Tagungsbericht 35/1995

Complex Geometry: Vector Bundles in Geometry and Physics

27.8. - 2.9.1995

Die Tagung fand unter Leitung von N. Hitchin (Cambridge) und K. Hulek (Hannover) statt, die Vorbereitung und Organisation wurde zusätzlich von R. Lazarsfeld (Los Angeles) mitgetragen. Die Teilnehmer kamen aus insgesamt zwölf Ländern. In den Vorträgen, bei denen insbesondere auch jüngere Mathematikerinnen und Mathematiker die Möglichkeit erhielten, über ihre Forschungsergebnisse zu berichten, und den anschließenden lebhaften Diskussionen stellten sich folgende Themenschwerpunkte heraus: Hitchin Hamiltonsche Systeme, Modulräume von Vektorbündeln, Hilbert Schemata, Hitchin-Kobayashi Korrespondenz, Verlinde Räume, symplektische Strukturen, Instantonen.

Vortragsauszüge

S. Ramanan

Vector bundles and the Schottky configuration

Let C be a smooth projective curve of genus $g \geq 3$, and $\mathcal{SU}_C(2)$ the moduli space of vector bundles of rank 2 with trivial determinant. Then, for every nontrivial element η of order 2, $\mathcal{SU}_C(2)$ contains 2 copies of the Kummer variety one can construct using the two-sheeted covering of C defined by η. It also contains the Kummer variety of the Jacobian of C. One can make an easy study of these images and write down where they intersect. On the other hand, $\mathcal{SU}_C(2)$ is imbedded in the 2Θ-linear system, at least for generic curves C. Now the above configuration of Kummer varieties, which we call Schottky configuration, is expected to give a sufficient condition for an abelian variety to be a Jacobian. I proved two theorems concerning these, one which relates the Verlinde space of level 4 with 3Θ's on the Pryms, and another, which says $\mathcal{SU}_C(2)$ is defined by quartics for generic C.

W. M. Oxbury

Verlinde spaces and theta functions

Let $\Theta \to M$ be the theta line bundle associated to the standard orthogonal representation of the group $\mathrm{Spin}(m, \mathbf{C})$, over the moduli space M of semistable $\mathrm{Spin}(m, \mathbf{C})$-bundles on a smooth curve. We observe that by the Verlinde formula $h^0(M, \Theta)$ is equal to the number of theta functions of level m on all the Pryms of the curve (the precise identity depends on whether m is odd or even), and accordingly construct duality pairings between $H^0(M, \Theta)$ and the corresponding direct sums. These pairings are conjectured to be nondegenerate; and examples in low rank are given where this is known.

Bert van Geemen

Hitchin's Hamiltonian system

Let M be the moduli space of semistable rank two bundles with trivial determinant on a curve C of genus g. Let $E \in M$ be stable, then $T_E^*M = \mathrm{Hom}_0(E, E \otimes K)$ (traceless homomorphisms, K is the canonical bundle). The Hitchin map is:

$$H : T^*M \to H^0(2K),\ \Phi \mapsto \det \Phi,$$

and Hitchin proved that it is a completely integrable Hamiltonian system. Associated to E is a divisor $D_E \in |2\Theta|$ with $\Theta \subseteq \mathrm{Pic}^{g-1}(C)$ the theta divisor. We construct rational maps ϕ_E, ψ_E such that the following diagram commutes:

$$\begin{array}{ccccc} & \overset{\phi_E}{\nearrow} & \mathbf{P}T_E^*M & \overset{\bar{H}}{\searrow} & \\ D_E & & & & \mathbf{P}H^0(2K) \\ & \underset{\psi_E}{\searrow} & \mathbf{P}H^0(K) & \underset{sq}{\nearrow} & \end{array}$$

We prove that ψ_E is the Gauss map. Applying our results to the case $g = 2$, we can determine $\bar{H}$ explicitly.
(joint work with Emma Previato)

Fabrizio Catanese

New results on nodal surfaces and surfaces with r.d.p.'s

$X^n_d \subset \mathbf{P}^{n+1}$ is said to be *nodal* if its only singularities are formally $\cong \{\sum x_i^2 = 0\}$. A still not understood function is $\mu(n,d) = \max_{X^n_d \text{ nodal}}(\#(\operatorname{Sing} X))$, though lower and upper bounds have been given by many authors in the non trivial case $n \geq 2$. After recalling several general results, I concentrated on the case $n = 2$ where $\mu(2,d) =: \mu(d)$ is known for $d \leq 6$ and for $d \leq 5$ the surfaces yielding the maximum are characterized ($d = 3$ Cayley-Clebsch diagonal surface, $d = 4$ Kummer surface, $d = 5$ Togliatti-Beauville surface). Recently Barth produced a 6-ic with 65 nodes, and Jaffe-Ruberman showed, by using coding theory, that $\mu(6) < 66$. We presented also a simplification of their proof. Coding theory enters the picture in the following way. Let $S \to X$ be a resolution of singularities, $A_1, \dots, A_\mu$ the exceptional (-2)-curves. Then the $c_1(A_i) (\operatorname{mod} 2)$ span an isotropic subspace of $H^2(S, \mathbf{Z}/2)$, therefore, if $\mu \geq \frac{b_2(S)}{2}$, there is a kernel K (which I described explicitly for the max. surfaces)

$$0 \to K \to \bigoplus_{i=1}^{\mu} (\mathbf{Z}/2) A_i \to H^2(S, \mathbf{Z}/2).$$

K is a binary code, and vectors $k \in K$ correspond to even sets Δ of nodes, i.e., such that there is a 2:1 cover $Y \xrightarrow{p} X$ branched exactly on Δ. One must generalize:

Def.: *Δ is $1/2(\delta)$-even ($\delta = 0, 1$) if $\sum A_i + H \equiv 0\,(2)$, H being the class of a plane section.*

In this situation one attaches to Δ a sheaf $\mathcal{F} = p_*(\mathcal{O}_Y)/\mathcal{O}_X$, with a perfect pairing $\mathcal{F} \times \mathcal{F} \to \mathcal{O}_X(-\delta)$ (one can define more generally such $\mathcal{F}$'s as *quadratic sheaves*).

Main Theorem *(Casnati, Catanese) Quadratic sheaves are exactly the cokernel of a symmetric map of vector bundles on $\mathbf{P}^{n+1}$*

$$0 \to \mathcal{E}^\vee(-\lambda) \xrightarrow{\alpha} \mathcal{E} \to \mathcal{F} \to 0 \quad (i.e.,\ \alpha = {}^t\alpha).$$

In $\mathbf{P}^3$ thus $X = \{\det \alpha = 0\}, \Delta = \{x |\operatorname{corank} \alpha = 2\}$.

The theorem (proved by the author in particular cases, and conjectured by Barth, Catanese) allows a classification of even sets for small d.

Motohico Mulase

Coverings of Riemann surfaces, vector bundles on them, and integrable systems

Let $\mathcal{U}_{n,d}$ be the moduli stack of arbitrary vector bundles of rank r and degree d on an algebraic curve C of genus $g \geq 0$ (with a point $p \in C$ and a formal coordinate on $\hat{C}_p$ fixed). Let Gr_n be the Sato Grassmannian parametrizing vector subspaces of $\mathbf{C}((z))^{\oplus n}$ that are commensurable with $\mathbf{C}[z^{-1}]^{\oplus n}$. Then there is a natural embedding $\mathcal{U}_{n,d} \hookrightarrow \mathrm{Gr}_n / \mathrm{Gl}_n(\mathbf{C}[\![z]\!])$. Now let k be the diagonal subalgebra of $\mathrm{gl}_n(\mathbf{C}((z)))$,and let $\mathrm{k}_- = \mathrm{k} \cap \mathrm{gl}_n(\mathbf{C}[\![z]\!]z)$. Then

Theorem *1. There is a rational injective morphism i*

$$\begin{array}{ccc} T^*\mathcal{U}_{n,d} & \overset{i}{\dashrightarrow} & \mathrm{Gr}_n / \exp(\mathrm{k}_-) \\ \downarrow & \curvearrowright & \downarrow \\ \mathcal{U}_{n,d} & \hookrightarrow & \mathrm{Gr}_n / \mathrm{Gl}_n(\mathbf{C}[\![z]\!]) \end{array}$$

2. The Hitchin Hamiltonian system on $T^*\mathcal{U}_{n,d}$ are equivalent with the n-component KP flows (i.e., the k-action on $\mathrm{Gr}_n / \exp(\mathrm{k}_-)$) via the morphism i.

Emma Previato

Poncelet's theorem in space

Poncelet proved that a plane polygon P inscribed in a conic C and circumscribed to a conic D can be continuously "rotated", as it were. One of the many proofs consists in identifying the passage from each side of P to the next with addition of a torsion point on an elliptic curve. In the n-space version, involving torsion points of hyperelliptic Jacobians, there is a $g = (n-1)$ dimensional manifold of allowable rotations, where g is the genus of the hyperelliptic curve; the polygon is now inscribed in one and circumscribed to $(n-1)$ quadrics. The polygon can be viewed as a periodic orbit of an integrable system (the billiard or geodesic flow on an ellipsoid). H. Knörrer's work (linking Jacobi's coordinates with the Desale-Ramanan/Newstead/Reid projective model for hyperelliptic Jacobians) gives Poncelet's configurations in $(2g+1)$ space, some of which were classically known.

Eyal Markman

Cubics and the Lagrangian relative Picard

We construct a symplectic structure on the relative Picard over the Hilbert scheme of Lagrangian subvarieties of an algebraically symplectic variety. Equipped with the support map, it is a completely integrable system. This specializes to

1. Moduli spaces of Higgs pairs.
2. Mukai's symplectic structure on the relative Picard over a linear system of curves on a $K3$ surface.

The symplectic structure is extended to the smooth stable locus of the compactification of the Lagrangian relative Picard by Simpsons's moduli space of semi-stable sheaves. We discuss an example of Lagrangian subvarieties of the Fano variety of lines on a cubic 4-fold. The stable locus is compact. If smooth, it would be an irreducible symplectic projective variety with $h^{11} \geq 22$.

Hiraku Nakajima

Instantons and affine Lie algebras

I talked about mysterious links between affine Lie algebras and $4D$ gauge theory.
Let X be a projective surface/$\mathbf{C}$, and $X^{[n]}$ the Hilbert scheme of n points in X. Let us introduce two correspondences $E_i(n)$ and $F_i(n)$ in $X^{[n-i]} \times X^{[n]}$ and $X^{[n+i]} \times X^{[n]}$ respectively given by

$$E_i(n) = \{(Z_1, Z_2) \in X^{[n-i]} \times X^{[n]} | \mathcal{J}_{Z_1} \supset \mathcal{J}_{Z_2}, \operatorname{supp} \mathcal{J}_{Z_1}/\mathcal{J}_{Z_2} = \{p\} \text{ for some } p \in X\}$$

$$F_i(n) = \{(Z_1, Z_2) \in X^{[n+i]} \times X^{[n]} | \mathcal{J}_{Z_1} \subset \mathcal{J}_{Z_2}, \operatorname{supp} \mathcal{J}_{Z_2}/\mathcal{J}_{Z_1} = \{p_0\}\}$$

where $p_0 \in X$ is a fixed point. The operators $E_i(\cdot), F_i(\cdot)$ acting on $\bigoplus_n H_*(X^{[n]})$ satisfy the defining relation of the Heisenberg algebra.
Similarly, when X is an ALE space, that is the minimal resolution of simple

singularities, we use the Hecke correspondences instead of the above correspondences, then get a representation of the affine Lie algebra. Here the Hecke correspondence is the moduli space of parabolic bundles where the parabolic structure is given on an irreducible component of the exceptional set.

V. Balaji

Cycles on moduli spaces of vector bundles

Let C be a smooth projective curve of genus $g \geq 2$ over the complex numbers $\mathbf{C}$ and let $\mathbf{M}_C(2, L)$ be the moduli space of vector bundles of rank 2, $\det E \simeq L$, $\deg L$ odd. One knows that $\mathbf{M}_C(2, L)$ is a smooth projective variety of $\dim 3g-3$. The aim of the talk was to give an idea of proof of the following theorem which studies the cycles (algebraic) on $\mathbf{M}_C$ from the standpoint of the Hodge conjecture and the standard conjectures.

Theorem

1. *(V. Balaji, A. King, P. Newstead) The algebraic cohomology ring $H^*_A(\mathbf{M}_C)$ can be described as $\nu(\mathbf{Q}[\alpha, \beta] \otimes H^*_A(J_c)) \simeq H^*_A(\mathbf{M}_C)$ where ν is the well-understood ring hom. $\nu : \mathbf{Q}[\alpha, \beta] \otimes H^*(J_C) \to H^*(\mathbf{M}_C)$. Further, the algebraic Poincaré polynomial of $\mathbf{M}_C$ is*

$$P_A(\mathbf{M}_C, t) = \frac{P_A(J_C, t^3) - t^9 P_A(J_C, t)}{(1-t)(1-t^2)}.$$

2. *(V. Balaji) Let $\mathrm{Griff}^p(X) = \dfrac{\mathrm{CH}^p_{\mathrm{hom}}(X)}{\mathrm{CH}^p_{\mathrm{alg}}(X)}$ be the Griffiths group of codim p cycles. Then one has:*

$$\mathrm{Griff}^k(\mathbf{M}_C) = \begin{cases} 0 & k < 5, g < 4 \\ \textit{non-zero} & k = 5, g \geq 4 \ (C \textit{ generic}) \end{cases}$$

Hans U. Boden

Rationality of moduli spaces of vector bundles over a Riemann surface

Let X be a Riemann surface of genus $g \geq 1, L$ a line bundle of degree d over X, $\mathcal{M}_{r,L}$ the moduli space of semistable bundles E of rank r with determinant L.

Conjecture $\mathcal{M}_{r,L}$ *is rational, i.e. it is birational to a projective space.*

Despite some positive results (mainly due to Newstead), this is still an open problem, even for $(r, d) = 1$.

In this talk we discuss Newstead's method of solving this conjecture, and why it doesn't work in general. We use it to illustrate a way to study a closely related problem, namely the birational classification of moduli spaces of parabolic bundles over X. The result is that these moduli spaces are rational whenever one of the multiplicities associated to the quasi-parabolic structure is equal to one. This implies that $\mathcal{M}_{r,L}$ is *stably rational*, which in turn can be used to prove the conjecture for $(r, d) = 1$ and either $(d, g) = 1$ or $(r - d, g) = 1$.

Atsushi Moriwaki

Non-Archimedean Arakelov theory on algebraic surfaces

In this lecture, I introduced "Non-Archimedean Arakelov theory on algebraic surfaces", which is an analogue of Arakelov theory on arithmetic surfaces. This theory is obtained by considering Green functions on dual graphs of singular fibres of a semistable curve over a smooth projective curve. As application of this theory, I discussed the following Bogomolov conjecture.

Let K be a function field of one variable; C a smooth projective curve over K of genus $g \geq 2$ and $J(C)$ the Jacobian of C. Let $j : C(\bar{K}) \to J(C)(\bar{K})$ be an embedding given by $j(x) = (2g - 2)x - \omega_C$. Then the conjecture claims that if C is non-isotrivial, $j(C(\bar{K}))$ is discrete in terms of the Neron-Tate norm $\|\cdot\|_{NT}$ on $J(C)(\bar{K})$. I explained that this conjecture holds if i) C has a global stable model with only irreducible fibres or ii) $g = 2$ and $\mathrm{char}(K) \neq 2, 3, 5$.

Francesco Bottacin

Poisson structures on moduli spaces of sheaves over certain surfaces

This work was inspired by a paper by Mukai (Invent. Math. 77, 1984, 101-116), where he proved that the moduli space of simple sheaves over an abelian or $K3$ surface has a canonical symplectic structure. We consider a more general case, namely the case of a surface S endowed with a Poisson structure, i.e. with a Lie algebra structure $\{\cdot,\cdot\}$ defined on the sheaf of regular functions on S, that is a derivation in each entry. After giving a characterization of these surfaces, which we shall call Poisson surfaces, we prove that the choice of a Poisson structure on S determines a Poisson structure on the moduli space $\mathbf{M}$ of stable vector bundles on S. In the symplectic case, our approach gives a direct proof of the closure of the 2-form ω introduced by Mukai. In the end we show how the dimension of the symplectic leaf through a point $E \in \mathbf{M}$ is related to the dimension of the space of global sections of the sheaf $\underline{\mathrm{End}}(E_{|D})$, where D is a divisor on S determined by the Poisson structure of S.

Steven Bradlow

A Hitchin-Kobayashi correspondence for holomorphic extensions

In this talk I introduced equations for special metrics and a notion of stability for holomorphic extensions. The equations can be viewed as a deformation of the Hermitian-Einstein equations, and the notion of stability is analogous to slope-stability. A Hitchin-Kobayashi correspondence between the two was discussed. I also discussed the relation between holomorphic extensions and various other types of augmented bundles. In particular, the relation with so-called holomorphic triples (consisting of two bundles and a map between them) was pointed out.

Rogier Brussee

The canonical class and the Seiberg-Witten classes of Kähler surfaces

Let X be a Kähler surface, of Kodaira dimension $\mathrm{kod}(X) \geq 0$, $X \xrightarrow{\sigma} X_{\min}$ the contraction to the minimal model, and $E_1, \ldots, E_n$ the (-1)-curves. Define $K_{\min} := \sigma^* c_1(X_{\min}) \in H^2(X, \mathbf{Z})$. A (-1)-sphere is an oriented smoothly embedded 2-sphere e with $e^2 = -1$.

Theorem *(Witten and Friends ($p_g > 0$), Friedman Morgan Brussee ($p_g = 0$))*

1. *$K_{\min}$ is determined by the underlying smooth oriented manifold*
2. *for every (-1)-sphere e, there is a (-1)-curve E such that $[e] = \pm[E] \in H_2(X, \mathbf{Z})$*

This theorem implies the oriented differentiable invariance of the Kodaira dimension kod (Van de Ven conjecture proved by Friedman Morgan, Friedman Qin, and Pidstrigatch Tyurin) and the plurigenera P_n (generalized Van de Ven conjecture).

Ziv Ran

Vector bundles and Fano manifolds

We discuss generic semipositivity properties for sheaves of differential operators on a manifold, and use these to show that for any Fano manifold X^n of Picard number 1, we have

$$(-K_X)^n \leq (n+1)^n,$$

a bound which is obviously sharp.

Andreas Steffens

Remarks on Seshadri constants

Let X be a smooth proj. variety and let L be an ample line bundle on X. Fix a point $x \in X$. Demailly introduced a very interesting measure of the local positivity at a point x of L, namely the real number $\epsilon(L, x) = \inf_{C \ni x} \frac{L.C}{m_x(C)}$, which is called the Seshadri constant of L at x. Here the infimum is taken over all irreducible curves C passing through x and $m_x(C)$ is the multiplicity of C at x.

There has been recent interest in trying to give lower bounds for this invariant at a very general point $x \in X$. Ein and Lazarsfeld show for $n = 2 \Rightarrow \epsilon(L, x) \geq 1$ for a very general point $x \in X$. In higher dimension $n \geq 3$ Ein, Küchle and Lazarsfeld proved that $\epsilon(L, x) \geq \frac{1}{n}$ for a very general point $x \in X$.

In this talk we consider the question, are there conditions which guarantee the maxima of the Seshadri constant of L at x, i.e. $\epsilon(L, x) = \sqrt[n]{L^n}$? We give a partial answer for surfaces and find examples where the answer to our question is negative. If (X, Θ) is a general principal polarized abelian surface, then $\epsilon(\Theta, x) = \frac{3}{4} < \sqrt{2} = \sqrt{\Theta^2}$ for all $x \in X$.

Rosa M. Miró-Roig

On the smoothness of the moduli space of mathematical instanton bundles

Let $\mathrm{MI}_{\mathbf{P}^{2n+1}}(k)$ be the moduli space of mathematical instanton bundles on $\mathbf{P}^{2n+1}$ with second Chern class k. Related to the smoothness of $\mathrm{MI}_{\mathbf{P}^{2n+1}}(k)$ we have two important conjectures:

Conjecture 1 *The moduli spaces* $\mathrm{MI}_{\mathbf{P}^3}(k)$ *are smooth of dimension* $8k-3$.

It is well known that the moduli spaces $\mathrm{MI}_{\mathbf{P}^3}(k)$ are irreducible and smooth of dimension $8k-3$ for any integer $k \leq 4$; and as far as I know conjecture 1 remains open for $k > 4$. For $n \geq 2$ the situation is quite different and we have:

Conjecture 2 *For all integers* $n \geq 2$ *and* $k \geq 3$, *the moduli spaces* $\mathrm{MI}_{\mathbf{P}^{2n+1}}(k)$ *are singular.*

The first contribution to conjecture 2 is due to Ancona and Ottaviani. In 1994 they proved that $\mathrm{MI}_{\mathbf{P}^5}(3)$ and $\mathrm{MI}_{\mathbf{P}^5}(4)$ are singular. The goal of my lecture is to show that conjecture 2 is indeed true.

(References: R. M. Miró-Roig and J. Orus-Lacort: On the smoothness of the moduli space of mathematical instanton bundles. Preprint 1995)

Daniel Huybrechts

Vector bundles on $K3$ surfaces and birational symplectic manifolds

Let X be a $K3$ surface. The Hilbert schemes of points on X provide higher dimensional examples of irreducible symplectic manifolds (Beauville). Moduli spaces of rank two sheaves on X also admit a symplectic structure (Mukai). As it turns out, most of them are indeed irreducible symplectic. This is a by-product of the proof of a theorem (joint work with L. Göttsche) saying that Hilbert scheme and moduli space have the same Hodge numbers; some evidence is given, that they are in fact deformation equivalent: Let $P \in X$ be a $\mathbf{P}_n$-bundle of codimension n in a symplectic manifold. The elementary transformation of X along P gives a new symplectic manifold X'. Answering a question of Mukai we show that one can construct two smooth proper families over the disc which are isomorphic over the punctured disc and have special fibres X and X', resp. More results concerning projective manifolds are discussed.

Olivier Biquard

Logarithmic Higgs bundles and integrable connections

We extend the correspondence between Higgs bundles and integrable connections (Simpson, Corlette) to the case of $X - D$, where X is a compact Kähler manifold and $D \subset X$ a smooth divisor (the case $\dim X = 1$ has already been considered by Simpson).
We use a Poincaré type metric on $X - D$ (complete with finite volume). The holomorphic bundle has a parabolic structure over D and the Higgs field may have logarithmic singularities (and similarly on the side of integrable connections). We get a complete correspondence between the two objects, with precise asymptotics of the Hermite-Einstein or harmonic metric around D; and the correspondence has a nice specialization over D.
We relate also natural hypercohomologies on both sides via L^2-cohomology. The analysis is sufficiently precise to get a connection construction of moduli spaces. As an example of consequences the moduli space of representations in $Gl_n\mathbf{C}$ of the fundamental group of a punctured Riemann surface has a holomorphic Poisson structure; one gets the symplectic leaves by fixing the

conjugacy class of the monodromies around the punctures and each such leaf has a hyperkähler structure.

Ines Quandt

Vector bundles on higher dimensional varieties and formal power series

Let $X \subseteq \mathbf{P}^m$ be an algebraic variety of dimension n, where either $n \geq 3$ or $X = \mathbf{P}^2$, let $Y \subseteq X$ be a reduced, irr. hyperplane section and $p \in Y$ be a smooth k-rat. point.
Given a torsionfree sheaf $\mathcal{G}$ of rank r on Y which is free at p and a trivialization of $\hat{\mathcal{G}}_p$ one can associate to these objects a subspace $V \subseteq k[\![x_1, \ldots, x_{n-1}]\!]^{\oplus r}_{X_{n-1}}$
Starting from this one can establish a one-to-one correspondence between extensions of $\mathcal{G}$ to torsionfree sheaves on X being free at p and extensions of the local trivialization at the one side and subspaces $\omega \subseteq k[\![x_1, \ldots, x_n]\!]^{\oplus r}_{X_n}$ s.t.

1. $\dim_k \omega \cap k[\![x_1, \ldots, x_n]\!]^{\oplus r}_{X_n} < \infty$
2. ω has the structure of a module over a given ring $A \subseteq k[\![x_1, \ldots, x_n]\!]_{X_n}$ coming from the varieties X and Y.
3. $V = pr((\omega)_{\frac{X_{n-1}}{X_n}} \cap k[\![x_1, \ldots, x_n]\!]^{\oplus r}_{X_{n-1}})$

at the other one.
This correspondence is used for the study of moduli of framed sheaves.

H. Kurke

Marked vector bundles on rational surfaces and framed local jumps

We consider surfaces X with a birational morphism $X \xrightarrow{\sigma} \mathbf{P}^2$ and a distinguished point P_0 or (blown up) rational ruled surfaces $X \xrightarrow{\pi} \mathbf{P}^1$ with a section, $E \subset X$.
We study rank 2 vector bundles V on X together with a marking $V(P_0) \simeq k^2$ or $V/E \simeq \mathcal{O}^2_E$ which are trivial on $\sigma^{-1}(l)$, l a generic line through $\sigma(P_0)$,

or on generic fibres of $X \to \mathbf{P}^1$.
The first case is reduced to the second case by blowing up in P_0. Then $\det(R^*\pi_*V(-E))^{-1}$ has a distinguished section which defines the divisor Δ of jumping fibres and $\deg(\Delta) = n = c_2(V)$. This gives a fibration of the moduli space of these bundles over $\mathrm{Div}_n(\mathbf{P}^1) = \mathbf{P}^n$, and we give a description of the fibres, which are called (after J. Hurtubise) "framed local jumps".

Günther Trautmann

Recent results on instantons

Let $\mathrm{MI}_s(r,n)$ denote the moduli space of semi-stable bundles E on projective space $\mathbf{P}_s$ of rank r, Chern polynomial $c(E) = (1-h^2)^{-n}$ and of trivial splitting type. By results of Ancona/Ottaviani and Miró-Roig the spaces $\mathrm{MI}_s(s-1,n)$ are neither irreducible nor smooth for odd $s \geq 5$ and $n \geq 4$. As a contribution to the problem, whether $\mathrm{MI}(n) = \mathrm{MI}_3(2,n)$ is irreducible and smooth, a proof of the irreducibility of MI(5) is sketched, based on common work with A. Tikhomirov.

Berichterstatter: M. Lönne

V. Balaji	balaji@ssf.ernet.in
S. Bauer	bauer@mathematik.uni-bielefeld.de
Th. Bauer	bauerth@mi.uni-erlangen.de
A. Beauville	beauville@dmi.ens.fr
O. Biquard	biquard@cmat.polytechnique.fr
H. Boden	hboden@mpim-bonn.mpg.de
F. Bottacin	bottacin@pdmat1.math.unipd.it
S. Bradlow	bradlow@uinc.edu
R. Brussee	brussee@mathematik.uni-bielefeld.de
F. Catanese	catanese@dm.unipi.it
W. Ebeling	ebeling@math.uni-hannover.de
G. Ellingsrud	ellingsr@math.uio.no
C. Gantz	gantz@maths.ox.ac.uk
O. Garcia-Prada	ogprada@ccuam3.sdi.uam.es
B. van Geemen	geemen@dm.unito.it
N. Hitchin	n.hitchin@pmms.cam.ac.uk
K. Hulek	hulek@math.uni-hannover.de
D. Huybrechts	huybrech@mpim-bonn.mpg.de
H. Kurke	kurke@mathematik.hu-berlin.de
J. le Potier	jlp@mathp7.jussieu.fr
M. Lehn	lehn@mathematik.uni-bielefeld.de
M. Lönne	loenne@math.uni-hannover.de
M. Lübke	lubke@rulwinw.leidenuniv.nl
E. Markman	markman@math.mcgill.ca
R. M. Miró-Roig	miro@cerber.ub.es
A. Moriwaki	moriwaki@kusm.kyoto-u.ac.jp
M. Mulase	mulase@math.ucdavis.edu
H. Nakajima	nakajima@math.tohoku.ac.jp
W. M. Oxbury	w.m.oxbury@durham.ac.uk
E. Previato	ep@math.bu.edu
I. Quandt	quandt@mathematik.hu-berlin.de
S. Ramanan	ramanan@tifrvax.tifr.res.in
Z. Ran	ziv.ran@durham.ac.uk
F. Schreyer	schreyer@btm8x2.mat.uni-bayreuth.de
A. Steffens	steffens@mpim-bonn.mpg.de
S. A. Strømme	stromme@mi.uib.no
G. Trautmann	trm@mathematik.uni-kl.de
A. N. Tyurin	tyurin@tyurin.mian.su
A. Van de Ven	ven@rulcri.leidenuniv.nl

Tagungsteilnehmer

Prof.Dr. Vikraman Balaji
School of Mathematics
SPIC Mathematical Institute
92, G.N. Chetty Rd., T. Nagar

Madras 600 017
INDIA

Prof.Dr. Hans Boden
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

53225 Bonn

Prof.Dr. Stefan Alois Bauer
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131

33501 Bielefeld

Prof.Dr. Francesco Bottacin
Dipartimento di Matematica Pura
e Applicata
Universita degli Studi
Via Belzoni, 7

I-35131 Padova

Dr. Thomas Bauer
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2

91054 Erlangen

Prof.Dr. Steve Bradlow
Department of Mathematics
University of Illinois

Urbana ,IL 61801
USA

Prof.Dr. Arnaud Beauville
Departement de Mathematiques et
d'Informatiques
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05

Dr. Rogier Brussee
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25

33615 Bielefeld

Dr. Olivier Biquard
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Fabrizio Catanese
Dipartimento di Matematica
Universita di Pisa
Via Buonarroti, 2

I-56127 Pisa

Prof.Dr. Wolfram Decker
Fachbereich 9 - Mathematik
Universität des Saarlandes
Postfach 151150

66041 Saarbrücken

Prof.Dr. Bert van Geemen
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof.Dr. Wolfgang Ebeling
Institut für Mathematik
Universität Hannover
Postfach 6009

30060 Hannover

Prof.Dr.Dr.h.c. Hans Grauert
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

37073 Göttingen

Prof.Dr. Geir Ellingsrud
Institute of Mathematics
University of Oslo
P. O. Box 1053 - Blindern

N-0316 Oslo

Prof.Dr. Nigel J. Hitchin
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB-Cambridge , CB2 1SB

Christian Gantz
Mathematical Institute
Oxford University
24 - 29, St. Giles

GB-Oxford OX1 3LB

Prof.Dr. Klaus Hulek
Institut für Mathematik (C)
Universität Hannover
Postfach 6009

30060 Hannover

Prof.Dr. Oscar Garcia-Prada
Departamento de Matematicas
Universidad Autonoma de Madrid
Cantoblanco

E-28049 Madrid

Dr. Daniel Huybrechts
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

53225 Bonn

Prof.Dr. Herbert Kurke
Institut für Reine Mathematik
Humboldt-Universität Berlin

10099 Berlin

Prof.Dr. Eyal Markman
Dept. of Mathematics and Statistics
McGill University
805, Sherbrooke Street West

Montreal , P.Q. H3A 2K6
CANADA

Prof.Dr. Joseph Le Potier
U. F. R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Rose Maria Miro-Roig
Facultat de Matematiques
Universitat de Barcelona
Departament d'Algebra i Geometria
Gran Via 585

E-08007 Barcelona

Dr. Manfred Lehn
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25

33615 Bielefeld

Prof.Dr. Atsushi Moriwaki
Dept. of Mathematics
Faculty of Science
Kyoto University
Kitashirakawa, Sakyo-ku

Kyoto 606-01
JAPAN

Michael Lönne
Institut für Mathematik (C)
Universität Hannover
Welfengarten 1

30167 Hannover

Prof.Dr. Motohico Mulase
Dept. of Mathematics
University of California

Davis , CA 95616 8633
USA

Dr. Martin Lübke
Mathematisch Instituut
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. Hiraku Nakajima
Mathematical Institute
Tohoku University
Aramaki, Aoba-Ku

Sendai , 980-77
JAPAN

Dr. William M. Oxbury
Dept. of Mathematical Sciences
The University of Durham
Science Laboratories
South Road

GB-Durham , DH1 3LE

Prof.Dr. Frank-Olaf Schreyer
Mathematisches Institut
Universität Bayreuth

95440 Bayreuth

Prof.Dr. Emma Previato
Dept. of Mathematics
Boston University
111 Cummington Street

Boston , MA 02215
USA

Andreas Steffens
UCLA
Department of Mathematics

Los Angeles , CA 90025-1555
USA

Ines Quandt
Institut für Reine Mathematik
Humboldt-Universität Berlin
Ziegelstraße 13a

10117 Berlin

Prof.Dr. Stein Arild Stromme
Matematisk Institutt
Universitetet i Bergen
Allegaten 55

N-5007 Bergen

Prof.Dr. S. Ramanan
Tata Institute of Fundamental
Research
School of Mathematics
Homi Bhabha Road, Colaba

400 005 Bombay
INDIA

Prof.Dr. Günther Trautmann
Fachbereich Mathematik
Universität Kaiserslautern

67653 Kaiserslautern

Prof.Dr. Ziv Ran
Dept. of Mathematics
University of California

Riverside , CA 92521-0135
USA

Prof.Dr. Andrei N. Tyurin
Steklov Mathematical Institute
MIAN
Academy of Sciences
42, Vavilova str.

Moscow 117 966 GSP-1
RUSSIA

Prof.Dr. Antonius Van de Ven
Mathematisch Instituut
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden