MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Die Tagung fand unter der Leitung von Herrn G. Huisken (Tübingen), Herrn L. Simon (Stanford) und Herrn M. Struwe (Zürich) statt. Die Teilnehmer kamen aus der Bundesrepublik Deutschland, den USA, Russland, Frankreich und anderen Ländern. Sie vertraten einen breiten Themenkreis aus dem Gebiet der partiellen Differentialgleichungen, und es wurde vor allem jungen Mathematikern Gelegenheit geboten, ihre Forschungsergebnisse einem interessierten Fachpublikum vorzustellen.

Ein nicht allzu dicht gedrängtes Programm und die angenehme Atmosphäre des Instituts begünstigten den informellen Ideenaustausch unter den Teilnehmern.

Nonuniqueness in geometric evolution equations

by T. Ilmanen (Northwest University)

Geometric heat equations, including the mean curvature flow, the harmonic map heat flow, and the equation $u_t = \Delta u + u^p$ share a remarkable property: the initial value problem is not well posed. We give some examples and discuss the significance.

Minimization of conformally invariant energies in homotopy classes

by E. Kuwert (Bonn)

This is a report on joint work with F. Duzaar (Bonn). We study the problem of minimizing a conformally invariant integral $F(u) = \int_{S^n} f(du(\xi))\, d\xi$ among maps u from the standard sphere S^n $(n \geq 2)$ into a compact set $X \subset \mathbb{R}^N$ belonging to a fixed free homotopy class $\alpha \in \pi_n(X)$. It is well-known that minimizing sequences are not necessarily relatively compact in $H^{1,n}(S^n, \mathbb{R}^N)$ due to the possibility of "separation of spheres". For $n = 2$ and $f(A) = |A|^2$ this was studied, for example, by Sacks-Uhlenbeck, Struwe, and Jost.

Let $M^{1,n}(S^n, \mathbb{R}^N) = H^{1,n}(S^n, \mathbb{R}^N)/\mathrm{Conf}(S^n)$. We say that $(U^i)_{i \in I}$ is a *weak limit set* of $(U_k)_{k \in N}$ in $M^{1,n}$, iff

1. none of the U^i is constant;
2. $\exists u_k \in U_k$, $u^i \in U^i$, $h_k^i \in \mathrm{Conf}(S^n)$ such that $u_k \circ h_k^i \rightharpoonup u^i$ weakly and $(h_k^i)^{-1} h_k^j$ diverges for $i \neq j$.

Theorem Any minimizing sequence $(U_k)_{k \in \mathbb{N}}$ has a subsequence $(U_k)_{k \in \mathfrak{K}}$ such that this subsequence has a weak limit set $(U^i)_{i \in I}$ with

- $\sum_{i \in I} F(U^i) = \lim_{k \to \infty} F(U_k)$;
- Any U^i minimizes F in its own homotopy class α^i;
- $(\alpha^i)_{i \in I}$ is a decomposition of the given class α.

Uniqueness for the harmonic map flow in two dimensions

by A. Freire (Knoxville)

Let M be a compact two-dimensional manifold without boundary, $N \subset \mathbb{R}^d$ a k-dimensional compact submanifold with second fundamental form denoted by $A(X,Y)$. We consider weak solutions in $H^1(M \times [0,T]; N)$ of the harmonic map flow:

$$\begin{aligned} u_t - \Delta u &= \mathrm{tr}_M\, A(\nabla u, \nabla u) \qquad \text{on } M \times [0,T] \\ u(\cdot, 0) &= u_0 \colon M \to N\,. \end{aligned}$$

Existence of global weak solutions $v\colon M \times [0,\infty) \to N$ (for $u_0 \in H^1(M,N)$) was obtained by M. Struwe (1984). v has finite singular set. Denoting by $T_1 < \cdots < T_K$ the singular times, v is the unique solution such that $v \in L^2_{loc}((T_i, T_{i+1}), H^2(M;N))$, $i = 0, \ldots, k-1$, $T_0 = 0$. Our main result is the uniqueness of solutions in 'energy-class':

Theorem Let $u \in H^1(M \times [0,T]; N)$ be a weak solution of the harmonic map flow, whose total energy $E_u(t)$ is non-increasing in t. Then $u = v$ almost everywhere. Here v denotes M. Struwe's solution with the same initial data.

The main step in the proof is a lemma stating that a time $T' \in (0,T)$ exists such that $du \in L^2([0,T'], L^4)$. This is proved by observing that in an appropriate 'gauge', the nonlinearity in the harmonic map flow has a 'determinant structure', then appealing to Hélein's construction of a fundamental system of solutions to a linear $\bar\partial$ equation with small coefficients in $W^{1,1}$. The 'appropriate gauge' is obtained by finding an 'optimal' connection 1-form (modifying a construction of Hélein), for each fixed time $t \in (0,T')$.

Given the lemma, a perturbation argument (and linear parabolic theory) may be used to show that $u \in L^4([0,T'], W^{2,4/3})$, and hence $du \in L^4([0,T'] \times M)$. Then from the equation it follows that $u \in L^2([0,T'], H^2)$.

Reference: Calculus of variations (1995) – for $N = S^k$.

Stability in hydrodynamics

by W. von Wahl (Bayreuth)

We consider viscous steady incompressible fluid flows in an infinite layer. This flow as well as the perturbations of it are periodic with respect to the plane directions. The perturbations u are assumed to fulfill the boundary conditions $u = 0$ at the walls. We first study the case that monotonic energy stability is followed by instability. The occurrence of this situation is characterized completely. Our result is applied to the Taylor-Couette-problem in the narrow gap case. Finally we show that the basic flow in this problem is unconditionally stable against axisymmetric perturbations precisely as long as the Reynolds-number stays below or is equal to the critical one. The latter case is the marginal one, where asymptotic stability may not hold, only a suitable functional does not increase with time.

Comparison principles for elliptic operators of arbitrary order

by H.-C. Grunau (Bayreuth)

This talk concerns joint work with Guido Sweers from Delft.

The clamped plate equation

$$\begin{aligned} \Delta^2 u = f \qquad & \text{in } \Omega, \\ u = \frac{\partial u}{\partial \nu} = 0 \qquad & \text{on } \partial\Omega \end{aligned}$$

with $\Omega \subset \mathbb{R}^2$ may serve as a prototype for the kind of Dirichlet problems treated here. We are interested in the positivity question: For which shapes Ω of the plate does upwards pushing, i.e. $f \geq 0$, imply upwards bending, i.e. $u \geq 0$? Boggio proved this property in 1905 when $\Omega = B$ is a ball. He and Hadamard conjectured in 1901/1908 by physical intuition that positivity should hold in any "reasonable" bounded convex domain. Coffman, Duffin, Garabedian, and many others disproved this conjecture: even in an ellipse with ratio of half-axes approximately 2 a solution u with $f \geq 0$ may change sign.

We show that the positivity conjecture actually holds for domains that are in "$C^{4,\gamma}$-sense" close to the unit ball B, in particular for ellipses with ratio of half-axes close to 1.

In general, we show positivity results for small perturbations of the setting $((-\Delta)^m$, homogeneous Dirichlet boundary conditions, and $\Omega = B \subset \mathbb{R}^n)$ allowing variations of

- the domain and leading terms of the differential operator if $n = 2$,
- lower order terms if $n \geq 3$.

Boundary singularities of quasilinear elliptic and parabolic equations

by C. Bandle (Basel)

Consider problems of the form $Lu = f(u)$ in $D \subset \mathbb{R}^N$, $u(x) \to \infty$ as $x \to \partial D$, where L is a uniformly elliptic operator of the second order and f is a positive function.

It is well-known that such solutions exist in bounded domains provided f satisfies a certain growth condition at infinity. Under this condition and if the boundary is smooth we have $u(x)/\phi(d(x)) \to 1$ as $x \to \partial D$ where ϕ solves $\phi'' = f(\phi)$ for $t > 0$ and $\phi(t) \to \infty$ as $t \to 0$. The function $d(x)$ is the distance from x to ∂D measured in the metric induced by the principal part of L.

In this talk we ask under what conditions $u(x) - \phi(d(x)) \to 0$ as $x \to \partial D$. We have to impose additional assumptions on f to kill the effect of the boundary curvature. The results extend partly to quasilinear operators $Lu = \mathrm{div}(g(|\nabla u|)\nabla u)$.

Numerical solution of Bernoulli's free-boundary problem

by M. Flucher (Basel)

Bernoulli's free-boundary problem arises in ideal fluid dynamics, heat flow optimization, electro chemical drilling and galvanization.

Given the fixed boundary the problem is to find the free boundary for which the electrostatic field induced by a voltage applied to the two boundaries is constant in magnitude along the free boundary. Typically the interior Bernoulli problem has two solutions, an elliptic one close to the fixed boundary and a hyperbolic one far from it. Precise definitions of elliptic and hyperbolic solutions are given in terms of a variational formula for the free boundary subject to variations of the Dirichlet and Neumann data. Previous results mainly deal with elliptic solutions exploiting their monotonicity as discovered by A. Beurling. Hyperbolic solutions are more unstable and more delicate for analysis and numerical approximation.

Nevertheless we derive a second order trial free-boundary method, the *implicit Neumann scheme*, with equally good performance for both types of solutions. Superlinear convergence of a semi discrete variant is proved under a natural non-degeneracy condition. Numerical examples computed by this method confirm analytic predictions including questions of uniqueness, connectedness, elliptic and hyperbolic limits. This gives a survey on the rich qualitative theory of Bernoulli's free boundary problem.

Reference: Flucher M., Rumpf M.: Bernoulli's free-boundary problem, qualitative theory and the implicit Neumann method (To appear).

Singular solutions for a semilinear elliptic equation using asymptotic analysis

by F. Pacard (Noisy-le-Grand)

We prove existence of weak solutions to the equation $\Delta u + u^p = 0$ that are positive in a domain $\Omega \subset \mathbb{R}^n$ and that are singular along arbitrary smooth k-dimensional submanifolds in the interior of these domains provided p lies in the interval $(\frac{n-k}{n-k-2}, \frac{n-k+2}{n-k-2})$. Applications to the singular Yamabe problem are given.

Removable singularities of Monge-Ampère equations in dimensions $n \geq 2$

by R. Beyerstedt (Aachen)

Denote by B the ball with center $x_0 \in \mathbb{R}^n$ and radius $R > 0$ and let $u \in C^2(B \setminus \{x_0\})$ be a solution of the Monge-Ampère equation $\det D^2u(x) = f(x, u(x), Du(x))$ in $B \setminus \{x_0\}$. If $D^2u > 0$ in $B \setminus \{x_0\}$ and $f \in C^2(B \times \mathbb{R} \times \mathbb{R}^n)$ is positive, then the isolated singularity at x_0 is removable provided that some directional derivative $\gamma \cdot Du$, where $|\gamma| = 1$, has a continuous extension to the point x_0.

Curves and surfaces of least total curvature and fourth-order flows

by A. Polden (Tübingen)

The classical variational problems in geometry are described by second-order pde's whose analysis pivots on the maximum principle. Functionals based on the total squared curvature (of an immersion or a metric) lead instead to variational equations of *fourth* order. But the maximum principle stops short with second-order equations.

We can't fall back on the standard Sobolev space theory, either. In these *geometric* problems, where the metric leads a life of its own, the 'constants' in the standard integral inequalities depend on time.

In this talk, we present an approach to such higher-order equations, with reference to a number of geometrically natural model problems.

Implicit time discretization for gradient flows of functionals involving the area

by S. Luckhaus (Bonn)

The mean curvature flow can be viewed as the gradient flow of the area functional for hypersurfaces with respect to an L^2 distance metric. If one looks at this as the invariant formulation of the normal

L^2 metric and Dirichlet integral for functions, it becomes natural to approximate this by an implicit time discretization

$$\int \operatorname{dist}(\cdot, \partial\Omega(t-h))|\chi - \chi(t-h)| + h\int |\nabla\chi| \to \min\,,$$

where $\chi(t,x)$ is 1 if $x \in \Omega(t)$ and 0 otherwise. $\partial\Omega(t)$ is the evolving surface.

The main difference to the implicit discretization of the heat equation is that the square root of the first term is not a metric for sets of finite perimeter. In order to compare it to a metric, and so to get compactness of the sequence in h, one has to show a density estimate for $(\Omega(t)\Delta\Omega(t-h)) \cup B_\varrho(x)$, $x \in \Omega(t)\Delta\Omega(t-h)$, $\varrho < \operatorname{dist}(x, \partial\Omega(t-h))$.

Still, in general, convergence to a set of finite perimeter, such that $\partial\Omega(t)$ is a weak solution for the mean curvature flow, is not true, at least with an additional forcing term.

There is a result of T. Sturzenhecker and the author that this convergence is true under the additional assumption $\int\int|\nabla\chi_h| \to \int\int|\nabla\chi|$ as $h \to 0$.

The method can be extended to coupled systems where heat and mass diffusion is combined with a kinetic undercooling-Gibbs-Thomson law at the free boundary $\partial\Omega$, as a model for phase changes. This law can be viewed as the gradient flow for an entropy that takes the interface area into account. For this system the convergence result is not yet proved.

Free energy estimates for reaction-diffusion processes

by K. Gröger (Berlin)

This is a report on a joint work with A. Glitzky and R. Hünlich. We consider reaction-diffusion processes of electrically charged species $X_1, \ldots X_m$ in a bounded domain in $\mathbb{R}^N$ ($N = 2$ or $N = 3$). Let $u = (u_0, u_1, \ldots, u_m)$ be the vector the components of which are the charge density u_0 and the concentrations $u_1 \ldots, u_m$ of the species $X_1, \ldots, X_m$. We show that in the natural space of all u there is a linear subspace U characterized by the stoichiometric structure of the underlying reaction system which has the following property: For each possible initial value u^0 in the affine spaces $U + u^0$ there exists a unique stationary solution u^*, and this solution is in fact a thermodynamic equilibrium. We introduce the free energy $\psi(u)$ of a state u and formulate conditions under which $\psi(u) - \psi(u^*)$, $u \in U + u^0$, can be estimated from above by the corresponding dissipation rate. If such an estimate holds, then, for solutions to the instationary problem starting at a point of $U + u^0$, one can show that $\psi(u) - \psi(u^*)$ decays exponentially as $t \to \infty$. From that decay further global *a priori* estimates follow.

Partial differential equation of ergodic control and application to Schödinger operators

by J. Frehse (Bonn)

The above equation reads in simple cases

$$-\Delta u + H(x, \nabla u) + \lambda = q(x)\,,$$

which is treated in $\mathbb{R}^n$. H is the Hamiltonian and has quadratic growth and a coerciveness property. In a former paper, existence and uniqueness has been studied by Bensoussan and the author under the condition $q(\infty) = \infty$. A new contribution is the case that $q(\infty)$ is sufficiently large and that there are regions in $\mathbb{R}^n$ where $q(x)$ is sufficiently small. In the case $H = |\nabla u|^2$ one obtains via the transformation $v = e^{-u}$ the existence of bounded states for the Schrödinger operator $-\Delta u + qu$.

Some remarks on Non-Newtonian fluids

by M. Fuchs (Saarbrücken)

We consider quasistatic flows of certain viscoplastic materials for which the velocity field $v\colon \mathbb{R}^n \supset \Omega \to \mathbb{R}^n$, $n \geq 2$, can be discovered as a minimizer of the energy $I(u) = \int_\Omega W(\mathcal{E}u)\,dx$ in classes of functions u satisfying $\operatorname{div} u = 0$ and also appropriate boundary conditions. The density W is characteristic for the fluid under consideration, and $\mathcal{E}u$ denotes the symmetric gradient.

In case of a Bingham fluid we have for example

$$W(\mathcal{E}u) = \eta|\mathcal{E}u|^2 + g|\mathcal{E}u|$$

with positive constants η, g. We also consider various perturbations of the Bingham model which are not assumed to be convex so that we have to study the relaxed variational problem. Our main result states that in all these cases the strain velocity $\mathcal{E}v$ of a minimizer v is a locally bounded function. Further results describe partial C^1-regularity of the velocity field, in particular for the general Bingham model $W_m(\mathcal{E}u) = \eta|\mathcal{E}u|^m + g|\mathcal{E}u|$, where $m \geq 2$.

Theorem Suppose that v is a minimizer of $I(u) = \int_\Omega W_m(\mathcal{E}u)\,dx$ with respect to the condition $\operatorname{div} u = 0$. Let $D_1 = \{x \in \Omega : x$ is a Lebesgue point for $\mathcal{E}v\}$ and $D_2 = \{x \in D_1 : \mathcal{E}v(x) \neq 0\}$. Then there is an open set $R \subset D_2$ such that $v \in C^{1,\alpha}(R)$ for any $0 < \alpha < 1$ and $|D_2 - R| = 0$.

References:

M. Fuchs, J. Grotowski, J. Reuling: On variational models for quasi-static Bingham fluids. Math. Meth. Appl. Sciences (in press)

M. Fuchs, G. Seregin: Nonconvex perturbations of the Bingham and Powell-Eyring model for viscoplastic fluids. Preprint 1995

The regularity and some qualitative questions for equations of the type of slow, normal, and fast diffusion

by A. V. Ivanov (St. Petersburg)

We investigate a large class of degenerate or singular nonlinear parabolic equations. The prototype of such equations is

$$\frac{\partial u}{\partial t} - \operatorname{div}\left(|u|^l |\nabla u|^{m-2} \nabla u\right) = 0, \qquad m > 1, \quad l > 1 - m\,.$$

Similar equations have a variety of applications. In the case $m \neq 2$, $l \neq 0$ such equations are known as doubly nonlinear parabolic equations (DNPE). The latter equations arise in the study of turbulent filtration of a gas or a fluid through porous media and Non-Newtonian fluids.

The first regularity results for DNPE were obtained by the author in the end of the eighties for the case $m > 2, l > 0$. To the moment Hölder estimates for generalized solutions as well as existence and uniqueness of some regular solution of Cauchy-Dirichlet problem are established in the general case of DNPE. Our proof of such regularity results depends of a type of DNPE. We say that a DNPE is an equation of the type of slow, normal, or fast diffusion if respectively $m + l > 2$, $m + l = 2$, or $m + l < 2$. The methods of the proof of Hölder estimates are a basis for establishing some qualitative properties of solutions of DNPE. In particular we establish results which are concerned with the following questions:

1. Large time behaviour, asymptotic profile, and time expansion of positivity for non-negative weak solutions of homogeneous DNPE of the type of slow or normal diffusion.

2. Extinction for a finite time T_*, inner positivity in $\Omega \times (0, T_*)$ of non-negative solutions of the Cauchy-Dirichlet problem with zero boundary conditions on $\partial\Omega$, asymptotic behaviour of solutions near the extinction time T_* and near the boundary $\partial\Omega$ for homogeneous DNPE of the type of fast diffusion.

3. Asymptotic behaviour of solutions and their derivatives for equations of the type of slow diffusion in the case of blow up on the parabolic boundary.

On Leray's possible counterexample

by M. Růžička (Bonn)

We prove that the system of equations describing the (hypothetical) self-similar singular solutions of the three dimensional Navier-Stokes equation has no nontrivial solutions belonging to $L^3(\mathbb{R}^3)$.

Existence and nonexistence results of the exterior Dirichlet problem for the minimal surface equation in the plane

by N. Kutev (Köln)

Exterior Dirichlet problem for the minimal surface equation $Mu := (1+|Du|^2)\Delta u - D_i u D_j u \cdot D_{ij} u = 0$ in Ω; $u = f$ on $\partial\Omega$; $u(x) = \mu \log|x| + O(1)$ when $|x| \to \infty$ is considered in a smooth, unbounded domain $\Omega \subset \mathbb{R}^2$ with bounded complement. If the Lipschitz constant of the data f is strictly less than 1 on the concave part of the boundary $\partial\Omega$ and the oscillation of the data on the whole boundary $\partial\Omega$ is small enough, existence of a classical solution of the exterior Dirichlet problem is proved for some appropriate values of μ. For special smooth data with Lipschitz constant strictly greater than 1 in a neighbourhood of some boundary point where the boundary is strictly concave (with respect to the interior unit normal), nonexistence of a classical $C^2(\Omega) \cap C(\bar{\Omega})$ solution of the Dirichlet problem is proved independently of the asymptotic behaviour of the solution at infinity. As a consequence some instability results for the Dirichlet problem under small perturbation of the data are proved, too. This is a joint work with F. Tomi (Heidelberg).

The singular limit of a vector-valued reaction-diffusion process

by B. Stoth (Bonn)

In this talk we present *a priori* estimates and rigorous asymptotics for the singular limit $\varepsilon \to 0$ of a vector-valued Allen-Cahn equation for an order parameter φ ranging in $\mathbb{R}^2$

$$\varepsilon \partial_t \varphi_\varepsilon - \varepsilon \Delta \varphi_\varepsilon + \frac{1}{\varepsilon} W_{,\varphi}(\varphi_\varepsilon) = 0\,.$$

We study the case of a bistable potential $W \geq 0$ that only depends on the modulus of φ and that vanishes along two concentric spheres. We decompose φ as ue^{if} and show that only u develops a transition layer as $\varepsilon \to 0$. Consequently in the limit two phases are created, according to the two stable manifolds of the potential. In the radially symmetric case we show that the interface separating them moves by mean curvature, whereas the limit of the polar angle f satisfies the heat equation in both bulk regions with a jump condition for the derivative on the interface.

The results presented in this talk are joint work with L. Bronsard.

Weak solutions for the curve shortening flow

by K. Deckelnick (Freiburg)

In the parametric approach to the curve shortening flow one looks for a solution $x\colon S^1 \times (0,T) \to \mathbb{R}^n$ for $n \geq 2$ of the following system of equations

$$\begin{aligned} x_t &= \frac{x_{uu}}{|x_u|^2} && \text{in } S^1 \times (0,T) \\ x(\cdot,0) &= x_0 && \text{in } S^1\,, \end{aligned}$$

where $x_0\colon S^1 \to \mathbb{R}^n$ is a given regular parametrisation of a closed curve. In the case $n = 2$ Gage, Hamilton, and Grayson have proved that if the initial curve is embedded then the solution $x(\cdot,t)$ shrinks to a point in finite time staying smooth during the evolution. But if the initial curve has self-intersections, singularities may develop and a classical solution no longer exists. Altschuler and Grayson, as well as Angenent, have developed methods in order to continue the solution past singularities which are of a geometric nature and work in the case $n = 2$. In order to obtain a global solution also in the case of higher codimension we introduce the following regularisation: for $\varepsilon > 0$ consider

$$\begin{aligned} x_t^\varepsilon &= \frac{x_{uu}^\varepsilon}{|x_u^\varepsilon|^2 + \varepsilon} && \text{in } S^1 \times (0,\infty) \\ x^\varepsilon(\cdot,0) &= x_0 && \text{in } S^1\,. \end{aligned}$$

It can be shown that this problem has a global smooth solution. Using *a priori* estimates and a compactness argument one further obtains that

$$\begin{aligned} x^{\varepsilon_k}, x_u^{\varepsilon_k} &\overset{*}{\rightharpoonup} x, x_u \qquad &&\text{in } L^\infty(S^1 \times (0,\infty)) \\ x_{uu}^{\varepsilon_k} &\rightharpoonup x_{uu} &&\text{in } L^2(S^1 \times (0,\infty)) \end{aligned}$$

for some sequence $\varepsilon_k \to 0$. The limit function x is a solution of the original problem in a weak sense. This notion of a weak solution is obtained by appropriately modifying ideas of Brakke.

p-harmonic flow into homogeneous spaces

by N. Hungerbühler (Freiburg)

For $2 < p < \dim M$ we establish existence of global weak solutions of the p-harmonic flow between Riemannian manifolds M and N for arbitrary initial data having finite p-energy in the case when the target N is a homogeneous space. In particular we construct a solution $f : M \times [0,\infty) \to N$ which satisfies the energy inequality

$$\frac{1}{2}\int_0^T \int_M |\partial_t f|^2 d\mu\, dt + \frac{1}{p}\int_M |df(T)|^p d\mu \le \frac{1}{p}\int_M |df(0)|^p d\mu$$

for all $T > 0$. In the proof we combine a (time-) discrete scheme with certain compactness properties of the p-harmonic flow into homogeneous spaces. If f_0 is weakly p-harmonic but not stationary p-harmonic then there exist infinitely many weak solutions of the flow with f_0 as initial function.

Miscellanea on radial solutions

by W. Walter (Karlsruhe)

For a nonlinear second order differential operator of the from $Lu = (\varphi(x,u'))'$ maximum principles (MP) and comparison principles (CP) are deduced for the equation $Lu = f(x,u)$, e.g.

$$\left.\begin{array}{c} (\varphi(x,u'))' - f(x,u) \le 0 \\ u(a) \ge 0, \quad u(b) \ge 0 \end{array}\right\} \quad \Rightarrow u \ge 0 \text{ in } [a,b] \qquad \text{(MP)}$$

if $\varphi(x,s)s > 0$ and $f(x,s)s \geq 0$ for $s \neq 0$. This is a theorem for weak solutions, i.e. $u \in C([a,b])$ and $u, \varphi(x,u') \in AC_{loc}(J^0)$. Comparison theorems for the corresponding initial value problem are derived, too. Important examples for L are the radial Laplace operator $\Delta u = r^{-\alpha}(r^\alpha u')'$, where $\alpha = N-1$, the radial p-Laplace operator $\Delta_p u = r^{-\alpha}(r^\alpha u'|u'|^{p-2})'$, and also $Lu = (r(x)u'(1+u'2)_{-1/2})'$. The simple proofs are different from the classical proofs of (MP) and (CP).

Two applications are considered: Dead core problems for $\Delta u = f(x,u)$ and blow-up problems of the form $\Delta_p u = f(u)$ in $[a,b)$ with $u(b) = \infty$. For the latter, it is proved that the difference of two blow-up solutions is decreasing if $\liminf_{s\to\infty} \frac{sf'(s)}{s(s)} > p-1$. In the standard example $f(s) = s^q$, this condition is $q > p-1$, which is the condition for blow-up to occur.

Remarks on the numerical computation of the solution using the "Lohner-Algorithm" which provides exact bounds are given, together with numerical examples.

Harmonic measure on locally flat domains

by T. Toro (Chicago)

This is a report on a joint work with C. Kenig. We are concerned with the relationship between the regularity of the boundary of a domain and the regularity up to the boundary of solutions of some elliptic PDE's.

For $\delta > 0$ and $R > 0$ we say that $\Omega \subseteq \mathbb{R}^{n+1}$ is a (δ, R) flat domain if

$$\|n\|_*(R) = \sup_{\substack{Q \in \partial\Omega \\ 0 < r \leq R}} |B_r(Q) \cap \partial\Omega|^{-1} \int_{B_r(Q)\cap\partial\Omega} |n - n_{r,Q}|\, dx^n \leq \delta^2$$

where n denotes the unit normal to $\partial\Omega$ and

$$n_{r,Q} = |B_r(Q) \cap \partial\Omega|^{-1} \int_{B_r(Q)\cap\partial\Omega} n\, dx^n$$

Theorem 1 There exists $\delta(n) > 0$ such that if $\Omega \subseteq \mathbb{R}^{n+1}$ is a (δ, R) flat domain with $0 \leq \delta \leq \delta(n)$ then Ω is non-tangentially accessible (NTA).

In particular this implies, by a result of Jerison and Kenig (82), that the Dirichlet problem for the Laplacian con be solved in Ω and that the harmonic measure ω is doubling. Moreover we prove that:
Theorem 2 $\forall\varepsilon > 0\ \exists\delta > 0$ and $\exists\gamma > 0$ such that if Ω is (δ, R) flat, $\Delta = B_r(Q) \cap \partial\Omega$, $r \leq \gamma$, and $E \subset \Delta$ then

$$(1-\varepsilon)\left(\frac{\sigma(E)}{\sigma(\Delta)}\right)^{1/(2\mu)} \leq \frac{\omega(E)}{\omega(\Delta)} \leq (1+\varepsilon)\left(\frac{\sigma(E)}{\sigma(\Delta)}\right)^{2\mu} \qquad \mu \in (0, 1/2)\,.$$

Here σ denotes the surface measure.

This theorem generalizes Dahlberg's result for Lipschitz domains as well as Jerison's and Kenig's (82) result for C^1 domains. A corollary of Theorem 2 is the following: Let $\Omega \subset \mathbb{R}^{n+1}$ be a (δ, R) flat domain and assume that $\|n\|_*(r) \to 0$ as $r \downarrow 0$ then $\|\log k\|_*(s) \to 0$ as $s \to 0$; i.e. $n \in \mathrm{VMO}(\partial\Omega)$ implies $\log k \in \mathrm{VMO}(\partial\Omega)$.

Sobolev spaces and harmonic maps between singular spaces

by G. Gregori (Salt Lake City)

We generalize the theory of Sobolev spaces for maps into metric spaces due to Korevaar-Schoen. While they require the domain to be regular, i.e. C^2, we work with Lipschitz submanifolds of $\mathbb{R}^n$. In particular this result can be used to obtain an existence theorem for harmonic maps from domains in a Lipschitz submanifold to complete metric spaces of non-positive curvature.

Weak compactness of wave maps

by S. Müller (Freiburg)

A smooth map $u\colon \mathbb{R} \times \mathbb{R}^2 \to N \hookrightarrow \mathbb{R}^d$ from $(1 + 2)$ dimensional Minkowski space to a k dimensional Riemannian manifold is called a wave map if

$$\Box u := \frac{\partial^2 u}{\partial t^2} - \Delta u \perp T_u N\,. \qquad (*)$$

We show the following

Theorem (Freire-M.-Struwe). Let u^n be a sequence of smooth wave maps with uniformly bounded energy

$$\sup_n \sup_t \int_{\{t\}\times \mathbb{R}^2} |Du^n|^2 d^2(x) \leq C$$

and suppose $u^n \to u$ in $L^2_{loc}(\mathbb{R}^3)$. Then u is a weak solution of equation $(*)$.

The proof uses the underlying determinant structure of the equation (after the choice of good frames) in connection with the $\mathcal{H}^1$ estimates for determinants of Coifman-Lions-Meyer-Semmes and the $\mathcal{H}^1$-BMO duality. To pass to the limit one can then apply the following result in the spirit of concentration compactness:

Suppose $f^n \overset{*}{\rightharpoonup} f$ in BMO, $g^n \overset{*}{\rightharpoonup} g$ in $\mathcal{H}^1$ then $f^n g^n \rightharpoonup fg + \nu$ in distributional sense, where ν is a Radon measure supported on

$$S := \{z : \limsup_{R\to 0} \limsup_{n\to\infty} [f^n]_{\mathrm{BMO}(B(z,R))} > 0\} .$$

To finish the proof one exploits the fact that the singular set S has capacity zero in view of the local energy inequality.

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