shows, that the pattern classes are well separated from each other, neural nets may be omitted and with simple and cheap nearest distance classifiers the same classification rate may be obtained. As an application we finally show results from a system, which allows to retrieve images from an image database, which are "most similar" to a given one, "presented to the system". The similarity measure is based upon the methods presented above.

2. Signal compression

Recently signal coding with samplings from the wavelet transform, resulting from detecting local extrema, has been investigated. Let $\sum = \{(a_i, b_i), \xi_i\}_{i=1}^n$ be a sampling of the WT $(\varphi_{(0,0)}, f)$ of a one dimensional signal f, where $(\varphi_{(a_i,b_i)}, f) = \xi_i$. Consider for an arbitrary $\varphi \in L^2(\mathbb{R})$ the subspaces $M_\varphi = \overline{\{span\ \varphi_{(a_i,b_i)}\}_{i=1}^n}$. Then, generalizing a classical result of Grossmann at al (1985), an explicit formula for reconstructing a signal $f_{M_\xi} \in M_\varphi$ from the sampling $\sum$ is given.

Fast polynomial transforms

Gabriele Steidl, *Darmstadt*

Let $\{P_j\}$ be a sequence of "classical" polynomials (on functions satisfying a three-term-recurrence relation) and let $\{x_k\}$ be an arbitrary knot sequence. We ask for the fast evaluation of

$$\underline{a} := P\underline{c} \quad , \quad P := (P_k(x_j))_{j,k=0}^N \in \mathbb{R}^{N+1,N+1} \tag{1}$$

for given $\underline{c} \in \mathbb{R}^{N+1}$. The above problem as well as its transposed version $\underline{b} := P^T\underline{c}$ and the related inverse problems arise in connection with interpolation and quadrature, with FFT on nonequis-paced data, FFT on s^2 or distance transitive graphs. We propose several ways for the solution of (1) using the fact that for the Chebyshev-Polynomials T_k and their extrema (zeros) as knots x_j there exists an $0(N \log N)$ algorithm for (1). We follow the line

$$\begin{array}{ccc} T_k(\cos \frac{j\pi}{N}) & \overset{FMM}{\longrightarrow} & T_k(x_j) \\ DHA \downarrow & & \downarrow DHA \\ P_k(\cos \frac{j\pi}{N}) & \overset{FMM}{\longrightarrow} & \boxed{P_k(x_j)} \end{array}$$

The horizontal ways can be gone for example by fast multipole methods (DUH/Rokhlin '93) in $0(N \log \frac{1}{\epsilon})$ and the vertical ways by a modification of the Discroll-Healy-Algorithm ('94) in $0(N \log^2 N)$.

On Multivariate Affine Frames

Joachim Stoeckler, *Universität - GH - Duisburg*

An *affine frame* (or *wavelet frame*) is, by definition, a frame of $L_2(\mathbb{R}^d)$ which consists of matrix-dilated and translated copies of a finite set $\Psi \subset L_2(\mathbb{R}^d)$ of "generators". The usual approach in order to develop properties of the frame is by analyzing the frame operator $T_\Psi^* T_\Psi$ which is a positive definite selfadjoint operator on $L_2(\mathbb{R}^d)$. Here, the operator T_Ψ is defined as follows: it maps $f \in L_2(\mathbb{R}^d)$ to the sequence of inner products with all the frame elements. We take a different approach by using a second affine frame generated by $\Theta \subset L_2(\mathbb{R}^d)$ as a preconditioner of T_Ψ. Several interesting features can be observed by taking a closer look at the newly defined operator

$$S := T_\Psi \circ T_\Theta^* .$$

First of all, the dilation invariance of the frame operator is now expressed by the fact that S is a Laurent operator in the sense of Gohberg; i.e. S commutes with the bilateral shift on the space of biinfinite sequences $\ell_2(\mathcal{H})$ where $\mathcal{H}$ is a properly chosen Hilbert space. General results about such operators lead to a new sufficient condition for the existence of an upper frame bound. Two further results hold true in a special setting when Ψ and Θ generate a biorthogonal wavelet basis and when the sampling lattice for Ψ contains $\mathbb{Z}^d$. Then, the operator S can be represented by a pyramidal scheme which is similar to the well-known reconstruction algorithm. An explicit representation of the selfadjoint Laurent operator S^*S can be given, additionally. This is used for the construction of simple preconditioners which lead to a substantial acceleration of the associated frame algorithm.

Stöckler, J., Multivariate affine Frames, Habilitationsschrift, Universität Duisburg, 1995, 128 pp.

Some remark on the distribution of zeros of the Daubechies filters and on the 'wavelet crime'

Gilbert Strang, *MIT*

To study wavelets and filter banks of high order, we begin with the zeros of $B_p(y)$. This is the binomial series for $(1-y)^{-p}$, truncated after p terms. Its zeros give the $p-1$ zeros of the Daubechies filter inside the unit circle, by $z+z^{-1}=2-4y$. The filter has p additional zeros at $z=-1$, and this construction makes it orthogonal and maximally flat. Then orthogonal wavelets with p vanishing moments come through the dilation equation. Symmetric biorthogonal wavelets (generally better in image compression) come similarly from a subset of the zeros of $B_p(y)$.

We study the asymptotic behavior of these zeros. Matlab shows a remarkable plot for $p=70$. The zeros approach a limiting curve $|4y(1-y)|=1$ in the complex plane, which is $|z-z^{-1}|=2$. All zeros have $|2y|<1$ for $p>2$, and the rightmost zeros approach $y=\frac{1}{2}$ with speed $p^{-\frac{1}{2}}$. The curve $|4y(1-y)|=(4\pi p)^{\frac{1}{2p}}|1-2y|^{\frac{1}{p}}$ gives a very accurate approximation for finite p.

The wide dynamic range in the coefficients of $B_p(y)$ makes the zeros difficult to compute for large p. Rescaling y by 4 allows us to reach $p=80$ by standard codes. This is "spectral factorization" of high order. The zeros at $z=-1$ stabilize the iteration of the Daubechies filter in a multiresolution.

The 'wavelet crime',i.e. pretending the data sample values are wavelet coefficients on the finest scale is briefly discussed.

Time-scale frequency characterization of noisy chirps

Bruno Torresani, *Marseille*

Time-scale and/or time-frequency representations have been shown to be very useful for the detection and the characterization of amplitude and frequency modulated signals (the so-called "chirps") in the absence of noise (or in the presence of a weak noise), a lot of methods are available to characterize the frequency modulations of the signal. However, in the presence of strong noise, most of the algorithms become unstable.
We describe a couple of (Monte Carlo type) algorithms to address this problem. Both rely on the existence of curves in the time-frequency plane (the ridges) associated with the chirps.
The first algorithm (the corona method) is based on the minimization of an appropriate energy functional on the space of ridges. The minimization is achieved via a simulated

annealing method, avoiding the local minima of the energy functional.
The second algorithm (the crazy climbers method), adapted to situations in which many ridges are present, is based on a random walk on the time-frequency plane. With the random walk is associated an occupation measure, which happens to "concentrate" on the ridges.
Both algorithms achieve ridge detection for signals embedded in (white) noise, for significantly low (negative) values of the signal to noise ratio.

With these two methods (and variants) is associated another algorithm (a constrained optimization algorithm) for signal reconstruction from ridges.

Filter Banks and Wavelets

Martin Vetterli, *Berkeley/USA & EPF Lausanne*

As is well-known, a strong link exists between discrete-time linear expansions based on convolutions or filter banks, and wavelet bases, as pioneered by Daubechies. In this talk, we discuss three cases:

1. Multiwavelets based on time-varying filter banks.
   In this case, one is lead to the study of infinite products of matrices of trigonometric polynomials

2. Multidimensional filter banks
   Several recent results generalizing the one-dimensional case (completion, factorigation of unimodular and paraunitary matrices of Laurent polynomials) are reviewed. A counter example to paraunitary factorisation is shown

3. Binary friendly wavelet constructions.
   We discuss an application where filters with integer coefficients, 2^h scaling factor and regularity are required, and show such constructions.

Best Approximation with Walsh Atoms

Lars F. Villemoes, *Stockholm*

We consider the approximation in $L^2(\mathbb{R})$ of a given function using finite linear combinations of Walsh atoms, which are Walsh functions localized to dyadic intervals, also called Haar-Walsh wavelet packets.It is shown that up to a constant factor, a linear combination of k atoms can be represented to relative, error ϵ by a linear combination of $(K^2 \log(1/\epsilon)$ orthogonal atoms.
In finite dimension N, best approximation with K orthogonal atoms can be realized, with an algorithm of order $KN(1 + \log(N/K))$. A faster algorithm of order $N \log N$ solves the problem with indirect control over K. Therefore the above result connects algorithmic and theoretical best approximation.

L. F. Villemoes: "Best Approximation with Walsh Atoms", TRITA-MAT-1995-MA-13, Royal Inst. Technology, (June 1995)

C. M. Thiele and L. F. Villemoes: "A fast algorithm for adapted time-frequency tilings", Appl. Comput. Harmonic Anal. (to appear)

Sampling on Non periodic sets with Applications

David F. Walnut, *Fairfax/USA*

In this talk, we describe a result on sampling band limited functions on a certain class of non uniform sets. These sets have the form $\bigcup_{i=1}^{m}\{\frac{n}{2r_i} : n \in \mathbb{Z}\}$ on the line and are cross products of such sets in higher dimensions. The $\{r_i\}$ satisfy r_i/r_j is irrational if $i \neq j$. We apply these results to two situations:

1. Solving the equation $\sum_{i=1}^{m} \mu_i * r_i = \varphi$ where $\mu_i = 1_{[-r_i, r_i]}$ and r_i are compactly supported and φ is a C^∞ approximate delta.

2. An industrial tomography problem first described by Laurent Deshat in Inverse Problems (1993).

Walnut, D., Sampling and the multisensor deconvolution problem *in Fourier Analysis: Analytic and Geometric Aspects, Bray, et al., eds., Marcel Dekker, Inc., New York (1994).*

Casey, S., and Walnut, D., Systems of convolution equations, deconvolution, Shannon sampling and the Gabor and wavelet transform, *SIAM Review, Vol. 36, No. 4, 537–577 (1994).*

Walnut, D., Nonperiodic sampling of bandlimited functions on unions of rectangular lattices, *J. Fourier Anal. Appl., submitted.*

Wavelet Compression for still Images

M. Victor Wickerhauser, *St. Louis, USA*

A genuine transform coder for image compression has 3 parts: transform T, quantization Q, and reclundancy removal R. Schematically:

$$\text{in} \longrightarrow \overset{\boxed{\text{T}}}{\text{invertible}} \longrightarrow \overset{\boxed{\text{Q}}}{\text{lossy}} \longrightarrow \overset{\boxed{\text{R}}}{\text{invertible}} \longrightarrow \text{out}$$

These are interrelated, though we may treat them separately.
images describe functions which are much simpler then a general input element, so that this procedure gives (for images) an output description much shorter than the input. We concider the problem of choosing T so that Q, R is most efficient. Options are:

- decrrelate pixels over an expected class of images ⇒ use the Karhunen-Love transform for the class. This is computationally complex and requieres fore - knowledge of the class.

- concentrate energy into a few transform coefficients ⇒ find the "theoretical dimension" of the image and retain only that many degrees of freedom. For certain measures of concentration (i.e., additive information cost functions) this is a fast computation.

Using the second principle, we judge several candidates for T:

- "'JPEG"': 8 x 8 blocks, Discrete Cosine Transform (DCT) in 2-d, which approximate the K-L transform and is rapidly computable

- modifications of JPEG: 16x16 blocks, Malvas-Coifman-Meyer lapped transforms

- subband coding methods: iterated filters, orthogonal wavelets, custom subband decompositions such as the FBI's "WSQ" choice for fingerpoint images scanned at 500 dpi and the "best basis algorithm" which chooses a decomposition to maximize energy concentration.

We note that Nick Bennet has enlarged the class of subband decompositions to include arbitrary rectangular things at dyadic points (thesis, 1995). It is not known whether this will dramatically improve the efficiency of subsequent Q and R stages.

Efficiency of Q, R can be computed from the position of the rate- distortion curve for the compression algorithm.

But the interrepts of the rate-distortion curve can be estimated by

$d(\chi) = \sum_{k=1}^{M} \log |\chi_k|$ "theoretical dimension"

where $x = \{\chi_k : k = 1, \ldots, M\}$ is the sequence of coefficients produced by T, if the decreasing rearrangement χ^* of $\{|\chi_k|\}$ has sufficiently rapid decay (Aurelija Trgo and MVW, 1995). This, since d is an additive information cost, it can be used in the best basis algorithm to choose a decomposition making Q, R most efficient in the rate-distortion sense.

Finally, we point out that there are political issues in choosing T, such as patents on certain T's, costs of implementing new T's when not-so-bad old ones exist, and customer quality and robustness requirements.

Generalized Sampling and Quasiinterpolation

Georg Zimmermann, *Wien/A*

Both in wavelet bases and multiresolution analysis one needs to consider projection operators of the form

$$P : f \mapsto \sum_{n\in\mathbb{Z}} \underbrace{< f, \tau_n g >}_{f * \tilde{g}(n)} \tau_n h$$

These can be splitup in the two parts

$$S : f \mapsto (< f, \tau_n g >)_{n\in\mathbb{Z}} \text{ and } R : (a_n)_{n\in\mathbb{Z}} \mapsto \sum a_n \tau_n h =: a *' h.$$

To construct wavelet basis in Banach spaces, we therefore need to consider maps between Banach spaces of function (or distribution) on $\mathbb{R}$ and sequences on $\mathbb{Z}$ which compute with integer translates.

We denote $S : X(\mathbb{R}) \to Y(\mathbb{Z})$ with $\tau_n S = S_{\tau_n}$ a <u>generalized sampling operator</u> and $\mathbb{R} : Y(\mathbb{Z}) \to X(\mathbb{R})$ with $\tau_n R = R_{\tau_n}$ a <u>quasiinterpolation.</u>

The spaces of these operators are denoted $\mathcal{L}_{\tau_{\mathbf{Z}}}(X(\mathbb{R}), Y(\mathbf{Z}))$ and $\mathcal{L}_{\tau_{\mathbf{Z}}}(Y(\mathbf{Z}), X, (\mathbb{R}))$, and we demonstrate some general properties of these spaces.
The most interesting cases are those when $Y(\mathbf{Z})$ is a discrete version of $X(\mathbb{R})$.
We present isometric representation for the cases $\mathcal{L}_{\tau_{\mathbf{Z}}}(L^1(\mathbb{R}), l^1(\mathbf{Z}))$ and $\mathcal{L}_{\tau_{\mathbf{Z}}}(L^2(\mathbb{R}), l^2(\mathbf{Z}))$, and exhibit parallels to well-known results on multipliers of $L^1(\mathbb{R})$ and $L^2(\mathbb{R})$, respectively.

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