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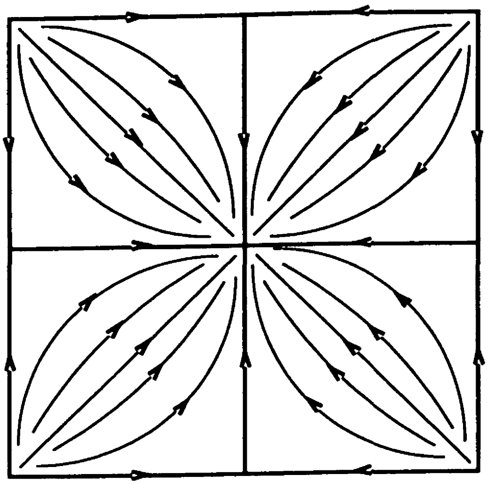

$BC_f \cong M$

The conference was organized by J.D.S. Jones (Warwick), I. Madsen (Aarhus), and E. Vogt (Berlin). 47 participants from Europe, the United States and East Asia attended the conference. Among the 20 lectures there was a series of three lectures by John D.S. Jones on Floer homotopy theory, a homotopy theoretical approach to infinite dimensional Morse theory. Other topics of interest were moduli spaces, low dimensional topology, relations between topology, geometry and group theory, stable and unstable homotopy theory, and K-theory.

Jon Berrick:

Localization of Non–nilpotent Groups and Spaces

I. Historical perspective

Inspired by localization of a ring $R \to R_\Sigma$, and thus R–modules, seen from different points of view, there have been various considerations of localization in mathematics. In particular, in the 1970's the localization of $\mathbb{Z}$–modules was generalized via central extensions to provide a widely used construction of localization of nilpotent groups. Similarly, localization of nilpotent spaces was defined via Postnikov decompositions. However, it was by no means clear how to extend these definitions beyond the nilpotent category.

Recently, originating with a viewpoint of Adams, one calls a **localization functor** an idempotent monad functor L (with natural transformation $\ell : I \to L$) s.t. $\forall$ object X

$$\begin{tikzcd} X \arrow[r, "\ell_x"] \arrow[dr, "\ell_x"'] & LX \arrow[d, equal] \arrow[dr, "\ell_{LX}\ \cong"] & \\ & LX \arrow[r, "L\ell_x \cong"'] & L^2X \end{tikzcd}$$

commutes. Then $\varphi : A \to B$ is called an L**–equivalence** when $L\varphi$ is an isomorphism, and X L**–local** when $LX \cong X$, whereupon $\varphi^* : \mathrm{Mor}(B, X) \to \mathrm{Mor}(A, X)$ is an isomorphism.

II. Localization of non–nilpotent groups at a set P of primes

For any discrete group G one now has the following functors extending localization of nilpotent groups (all of which, except (7), are localization functors).

$$\begin{array}{cccccccccccccc} G \to & G_p & \to & E^{\mathbb{Z}_p}G & = T^p_\alpha G \twoheadrightarrow & \ldots & \twoheadrightarrow T^p_\omega G & = & L_pG & \hookrightarrow & G^1_p \\ & (1) & & (2) & & (3) & & (4) & (5) & (6) & (7) \end{array}$$

(1) Ribenboim: the initial localization (2), (3) Bousfield (4) Tan*

(5) Berrick–Casacuberta–Frei–Tan*: the terminal localization

⑥ Casacuberta–Frei–Tan*: idempotent approximation: L_p and $(\ )^\wedge_p$ have the same equivalences, viz. $G \to H$ s.t. $\forall i\ (G/\Gamma_i G)_p \xrightarrow{\cong} (H/\Gamma_i H)_p$

⑦ Bousfield–Kan: P–completion.

The following fails without a finiteness condition on G.

Theorem [Berrick–Tan]*. (a) TFAE.

(i) $\Gamma_c G = \Gamma_{c+1}(G)(:= [\Gamma_c G, G])$;
(ii) $\forall_p\ E^{\mathbb{Z}_p} G$ is nilpotent of class $< c$;
(iii) $\forall_p\ L_p G$ is nilpotent of class $< c$;

(b) Suppose $H_1(G;\mathbb{Z})$ is finitely generated. TFAE.

(i) $\exists \subset$ s.t. $\Gamma_c G = \Gamma_{c+1} G$;
(ii) $\forall_p\ E^{\mathbb{Z}_p} G$ is nilpotent;
(iii) $\forall_p\ L_p G$ is nilpotent.

III. Localization with respect to a map

Dror–Farjoun has introduced the study of localization w.r.t. a map $f : A \to B$, where X is f**–local** iff $f^* :\ \mathrm{map}_*(B, X) \to \mathrm{map}_*(A, X)$ is a weak homotopy equivalence. When $B = pt.$, call this localization P_A.

Theorem [Berrick–Casacuberta]*. (a) A is acyclic iff P_A is Quillen's $X \to X^+_N$ for some perfect $N \trianglelefteq \pi_1 X$ (depending on $[A, X]$).

(b) $\exists$ perfect, locally free group F whose BF (a 2–complex) is the minimal A giving P_A as the plus–construction $X \to X^+$ (w.r.t. maximal perfect $\trianglelefteq \pi_1 X$).

*: to appear

C.–F. Bödigheimer:
Homological Stability of Mapping Class Groups with respect to Punctures

Let $\Gamma^s_{g,r}$ denote the mapping class group of an oriented surface of genus $g \geq 0$ with $r \geq 1$ boundary curves and $s \geq 0$ punctures; the diffeomorphisms are orientation preserving, the identity on the boundary, and permute punctures. There are inclusions

(1) $\rho : \Gamma^s_{g,r} \to \Gamma^s_{g,r+1}$, (2) $\sigma : \Gamma^s_{g,r} \to \Gamma^{s+1}_{g,r}$

and (3) $\gamma : \Gamma^s_{g,r} \to \Gamma^s_{g+1,r}$, by gluing to a chosen boundary curve a pair of pants (1), a cylinder with an extra puncture (2), or a torus with two boundary curves (3), resp. In the case (1) there is a left–inverse on the group level. In the case (2) and (3) Harer proved that σ and γ induce isomorphisms in $H_*(\ ;\mathbb{Z})$ for $* \leq g/3$, and $g \geq 3$. (This stability range was improved by Ivanov to $* \leq g/2$, and recently by Harer again to $* \leq 2g/3$ for $H_*(\ ;\mathbb{Q})$, $g \geq 3$ and $r \geq 1$.)

We show for the case (2):

Theorem 1. $\sigma_* : H_*(\Gamma^s_{g,r};\mathbb{Z}) \to H_*(\Gamma^{s+1}_{g,r};\mathbb{Z})$ is the injection of a direct summand in all degrees, for any $g \geq 0$, $s \geq 0$ and $r \geq 1$. □

This result follows from

Theorem 2. The map $B\sigma : B\Gamma^s_{g,r} \to B\Gamma^{s+1}_{g,r}$ stably splits, for $g \geq 0$, $s \geq 0$, $r \geq 1$. □

The proof uses a model for $B\Gamma^s_{g,r}$ which is a configuration space of "parallel slit domains" in the complex plane. This space is accessible to techniques from homotopy theory, developed to achieve stable splittings of loop spaces. We remark that actually a finite number of suspensions suffice.

Michel Boileau:

Gromov Volume and Circle Foliations

(joint work with **S. Druck** and **E. Vogt**)

The question of deciding which open 3–manifolds support a circle foliation is still widely open. The work of E. Vogt gave a method to build circle foliations on a number of unexpected 3–manifolds, in particular $\mathbb{R}^3$. The foliations built by Vogt are the simplest after Seifert fibrationns in the sense that the bad set (i.e. the set of leaves with infinite holonomy) is itself a generalized topological Seifert fibration. Call this type of foliation an Epstein length 1 circle foliation.

Our main result with regard to this problem is

Theorem. Let M be a compact orientable connected 3–manifold such that the manifold $\hat{M}$ obtained by capping off the boundary spheres by 3–balls is irreducible and ∂–irreducible. If Int (M) supports a circle foliation of Epstein length at most 1, then the Gromov volume $\|\hat{M}, \partial\hat{M}\| = 0$.

In particular, if $\hat{M}$ is sufficiently large, then $\hat{M}$ is a graph manifold.

As a corollary we obtain the following characterization of graph manifolds:

Corollary. Let M be a compact irreducible ∂–irreducible orientable 3–manifold. Then M is a 3–ball or a graph manifold iff there exists a link L in M such that $M \setminus L$ supports a circle foliation of Epstein length ≤ 1.

Martin R. Bridson:

Non–positively Curved Complexes and Knot Groups

We consider complete geodesic metric spaces which are non–positively curved in the sense of A.D. Alexandrov. The universal cover of such a space is a $CAT(0)$ space.

If a finitely generated group Γ acts properly and cocompactly on a $CAT(0)$ space, by isometries, then:

Theorems.

1) Γ is finitely presented of type FP_∞.

2) Γ has finitely many conjugacy classes of finite subgroups. If the action is free then Γ is torsion–free. There exist examples where Γ is not virtually torsion–free (Wise).

3) Γ has an efficient solution to the word problem (it has quadratic isoperimetric inequality).

4) Γ has a solvable conjugacy problem.

5) Every solvable subgroup of Γ is finitely generated and virtually abelian.

6) The centralizer $C_\Gamma(\gamma)$ of each $\gamma \in \Gamma$ is finitely presented. If γ has infinite order then $\exists H \leq C_\Gamma(\gamma)$ of finite index such that $H = K \times \langle\gamma\rangle$ (some K).

Remark. (6) can be used to prove the non–existence of, for example, actions of mapping class groups on $CAT(0)$ spaces (Mess, Kapovitch–Leeb) or $\mathrm{Aut}(F_n)$ (B–Vogtmann, Gersten).

Examples. (i) Davis and Janusciewicz show that for $n \geq 5$ there exist closed topological manifolds with no smooth structure which have geodesic metrics of non–positive curvature. In dim 3:

Conjecture. If a closed 3–manifold has a geodesic metric of non–positive curvature then it has a Riemannian metric of non–positive curvature.

Remark. Positive evidence comes from results of Bridson/Mosher and Leeb.

(ii) (Moussong) Every Coxeter group acts properly and cocompactly on a $CAT(0)$ space.

(iii) If Γ_1 and Γ_2 admit such actions, so does each $\Gamma_1 *_{\mathbb{Z}} \Gamma_2$.

(iv) Non–uniform lattices in $SO(n,1)$ admit such actions, non–uniform lattices in other rank 1 Lie groups do not (cf.5).

(v) One can build many interesting examples out of Euclidean squares. (D. Wise constructs many.)

(vi) **Theorem.** If X is a non–positively curved 2–complex then every f.p. subgroup of $\pi_1 X$ is the fundamental group of a compact non–positively curved 2–complex.

Remark. This result is spectacularly false in higher dimensions.

Knots. Theorem. If $K \subseteq \mathbb{S}^3$ is an alternating knot then $\pi_1(\mathbb{S}^3 \smallsetminus K)$ is the fundamental group of a compact non–positively curved 2–complex.

Remark (a). Following this theorem one applies (vi) and (1)–(6).

Remark (b). (iii) applies to torus knot groups and (iv) takes care of hyperbolic knots.

Octavian Cornea:

Functions with Few Critical Points

For any finite, 2–connected CW–complex X there is a smooth compact manifold $M \simeq X$ that supports a smooth, self–indexed function maximal, constant and regular on ∂M with less than $\mathrm{cat}(X) + 2$ critical points which are of a certain "reasonable" type. To such a critical point there corresponds, homotopically, the attachement of a cone. Conversely to a cone attachment we may associate, under certain dimensionality and connectivity conditions, a "reasonable" critical point.

John Greenlees:

THE COMPLETION THEOREM, THE LOCAL COHOMOLOGY THEOREM, AND THE MULTIPLICATIVE NORM MAP FOR EQUIVARIANT BORDISM

(joint work with **J.P. May**)

For finite groups and compact Lie groups with torus identity component the completion theorem for tomDieck's equivariant bordism $MU_G^*(\cdot)$ is true, as made precise in the theorem below. By finding the correct algebraic model for $MU_*^G(EG_+)$, and first proving the local cohomology theorem for calculating it, the structure of the proof becomes very simple. In fact, there are only two main points:

(a) by working in the category of highly structured modules (in the sense of Elmendorf, Kriz, Mandel & May) over MU it is easy to realise the algebra geometrically, provided the augmentation ideal $J = \ker(MU_G^* \to MU^*)$ can be replaced by a finitely generated ideal J'.

(b) To find a suitable ideal J' we need the existence of Thom isomorphisms (and hence of Euler classes) and the existence of a multiplicative norm.

Theorem 1. For any highly structured module m over the equivariant bordism spectrum MU (such as an equivariant form of K, ku, $K(n)$, $k(n)$, $E(n)$, etc.) there are spectral sequences

$$E_2^{**} = H_*^J(m_G^*(X)) \Rightarrow m_G^*(EG_+ \wedge X) \cong m^*(EG_+ \wedge X)$$

and

$$E^2_{**} = H_J^*(m_*^G(X)) \Rightarrow m_*^G(EG_+ \wedge X) = m_*(EG_+ \wedge X).$$

Here H_J^* is local cohomology in the sense of Grothendieck and H_*^J is local homology in a dual sense. Under finiteness hypotheses H_J^* calculates the right derived functors of the J–power torsion functor and H_*^J calculates the left derived functors of J–adic completion.

Theorem 2. There is a multiplicative norm map

$$\mathrm{norm}_H^G : MU_0^H(X) \to MU_0^G(X^{\wedge n})$$

where H is of index n in G. This is transitive, the identity if $H = G$, multiplicative, and satisfies the restriction formula

$$\mathrm{res}_K^G \, \mathrm{norm}_H^G(x) = \prod_{HgK \in H\backslash G/K} \mathrm{norm}_{K\cap {}^gH}^K \, \mathrm{res}_{K\cap {}^gH}^{{}^gH} c_g(x)$$

Theorem 2 is proved more generally for families of cohomology theories represented by "global $\mathcal{J}_*$ functors with smash product". This includes MU by virtue of the finite Thom space model of its representing pre–spectrum.

Jean–Claude Hausmann:

Polygon Spaces and Grassmannians

(joint work with **Allen Knutson**)

Let ${}^m\tilde{P}^k$ be the space of m–gons in $\mathbb{R}^k$ up to translation and positive homotheties. This space comes with several structures: an action of $O(k)$, an action of S_m permuting the edges, and a function $\ell : {}^m\tilde{P}^k \to \mathbb{R}^m$ taking a polygon to the lengths of its edges. The quotients of ${}^m\tilde{P}^k$ by SO_k (or O_k) are the moduli spaces ${}^m\tilde{P}_+^k$ (respectively, ${}^m\tilde{P}^k$). Fixing a reflection in $O(k)$ provides an involution on ${}^m\tilde{P}^k$ and ${}^m\tilde{P}_+^k$ whose fixed point sets are ${}^m\tilde{P}^{k-1}$ and ${}^mP^{k-1}$. The goal of this paper is to understand the topology of these various spaces and the geometric structures that they naturally carry when $k = 2$ or 3. They are closely related to more familiar objects (Grassmannians, projective spaces, Hopf bundles, etc.). The spaces ${}^mP^k(\alpha) := \ell^{-1}(\alpha)$ of polygons with given side–lengths $\alpha \in \mathbb{R}^m$ are of particular interest.

The great miracle occurs when $k = 3$, because $\mathbb{R}^3$ is isomorphic to the space $I\mathbb{H}$ of pure imaginary quaternions, and the 2–sphere in $\mathbb{R}^3$ is Kähler. The tools of symplectic geometry can then be used. Most prominent is a symplectic version of the Gel'fand–MacPherson correspondence identifying the spaces ${}^mP^3(\alpha)$ as symplectic quotients of the Grassmannian of 2–planes in $\mathbb{C}^m$.

While this paper illustrates many phenomena in symplectic geometry, the proofs are entirely polygon-theoretic and involve only classical differential topology. Nonetheless, many of the examples are new, interesting in their own right and instructive for both fields.

Hans–Werner Henn:

COMMUTATIVE ALGEBRA OF UNSTABLE K–MODULES, LANNES' T–FUNCTOR AND AN APPLICATION TO $H^*(GL(u,\mathbb{Z}[\frac{1}{2}]);\mathbb{F}_2)$

Let p be a fixed prime and let K be an unstable algebra over the mod $-p$ Steenrod algebra A such that K is finitely generated as graded $\mathbb{F}_p$–algebra. Let K_{fg}-$\mathcal{U}$ denote the abelian category of finitely generated K–modules with a compatible unstable A–module structure. We study various concepts of commutative algebra in this setting. The rôle of the prime ideal spectrum of a commutative ring is here taken by a category $R(K)$ which, roughly speaking, consists of the A–invariant prime ideals of K together with certain "Galois information"; sheaves will correspond to functors on this category, and the rôle of the sheaf associated to a module will be taken by the components of Lannes' T–functor. We discuss the notions of support, of $\mathfrak{a}$–torsion modules (for an invariant ideal $\mathfrak{a}$ of K) and of localization away from the Serre subcategory $\mathbf{Tors}(\mathfrak{a})$ of $\mathfrak{a}$–torsion modules in our setting. We show that the category K_{fg}-$\mathcal{U}$ has enough injectives and use these injectives to study these localizations and their derived functors; they are closely related to the derived functors of the $\mathfrak{a}$–torsion functor $F_{\mathfrak{a}}$. Our results are formally analogous to Grothendieck's results in the classical situation of modules over a noetherian commutative ring R.

Important for applications is the case $K = H^*BG$, the mod $-p$ cohomology of a classifying space of a compact Lie group (or a suitable discrete group), and $M = H^*_G X$ where X is a (suitable) G–CW–complex. In these cases the category $R(K)$ and the functor on $R(K)$ associated to $H^*_G X$ can be described in terms of group theoretic and geometric data, and our theory yields a far-reaching generalization of a result of Jackowski and McClure resp. of Dwyer and Wilkerson. As a concrete application of our theory we describe the size of the kernel of the restriction map from the unknown mod -2 cohomology of the S–arithmetic group $GL(n,\mathbb{Z}[1/2])$ to the known cohomology of its subgroup D_n of diagonal matrices.

Steven Hutt:

Poincaré Sheaves on Topological Spaces

To a normal map $(f, b) : M^n \to X^n$ from a manifold to a Poincaré space Wall associated a quadratic signature $\sigma(f, b) \in L_n(\mathbb{Z}\pi_1(X))$ whose triviality is necessary (and for $n \geq 5$ sufficient) for (f, b) to be normal bordant to a homotopy equivalence. The quadratic L groups were later interpreted by Ranicki as cobordism groups $L_n(R)$ of R–module chain complexes C with an n–dimensional quadratic Poincaré duality $\chi : C^{n-*} \cong C$. In particular the quadratic signature $\sigma(f, b)$ of Wall may be interpreted as the cobordism class of a certain n–dimensional algebraic quadratic Poincaré complex (C, χ) associated to (f, b).

We extend these notions to define objects called Poincaré sheaves on topological spaces, (generalizing a previous simplicial version of Ranicki). A Poincaré sheaf (A, ψ) on a space X is a complex A of R–module sheaves on X together with a quadratic Poincaré duality $\chi : A \cong \Sigma^n DA$ where D is the Verdier duality operator. Poincaré sheaves carry surgery invariants in a way that allows us to localise to open subsets. Application of the section functor returns us to the algebraic Poincaré complexes above so that the classical theory is recovered. Furthermore, the effect of geometric constructions on surgery invariants can be closely followed via such sheaves.

Many classical surgery invariants may be sheafified, including the quadratic invariant of a normal map, the construction of Quinn's invariant for a homology manifold and Ranicki's total surgery obstruction of a Poincaré space. Applications include a new proof of Novikov's result on the topological invariance of the rational Pontrjagin classes of a PL–manifold. This proof does not require the Bass–Heller–Swan calculation.

Klaus Johannson:

The Exceptional Nature of the Figure 8 Knot

A Heegaard string (tunnel) in a 3–manifold M is an arc $t \subset M$ such that $(M - U(t))^-$ is a handlebody. The fundamental group of a 3–manifold admitting Heegaard strings is a one–relator group. So the study of Heegaard strings may also be viewed as the study of geometric presentations of those one–relator groups. In this talk I discussed an approach towards the classification of Heegaard strings in Haken 3–manifolds, with special attention payed to surface bundles over S^1. I also discussed the special importance of these special 3–manifolds in the general scheme. As a result I showed that Heegaard strings in bundles over S^1 can be pushed into certain normal positions and that this reduces the classification of Heegaard strings in those bundles to the algebraic problem of classifying "filling elements" in surface groups with given automorphism. Applying this classification to torus bundles yields the result that only torus bundles with monodromy conjugate to $\begin{pmatrix} d+1 & 1 \\ d & 1 \end{pmatrix}$ can have Heegaard strings at all, and only those with monodromy conjugate to $\pm\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ give (exactly 2 resp. 4) different Heegaard strings. The matrix $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ is the monodromy for the figure 8 knot (and hence the title of the talk.)

John Jones:

Gauge Theory and Homotopy Theory

I Flow Categories and Morse Theory

II Floer Homotopy Theory and Quantum Cohomology

III Furuta's Work on the 11/8 Conjecture

The theme of these three lectures is the interaction between homology theory and that part of global analysis concerned with the study of the partial differential equations which arise in that part of mathematical physics known as gauge theory.

The first two lectures, which were on joint work with Ralph Cohen and Graeme Segal, were concerned with the study of the homotopy theory underlying Floer's infinite dimensionnal version of Morse theory. The first step is to re–examine finite dimensional Morse theory. Let $f : M \to \mathbb{R}$ be a Morse–Smale function (i.e. a generic Morse function) on a compact Riemannian manifold. The **flow category** C_f is the category whose objects are the critical points of f and the morphisms between two critical points are the piecewise flow lines (in the obvious sense) of the gradient flow of f joining these critical points. Associated to any category C is its classifying space BC.

Theorem. BC_f is homeomorphic to M.

This theorem gives a new method of processing the data provided by finite dimensional Morse theory. To use these flow categories in infinite dimensions it is necessary to introduce the notion of a framing. In the present finite dimensional situation, given a framing Λ of C_f it is possible to construct a virtual vector bundle $\zeta = \zeta(\Lambda)$ over $BC_f \cong M$. Furthermore, by following the usual construction it is possible to construct a chain complex $C(M, f, \Lambda)$ — this is the **Morse–Smale chain complex.**

Theorem. $H_*(C(M, f, \Lambda)) \cong H_*(M^\zeta)$
where M^ζ is the Thom space of the virtual bundle ζ.

This theorem is a mild generalization of the fundamental result of finite dimensional Morse theory.

In the infinite dimensional situations studied by Floer the flow category still makes sense. The appropriate notion of a framing gives a system of vector bundles $\zeta_a = \zeta_a(\Lambda)$ indexed by the critical points of the function. If a and b are critical points and there is a flow from a to b then ζ_a is a sub-bundle of ζ_b. This gives an inverse system of Thom spectra $(BC_f)^{-\zeta_a}$ indexed by the critical points. There is a chain complex $C(X, f, \Lambda)$ which depends on the function $f : X \to \mathbb{R}$ (here X is infinite dimensional and f is a "suitable function" which will be called a Floer function) and the framing Λ — this is the Floer complex.

Theorem. Suppose the space of piecewise flows between any two critical points is compact; then

$$H^*(C(X, f, \Lambda)) \cong \operatorname*{colim}_a \ H^*(BC_f^{-\zeta_a})$$

This inverse system of Thom spectra $BC_f^{-\zeta_a}$ is the **Floer homotopy type** associated to the Floer function f and the framing Λ.

The first example is as follows. Let V be the vector space over $\mathbb{R}$ with basis e_i, $i \in \mathbb{Z}$, topologised as the direct limit of its finite dimensional subspaces. The preceding theory can be applied to the function $f : \mathbb{P}(V) \to \mathbb{R}$ defined by $f[x] = (\sum_{n=-\infty}^{\infty} n x_n^2)/(\sum_{n=-\infty}^{\infty} x_n^2)$. It yields the inverse system

$$\mathbb{R}P^\infty \leftarrow (\mathbb{R}P^\infty)^{-\eta} \leftarrow (\mathbb{R}P^\infty)^{-2\eta} \leftarrow \cdots$$

often known as $\mathbb{R}P^\infty_{-\infty}$. Here η is the real Hopf line bundle. Thus the "Floer homotopy groups" of this function are, by a deep theorem of W.H. Lin, given by the 2-adic completion of the stable homotopy groups of spheres.

The second example is the area function, or symplectic action on $\tilde{L}(\mathbb{C}P^n)$, the universal cover of the free loop space of $\mathbb{C}P^n$. After compactifying this leads to the inverse system

$$\mathbb{C}P^\infty \leftarrow (\mathbb{C}P^\infty)^{-(n+1)\zeta} \leftarrow (\mathbb{C}P^\infty)^{-2(n+1)\zeta} \leftarrow \cdots$$

Rather remarkably the Floer cohomology groups in this example are the same as the quantum cohomology of $\mathbb{C}P^n$. This quantum cohomology $QH^*(V)$ is defined for a class of Kähler manifolds. It depends on a parameter q and if q is set to be zero the quantum cohomology reduces to ordinary cohomology. This relation between quantum cohomology and Floer cohomology seems to be a general phenomenon; but as yet there are no theorems to this effect.

The third lecture was a report on work of Furuta.

Theorem. Let M be a closed simply connected smooth 4–manifold with intersection form Q_M given by

$$Q_M = 2kE_8 \oplus \ell H\,.$$

Then $\ell \geq 2k + 1$.

Here E_8 is the usual quadratic form with rank 8 and signature 8 and H is given by the matrix $\left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right)$.

The conjecture $\ell \geq 3k$ is known as the 11/8 conjecture, and is a very important unsolved problem in the theory of 4–manifolds.

Furuta uses the Seiberg–Witten equations to show that if M exists then there is a G–equivariant map $S(V) \to S(W)$ where G is the subgroup of $Sp(1)$ generated by the unit complex numbers and the quaternion j and V and W are specific representations of G which depend on k and ℓ. Now using equivariant K-theory K_G Furuta shows that if such an equivariant map exists then necesssarily $\ell \geq 2k + 1$.

Stephan Klaus:

The Ochanine k–Invariant is a Brown–Kervaire invariant

For closed $(8m + 2)$–dimensional spin manifolds one has on the one hand the Brown–Kervaire invariants and on the other hand Ochanine k–invariant.

The first ones have a homotopy theoretic interpretation as Arf invariants of a certain quadratic refinement of the $\mathbb{Z}/2$ intersection form and they form for each m a finite set. The second one has an expression by KO–characteristic numbers and has an analytic meaning. I proved in my dissertation that the Ochanine k–invariant is in fact a Brown–Kervaire invariant. This result is an analogue of Hirzebruch's signature theorem.

The proof uses:

1. The integral elliptic homology of Kreck and Stolz which characterizes in particular multiplicative invariants in $\mathbb{H}P^2$–bundles.
2. Kristensen's theory of cochain transformations and secondary cohomology operations which gives us a Cartan formula for such operations in $\mathbb{H}P^2$–bundles.

Christine Lescop:

The Casson–Walker Invariant and the Mapping Class Group

We study the following question: How does the Casson–Walker invariant λ of a rational homology 3–sphere obtained by gluing two pieces along a surface depend on the two pieces? Our partial answer may be stated as follows. For a compact oriented 3–manifold A with boundary ∂A, the kernel $\mathcal{L}_A$ of the map from $H_1(\partial A; Q)$ to $H_1(A; Q)$ induced by the inclusion is called the Lagrangian of A. Let Σ be a closed oriented surface, and let A, A', B and B' be four rational homology handlebodies such that ∂A, $\partial A'$, $-\partial B$ and $-\partial B'$ are identified via orientation-preserving homeomorphisms with Σ. Assume that $\mathcal{L}_A = \mathcal{L}_{A'}$ and $\mathcal{L}_B = \mathcal{L}_{B'}$ inside $H_1(\Sigma; Q)$ and also assume that $\mathcal{L}_A$ and $\mathcal{L}_B$ are transverse. Then we express

$$\lambda(A \cup_\Sigma B) - \lambda(A' \cup_\Sigma B) - \lambda(A \cup_\Sigma B') + \lambda(A' \cup_\Sigma B')$$

in terms of the form induced on $\Lambda^3 \mathcal{L}_A$ by the algebraic intersection on $H_2(A \cup_\Sigma -A')$ paired to the analogous form on $\Lambda^3 \mathcal{L}_B$ via the intersection form of Σ. The simple formula that we obtain naturally extends to the extension of the Casson–Walker invariant of the author.

This formula applies to generalize a result of Monta which computes the coboundary of some functions induced naturally on the Torelli group by the Casson invariant in terms of the Johnson homomorphism. Our proof leads us to study a (tautological, but interesting) extension of the Reidemeister torsion to compact 3–manifolds with arbitrary boundary.

Wolfgang Lück:

L^2–Invariants and Towers of Coverings

Let M be a closed Riemannian manifold with residually finite fundamental group π, i.e. π has a tower $\pi = \Gamma_0 \supset \Gamma_1 \supset \Gamma_2 \supset \ldots$ consisting of normal subgroups Γ_m of finite index such that $\bigcap \Gamma_m = \{1\}$. Let $M_m \to M$ be the covering of M associated to Γ_m. Denote by $b_p(M_m)$ its p-th Betti number. Let $b_p^{(2)}(M)$ be the p-th L^2-Betti number of M which is defined as

$$b_p^{(2)}(M) = \dim_{\mathcal{N}(\pi)} H^p_{(2)}(\tilde{M}) = \lim_{t\to\infty} \int_{\mathcal{F}} e^{-t\tilde{\Delta}_p}(\tilde{x},\tilde{x})\mathrm{d}\, vol$$

Here $H^p_{(2)}(\tilde{M})$ is the space of harmonic L^2–integrable p–forms on $\tilde{M}$ and $e^{-t\tilde{\Delta}_p}(\tilde{x},\tilde{x})$ the heat kernel of the universal covering. We discuss the proof and applications of the following theorem which was conjectured by Gromov:

Theorem. $\lim\limits_{m\to\infty} \frac{b_p(M_m)}{[\pi:\Gamma_m]} = b_p^{(2)}(M)$.

Mark Mahowald:

The Elliptic Curve Hopf Algebroid

(joint work with **M. Hopkins**)

The Weierstrass form of an elliptic curve is usually written $y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$. Coordinate transformations do not change anything, so $x = x' + r$, $y = y' + sx' + t$ give the same curve.

Let K be a field. Then an elliptic curve over K is given by a ring homomorphism $\mathbb{Z}[a_1, a_2, a_3, a_4, a_6] \to K$. The automorphisms are given by $\mathbb{Z}[a_1, a_2, a_3, a_4, a_6, s, r, t]$. If we include the transformation in the coefficients induced by the coordinate transformation we get the structure maps for a Hopf algebroid (Ravenel, Complex Cobordism and Stable Homotopy). For example

$$a_1^1 = a_1 + 2s$$
$$a_2^1 = a_2 - a_1 s + 3r - s^2$$

Let $n_L a_i = a_i$ and $n_R a_i = a_i^1$, then these are the structure maps to give for $R = \mathbb{Z}[a_1, \ldots, a_4, a_6]$ and $\Lambda = R[s, r, t]$. We then have a resolution

$$R \; \substack{\rightarrow \\ \leftarrow \\ \rightarrow} \; \Lambda \; \substack{\leftarrow \\ \rightarrow \\ \leftarrow \\ \rightarrow} \; \Lambda \otimes_R \Lambda \; \substack{\rightarrow \\ \leftarrow \\ \rightarrow \\ \leftarrow \\ \rightarrow} \; \Lambda \otimes_R \Lambda \otimes_R \Lambda \rightarrow \cdots .$$

This resolution is then calculated by observing that it is the same as the one constructed by letting T be the Thom complex of the natural bundle over $\Omega SU(4)$. Then take the standard T resolution

$$T \; \substack{\rightarrow \\ \leftarrow \\ \rightarrow} \; T \wedge T \; \substack{\rightarrow \\ \leftarrow \\ \rightarrow \\ \leftarrow \\ \rightarrow} \; T \wedge T \wedge T \ldots$$

and apply $eo_{2\bullet}$. $eo_{2\bullet}(T) \cong R$ and $eo_{2\bullet}(T \wedge T) \simeq \Lambda$ and the structure maps agree.

John Rognes:

The Fiber of the Linearization Map $A(*) \rightarrow K(\mathbb{Z})$

(joint work with **John Klein**)

Waldhausen's algebraic K-theory of spaces satisfies $A(*) \simeq Q(S^0) \times B^2\mathcal{P}(*)$ when applied to a point. Here $\mathcal{P}(*) = \mathrm{hocolim}_N \mathrm{DIFF}(D^{N+1} \mathrm{rel} D^N)$ is the stable smooth pseudoisotopy space of a point. $A(*)$ can be viewed as the algebraic K-theory of the ring up to homotopy represented by the sphere spectrum, with underlying space $Q(S^0)$. The linearization map to path components $Q(S^0) \rightarrow \mathbb{Z}$ then induces a map on K-theory $L : A(*) \rightarrow K(\mathbb{Z})$.

Theorem. *Let p be an odd prime. After completion at p the homotopy fiber $\mathcal{F}$ of the linearization map $L : A(*) \rightarrow K(\mathbb{Z})$ has homotopy groups*

$$\pi_* \mathcal{F} \cong \begin{cases} \mathbb{Z}/p & \text{if } * = 2n \text{ with } k(p-1) \leq n < kp \text{ for some } 1 \leq p < k, \\ 0 & \text{otherwise,} \end{cases}$$

for $* < 2p(p-1) - 2$.

This describes $\mathcal{F}$ in the initial range of degrees where the stable homotopy groups of spheres only consist of the image of the J-homomorphism.

The proof is an application of the cyclotomic trace map from K-theory to topological cyclic homology, and a recent theorem of Bjørn Dundas.

Corollary. *There are torsion classes of order p in*

$$\pi_{2n-2}\mathcal{P}(*) \cong \pi_{2n-2}\mathrm{DIFF}(D^{N+1}\mathrm{rel}D^N)$$

for $N \gg n$, when $k(p-1) < n < kp$ for some $1 < k < p$. These classes come from $\pi_{2n}A(*)$ and map to zero under linearization to $\pi_{2n}K(\mathbb{Z}) = K_{2n}(\mathbb{Z})$.

Stephan Stolz:

A CONJECTURE CONCERNING POSITIVE RICCI CURVATURE AND THE WITTEN GENUS

We discuss evidence for the following

Conjecture. Let M be a 4k–dimensional spin manifold (smooth and closed) with $\frac{p_1}{2}(M) = 0$. If M admits a metric of positive Ricci curvature, then the Witten genus $\phi_w(M)$ vanishes.

Here $p_1(M)$ is the first Pontrjagin class of M (which for spin manifolds is canonically divisible by 2), and $\phi_w(M)$ is a collection of characteristic numbers of M, including the $\hat{A}$–genus $\hat{A}(M)$. The conjecture is true for complete intersections, homogeneous spaces G/H for G compact, semi-simple, and fiber bundles with fiber G/H (G as above) and structure group G. This conjecture is analogous to the result of Lichnerowicz which says that the $\hat{A}$–genus of spin manifolds with positive scalar curvature vanishes. In fact, the author hopes that applying Lichnerewicz' argument to a hypothetical "Dirac" operator on the free loop space of M might lead to a proof of the conjecture.

Peter Teichner:

NEW GOOD GROUPS FOR TOPOLOGICAL 4–MANIFOLDS

(joint work with **Mike Freedman**)

The classification of manifolds in dimension ≥ 5 uses two main theorems, the s–cobordism and the surgery theorems. They translate homotopy data into actual homeomorphism information.

In dimension 4, the basic tool in the proof of the above theorems (namely the Whitney trick) is a priori not available. In the smooth category this in fact leads to the failure of both theorems but the obstructions can only be constructed using Gauge–theory [Donaldson 81].

In the topological category, [Freedman 81] proved that the two theorems do hold under certain restrictions on the fundamental group. ("Good groups" are those for which the theorems hold). More precisely, he showed that all elementary amenable groups are good.

Theorem. Groups of subexponential growth are good. [Freedman.–T. 94]

It is known that an elementary amenable group has either polynomial or exponential growth. Moreover, [Grigorchuk] constructed uncountably many finitely generated groups of intermediate growth (i.e. neither polynomial nor exponential). Therefore, the above theorem constructs in fact new good groups.

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